# Uncertainty and Reliability Analysis in Water Resources Engineering

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## UNCERTAINTIES IN WATER RESOURCES ENGINEERING

Water resources engineering design and analysis deal with the occurrence of water in various parts of a hydrosystem and its effects on environmental, ecological, and socio-economical settings. Due to the extreme complex nature of the physical, chemical, biological, and socio-economical processes involved, tremendous efforts have been devoted by researchers attempting to have a better understanding of the processes. One beneficial product of these research efforts is the development of a model which describes the interrelationships and interactions of the components involved in the processes. Herein, the term 'model' is used in a very loose manner, referring to any structural or nonstructural ways of transforming inputs to produce some forms of outputs. In water resources engineering, most models are structural which take the forms of mathematical equations, tables, graphs, or computer programs. The model is a useful tool for engineers to assess the system performance under various scenarios based on which efficient designs or effective management schemes can be formulated. Despite numerous research efforts made to further our understanding of various processes in hydrosystems, there is still much more that are beyond our firm grasp. Therefore, uncertainties exist due to our lack of perfect - knowledge concerning the phenomena and processes involved in problem definition and resolution.

In general, uncertainty due to inherent randomness of physical processes cannot be eliminated. On the other hand, uncertainties such as those associated with lack of complete knowledge about the process, models, parameters, data, and etc. could be reduced through research, data collection, and careful manufacturing. In water resources engineering, uncertainties involved can be divided into four basic categories: hydrologic, hydraulic, structural, and economic. More specifically, in water resources engineering analyses and designs uncertainties could arise from the various sources including natural uncertainties, model uncertainties, parameter uncertainties, data uncertainties, and operational uncertainties.

Natural uncertainty is associated with the inherent randomness of natural processes such as the occurrence of precipitation and flood events. The occurrence of hydrological events often display variations in time and in space. Their occurrences and intensities could not be predicted precisely in advance. Due to the fact that a model is only an abstraction of the reality, which generally involves certain degrees of simplifications and idealizations. Model uncertainty reflects the inability of a model or design technique to represent precisely the system's true physical behavior. Parameter uncertainties resulting from the inability to quantify accurately the model inputs and parameters. Parameter uncertainty could also be caused by change in operational conditions of hydraulic structures, inherent variability of inputs and parameters in time and in space, and lack of sufficient amounts of data.

Data uncertainties include (1) measurement errors, (2) inconsistency and non-homogeneity of data, (3) data handling and transcription errors, and (4) inadequate representation of data sample due to time and space limitations. Operational uncertainties include those associated with construction, manufacture, deterioration, maintenance, and human. The magnitude of this type of uncertainty is largely dependent on the workmanship and quality control during the construction and manufacturing. Progressive deterioration due to lack of proper maintenance could result in changes in resistance coefficients and structural capacity reduction.

The purpose of this article is to briefly summarize the state-of-the-art of uncertainty and reliability analyses procedures in water resources engineering. For more detailed descriptions of the various techniques and applications can be found in the two references at the end of this article.

## IMPLICATIONS OF UNCERTAINTY AND PURPOSES OF UNCERTAINTY ANALYSIS

In water resources engineering design and analysis, the decisions on the layout, capacity, and operation of the system largely depend on the system response under some anticipated design conditions. When some of the components in a hydrosystem are subject to uncertainty, the system responses under the design conditions cannot be assessed with certainty. Therefore, the conventional deterministic design practice is inappropriate because it is unable to account for possible variation of system responses. In fact, the issues involved in the design and analysis of hydrosystems under uncertainty are multi-dimensional. An engineer has to consider various criteria including, but not limited to, cost of the system, probability of failure, and consequence of failure so that a proper design can be made for the system.

In water resources engineering design and modeling, the design quantity and system output are functions of several system parameters not all of them can be quantified with absolute accuracy. The task of uncertainty analysis is to determine the uncertainty features of the system outputs as a function of uncertainties in the system model itself and the stochastic variables involved. It provides a formal and systematic framework to quantify the uncertainty associated with the system output. Furthermore, it offers the designer useful insights regarding the contribution of each stochastic variable to the overall uncertainty of the system outputs. Such knowledge is essential to identify the 'important' parameters to which more attention should be given to have a better assessment of their values and, accordingly, to reduce the overall uncertainty of the system outputs.

