A METHODOLOGICAL INTEGRATION OF GENERALIZED EQUIVALENCE CLASSES, NATURAL CATEGORIES, AND CROSS-MODAL PERCEPTION

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A generalized equivalence class contains some stimuli that are perceptually disparate and others that resemble one another. They all function as members of a generalized equivalence class when all of them occasion the selection of the others in the set, and a response trained to one or a few are occasioned by all other stimuli in the set. This article formalizes the structural relations among the stimuli in these classes and identifies the generalization tests needed to fully document the range of stimuli that function as members of such a class. These include tests conducted in variant-to-base, base-to-variant, and variant-to-variant formats. The structural and functional properties that characterize generalized equivalence classes also characterize naturally occurring categories, natural kinds, semantic superordinate categories, and cross-modal perceptual classes. Thus, the procedures used to measure generalized equivalence classes could also be used to document these other classes.

When a set of stimuli function as a class, each stimulus in the set occasions the selection of the other stimuli in the set. This occurs even though only a few relations among them had been established by training (Fields & Reeve, in press; Sidman, 1994; Wasserman, Keidinger, & Bhatt, 1988). Alternatively, a set of stimuli function as a class when a response trained to occur in the presence of a few stimuli in the set is also occasioned by all of the stimuli in the set without benefit of direct training (Goldiamond, 1962; Keller & Schoenfeld, 1950). In either case, the control of behavior by stimulus classes is adaptive because it results in the emergence of untrained performances that are appropriate to new circumstances. Thus, it is important to understand the relations that exist among the stimuli in a class, to have tests that can be used to confirm the

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existence of a stimulus class, and to measure the range of stimuli that function as class members.

Some stimulus classes are said to be close-ended because they contain a finite number of stimuli. In one type called an equivalence class (Fields & Verhave, 1987; Sidman, 1994; Sidman & Taliby, 1982), the stimuli that are class members can not be arrayed along some dimension and do not bear any perceived similarity to each other. After appropriate training, however, they become functionally interchangeable. To illustrate, a set of three stimuli can give rise to nine relations. If the stimuli in the set are represented by A, B, and C, a class can be established by training AB and BC. If all of the stimuli in the set then occasion the mutual selection of the other stimuli in the set, all of them are functioning as members of an equivalence class.

The relations among the stimuli in an equivalence class have the properties of reflexivity, symmetry, and transitivity, alone and in combination. The emergence of the new relations AA, BB, and CC demonstrate the property of reflexivity. The emergence of the new relations BA and CB demonstrate the symmetrical property of each trained relation. The emergence of the new relation AC demonstrates the transitive property of the A and C stimuli. Finally, the emergence of the new relation CA demonstrates the combined properties of symmetry and transitivity, or equivalence. These tests constitute a well developed set of procedures for the documentation of equivalence classes.

In contrast to equivalence classes, other stimulus classes contain stimuli that can be arrayed along some physically defined, mathematically derived (Hrycenko & Harwood, 1980), or psychometrically defined dimensions, and they can be scaled in terms of perceived similarity. These classes, which are open-ended because they contain an infinite number of stimuli, have been called dimensional (Fields & Reeve, in press), feature-based (McIlvane, Dube, Green, & Serna, 1993), relational (Wright, Cook, Rivera, Sands, & Delius, 1988), perceptual (Fields & Reeve, in press), similarity-based (Wasserman & DeVolder, 1993), fuzzy (Herrnstein, 1990; Wittgenstein, 1953), ill-defined (Homa & Little, 1985), probabilistic (Medin & Smith, 1984), or polymorphous (Jitsumori, 1993, 1994; Lea & Harrison, 1978) depending on the dimension along which potential class members can be arrayed.

Open-ended classes can be established by use of multiple exemplar training. After reinforcing the selection of a few stimuli in a set in the presence of a few other stimuli in the set, the presentation of any stimulus in the set may occasion the selection of any other stimulus in the set. In addition, if many of the stimuli in the set are also discriminable from each other (Fields & Reeve, in press; Lea, 1984; Wasserman et al., 1988), all of the stimuli would be functioning as members of an open-ended class (Fields & Reeve, in press; Goldiamond, 1962; Herrnstein, 1990; Keller & Schoenfeld, 1950; Lea, 1984; Wasserman et al., 1988; Wright et al., 1988). The procedures needed to document the existence of these classes have also been well developed. As will be demonstrated,
however, additional tests may have to be conducted to fully document the existence, and extent, of open-ended classes.

