Violence and Capital

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Recommended Citation

http://opensiuc.lib.siu.edu/econ_dp/71
Abstract: We introduce a survival contest with lethal violence into a simple overlapping generations model with production and characterize how such violence interacts with capital along the equilibrium path. We find that shocks to economic fundamentals can have a variety of effects on the steady state levels of capital and violence, thus, explaining observed variation in development-violence outcomes. We also show that violence and capital are always negatively related along the adjustment path to steady state.

JEL Classification: D90, D74, O11, O40, C72

Keywords: Violence, Mortal Conflict, Capital Accumulation, Overlapping Generations, Nash Equilibrium, Contest.

Acknowledgement: We would like to thank Rick Grabowski, Sunnar Hohlfeld, Leslie John, Sajal Lahiri, Kevin Sylvester, Costas Syropolous, Ping Wang, and seminar participants at SIU, the Missouri Economics Conference, and the Midwest Macroeconomics Meetings.

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1 Introduction

This paper introduces mortal conflict and violence into Diamond’s (1965) overlapping generations model. Past work has focussed on more benign forms of conflict in dynamic economies, such as rent-seeking and predation of property, and has failed to consider the socially far more costly types of conflicts that result in lost lives.\textsuperscript{1,2} Apart from its social significance, mortal conflict presents an interesting theoretical challenge because it introduces an endogenous risk of death that depends on the actions of all the adversaries engaged in conflict. As Chakraborty (2004) and others have shown, having endogenous survival probability can have significant effects on capital accumulation. However, this work has not recognized the strategic aspect of survival that arises naturally with mortal conflict and that may complicate our understanding of dynamic economies.

We develop a simple dynamic general equilibrium model in order to address various issues related to violence, which we narrowly define as actions that are intended to result in death. Thus, we do not distinguish between personal and collective violence to focus on elements common to both. Our focus is motivated by findings that the scale of killing from homicide or civil war is comparable and that different forms of lethal violence are driven by some of the same economic processes (Neumeyer, 2003, and Collier and Hoeffler, 2004).\textsuperscript{3} Our study is further motivated by conventional opinion that violence inhibits development or that development inhibits violence. Because such thinking is not based on any formal analytical framework, it is difficult to fathom the ultimate outcome of corrective measures and proposals for conflict resolution. Such sentiments are also insufficient to explain the

\textsuperscript{1}For recent examples see Grossman and Kim (1996), Tornell and Lane (1999), and Gonzalez (2006). A large literature builds on the seminal contribution by Tullock (1980) and models conflict and rent-seeking as static contests in which individuals compete for a prize by spending resources, where the prize is either exogenous or endogenous. The conflict literature is reviewed by Garfinkel and Skaperdas (2006), while the rent-seeking literature is surveyed by Congleton (2006).

\textsuperscript{2}The social cost of wasted resources has been previously emphasized in the conflict literature. But this welfare loss probably pales in comparison to the social cost of violent deaths, which can be substantial, as indicated in empirical studies by Hess (2003) and Soares (2006).

\textsuperscript{3}Because we are primarily interested in forces that affect all types of violence, we do not consider forces that only appear to affect homicide or civil war individually such as ethno-linguistic fractionalization or inequality (Collier and Hoeffler, 2004). While such variables may be of further interest, the empirical literature does not seem to have settled on an answer whether, and if so how, inequality affects crime or ethnicity affects civil war, according to Dahlberg and Gustavsson (2005) and Esteban and Ray (2006).
large variety of outcomes across time and space noted by, for instance, Glaeser, Sacerdote, and Scheinkman (1996), and documented in the recent World Report on Violence and Health (Krug et al., 2002). It is, furthermore, not clear whether these sentiments refer to short-run or long-run movements or what the underlying economic forces are that drive them.

The setting we study is a conventional production economy with a single consumption good. This economy is inhabited by overlapping generations of two-period lived individuals who work when young and save for old age. Not everyone survives to old-age because of a survival contest among the young that takes the form of a simple Tullock (1980) contest. Thus, we take as a starting point the Hobbesian jungle with “continual fear and danger of violent death” and are able to compare it with the peaceful environments encountered in the growth and development economics literature (as surveyed recently in Aghion and Durlauf, 2005). Not only does mortal conflict lead to deaths, it also uses up resources that could otherwise be consumed or saved. For the individual these resource costs are the price of survival, while the prize of survival is the ability to consume the fruits of one’s past savings. Savings thus acts as a source of future consumption and effectively as a prize in the survival game and it is this dual role that provides the essential tension in the model. Not only do savings and capital influence violence, but they are also influenced by violence through the endogenous survival probability. The tension is resolved simultaneously with the Nash equilibrium of the violent survival contest and with the clearing of the capital market.

To foreshadow some of our results, in the early part of this paper we establish existence and uniqueness of the dynamic Nash equilibrium. We also characterize the Nash equilibrium trajectory for this economy and how lethal violence interacts with capital along the equilibrium path. We show that violence and development are always negatively related along the adjustment path to steady state, and thus we provide a formal analytical framework consistent with the conventional opinion that motivates our paper. However, when the focus is on the steady state effects of shocks to economic fundamentals, we find that shocks that encourage capital deepening can have a variety of effects on violence. For example, some types of shocks that effectively lengthen an individuals horizon yield growth accompanied by violence, whereas other types of shocks related to the pressures for violent appropriation can yield growth accompanied by a reduction in violence. By pressure variables we mean
changes in the population growth rate or the degree winners can plunder or appropriate resources from losers, where the first can loosely be interpreted as population crowding and the second as varying degrees of property rights enforcement (or weak governance), though strictly speaking we take political and social institutions as given. Our results also indicate that dynamically efficient economies with violence may benefit from a different and more nuanced set of anti-violence measures than dynamically inefficient economies and we discuss how our work might be translated into real world counterparts.