## MEASURES OF UNCERTAINTY

Several expressions have been used to describe the degree of uncertainty of a parameter, a function, a model, or a system. In general, the uncertainty associated with the latter three is a result of combined effect of the uncertainties of the contributing parameters.

The most complete and ideal description of uncertainty is the probability density function (PDF) of the quantity subject to uncertainty. However, in most practical problems such a probability function cannot be derived or found precisely.

Another measure of the uncertainty of a quantity is to express it in terms of a reliability domain such as the confidence interval. A confidence interval is a numerical interval that would capture the quantity subject to uncertainty with a specified probabilistic confidence. Nevertheless, the use of confidence intervals has a few drawbacks: (1) the parameter population may not be normally distributed as assumed in the conventional procedures and this problem is particularly important when the sample size is small; (2) no means is available to directly combine the confidence intervals of individual contributing random components to give the overall confidence interval of the system.

A useful alternative to quantify the level of uncertainty is to use the statistical moments associated with a quantity subject to uncertainty. In particular, the variance and standard deviation which measure the dispersion of a stochastic variable are commonly used.

## AN OVERVIEW OF UNCERTAINTY ANALYSIS TECHNIQUES

Several techniques can be applied to conduct uncertainty analysis of water resources engineering problems. Each technique has different levels of mathematical complexity and data requirements. Broadly speaking, those techniques can be classified into two categories: analytical approaches and approximated approaches. The selection of an appropriate technique to be used depends on the nature of the problem at hand including availability of information, resources constraints, model complexity, and type and accuracy of results desired.

### Analytical Techniques

This section briefly describes several analytical methods that allow an analytical derivation of the exact PDF and/or statistical moments of a model as a function of several stochastic variables. Several useful analytical techniques for uncertainty analysis including derived distribution technique and various integral transform techniques. Although these analytical techniques are rather restrictive in practical applications due to the complexity of most models, they are, nevertheless, powerful tools for deriving complete information about a stochastic process, including its distribution, in some situations. The analytical techniques described herein are straightforward. However, the success of implementing these procedures largely depends on the functional relation, forms of the PDFs involved, and analyst's mathematical skill.

<u>Derived Distribution Technique</u>- This derived distribution method is also known as the transformation of variables technique. Example applications of this technique can be found in modeling the distribution of pollutant decay process and rainfall-runoff modeling.

<u>Fourier Transform Technique</u> - The Fourier transform of the PDF of a stochastic variable X results in the so-called the characteristic function. The characteristic function of

a stochastic variable always exists and two distribution functions are identical if and only if the corresponding characteristic functions are identical. Therefore, given a characteristic function of a stochastic variable, its PDF can be uniquely determined through the inverse Fourier transform. Also, the statistical moment of the stochastic variable X can be obtained by using the characteristic function. Fourier transform is particularly useful when stochastic variables are independent and linearly related. In such cases, the convolution property of the Fourier transform can be applied to derive the characteristic function of the resulting stochastic variable.

<u>Laplace and Exponential Transform Techniques</u> - The Laplace and exponential transforms of the PDF of a stochastic variable lead to the moment generating function. Similar to the characteristic function, statistical moments of a stochastic variable X can be derived from its moment generating function. There are two deficiencies associated with the moment generating functions: (1) the moment generating function of a stochastic variable may not always exist, and (2) the correspondence between a PDF and moment generating function may not necessarily be unique. However, the existence and unique conditions are generally satisfied in most situations. Fourier and exponential transforms are frequently used in uncertainty analysis of a model that involves exponentiation of stochastic variables. Examples of their applications can be found in probabilistic cash flow analysis and probabilistic modeling of pollutant decay.

<u>Mellin Transform Technique</u> - When the functional relation of a model satisfies the product form and the stochastic variables are independent and non-negative, the exact moments for model output of any order can be derived analytically by the Mellin transform. The Mellin transform is particularly attractive in uncertainty analysis of hydrologic and hydraulic problems because many models and the involved parameters satisfy the above two conditions. Similar to the convolution property of the Laplace and Fourier transforms, the Mellin transform of the convolution of the PDFs associated with independent stochastic variables in a product form is simply equal to the product of the Mellin transforms of individual PDFs. Applications of the Mellin transform can be found in economic benefit-cost analysis, and hydrology and hydraulics. One caution about the use of the Mellin transform is that under some combinations of distribution and functional form, the resulting transform may not be defined. This could occur especially when quotients or variables with negative exponents are involved.