A third type of stimulus class consists of some stimuli that are perceptually disparate and others that are physically similar. When all of these stimuli become related to each other, they constitute a generalized equivalence class (Adams, Fields, & Verhave, 1993). A real world example of such a class would be the written words DOG, PERRO, and CHIEN, a picture of a dog and many variants of that picture, the many sounds of barking dogs, and the many odors of dogs. In a generalized equivalence class, the perceptually disparate stimuli function as members of an equivalence class, and the physically similar stimuli function as members of an open-ended class. All of the stimuli in both of these classes become related to each other because one stimulus serves as a member of both the equivalence class and the open-ended class (Fields, Adams, Brown, & Verhave, 1993).

A number of generalized equivalence classes have been established in laboratory settings with visual stimuli only (Barnes & Keenan, 1993; Fields et al., 1993; Fields, Reeve, Rosen, et al., 1997; Stromer, Mackay, Howell, & McVay, 1996), visual and auditory stimuli (Lane, Clow, Innis, & Critchfield, 1998; Lane, Innis, Clow, & Critchfield, 1998; Rehfeldt, Hayes, & Steele, 1998), tactual and visual stimuli (Belanich & Fields, 1999), and visual and drug-induced interoceptive stimuli (deGrandpre, Bickel, & Higgins, 1992). Thus, membership of stimuli in a generalized equivalence class does not appear to be bounded by sensory modality. Generalized equivalence classes, then, may provide a good laboratory analog of most categories that influence behavior in natural settings.

Unlike equivalence classes and open-ended classes, however, there is as yet neither a full characterization of the relations that exist among the stimuli in generalized equivalence class, nor a complete specification of the tests needed to fully document these relations. The purpose of the remainder of the current article is to clarify each of these issues.

**Minimal generalized equivalence classes.** To date, generalized equivalence classes have been studied in laboratory settings using minimal preparations; the classes have consisted of a small number of disparate stimuli (3 or 4) that function as members of a basal equivalence class along with variants of only one member of the basal equivalence class. Figure 1 illustrates such a minimal generalized equivalence class of a marsh. The class contains three disparate representations of a marsh: a topographical map, its orienteering symbol, and a picture of a marsh. It also contains many images of the basal picture of the marsh that can be arrayed along a continuum, in this case, one of increased masking (C').

Figure 2 contains a symbolic representation of the same minimal generalized equivalence class. The three stimuli that are members of the basal equivalence class are represented by the letters A, B, and C. The stimuli that are variants of one member of the basal equivalence class are represented collectively by C'. C't defines the variant that is most removed from C along some dimension that always occasions selection of another
Figure 1. A minimal generalized equivalence class that contains variants of one member of a basal equivalence class.

stimulus in the set. C't, then, defines the endpoint of the range of variants that functions as a member of the generalized equivalence class.

A generalized equivalence class can be established by the formation of an equivalence class among the physically disparate stimuli, the formation of an open-ended class among the stimuli that are dimensional variants of each other, and the inclusion of one stimulus as a member of both classes (Fields, Reeve, Adams, & Brown, & Verhave, 1997). Thus,
the three disparate images of the marsh can become members of a basal equivalence class by training conditional discriminations between the map of the marsh and word MARSH (AB) along with the word MARSH and the picture of a marsh (BC). The formation of the equivalence class of marsh would be confirmed by demonstrations of class-consistent selections in the presence of all of the emergent relations probes that can be derived from the potential class members. Many of the masked images of the picture of the marsh (variants of C), either function as members of an open-ended class without training, or come to function as members of an open-ended class by use of multiple exemplar training and testing (Fields & Reeve, in press; Hull, 1920; Herrnstein, Loveland, & Cable, 1975). If training is required, the selection of one stimulus would be reinforced in the presence of some of the stimuli in the same set. The emergence of an open-ended class would be shown if most of the stimuli in the set occasioned the selection of other stimuli in the set without additional training. In addition, it would also be necessary to show that many of the stimuli in the set were discriminable from each other (Fields & Reeve, in press; Keller & Schoenfeld, 1950; Lea, 1984; Wasserman et al., 1988). Finally, to ensure that the stimuli in both classes were functioning as members of the generalized equivalence class of marsh, one picture of the marsh would have to be a member of both stimulus classes.