One advantage of our approach is that it yields a more realistic description of growth and development that has antecedents in the writings of Malthus, who linked violence to population pressures, and Hobbes, who linked a natural state of violent anarchy to the rise of absolutist government (Kavka, 1983). Another advantage is that we can move beyond the non-violent interactions analyzed in the conflict and crime literatures. Because the conflicts analyzed are typically ones in which there are no human casualties, the literature has had little to say about the human tragedy and social cost of lost lives that is a central concern for the public and that periodically mobilizes policymakers. Similarly, the crime literature following the work of Becker (1968) has mainly focussed on property crimes. Homicides have been treated separately, partly because such crimes were not thought to be economically motivated. One exception is Donohue and Levitt (1998), who also treat violence as a contest, but without the deadly consequences and without the link to a broader intertemporal framework considered in the present paper.

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4 Malthus (1798) identifies preventative and positive checks on population growth. Recent work on the demographic transition has emphasized the preventative checks by focussing primarily on endogenous fertility as a check on excess population growth. Our work fits more in the category of a Malthusian positive check, though we are not primarily concerned with the effect on population. As Wolfe (1942) writes “War is one of the positive checks to population growth, but it may be doubted whether modern wars, despite the high mortality that they cause, have more than a momentary effect on the balance between population and productive resources. Unless the warring peoples are already at a sheer subsistence level, it is doubtful whether war losses will much reduce the felt pressure of population. It is extremely difficult to judge the effects of war on standards of living afterwards, because of the large number of dynamic economic, political, technological, and social factors involved.” It is the effects of lethal violence on standards of living (and vice versa) that we are primarily concerned with.

5 Starting with work by İmrohoroğlu, Merlo, and Rupert (2000), and continuing with Burdett, Lagos, and Wright (2003) and Huang, Laing and Wang (2004), there has been much recent interest in analyzing property crime in dynamic general equilibrium models. Our focus on violence with varying degrees of property crime or plunder may be thought of as a complement to this recent work.
2 The Environment

The setting is Diamond’s (1965) overlapping generations model with production. Time is discrete and in each period $t \geq 1$ there is a population $N_t$ that grows at a rate of $n$. Individuals are born in periods $t \geq 1$ and live and consume for two periods, working and saving in the first period of their life. Not everyone survives to old age because of mortal conflict. Mortal conflict is modeled as a Tullock (1980) contest where adversaries must devote resources in the first period to survive to the second period of their lives. Production is standard with perfectly competitive firms combining labor and capital to produce a single consumption good.

We assume a Cobb-Douglas production technology and that there is complete depreciation of capital. If we define $k_t$ as the capital-labor ratio and $y_t$ as output per capita, then the Cobb-Douglas form with Hicks-neutral technological progress can be represented by $y_t = A (k_t)^\alpha$ where $0 < \alpha < 1$ and $A > 0$ is the technology parameter. Perfectly competitive firms maximize their profits in every period which implies optimality conditions of

$$w_t = (1 - \alpha) A (k_t)^\alpha$$
$$r_t = \alpha A (k_t)^{\alpha - 1}$$

where $w_t$ is the wage rate in period $t$ and $r_t$ is the real interest rate in $t$.

The population is divided into two equal sized groups of identical individuals. All individuals born in periods $t \geq 1$ may live for two periods. In the first period of their life, they inelastically supply a unit of labor and earn $w_t$. Earnings are allocated to consumption, savings for old age, and conflict. Specifically, individuals spend a fraction of wages on mortal conflict $x_i^t w_t$, where $x_i^t$ is the propensity for violence which we will call violence for short with the understanding that this refers to activities with the intent to kill. Consumption in the first period equals net wages $w_t (1 - x_i^t)$ less savings $s_i^t$. Consumption in the second period of life equals the return to savings $r_{t+1} s_i^t$ (an assumption we will relax later). Finally, we note that in period $t = 1$, there also exists an old generation of survivors that consumes its past savings.
Individuals choose savings and violence to maximize expected lifetime utility. Lifetime utility is a discounted sum of first and second period utilities, where the second period utility is discounted by $\beta \in (0, 1)$ and is multiplied by the endogenous probability of survival $\pi_i^t$. Utility in every period is assumed to have constant intertemporal substitution so that lifetime utility is represented by

$$\frac{(w_t (1 - x_i^t) - s_i^t)^{1-\theta^{-1}}}{1 - \theta^{-1}} + \pi_i^t \beta \frac{(r_{t+1} s_i^t)^{1-\theta^{-1}}}{1 - \theta^{-1}}$$

(3)

We assume that the intertemporal elasticity of substitution $\theta$ is greater than one, an assumption that will be helpful in several ways. First, the assumption guarantees the existence of an interior Nash equilibrium for the survival contest. Second, the assumption implies a unique interior intertemporal equilibrium so that we can focus on capital-violence dynamics without the interesting complication of multiple equilibria and development traps (see Azariadis, 1996).