<u>Estimations of Probabilities and Quantiles Using Moments</u> - Although it is generally difficult to analytically derive the PDF from the results of the integral transform techniques described above and the approximation techniques in the next section, it is, however, rather straightforward to obtain or estimate the statistical moments of the stochastic variable one is interested in. Based on the computed statistical moments, one is able to estimate the distribution and quantile of the stochastic variable. One possibility is to base on the asymptotic expansion about the normal distribution for calculating the values of CDF and quantile, and the other is to base on the maximum entropy concept.

**Approximation Techniques**

Most of the models or design procedures used in water resources engineering are nonlinear and highly complex. This basically prohibits any attempt to derive the probability distribution or the statistical moments of model output analytically. As a practical alternative, engineers frequently resort to methods that yield approximations to the statistical properties of uncertain model output. In this section, several methods that are useful for uncertainty analysis are briefly described.

<u>First-order variance estimation (FOVE) method</u> - The method, also called the variance propagation method, estimates uncertainty features associated with a model output based on the statistical properties of model's stochastic variables. The basic idea of the method is to approximate a model by the first-order Taylor series expansion. Commonly, the FOVE method takes the expansion point at the means of the stochastic variables. Consider a hydraulic or hydrologic design quantity W which is related to N stochastic variables $\mathbf{X}=(X_1, X_2, \ldots, X_N)$ as

$$W = g(X_1, X_2, \ldots, X_N)$$

The mean of W, by the FOVE method, can be estimated as

$$E[W] \quad g(\ _1, \ _2, \ldots, \ _N)$$

in which $\ _i$ is the mean of the i-th stochastic variable. When all stochastic variables are independent, the variance of the design quantity W can be approximated as

$$Var[W] \quad s_1^2 \ _1^2 + s_2^2 \ _2^2 + \ldots + s_N^2 \ _N^2$$

in which $s_i$ is the first-order sensitivity coefficient of the i-th stochastic variable and $\ _i$ represents the corresponding standard deviation. From the above equation, the ratio,

$s_i^2\ {}_i^2/\mathrm{Var}[W]$, indicates the proportion of overall uncertainty in the design quantity contributed by the uncertainty associated with the stochastic variable $X_i$.

In general, $E[g(\mathbf{X})]\quad g(\quad)$ unless $g(\mathbf{X})$ is a linear function of $\mathbf{X}$. Improvement of the accuracy can be made by incorporating higher-order terms in the Taylor expansion. However, the inclusion of the higher-order terms rapidly increases not only the mathematical complication but also the required information. The method can be expanded to include the second-order term to improve estimation of the mean to account for the presence of model non-linearity and correlation between stochastic variables. The method does not require knowledge of the PDF of stochastic variables which simplifies the analysis. However, this advantage is also the disadvantage of the method because it is insensitive to the distributions of stochastic variables on the uncertainty analysis.

The FOVE method is simple and straightforward. The computational effort associated with the method largely depends on the ways how the sensitivity coefficients are calculated. For simple analytical functions the computation of derivatives are trivial tasks. However, for functions that are complex and/or implicit in the form of computer programs, or charts/ figures, the task of computing the derivatives could become cumbersome or difficult. In such cases probabilistic point estimation techniques can be viable alternatives.

There are many applications of the FOVE method in the literature. Example applications of the method can be found in open channel flow, groundwater flow, water quality modeling, benefit-cost analysis, gravel pit migration analysis, storm sewer design, culverts, and bridges.