If all of the images in the basal equivalence class and the open-ended class occasion the mutual selection of all other members of both classes, collectively, they would constitute a generalized equivalence class. In all of the studies conducted to date, the tests used to document the range of stimuli that function as members of a generalized equivalence class have all had the same format. Tests were conducted using a matching-to-sample procedure. Each test trial involved the presentation of a variant as a sample stimulus with members of the basal equivalence classes as comparisons. Thus, these tests were conducted in a variant-to-base format.
Variant-to-base tests. After AB and BC training to establish an equivalence class, C'-A generalization tests of equivalence involve the presentation of variants of C as samples with the A1 and A2 stimuli as comparisons. C'-B generalization tests of symmetry involve the presentation of the variants of C as samples with the B1 and B2 stimuli as comparisons. Finally, the C'-C tests of primary generalization involve the presentation of the variants of C as samples with the C1 and C2 stimuli as comparisons. These tests identify the range of stimuli that are functioning as members of an open-ended stimulus class, the members of which are arrayed along some measureable dimension.

If one comparison is available for each potential class, a subject does not have access to a response that could be used if a test stimulus was not a member of any of the classes. The absence of such a response, then, could unnecessarily broaden the range of variants that would appear to be functioning as members of a given open-ended class. Thus, procedures for measuring the range of variants that function as members of open-ended classes should include a response option that permits a subject to signal that a test stimulus is not a member of any of the available classes. In the context of a matching-to-sample procedure, that response is called a neither comparison (Fields et al., 1993; Fields, Adams, Buffington, Yang, & Verhave, 1996; Fields, Reeve, Adams, et al., 1997) or a default option (Innis, Lane, Miller, & Critchfield, 1998). The inclusion of a neither comparison, then, allows the identification of variants that are not functioning as members of any of the classes established by the contingencies in a particular experiment.

Variants that always occasion the selection of one comparison would be functioning as members of the class of which the selected comparison was a member. A variant that occasions the selection of the neither comparison would not be functioning as a member of any of the classes designated by the other comparison stimuli. In general, some range of variants that are most similar to a stimulus that is a member of one of the basal equivalence classes would occasion the selection of all of the stimuli in that class. As such, those variants would be functioning as members of that class. Variants that deviate to a greater degree from the value of the stimulus that was a member of the basal equivalence class would occasion a decline in the selection of stimuli from that class, and a complementary increase in the selection of the neither comparison. These variants would not be functioning as members of either of the basal equivalence classes. The point of inflection in the positive comparison (Co+) function would define the endpoint of the range of variants that functioned as members of a generalized equivalence class.

Table 1 summarizes the baseline conditional discriminations and emergent relations tests that would be used to document the basal equivalence class. It also lists the generalization tests of emergent relations that would be presented in the variant-to-base format to identify the variants that function as members of a generalized equivalence class.

Base-to-variant tests. As can be seen in the second column of Table
Table 1
Tests to Assess Minimal Generalized Equivalence Class

<table>
<thead>
<tr>
<th>Probe Type</th>
<th>Base-to-Base</th>
<th>Test Formats</th>
<th>Base-to-Variant</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL</td>
<td>A-B</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B-C</td>
<td></td>
<td>B-C'</td>
</tr>
<tr>
<td>Rfx/Pgen</td>
<td></td>
<td>C-C</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C'-C</td>
<td>C-C'</td>
</tr>
<tr>
<td>Sym</td>
<td>B-A</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C-B</td>
<td>C'-B</td>
<td></td>
</tr>
<tr>
<td>Tty</td>
<td>A-C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eqv</td>
<td>C-A</td>
<td>C'-A</td>
<td></td>
</tr>
</tbody>
</table>

Note. All of the baseline conditional discriminations, emergent relations probes and generalization tests of the baseline conditional relations and emergent relations in base-to-base, variant-to-base, and base-to-variant formats after training with AB and BC, when using variants of C, denoted by C'.