Individuals compete for survival assuming a contest success function introduced by Tullock (1980) and later axiomatized by Skaperdas (1996). We adopt a simple form of this contest success function, an assumption that is not critical but makes for uncomplicated comparisons with later sections. Individual $i$ spends resources $x_i^t w_t$ to increase $i$’s probability of survival $\pi_i^t$ which is determined by the actions of all adversaries $i$ and $j$ according to:

$$\pi_i^t = \frac{\sum_j \pi_j^t (x_i^t w_t)}{\sum_j \pi_j^t (x_i^t w_t) + \frac{N}{2} (x_i^t w_t)}$$

(4)

where $\sum_i \pi_i^t = \psi \leq 1$ and $1 - \psi$ is the deadliness of the conflict in the sense that it gives the probability of mutual destruction. Though we are mainly concerned with interior solutions, we assume that $\pi_i^t = \frac{1}{2} \psi$ when $x_i^t = 0$ for all adversaries $i$. The contest between two identical groups can be interpreted either as a war between balanced forces or as individual one-on-one survival contests where individuals are randomly matched from the two sides.\(^6\) As mentioned in the introduction, we do not distinguish between personal and collective violence in order to focus on elements common to both. Thus, we sidestep the important question of why

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\(^6\)Another interpretation for our survival contest is that of an extreme form of self-protection or vigilantism. Rather than taking purely defensive protective actions as in the crime literature, individuals take offensive and potentially lethal survival actions.
there is violence in the first place and how violent groups form and take the Hobbesian view of violence as a pre-existing condition and fact of life. We note that agent \(i\)'s contest success function increases in \(x^i_t\) and decreases in \(x^j_t\) according to

\[
\frac{d\pi^i_t}{dx^i_t} = \pi^i_t (1 - \pi^i_t/\psi) > 0, \quad \frac{d\pi^i_t}{dx^j_t} = -\pi^i_t (1 - \pi^i_t/\psi) < 0
\]

Individuals choose violence and savings to maximize (3) given the contest success function defined in (4). There are two first order conditions from lifetime utility maximization. First, there is the Euler condition that equates the marginal cost from saving (on the left hand side) and the marginal benefit:

\[
(w_t (1 - x^i_t) - s^i_t)^{-\theta-1} = \pi^i_t \beta r_{t+1} (r_{t+1} s^i_t)^{-\theta-1}
\]

Also, there is a conflict condition that equates the marginal benefit of conflict (on the left hand side) to the the marginal cost of conflict:

\[
\frac{d\pi^i_t \beta (r_{t+1} s^i_t)^{1-\theta-1}}{dx^i_t} = w_t (w_t (1 - x^i_t) - s^i_t)^{-\theta-1}
\]

Using the Euler condition to substitute for the marginal cost of saving in (6) implies a conflict condition that resembles those of contest literature:

\[
\frac{d\pi^i_t V^i_t}{dx^i_t} = \pi^i_t \quad \text{where} \quad V^i_t = \frac{\theta}{\theta - 1} \frac{s^i_t}{w_t}
\]

Consistent with the contest literature we interpret \(V^i_t\) as the endogenous prize in the conflict. Thus, the prize in the survival contest is living until old age and being able to consume the savings of youth. Normally, in the contest literature the marginal cost of conflict is one, not \(\pi^i_t\) as it is here. The difference is not essential since the outcome is still a best-response function \(x^i_t = x(x^j_t, V^i_t)\), where \(x^i_t\) reacts positively to changes in \(x^j_t\).

Finally, we close the model by considering the aggregate resource constraint. The aggregate resource constraint is satisfied when output is fully allocated to consumption by the young and the old as well as to investment. The resource constraint is equivalent to a capital accumulation equation that equates investment to savings, or resources not consumed:

\[
(1 + n) k_{t+1} = s_t = \Sigma_i \frac{1}{2} s^i_t
\]
3 Dynamic Nash Equilibrium

To characterize the equilibrium in the model, we first derive the within period Nash equilibrium and then use this information to derive the intertemporal equilibrium. The first-order conditions of the individual imply conditional solutions for savings and violence:

\[ x_t^i = \Lambda_t^i \frac{s_t^i}{w_t} \]  
\[ s_t^i = J_{t+1}^i w_t (1 - x_t^i) \]

where we define \( \Lambda_t^i = (1 - \pi_t^i/\psi)(1 - \theta^{-1})^{-1} \), \( J_t^i = (1 + Q_t^i)^{-1} \), as well as \( Q_t^i = (\pi_t^i\beta)^{-\theta} (r_t^i)^{1-\theta} \). These conditional solutions are interdependent and will be solved out after the Nash equilibrium has been determined.

The Nash equilibrium is solved for by forming marginal benefit-cost ratios from the conflict condition (6) and equating them for all individuals. This implies that violence is identical for all individuals, or \( x_t = x_t^i = x_t^j \), and that the survival probabilities are also identical, or \( \pi = \pi_t^i = \frac{1}{2}\psi \). Because violence and survival probabilities are identical, savings also must be identical according to the conditional savings function, or \( s_t = s_t^i = s_t^j \).