<u>Probabilistic Point Estimation (PE) Methods</u> - Unlike the FOVE methods, probabilistic PE methods quantify the model uncertainty by performing model evaluations without computing the model sensitivity. The methods generally is simpler and more flexible especially when a model is either complex or non-analytical in the forms of tables, figure, or computer programs. Several types of PE methods have been developed and applied to uncertainty analysis and each has its advantages and disadvantages. It has been shown that the FOVE method is a special case of the probabilistic PE methods when the uncertainty of stochastic variables are small.
Rosenblueth in 1975 developed a method for handling stochastic variables that are symmetric and the method is later extended to treat non-symmetric stochastic variables in 1981. The basic idea of Rosenblueth's PE method is to approximate the original PDF or PMF of the stochastic variable by assuming that the entire probability mass is concentrated at two points. The four unknowns, namely, the locations of the two points and the corresponding probability masses, are determined in such a manner that the first three moments of the original stochastic variable are preserved. For problems involving N stochastic variables, the two points for each variable are computed and permutated to produce a total of $2^N$ possible points of evaluation in the parameter space based on which the statistical moments of the model outputs are computed. The potential drawback of Rosenblueth's PE method is its practical application due to explosive nature of the computation requirement. For moderate or large N, the number of required model evaluations could be too numerous to be implemented practically, even on the computer. Example applications of Rosenblueth's PE method for uncertainty analysis can be found in groundwater flow model, dissolved oxygen deficit model, and bridge pier scouring model.

To circumvent the shortcoming in computation, Harr developed an alternative PE method that reduces the $2^N$ model evaluations required by Rosenblueth's method down to 2N. Harr's method utilizes the first two moments (that is, the mean and covariance) of the involved stochastic variables. The method is appropriate for treating stochastic variables that are normal. The theoretical basis of Harr's PE method is built on the orthogonal transformation using eigenvalue-eigenvector decomposition which maps correlated stochastic variables from their original space to a new domain in which they become uncorrelated. Hence, the analysis is greatly simplified. Harr's PE method has been applied to uncertainty analysis of a gravel pit migration model, regional equations for unit hydrograph parameters, groundwater flow models, and parameter estimation of a distributed hydrodynamic model.

Recently, Li proposed a computationally practical PE method that allows incorporation of the first four moments of correlated stochastic variables. In fact, Rosenblueth's solutions are a special case of Li's solution. Li's method requires $(N^2+3N+2)/2$ evaluations of the model. When the polynomial order of the model is four or less, Li's method yields the exact expected value of the model.

Among the three probabilistic PE algorithms described above, Harr's method is the most attractive from the computational viewpoint. However, the method cannot incorporate additional distributional information of the stochastic variables other than the first two moments. Such distributional information could have important effects on the results of uncertainty analysis. To incorporate the information about the marginal distributions of involved stochastic variables, a transformation between non-normal parameter space and a multivariate standard normal space has been incorporated into Harr's method. The resulting method preserves the computational efficiency of Harr's PE method while extends its capability to handle multivariate non-normal stochastic variables.

Monte-Carlo Simulation - Simulation is a process of replicating the real world based on a set of assumptions and conceived models of reality. Because the purpose of a simulation model is to duplicate reality, it is a useful tool for evaluating the effect of different designs on system performance. The Monte Carlo procedure is a numerical simulation to reproduce stochastic variables preserving the specified distributional properties.

Several books have been written for generating univariate random numbers. A number of computer programs are available in the public domain. The challenge of Monte Carlo simulation lies in generating multivariate random variates. Compared with univariate random variate generators, algorithms for multivariate random variates are much more restricted to a few joint distributions such as multivariate normal, multivariate lognormal, multivariate gamma, and few others. If the multivariate stochastic variables involved are correlated with a mixture of marginal distributions, the joint PDF is difficult to formulate. Rather than preserving the full multivariate features, practical multivariate Monte Carlo simulation procedures for problems involving mixtures of non-normal stochastic variables have been developed to preserve the marginal distributions and correlation of involved stochastic variables.

In uncertainty analysis, the implementation of brutal-force type of simulation is straightforward but can be very computationally intensive. Furthermore, because the Monte Carlo simulation is a sampling procedure, the results obtained inevitably involve sampling errors which decrease as the sample size increases. Increasing sample size for achieving higher precision generally means an increase in computer time for generating random variates and data processing. Therefore, the issue lies on using the minimum possible computation to gain the maximum possible accuracy for the quantity under estimation. For this, various variance reduction techniques have been developed.

Applications of Monte Carlo simulation in water resources engineering are abundant. Examples can be found in groundwater, benefit-cost analysis, water quality model, pier-scouring prediction, and open channel.