1, however, a number of emergent relations probes involve the presentation of the C stimuli as comparisons. These include the reflexivity probe (C-C), the baseline conditional discrimination (B-C), and the transitivity probe (A-C). To date, no studies have used these types of probes to document the extent of generalized equivalence classes. Because the basal stimuli from a class would be presented as samples prior to the presentation of variants as comparisons, these tests would be conducted in a base-to-variant format.

The right-most column of Table 1 lists symbolic representations of generalization tests conducted in the base-to-variant format. The C-C' probes would assess primary generalization of the reflexive relation, the B-C' probes would assess generalization of one of the baseline conditional discriminations, and the A-C' probes would assess generalization of transitivity.

A base-to-variant test trial would contain one member of a basal equivalence class as a sample stimulus with positive and negative comparisons drawn from the dimension along which the variants are arrayed. The negative comparison would be a member of the basal equivalence class that differs from the sample. The other variants would be presented as the potential positive comparisons. Finally, each trial should also contain a neither comparison. The neither comparison could be selected if the subject could not determine affinity of either comparison with the sample. For example, some A-C' test trials involve the presentation of A1 as a sample, with C2 as a negative comparison and a variant of C1 as the potential positive comparison. Selection of a variant indicates that it is functioning as a member of the same class that contains the sample stimulus on that trial. Selection of the neither comparison would indicate that the variant stimulus on that trial was not functioning as a member of the class that contained the sample stimulus on that trial.
In these A-C' test trials, the A stimulus from one class would occasion the selection of variants that were closest in value to the variant that was a member of the basal equivalence class. The variant that was always selected in the presence of the sample stimulus from a basal equivalence class, and was most removed from its corresponding member of the basal equivalence class, would define the outer limit of the range of variants that functioned as members of the generalized equivalence class. Variants that were beyond that boundary stimulus would be selected with decreasing likelihood. This decline would also be accompanied by an increase in the percentage of trials that occasioned the selection of the neither comparison.

To date, all of the experiments that have studied generalized equivalence classes that contained variants of one class member have used only variant-to-base tests. According to the current analysis, however, variant-to-base tests alone do not provide a complete documentation of membership of all stimuli in a given generalized equivalence class. Thus, complete documentation of the range of stimuli that function as members of a generalized equivalence class with variants of one basal class member requires the use of generalization tests conducted in the base-to-variant format for some relations and generalization tests conducted in the variant-to-base format for other relations.

Generality across locus of variants and training structures. The example presented above included variants of the C stimulus after training of AB and BC to form basal equivalence classes. A similar analysis can be conducted with variants of any member of a basal equivalence class. For example, if AB and BC are the baseline relations, and variants of the A stimulus were used to form a generalized equivalence class, probes presented in the variant-to-base format would assess primary generalization with A'-A trials, generalization of a baseline conditional relation with A'-B trials, and generalization of transitivity with A'-C trials. Probes presented in the base-to-variant format would be used to assess generalization of equivalence with C-A' trials, generalization of symmetry with B-A' trials, and primary generalization with A-A' trials.

The class considered thus far was formed by training AB and BC. Equivalence classes can be formed by use of other training clusters (Fields & Verhave, 1987), also called training structures (Saunders & Green, 1999). For example, basal equivalence classes could be formed by the establishment of AB and AC relations which is called sample-as-node training (Fields, Hobbie-Reeve, Adams, & Reeve, 1999; Saunders, K. J., Saunders, Williams, & Spradlin, 1993; Saunders, Drake, & Spradlin, 1999). After class formation of this sort, variants of the C stimulus could be used to evaluate the extension of the equivalence classes by generalization. Under these conditions, tests conducted in the variant-to-base format would include assessments of primary generalization with C'-C tests, generalization of symmetry with C'-A trials, and generalization of equivalence with C'-B trials. The tests that could be conducted in the base-to-variant format would include generalization tests of equivalence with B-C' trials, of a baseline conditional relation with A-C' trials, and primary
generalization with C-C' probes. Although Lane, Clow, et al. (1998) studied the formation of generalized equivalence classes that had the above mentioned structure, that study included generalization tests of emergent relations which were conducted in the variant-to-base format only.