Next we remove the interdependence of the conditional solutions by solving (9) and (10) for \( x_t \) and \( s_t \). This implies that in a symmetric Nash equilibrium violence and savings are

\[ x_t(r_{t+1}) = \frac{J(r_{t+1})\Lambda}{1 + J(r_{t+1})\Lambda} \]  
\[ s_t(w_t, r_{t+1}) = \frac{J(r_{t+1})}{1 + J(r_{t+1})\Lambda} w_t \]

where \( x_t(r_{t+1}) \in [0, 1] \) and \( s_t(w_t, r_{t+1}) \in [0, w_t] \) and where we define

\[ \Lambda = \frac{1}{2\theta - 1} \quad \text{and} \quad J(r_{t+1}) = \frac{1}{1 + Q_{t+1}} \quad \text{and} \quad Q_{t+1} = (\frac{1}{2}\psi\beta)^{-\theta} (r_{t+1})^{1-\theta} \]

To understand the forces that influence violence and savings, we totally differentiate \( x_t(r_{t+1}) \) in (11) and \( s_t(w_t, r_{t+1}) \) in (12), where to avoid clutter we have left out time subscripts:

\[ dx = x_w dw + x_r dr + x_{\beta} d\beta + x_{\psi} d\psi \]  

7
where

\[ x_w = 0 \]
\[ x_r = \frac{x}{r} (1 - x) (1 - J) (\theta - 1) > 0, \]
\[ x_\beta = \frac{x}{\beta} (1 - x) (1 - J) \theta > 0, \]
\[ x_\psi = \frac{x}{\psi} (1 - x) (1 - J) \theta > 0 \]

and

\[ ds = s_w dw + s_r dr + s_\beta d\beta + s_\psi d\psi \]

The intuition for the comparative static results is straightforward. First, higher wages have an income effect that increases savings. At the same time, the marginal benefit and cost of conflict do not change, resulting in no change in violence. This neutrality result hinges on the fact that higher wages leave the prize \( s_w \) unchanged, which is a direct result of our assumption of homothetic utility. Second, a higher interest rate increases savings since the substitution effect outweighs the wealth effect for \( \theta > 1 \). Increased savings leads to a higher prize and so increases violence by increasing the marginal benefit of conflict relative to the marginal cost. Third, a higher utility discount factor has a similar effect to a higher interest rate except that there is only a substitution effect on savings as the subjective valuation of youthful consumption falls. In other words, greater patience increases the value of the second period prize and, thus, violence. Finally, greater deadliness of conflict has an effect that is entirely analogous to greater impatience, both tending to shorten the effective lifetime horizon of the individual and reducing the value of the prize and the return to saving.

We summarize our findings in
**Proposition 1. (Nash Equilibrium)** There exists a unique interior Nash equilibrium for violence $x_t(r_{t+1})$ and savings $s_t(w_t, r_{t+1})$. A higher wage rate increases savings but does not affect violence. A higher interest rate, a higher rate of patience, or a lower deadliness of conflict increases savings and violence.

For comparison with the Hobbesian jungle considered here, individuals living in a conflict-free society have $x_t^p = 0$ and $\Lambda^p = 0$ so that $s_t^p = Jw_t$ in (12). Thus, the marginal propensity to save out of wages is higher in the peaceful society than for individuals who face mortal conflict. This is sensible, because no resources have to be devoted to conflict. Also, the response of savings to interest rates $s_r$ is higher in the peaceful society because there is no additional siphoning of resources from violence rising with the interest rate.

The dynamics of this economy are described by the path of capital. In equilibrium, the dynamics are characterized by a set of equations

\[
(1 + n) k_{t+1} = s_t (w_t, r_{t+1}) = \frac{J(r_{t+1})}{1 + J(r_{t+1})} w_t
\]

(16)

\[
J(r_{t+1}) = \frac{1}{1 + Q(r_{t+1})}
\]

with

\[
Q(r_{t+1}) = \left(\frac{1}{2} \psi \beta \right)^{-\theta} (r_{t+1})^{1-\theta}
\]

(17)

\[
w_t = (1 - \alpha) A (k_t)^{\alpha}
\]

(18)

\[
r_{t+1} = \alpha A (k_{t+1})^{\alpha-1}
\]

(19)

where $\Lambda$ is defined in (13). Violence $x_t(r_{t+1})$ can be determined recursively using (11) once the equilibrium time path for $k_t$ has been determined.

It is straightforward to show that there exists a unique interior equilibrium trajectory for capital ($k_t$) for any given initial capital stock $k_0 > 0$. Proof of this assertion follows Galor and Ryder (1989). Existence is demonstrated by defining a continuous excess demand function for capital $e_t \equiv (1 + n) k_{t+1} - s_t (w_t, r_{t+1})$ and letting $k_{t+1}$ vary continuously from zero to infinity as $k_t$ and thus $w_t$ are held constant. When $k_{t+1} \to \infty$, $e_t > 0$ because $s_t < w_t$ and $(w_t/k_{t+1}) \to 0$. When $k_{t+1} \to 0$, $e_t < 0$ because $s_r > 0$ so that $s_t \to \infty$. The intermediate value theorem then implies that there exists a positive $k_{t+1}$ such that the excess demand for capital is zero. Uniqueness follows from the fact that $e_t$ is a monotone
function of $k_{t+1}$ because $s_r > 0$. Because of the monotonicity, we can invert (16) and form a relationship

$$k_{t+1} = g(k_t) \quad \text{with} \quad g'(k_t) = \frac{-s_w k_t f''(k_t)}{1 + n - s_r f''(k_{t+1})}$$

where the latter follows by the implicit function theorem. Later, we will explicitly show that $g(k_t)$ is concave when the structure of conflict is varied. For now, we conclude with

**Proposition 2. (Dynamic Equilibrium)** Given an initial capital stock $k_0 > 0$, there exists a unique interior dynamic equilibrium path for capital $(k_t)$ for $t \geq 1$.