Resampling Techniques - Note that Monte Carlo simulations are conducted under the condition that the probability distribution and the associated population parameters are known for the stochastic variables involved in the system. The observed data are not directly utilized in the simulation. Unlike the Monte Carlo simulation approach, resampling techniques reproduce random data exclusively on the basis of observed ones. The two resampling techniques that are frequently used are jackknife method and bootstrap method.

## RELIABILITY ANALYSIS

In many water resource engineering problems, uncertainties in data and in theory, including design and analysis procedures, warrant a probabilistic treatment of the problems. The failure associated with a hydraulic structure is the result of the combined effect from inherent randomness of external load and various uncertainties involved in the analysis, design, construction, and operational procedures described previously.
Failure of an engineering system occurs when the load (external forces or demands) on the system exceeds the resistance (strength, capacity, or supply) of the system. In hydraulic and hydrologic analyses, the resistance and load are frequently functions of a number of stochastic variables. Without considering the time-dependence of the load and resistance, static reliability model is generally applied to evaluate the system performance subject to a single worst load event.

However, a hydraulic structure is expected to serve its designed function over an expected period of time. In such circumstances, time-dependent reliability models are used to incorporate the effects of service duration, randomness of occurrence of loads, and possible change of resistance characteristics over time.

In reliability analysis, the load and resistance functions are often combined to establish a performance function, $W(\mathbf{X})$, which divides the system state into a safe (satisfactory) set defined by $W(\mathbf{X})$ 0 and a failure (unsatisfactory) set defined by $W(\mathbf{X})<0$. The boundary separating the safe set and failure set is a surface defined by W(**X**)=0 which is called the failure surface or limit-state surface. The commonly used safety factor and safety margin are the special cases of the performance function. Alternatively, the reliability index, defined as the ratio of the mean to the standard deviation of the performance function, is another frequently used reliability indicator.

**Computation of Reliability**

The computation of reliability requires knowledge of probability distributions of load and resistance, or the performance function, W. This computation of reliability is called load-resistance interference.

Direct Integration Method - The method of direct integration requires the PDFs of the load and resistance or the performance function be known or derived. This information is seldom available in practice, especially for the joint PDF, because of the complexity of hydrologic and hydraulic models used in design. Explicit solution of direct integration can be obtained for only a few PDFs. For most PDFs numerical integration may be necessary. When using numerical integration, difficulty may be encountered when one deals with a multivariate problem.

Mean-Value First-Order Second-Moment (MFOSM) Method - The MFOSM method for reliability analysis employs the FOVE method to estimate the mean and standard deviation of the performance function W(**X**) from which the reliability index is computed. Several studies have shown that reliability is not greatly influenced by the choice of distribution for the performance function and the assumption of a normal distribution is quite satisfactory, except in the tail portion of a distribution. The MFOSM method has been used widely in various hydraulic structural and facility designs such as storm sewers, culverts, levees, flood plains, and open channel hydraulics.

The applications of the MFOSM method is simple and straightforward. However, it possesses certain weaknesses in addition to the difficulties with accurate estimation of extreme failure probabilities as mentioned above. These weaknesses include: (1) Inappropriate choice of the expansion point; (2) Inability to handle distributions with large skew coefficient; (3) Generally poor estimation of the mean and variance of highly nonlinear functions; (4) Sensitivity of the computed failure probability to the formulation of the performance function W; (5) Inability to incorporate available information on probability distributions. The general rule of thumb is not to rely on the result of the MFOSM method if any of the following conditions exist: (a) high accuracy requirements for the estimated reliability or risk; (b) high nonlinearity of the performance function; © many skewed random variables are involved in the performance function.

Advanced First-Order Second-Moment (AFOSM) Method - The main thrust of the AFOSM method is to reduce the error of the MFOSM method associated with the nonlinearity and non-invariability of the performance function, while keeping the advantages and simplicity of the first-order approximation. The expansion point in the AFOSM method is located on the failure surface defined by the limit-state equation.

Among all the possible values of **x** that fall on the limit-state surface one is more concerned with the combination of stochastic variables that would yield the lowest reliability or highest risk. The point on the failure surface with the lowest reliability is the one having the shortest distance to the point where the means of the stochastic variables are located. This point is called the design point or the most probable failure point. With the mean and standard deviation of the performance function computed at the design point, the AFOSM reliability index can be determined. At the design point, the sensitivities of the failure probability with respect to each of the stochastic variable can be computed easily. Methods for treating non-normal and correlated stochastic variables have been developed for the AFOSM method.