**Differential effects of variant-to-base and base-to-variant tests.** As can be seen in Table 1, the same pairs of stimuli can be used in generalization tests conducted in a variant-to-base and a base-to-variant format. For example, after training AB and BC with variants of C, the pairs of tests would include C'-A and A-C', as well as B-C' and C'-B. Theoretically, the same set of stimuli used in tests with different formats could show either the same or different ranges of variants that function as members of a generalized equivalence class. Each outcome would have different implications with regard to a broad characterization of generalized equivalence classes. Thus, if the same set of variants function as class members for all test formats, class membership would not be influenced by testing format. On balance, differences in class membership would show that membership in a generalized equivalence class is dependent on test format. Preliminary data showing both outcomes have been reported (Varelas, Fields, Wadhwa, DeRosse, & Belanich, 1999; Wadhwa, Fields, DeRosse, Varelas, & Belanich, 1999). Once replicated, these data would demonstrate the necessity of conducting variant-to-base and base-to-variant generalization tests to identify the range of variants that function as members of a generalized equivalence class containing variants of only one basal class member. These results, then, suggest that the procedures used to measure the extent of a generalized equivalence class may be one of its defining properties.

**Partially elaborated generalized equivalence classes.** The generalized equivalence classes that have been studied in laboratory settings have contained variants of only one member of a basal equivalence class. Theoretically, however, a generalized equivalence class can have variants of any or all members of the basal equivalence class. Indeed, such an occurrence is probably the rule rather than the exception in most real world settings. How, then, can one measure all of the relations that exist among the members of a such a generalized equivalence class?

Figure 3 contains an example of a partially elaborated generalized equivalence class containing variants of two of three basal class members. The basal class consists of a map and a picture of a marsh, and the written word marsh. The picture and the map, which are members of the basal equivalence class, also function as endpoint stimuli for the variants that are illustrated in the ordered arrays in the columns beneath each basal stimulus. Such a class is represented in symbolic form in Figure 4. The members of the basal equivalence class consist of A, B, and C. Two of the basal stimulus function as anchors for dimensional variants represented by A' and C'. The most extreme variants that function as members of the partially elaborated generalized equivalence class are represented by A't and C't.

Table 2 lists all of the generalization tests that are required to document a generalized equivalence class containing variants of two
Figure 3. A partially elaborated generalized equivalence class containing three basal members of an equivalence class with dimensional variants of two of the basal class members.

basal class members. After training the baseline conditional AB and BC, the emergent relations probes listed in the first column would be used to demonstrate the formation of the basal equivalence class. The relations among the variants of one class member with the other basal members of the equivalence class can be identified by use of the variant-to-base tests in Column 2 and the base-to-variant tests in Column 3. These tests,
A1----> B1----> C1
•
•
•
•
A1'  
•
•
•
C1'
•
•
•
A1't
•
•
•
C1't

Figure 4. A symbolic representation of a partially elaborated generalized equivalence class containing three basal members of an equivalence class (A, B, and C) with dimensional variants of two of the basal class members (A' and C'). The basal equivalence class is established by training AB and BC. A1' and C1' represent the dimensional variants of the basal stimuli A1 and C1, respectively. A1't and C1't represent the most extreme variant of A1 and C1 that function as members of generalized equivalence class 1.

Table 2
Tests to Assess Partially Elaborated Generalized Equivalence Class

<table>
<thead>
<tr>
<th>Probe Type</th>
<th>Test Formats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base-to-Base</td>
<td>Variant-to-Base</td>
</tr>
<tr>
<td>BL</td>
<td>A-B</td>
</tr>
<tr>
<td>Rfx/Pgen</td>
<td>A-A</td>
</tr>
<tr>
<td>Sym</td>
<td>B-C</td>
</tr>
<tr>
<td>Tty</td>
<td>C-C'</td>
</tr>
<tr>
<td>Eqv</td>
<td>B-A</td>
</tr>
<tr>
<td></td>
<td>A-C</td>
</tr>
</tbody>
</table>

Note. All baseline conditional discriminations, and generalization tests of emergent relations and of the baseline conditional discriminations for a generalized equivalence class that contains three basal class members (A, B, and C) and variants of two basal class members (A' and C') when the class is formed by training AB and BC.
however, are not comprehensive because they do not measure the
relations among the variants of two different members of the basal
equivalence class.