### 4 Equilibrium Analysis

In this section we study the transition to steady state and how the steady state equilibrium responds to various shocks. As a first step in the analysis, we show that with mortal conflict $k_{t+1}$ is a concave function of $k_t$ and that $x_t$ is a convex function of $k_{t+1}$. Specifically, we find

$$g'(k_t) = \frac{J_\alpha w_k}{(1 + n)(1 + \Upsilon(1 - x))} > 0$$

$$g''(k_t) = -(g')^2 \frac{\Upsilon}{g} \left[ 1 + \frac{\Upsilon(1 - x)}{1 + \Upsilon(1 - x)} \frac{J + (1 - J)x}{1 - J} \right] - \frac{1 - \alpha}{k_t} (g') < 0$$

where $\Upsilon = (1 - J)(\theta - 1)(1 - \alpha)$ is a bounded function of $g(k_t)$. Note that to get peaceful analogs $g'_P$ and $g''_P$, we must set $x = 0$ in $g'$ and $g''$. Also, we can see that $x_t = x(k_{t+1}) \in [0, \frac{\Lambda}{1+\Lambda}]$. Violence falls over its range as $k_{t+1}$ rises from zero towards infinity according to

$$x'(k_{t+1}) = - (1 - x) \frac{\Upsilon}{k_{t+1}} < 0$$

$$x''(k_{t+1}) = - \left[ \frac{1 + Q(1 - \Lambda)}{1 + Q + \Lambda} + \frac{1}{Q} \right] \frac{\Upsilon}{k_{t+1}} - \frac{1}{k_{t+1}} < 0$$

Having verified the shape of the capital and violence functions, we are now in position to graph them in Figure 1. The equilibrium capital accumulation relationship $k_{t+1} = g(k_t)$ is graphed in the right quadrant of Figure 1 and is called the KK-curve. The associated Nash equilibrium level of violence $x_t = x(k_{t+1})$ is graphed in the left quadrant and is called the XK-curve. Also, the conflict-free steady-state equilibrium is at point A, while the
steady-state equilibrium with mortal conflict is at points A’ and B’. It is readily apparent that moving from a conflict-free steady state to one with mortal conflict lowers the steady state capital stock.

To formally show that mortal conflict lowers steady state capital, assume that $k_{t+1}^p = k_{t+1}$ with $0 < k_{t+1} < \infty$ so that $J(r_{t+1}^p) = J(r_{t+1})$, where the superscript $p$ denotes the peaceful world. Because $k_{t+1}^p = k_{t+1}$, we also have $s_t^p = s_t$ in (12) where $\Lambda^p = 0$ so that $s_t^p = Jw_t^p$. Because $J(r_{t+1}^p) = J(r_{t+1})$, we see that $w_t^p < w_t$. In other words, wages during peace are less than wages with conflict when $k_{t+1}^p = k_{t+1}$, which implies that $k_t^p < k_t$ when $k_{t+1}^p = k_{t+1}$. Thus, $g_p(k_t^p) < g(k_t)$ or that the KK curve with mortal conflict lies to the right of the KK curve with peace when $k_{t+1}^p = k_{t+1}$. This implies that the steady state level of capital with peace must be greater than the steady state value with mortal conflict. Graphically, this proof compares point A and point a in Figure 1.

We summarize how the introduction of lethal violence alters the steady state capital:
Proposition 3. (Violence and Steady State Capital) Violence from mortal conflict lowers steady state capital compared with the capital in a conflict-free society.

Over time, violence and capital adjust monotonically towards their long-run steady states. For instance, consider the transition from a conflict-free steady state (point A) to one with mortal conflict (points A’ and B’). Once mortal conflict is introduced, capital is initially above the new steady state equilibrium capital stock. Because the interior steady state for capital is globally stable, capital will fall over time towards the new steady state. As capital falls along the KK’ curve, violence rises along the XK’ curve. The same sort of adjustment occurs anytime the initial capital stock is above its steady state level, while the reverse happens when capital is below its steady state. This adjustment over time of violence and capital to their steady state levels provides a rigorous foundation for the negative relationship expressed in the conventional opinion mentioned in the introduction that violence inhibits development or that development inhibits violence. We express these sentiments more formally as:

Proposition 4. (Adjustment to Long-Run Equilibrium) Capital and violence are inversely related along the path to steady state.

So far, we have shown how mortal conflict affects steady state violence and capital compared to the peaceful steady state. Next we analyze how the stationary equilibrium reacts to changes in economic fundamentals such as technology $A$, patience $\beta$, the deadliness of conflict $\psi$, and the population growth rate $n$. To analyze steady state behavior, we drop time subscripts in equations (16) through (19) characterizing the dynamic equilibrium. Totally differentiating this system implies first that

$$\Delta_k dk = (-k) dn + \left(\frac{k}{A} \frac{\Delta_k}{A - 1}\right)dA + s_\beta d\beta + s_\psi d\psi$$

\(^7\)We note that because XK is more elastic when capital is low, low income locations will see greater changes in violence when adjusting to steady state than locations with high income.
where $s_\beta$ and $s_\psi$ are defined in (15) and where we define
\[
\Delta_k = 1 + n + (1 - \alpha) \frac{rs_r}{k} - \alpha \frac{ws_w}{k} = (1 + n) (1 - \alpha) (1 + (1 - x) (1 - J) (\theta - 1)) > 0
\]

Another implication of total differentiation is
\[
dx = \left(-x_r (1 - \alpha) \frac{r}{k}\right) dk + \left(x_r \frac{r}{A}\right) dA + x_\beta d\beta + x_\psi d\psi
\tag{21}
\]
where $x_r$, $x_\beta$, and $x_\psi$ are defined in (14). Using (20) to substitute for $dk$ in (21) yields
\[
dx = X_n dn + X_A dA + X_\beta d\beta + X_\psi d\psi
\tag{22}
\]
where
\[
X_n = x_r (1 - \alpha) r > 0,
X_A = 0
X_\beta = x_\beta \frac{1}{1 + (1 - x) (1 - J) (\theta - 1)} > 0
X_\psi = x_\psi \frac{1}{1 + (1 - x) (1 - J) (\theta - 1)} > 0
\]