Due to the nature of nonlinear optimization, the algorithm AFOSM does not necessarily converge to the true design point associated with the minimum reliability index. Therefore, different initial trial points be used and the smallest reliability index be selected to compute the reliability.

**Time-to-Failure Analysis**

Any system will fail eventually; it is just a matter of time. Due to the presence of many uncertainties that affect the operation of a physical system, the time that the system fails to satisfactorily perform its intended function is a random variable. Instead of considering detailed interactions of resistance and loading over time, a system or its components can be treated as a black box or a lumped-parameter system and their performances are observed over time. This reduces the reliability analysis to a one-dimensional problem involving time as the only random variable. The term 'time' could be used in a more general sense. In some situations other physical scale measures, such as distance or length, may be appropriate for system performance evaluation.

Failure and Repair Characteristics - The time-to-failure analysis is particularly suitable for assessing the reliability of systems and/or components which are repairable. For a system that is repairable after its failure, the time period it would take to have it repaired back to the operational state is uncertain. Therefore, the time-to-repair (TTR) is also a random variable.

For a repairable system or component, its service life can be extended indefinitely if repair work can restore the system as if it was new. Intuitively, the probability of a repairable system available for service is greater than that of a non-repairable system.

The failure density function is the probability distribution that governs the time occurrence of failure and it serves as the common thread in the reliability assessments in time-to-failure analysis. Among them, the exponential distribution perhaps is the most widely used. Besides its mathematical simplicity, the exponential distribution has been found, both phenomenologically and empirically, to adequately describe the time-to-failure distribution for components, equipment, and systems involving components with mixtures of life distributions.

In general, the failure rate for many systems or components has a bathtub shape in that three distinct life periods, namely, early life (or infant mortality) period, useful life period, and wear-out life period are identified. A commonly used reliability measure of system performance is the mean-time-to-failure (MTTF) which is the expected time-to-failure.

For repairable water resources systems, such as pipe networks, pump stations, storm runoff drainage structures, failed components within the system can be repaired or replaced so that the system can be put back into service. The time required to have the failed system repaired is uncertain and, consequently, the total time required to restore the system from its failure to operational state is a random variable.

Like the time-to-failure, the random time-to-repair (TTR) has the repair density function describing the random characteristics of the time required to repair a failed system when failure occurs at time zero. The repair probability is the probability that the failed system can be restored within a given time period and it is sometimes used for measuring the maintainability. The mean-time-to-repair (MTTR) is the expected value of time-to-repair of a failed system which measures the elapsed time required to perform the maintenance operation. The MTTF is a proper measure of the mean life span of a non-repairable system. For a repairable system, a more representative indicator for the fail-repair cycle is the mean-time-between-failure (MTBF) which is the sum of MTTF and MTTR.

Availability and Unavailability - A repairable system experiences a repetition of repair-to-failure and failure-to-repair processes during its service life. Hence, the probability that a system is in operating condition at any given time t for a repairable system is different than that of a non-repairable system. The term availability is generally used for repairable systems to indicate the probability that the system is in operating condition at any given time t. On the other hand, reliability is appropriate for non-repairable systems indicating the probability that the system has been *continuously* in its operating state starting from time zero up to time t.
Availability can also be interpreted as the percentage of time that the system is in operating condition within a specified time period. On the other hand, unavailability is the percentage of time that the system is not available for the intended service in a specified time period, given it is operational at time zero.

## SYSTEM RELIABILITY

Most systems involve many sub-systems and components whose performances affect the performance of the system as a whole. The reliability of the entire system is affected not only the reliability of individual sub-systems and components, but also the interaction and configuration of the subsystems and components. Furthermore, water resources systems involve multiple failure modes, that is, there are several potential modes of failure in which the occurrence of any or a combination of such failure modes constitute the system failure. Due to the fact that different failure modes might be defined over the same stochastic variables space, the failure modes are generally correlated.