**Variant-to-variant tests.** The generalization tests needed to identify
the variants of one basal class member that are related to the variants of
the second basal class member are listed symbolically in the last column
of Table 2. Because variants are presented as both samples and
comparisons, these tests are presented in a *variant-to-variant* format. For
example, the C'-A' test for Class 1 would present variants of the basal
class member C1 as samples with variants of the basal class member A1
as a nominal positive comparison, and A2 as a negative comparison. In
addition, test trials would also contain a neither comparison. For a given
variant of C, some range of variants of A should always be selected by a
subject. For variants that differ more from A1, the selection of the variants of
A should decrease and be accompanied by a complementary increase in
the selection of the neither comparison. The variant of C presented as a
sample and the range of variants that were always selected would then be
members of the same generalized equivalence class. The results of many of
these tests would define the range of variants of C and of A that functioned
as members of a partially elaborated generalized equivalence class.

**Potential impact of test format on class membership.** One obvious
question emerges from this analysis. Would the range of variants that
function as members of a generalized equivalence class differ with testing
format? This question had been addressed when we considered
differences that might arise in the performances occasioned by variant-to-
base and base-to-variants tests. Now the question can be expanded to
include a consideration of these test formats along with tests conducted
in a variant-to-variant format.

To place the question in a concrete experimental framework, consider a
3-node 5-member equivalence class established by training AB, BC, CD,
and DE, where the variants of A and E are used to assess extension of class
membership by generalization. The generalized equivalence class can be
represented symbolically by . . . A't A → B → C → D → E → E't . . . . After the
establishment of the basal equivalence class, variant-to-base tests would
identify A't as the variant of A most distant from A that always occasioned the
selection of the basal class members A-E. In addition, similar tests would
identify E't as the variant of E most distant from E that always occasioned the
selection of the basal class members A-E. Thus, variants that range from A
to A't and E to E't would be functioning as members of the generalized
equivalence class. When variant-to-variant tests are conducted, however,
two outcomes are possible. The A't stimulus might always occasion the
selection of E't, or A't might occasion the selection of E't with a lower relative
frequency. Arguments predictive of either outcome can be presented and
depend on assumptions made regarding the strength of the relations among
the members of an equivalence class and the strength of relations among
the members of an open-ended class.

On one hand, it can be assumed that all of the stimuli are equally
related to each other in an equivalence class (Sidman, 1992, 1994). Specifically, this assumption is supported by the observation that all members of a potential equivalence class are selected with equal likelihood in the presence of the other members of the potential class during emergent relations tests. It can also be assumed that all of the stimuli in an open-ended class are equally related to each other (Goldiamond, 1962). This assumption is supported by the observation that a response trained to occur in the presence of a few members of a

![Figure 5](image_url). A fully elaborated generalized equivalence class containing three basal members of an equivalence class with dimensional variants of the three basal class members.
potential class is evoked with the same probability by the other members of the class (Hull, 1920; Keller & Schoenfeld, 1950; Wasserman et al., 1988; Wright et al., 1988). According to these assumptions, $E^i$ and $A^i$ should always occasion selection of each other.

Alternatively, the assumption that stimuli in a class are equally related to each other may not be accurate. Specifically, there is evidence that the strength of relations among the stimuli in an equivalence class is an inverse function of nodal distance (Bentall, Dickins, & Fox, 1993; Dube, Green, & Serna, 1993; Fields, Landon-Jlmenez, Buffington, & Adams, 1995; Fields, Reeve, Rosen, et al., 1997; Fields & Verhave, 1987; Kennedy, 1991; McDonagh, McIlvane, & Stoddard, 1984; Meehan & Fields, 1995; Saunders, Wachter, & Spradlin, 1988; Sidman, Kirk, & Willson-Morris, 1985; Spencer & Chase, 1996), and the strength of relations among stimuli in an open-ended class is an inverse function of the physical or psychometric distance that separates stimuli in a domain (Medin & Smith, 1984; Rosch & Mervis, 1975). According to these assumptions, $E^i$ and $A^i$ should be less likely to occasion the mutual selection of each other in our hypothetical generalized equivalence class.

Because plausible arguments can be made to predict either outcome, the question cannot be answered by a logical or theoretical analysis. Empirical data, then, are needed to determine the relations that exist between the boundary variants of two members of a generalized equivalence class. On an empirical level, those data would show how all test formats influence membership in a generalized equivalence class. On a theoretical level, those data should extend our understanding of the strength of relations that exist among stimuli that are members of open-ended classes, equivalence classes, and generalized equivalence classes.