We find that shocks that promote capital deepening can have a variety of effects on violence. To see this, note that shifts to the KK curve are given in (20) and shifts to the XK curve are given in (21). The complete effect on violence of both curves shifting at same time is given by (22). Graphically, the KK curve shifts up in Figure 1 when $A$, $\beta$, or $\psi$ increase or when $n$ falls. The increase in the steady state capital is associated with a movement along the XK curve and a reduction in violence. Separately, the XK curve shifts leftward when either $A$ or $\beta$ or $\psi$ increase. The complete effect on violence from both curves shifting is zero for productivity shocks and positive for patience shocks. That is, the effect on violence from both curves shifting is exactly offsetting for a productivity shock, but for a patience shock the shift of the XK curve dominates.

We summarize the response of capital and violence in:

**Proposition 5. (Comparative Steady States)** Steady state capital rises when the population growth rate or the deadliness of conflict falls or when productivity or the rate of
patience rises. However, the steady state level of violence falls when the population growth rate or patience falls, or when the deadliness of conflict rises, but remains unaffected by productivity shocks.

That impatience or deadliness implies lower violence and capital is fairly striking. Indeed the result appears almost counter-intuitive, because it seems to be saying that youth or more deadly weaponry does not lead to more lethal violence. But this result can be understood by remembering that impatience or deadliness cause savings to fall and thus reduces the prize in the survival contest. In turn, a smaller prize reduces the incentive for violence and also leads to lower capital in the long run. Another striking finding is that not all sources of growth are a remedy for violence as illustrated by the neutrality of production technology shocks. Though this result is due to functional form assumptions, it does suggest that one can not infer that growth inevitably is associated with a reduction in violence.\(^8\)

5 Does Plunder Matter?

So far, mortal conflict has only involved survival and the opportunity to consume savings in old age. More realistically, violent conflict often involves some element of plunder. To capture plunder, we now allow winners in the survival contest to also appropriate the savings of the dead losers. This effectively increases the prize of the survival contest and thus should increase violence. On the other hand, because there is now an external source of old-age income, savings by all should fall. The reason is that the type of plunder we consider functions like social security, both processes redistributing from the young that die early to old survivors.

With plunder lifetime old age consumption for agent \(i\) changes from \(r_{t+1}s_i^t\) to \(r_{t+1}[s_i^t + s_j^t]\) and the individual optimality conditions are modified accordingly. The Euler condition

\(^8\)Resources devoted to violence \(xw\) behave somewhat differently than the propensity for violence \(x\), which we have denoted as violence. Though technology shocks do not alter violence, they do increase resources devoted to violence, which rise together with capital and the wage rate. Similarly, though a higher population growth rate encourages violence, resources devoted to violence may fall if the decline of wages is sufficiently strong. We note for later that the degree of plunder also tends to move violence and wages in opposite directions, so that violence and resources devoted to violence need not move in the same direction.
becomes
\[
(w_t (1 - x^i_t) - s^i_t)^{-\theta - 1} = \pi^i_t \beta r_{t+1} (r_{t+1} [s^i_t + s^j_t])^{-\theta - 1}
\]  
and the conflict condition is now
\[
\frac{d\pi^i_t}{dx^i_t} V_t = \pi^i_t \text{ where } V_t = \frac{\theta}{\theta - 1} \frac{s^i_t + s^j_t}{w}
\]  
The conditional violence expression derived from the conflict condition looks just like before, except that the new definition for \( V_t \) contains the sum of savings. In other words, we now have a contest with a homogenous prize rather than the heterogeneous prizes we considered previously. More importantly, the conditional savings expression derived from the Euler equation looks different than before, because it now includes the plunder from the adversary’s savings:
\[
s^i_t = J^i_{t+1} w_t (1 - x^i_t) - J^i_{t+1} Q^i_{t+1} s^j_t
\]  
where just like before \( J^i_{t+1} = (1 + Q^i_{t+1})^{-1} \) and \( Q^i_{t+1} = (\pi^i_t \beta)^{-\theta} (r_{t+1})^{1-\theta} \).

We see that violence remains a strategic complement for individuals. That is, one agent’s increase in violence will produce a conflict externality and a positive reaction in the violence of the adversary. However, with plunder, the saving decisions of the adversaries now also have a strategic element. Because higher savings by one will add to the prize of the survival contest and so induce an offsetting reduction of the savings by the adversary, savings choices in effect become strategic substitutes.

Proceeding as before, we find that the Nash equilibrium yields identical survival probabilities \( \pi^i_t = \pi = \frac{1}{2} \psi \) and identical violence levels \( x^i_t = \bar{x}_t \) and, thus, identical savings \( s^i_t = \bar{s}_t \). But now \( \bar{x}_t \) is higher than before, because the prize \( V_t \) includes the plunder from the conflict
\[
\bar{x}_t = \left(1 - \frac{\pi}{\psi}\right) V_t \text{ where } V_t = \frac{\theta}{\theta - 1} \frac{2s_t}{w_t}
\]  
Setting \( s^i_t = \bar{s}_t \) in (25) and solving the interdependences in (25) and (26), we find that the Nash equilibrium values for savings and violence are
\[
\bar{s}_t (w_t, r_{t+1}) = \frac{J(r_{t+1})}{1 + 2J(r_{t+1}) \Lambda + J(r_{t+1}) Q(r_{t+1})} w_t
\]
\[ \bar{x}_t(r_{t+1}) = \frac{2J(r_{t+1})\Lambda}{1 + 2J(r_{t+1})\Lambda + J(r_{t+1})Q(r_{t+1})} \] 

Comparing actions with plunder (\(\bar{x}_t\) and \(\bar{s}_t\)) to those without (\(x_t\) and \(s_t\)), we see that \(\bar{s}_t < s_t\) and \(\bar{x}_t > x_t\). The comparison is misleading, because it assumes that wages and interest rates are constant - something that is not necessarily true when comparing equilibria. We offer a fuller statement below when we compare stationary equilibria.