For a complex system involving many sub-systems, components and contributing stochastic variables, it is generally difficult, if not impossible, to directly assess the reliability of the system. In dealing with a complex system, the general approach is to reduce the system configuration, based on its component arrangement or modes of operation, to a simpler system for which the analysis can be performed easily. However, this goal may not be achieved for all cases necessitating the development of a special procedure. Some of the potentially useful techniques for water resources system reliability evaluation are briefly described below.

State Enumeration Method - The method lists all possible mutually exclusive states of the system components that define the state of the entire system. In general, for a system containing M components in which each can be classified into N operating states, there will be $N^M$ possible states for the entire system. Once all the possible system states are enumerated, the states that result in successful system operation are identified and the probability of the occurrence of each successful state is computed. The last step is to sum all of the successful state probabilities which yield the system reliability.

Path Enumeration Method - A path is defined as a set of components or modes of operation which lead to a certain state of the system. In system reliability analysis, the system states of interest are those of failed state and operational state. The tie-set analysis and cut-set analysis are the two well-known techniques.

The cut-set is defined as a set of system components or modes of operation which, when failed, causes failure of the system. Cut-set analysis is powerful for evaluating system reliability for two reasons: (1) it can be easily programmed on digital computers for fast and efficient solutions of any general system configuration, especially in the form of a network, and (2) the cut-sets are directly related to the modes of system failure. The cut-set method utilizes the minimum cut-set for calculating the system failure probability. A minimum cut-set implies that all components of the cut-set must be in the failure state to cause system failure. Therefore, the components or modes of operation involved in the minimum cut-set are effectively connected in parallel and each minimum cut-set is connected in series.

As the complement of a cut-set, a tie-set is a minimal path of the system in which system components or modes of operation are arranged in series. Consequently, a tie-set fails if any of its components or modes of operation fail. The main disadvantage of the tie-set method is that failure modes are not directly identified. Direct identification of failure modes is sometimes essential if a limited amount of a resource is available to place emphasis on a few dominant failure modes.

Conditional Probability Approach - The approach starts with a selection of key components and modes of operation whose states (operational or failed) would decompose the entire system into simple series and/or parallel subsystems for which the reliability or failure probability of subsystems can be easily evaluated. Then, the reliability of the entire system is obtained by combining those of the sub-systems using conditional probability rule.

Fault Tree Analysis - Conceptually, fault-tree analysis traces from a system failure backward, searching for possible causes of the failure. A fault tree is a logical diagram representing the consequence of component failures (basic or primary failures) on system failure (top failure or top event). The fault tree consists of event sequences that lead to system failure.

## RISK-BASED DESIGN OF WATER RESOURCES SYSTEMS

Reliability analysis can be applied to design of various hydraulic structures with or without considering risk costs which are the costs associated with the failure of hydraulic structures or systems. The risk-based least cost design of hydraulic structures promises to be, potentially, the most significant application of reliability analysis.

The risk-based design of water resources engineering structures integrates the procedures of economic, uncertainty, and reliability analyses in the design practice. Engineers using a risk-based design procedure consider trade-offs among various factors such as risk, economics, and other performance measures in hydraulic structure design. When risk-based design is embedded into an optimization framework, the combined procedure is called optimal risk-based design.

Because the cost associated with the failure of a hydraulic structure cannot be predicted from year to year, a practical way to quantify it is to use an expected value on the annual basis. The total annual expected cost is the sum of the annual installation cost and annual expected damage cost.

In general, as the structural size increases, the annual installation cost increases while the annual expected damage cost associated with failure decreases. The optimal risk-based design determines the optimal structural size, configuration, and operation such that the annual total expected cost is minimized.

In the optimal risk-based designs of hydraulic structures, the thrust of the exercise is to evaluate annual expected damage cost as the function of the PDFs of loading and resistance, damage function, and the types of uncertainty considered. The conventional risk-based hydraulic design considers only the inherent hydrologic uncertainty due to the random occurrence of loads. It does not consider hydraulic and economic uncertainties. Also, the probability distribution of the load to the water resources system is assumed known which is generally not the case in reality. However, the evaluation of annual expected cost can be made by further incorporating the uncertainties in hydraulics, hydrological model and parameters.

To obtain an accurate estimation of annual expected damage associated with structural failure would require the consideration of all uncertainties, if such can be practically done. Otherwise, the annual expected damage would, in most cases, be underestimated, leading to inaccurate optimal design.

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