**Fully elaborated generalized equivalence classes.** A fully elaborated generalized equivalence class has variants of every member of a basal equivalence class, as illustrated in Figure 5. The basal members of this class consist of a map of a marsh, a picture of a marsh, and the written word, MARSH. Each basal class member functions as an anchoring stimulus for the variants that are illustrated in the ordered arrays in the columns beneath each basal stimulus.

Figure 6 represents this generalized equivalence class in symbolic form. The members of the basal equivalence class consist of A, B and C. Each basal stimulus functions as an anchor for dimensional variants represented by $A'$, $B'$, and $C'$. The most extreme variants that function as a member of the fully elaborated generalized equivalence class are represented by $A''$, $B''$, and $C''$.

Table 3 lists all of the generalization tests required to document a generalized equivalence class containing variants of all basal class members. After training the baseline conditional AB and BC (listed in the first column), the emergent relations probes listed in the first column would be used to demonstrate the formation of the basal equivalence class. The relations among the variants of one class member with the other basal members of the underlying equivalence classes can be
Figure 6. A symbolic representation of a fully elaborated generalized equivalence class containing three basal members of an equivalence class (A, B, and C) with dimensional variants of all basal class members (A', B', and C'). The basal equivalence class is established by training AB and BC. A1', B1', and C1' represent the dimensional variants of the basal stimuli A1, B1, and C1, respectively. A1't, B1't, and C1't represent the most extreme variants of A1, B1, and C1 that function as members of generalized equivalence class 1.

Table 3

Tests to Assess Fully Elaborated Generalized Equivalence Class

<table>
<thead>
<tr>
<th>Probe Type</th>
<th>Base-to-Base</th>
<th>Variant-to-Base</th>
<th>Base-to-Variant</th>
<th>Variant-to-Variant</th>
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<tbody>
<tr>
<td>BL</td>
<td>A-B</td>
<td>A'-B</td>
<td>A-B'</td>
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<td>C-C</td>
<td>C'-C</td>
<td>C-C'</td>
<td>C'-C'</td>
</tr>
<tr>
<td>Sym</td>
<td>B-A</td>
<td>B'-A</td>
<td>B-A'</td>
<td>B'-A'</td>
</tr>
<tr>
<td></td>
<td>C-B</td>
<td>C'-B</td>
<td>C-B'</td>
<td>C'-B'</td>
</tr>
<tr>
<td>Tty</td>
<td>A-C</td>
<td>A'-C</td>
<td>A-C'</td>
<td>A'-C'</td>
</tr>
<tr>
<td>Eqv</td>
<td>C-A</td>
<td>C'-A</td>
<td>C-A'</td>
<td>C'-A'</td>
</tr>
</tbody>
</table>

Note. All baseline conditional discriminations, and generalization tests of emergent relations and of the baseline conditional discriminations for a generalized equivalence class that contains three basal class members (A, B, and C) and variants of all basal class members (A', B', and C') when the class is formed by training AB and BC.
assessed with the variant-to-base tests (in Column 2) and the base-to-variant tests (in Column 3). The relations among the variants of one basal class member with the variants of all other basal class members can be assessed with the variant-to-variant tests listed in the last column. All of these tests would be needed to identify all of the stimuli that were functioning as members of a fully elaborated generalized equivalence class, that is, a basal equivalence class that contained variants of all class members.

**Natural categories.** Naturally occurring categories, natural kinds (Gelman, 1988a, 1988b; Gelman & Markham, 1986, 1987), and semantic superordinate categories (Medin & Smith, 1984; Rosch & Mervis, 1975; Wittgenstein, 1953) all consist of some stimuli that are physically similar to each other, and other stimuli that do not bear any physical similarity. For each of these classes, however, all of the stimuli are functionally interchangeable. These are the same structural and functional properties that characterize generalized equivalence classes. Thus, all of these different names may be denoting the same emergent behavioral phenomenon. If so, the measurements that document the existence and extent of a generalized equivalence class should also document the existence and extent of these other categories.