Totally differentiating \(\bar{s}_t(w_t, r_{t+1})\) and \(\bar{x}_t(r_{t+1})\) in (27) and (28), we see that the responses are qualitatively similar to the responses of violence and savings without plunder. First, we find that for violence

\[ d\bar{x} = \bar{x}_wdw + \bar{x}_rdr + \bar{x}_\beta d\beta + \bar{x}_\psi d\psi \] 

where

\[ \bar{x}_w = 0 \]
\[ \bar{x}_r = \frac{x}{r} (1 - x) \frac{2 (1 - J)}{(2 - J)} (\theta - 1) > 0, \]
\[ \bar{x}_\beta = \frac{x}{\beta} (1 - x) \frac{2 (1 - J)}{(2 - J)} \theta > 0, \]
\[ \bar{x}_\psi = \frac{x}{\psi} (1 - x) \frac{2 (1 - J)}{(2 - J)} \theta > 0 \]

and for savings

\[ d\bar{s} = \bar{s}_wdw + \bar{s}_rdr + \bar{s}_\beta d\beta + \bar{s}_\psi d\psi \] 

where

\[ \bar{s}_w = \frac{s}{w} \in (0, 1), \]
\[ \bar{s}_r = \frac{s}{r} (1 - x) \frac{2 (1 - J)}{(2 - J)} (\theta - 1) > 0, \]
\[ \bar{s}_\beta = \frac{s}{\beta} (1 - x) \frac{2 (1 - J)}{(2 - J)} \theta > 0, \]
\[ \bar{s}_\psi = \frac{s}{\psi} (1 - x) \frac{2 (1 - J)}{(2 - J)} \theta > 0 \]

The main difference compared to mortal conflict without plunder is that now most derivatives increase by a factor of \(2/(2 - J)\), but qualitatively there is no difference.

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We characterize the new dynamic Nash equilibrium in

**Proposition 6. (Dynamic Nash Equilibrium with Plunder)** When there is plunder, there exists a unique Nash equilibrium solution for violence and savings with \( \tilde{x}_t (r_{t+1}) \in [0, 1] \) and \( \tilde{s}_t (w_t, r_{t+1}) \in [0, w_t] \). The response of violence and savings to changes in wages, interest rates, patience, and the deadliness of conflict is qualitatively similar to the responses without plunder. There also exists a unique interior intertemporal equilibrium described by the capital dynamics of the previous section with \( \tilde{s}_t (w_t, r_{t+1}) \) replacing \( s_t (w_t, r_{t+1}) \) in (16).

Next, we look at how the graph introduced in Figure 1 changes with the introduction of plunder. We first need to verify that the KK and the XK curves retain their shapes with plunder. The KK curve stays concave because:

\[
\tilde{g}'(\tilde{k}_t) = \frac{J \alpha w_t}{(1 + n) (1 + \Upsilon 1 - \frac{x}{2 - J})} > 0
\]

\[
\tilde{g}''(\tilde{k}_t) = - (\tilde{g}')^2 \frac{\Upsilon}{g} \left[ 1 + \frac{\Upsilon 1 - \frac{x}{2 - J} J (2 - J) + (2 - x - J) (1 - J)}{(1 - J) (2 - J)} \right] - \frac{1 - \alpha}{\tilde{k}_t} (\tilde{g}') < 0
\]

where for comparison with the earlier case \( \Upsilon 1 - \frac{x}{2 - J} \) replaces \( \Upsilon (1 - x) \) where just like earlier \( \Upsilon = (1 - J) (\theta - 1) (1 - \alpha) \). Also looking at the XK curve, we can see that violence is bounded or \( \tilde{x}(k_{t+1}) \in [0, \frac{2\Lambda}{1 + 2\Lambda}] \). Violence falls over its range as \( k_{t+1} \) rises from zero towards infinity according to

\[
\tilde{x}'(\tilde{k}_{t+1}) = -(1 - \tilde{x}) \frac{\Upsilon}{\tilde{k}_{t+1}} \frac{2}{1 - J} < 0
\]

\[
\tilde{x}''(\tilde{k}_{t+1}) = - \left[ 1 + Q (1 - \Lambda) + \frac{1}{Q} + \frac{J}{2 - J} \right] \frac{\Upsilon}{\tilde{k}_{t+1}} - \frac{1}{\tilde{k}_{t+1}} < 0
\]

Having verified that the shapes of the KK and XK curves are the same as in Figure 1, we show in Figure 2 how plunder affects equilibrium violence and capital. Introducing plunder makes the KK curve shift downward (from KK to KK') and also makes the XK curve shift to the left (from XK to XK') with the x-intercept rising from \( (1 + (\Lambda)^{-1})^{-1} \) to \( (1 + (2\Lambda)^{-1})^{-1} \).
The net effect of both shifts is reinforcing for violence and steady state violence rises from point B to B' at the same time steady state capital falls from point A to A'.