**Cross-modal perception.** Cross-modal perception and intersensory integration (Bahrick & Pickens, 1994; Lewkowicz, 1994) are terms that denote the presence of relations between stimuli in two different sensory modalities.

\[
A_1 \rightarrow A_1' \rightarrow A_1'' \rightarrow \ldots \rightarrow A_1' \rightarrow A_1'' \rightarrow A_1''' \rightarrow \ldots \rightarrow C_1' \rightarrow C_1'' \rightarrow C_1''' \rightarrow \ldots
\]

*Figure 7.* A symbolic representation of a cross-modal class containing two basal members (A and B), each of which is from a different sensory modality. Dimensional variants of stimuli in each domain are represented by A' and B' respectively. A1't and B1't represent the most extreme variants of A1 and B1 that function as members of a cross-modal class.
For example, intersensory integration has occurred when many pictures of a mother occasion the selection of many sounds of that mother's voice.

Figure 7 represents the phenomenon in a form like that used to characterize generalized equivalence classes. Stimuli in two sensory modalities (A' and C') become linked by training of a conditional discrimination between one stimulus from each of two sensory modalities (A→C).

Cross-modal perception can be integrated into the framework that describes generalized equivalence classes in the following way. As summarized in Table 4, after establishment of AC, the variants within a sensory modality that function as members of separate open-ended classes

<table>
<thead>
<tr>
<th>Probe Type</th>
<th>Test Formats</th>
<th>Test Formats</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base-to-Base</td>
<td>Variant-to-Base</td>
</tr>
<tr>
<td>BL</td>
<td>A-C</td>
<td>A'-C</td>
</tr>
<tr>
<td>Rfx/Pgen</td>
<td>A-A</td>
<td>A'-A</td>
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<tr>
<td></td>
<td>B-C</td>
<td>C-C'</td>
</tr>
<tr>
<td>Sym</td>
<td>C-A</td>
<td>C'-A</td>
</tr>
</tbody>
</table>

Note: The conditional discrimination and generalization tests of emergent relations and of the baseline conditional discriminations needed to assess all stimulus-stimulus relations among the members of a cross-modal perceptual class that contains the basal class members (A and C) and their variants (A' and C')

can be measured with A'-A and C'-C probes, which are variant-to-base tests, and A-A' and C-C' probes, which are base-to-variant tests. The relations between the stimuli in the two modalities can be assessed with A'-C and C'-A probes which are variant-to-base tests, A→C and C→A probes which are base-to-variant tests, as well as A'-C' and C'-A' probes which are variant-to-variant tests. The cross-modal class, then, would be composed of all of the variants of A and C that always occasion the mutual selection of each other in all of the above-mentioned tests (Lewkowicz, 1994).

Although cross-modal and generalized equivalence classes can be placed in the same analytic framework, the two phenomena differ in one important regard. A partially elaborated generalized equivalence class (Figure 4) consists of a basal equivalence class that contains at least three stimuli, A, B, and C. If the class is established by training AB and BC, the variants of A would become related to the variants of C through two intervening conditional discriminations (AB and BC) or one intervening nodal stimulus, B. This set of relations is represented symbolically as A'→...→B→C→...→C'. In contrast, a cross-modal class (Figure 7) consists of stimuli in two different sensory modalities (A' and C'). Therefore, relations between variants of A and C would be linked by the directly trained relation (A→C), and no intervening nodal stimuli. At most, the relation between C' and A' would represent the generalization of symmetry. This set of relations is represented symbolically as A'→...→A→C→...→C'. Thus, a cross-modal class would appear to
be only a component of a generalized equivalence class. Regardless of these differences, however, the measures needed to fully document generalized equivalence classes would also be used to document the range of stimuli that function as members of cross-modal classes.

**Summary.** Broad ranges of emergent human behavior are controlled by generalized equivalence classes, which are also called naturally occurring categories, natural kinds, semantic superordinate categories, or cross-modal classes. To fully understand these classes and their effects on behavior, it is necessary to identify all of the stimuli that function as members of such classes. In addition, such measurements are critical for the identification of variables that influence the likelihood of forming such classes and their extent. The present methodological analysis suggests that generalization tests conducted in variant-to-base, base-to-variant, and variant-to-variant formats are needed to fully document the stimuli that function as members of those classes. To date, however, the breadth of these classes has been documented with the presentation of only one of these test formats. Because stimuli that function as members of a generalized equivalence class may covary with test format, the entire battery of tests is critical to fully characterize the extent of these classes.

**References**