More formally, to show that plunder increases violence and decreases capital, assume that $\tilde{k}_{t+1} = k_{t+1}$ so that $J(\tilde{r}_{t+1}) = J(r_{t+1})$ with $0 < k_{t+1} < \infty$. Then comparing (11) and (28) we see that $x_t > \tilde{x}_t$. Also, $\tilde{k}_{t+1} = k_{t+1}$ implies $\tilde{s}_t = s_t$ in (27) and (12). Because $J(\tilde{r}_{t+1}) = J(r_{t+1})$, it must be that the wage associated with $\tilde{k}_t$ is larger than the wage associated with $k_t$, which implies $\tilde{k}_t > k_t$ when $\tilde{k}_{t+1} = k_{t+1}$. Thus, $\tilde{g}(\tilde{k}_t) > g(k_t)$ when $\tilde{k}_{t+1} = k_{t+1}$, or that the KK curve associated with $\tilde{k}_t$ lies to the right of the KK curve associated with $k_t$ along a line that holds $k_{t+1}$ fixed. From this we infer that the steady state capital with plunder is less than the steady state capital without plunder. Graphically, the proof compares points B' and b for violence and points a and A' for capital in period $t$ along a line that holds capital in period $t + 1$ fixed. The argument identifies the XK and KK curves with plunder and the XK' and KK' curves without plunder in Figure 2. Thus,
we have shown that

**Proposition 7. (Stationary Equilibria With and Without Plunder)** In the long run, plunder aggravates violence and weakens capital compared with the violence and capital without plunder.

So far we have contrasted the scenario where all of the opponent’s savings is plundered to that where none gets plundered. Rather than have plunder be all or nothing, we could easily let adversaries take only a fraction of the opponents savings and so parameterize the degree of plundering. If we let \( \sigma \) be the share of savings that is appropriated when one side wins, then old age consumption for agent \( i \) changes to \( r_{t+1}[s^i_t + \sigma s^j_t] \). This implies a conflict condition where the prize for the individual \( V^i_t \) now is proportional to \( s^i_t + \sigma s^j_t \). Thus, the prizes are heterogeneous rather than homogeneous as in the conflict condition in equation (24). Having heterogeneous prizes introduces a potential difficulty, that is resolved since the Nash equilibrium involves symmetric players. It can be easily verified that the symmetric Nash equilibrium is described by the earlier equations except that \( 1 + \sigma \) replaces 2 in equations (27) and (28).

It is now straightforward to see that an increase in the degree of plundering (\( \sigma \)) leads to a fall in steady state level of capital and an increase in steady state violence. As for the shocks considered earlier (technology \( A \), tastes \( \beta \), deadliness \( \psi \), and the population growth rate \( n \)), it is clear that their steady state effects have not changed qualitatively. Thus, we conclude with the following proposition that collects all the previous comparative steady state effects:

**Proposition 8. (Shocks to the Stationary Capital Equilibrium with Violence)** Greater impatience or deadliness of conflict reduces steady state capital and violence. A lower population growth rate or a lower degree of plunder leads to a higher steady state level of capital and a lower steady state level of violence. An exogenous technological advance raises steady state capital without any effect on violence.

This final proposition suggests that there may be a variety of fundamental reasons why different regions have such disparate development-violence outcomes. Of course, economic fundamentals are only part of the story. Government and social institutions also matter.
In the present context, these institutions could be thought of as producing forces that either mimic the factors that change the expected horizon (impatience and deadliness) or that mimic factors that alter the pressure for violent appropriation (population and degree of appropriation). The latter factors appear as incentives that shift resources from violence to capital formation, where loosely speaking, one could interpret the degree of plundering as how well property rights are enforced by governments or encouraged by social norms. Or else, government and social institutions could create forces that mimic technology changes, which fit in neither camp since they are neutral with respect to violence. Alternatively, one could think of government and social institutions as creating forces that may be thought of as primarily shifting the XK curve, or the KK curve, or both. An important question for the future is whether these forces are violence reducing and development enhancing, or whether they produce less desirable outcomes.

The final proposition, also, suggests that not all proposals for violence reduction are equally desirable in a social welfare sense. The key lies in recognizing that the recipe for violence reduction varies depending on whether or not the competitive equilibrium is dynamically inefficient. Indeed one can show that dynamic inefficiency restricts the set of acceptable measures for violence reduction, while dynamic efficiency implies that Pareto improving violence reduction must leave capital unaffected to unambiguously increase welfare or else it will cause some generation to suffer during the transition to a new steady state. Alternatively, if transitional welfare receives low weight, anti-violence measures would also have to encourage capital formation.

To see this most simply, denote the stationary Golden Rule level of capital as $k^{GR}$ and recall that $k^p$ is the peaceful or conflict-free equilibrium level of capital. Using standard arguments one can easily show that the socially optimal level of violence is zero. The issue

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9 As mentioned previously, moving away from homothetic preferences would allow the possibility that wage-increasing technology shocks lead to higher or lower violence.

10 The socially optimal level of violence is zero, because it represents a pure resource loss to society that is minimized when all adversaries foreswear violence. Given the optimal level of violence, we note that a Millian social welfare function with equal treatment of all generations produces the standard Golden Rule capital level in our framework. We adopt this welfare criterion because population growth remains exogenous as lethal violence only affects survival to old age. If lethal violence killed youths or if fertility were endogenous, one would have to address optimal population considerations when determining the socially optimal levels of capital and violence, which depend crucially on the choice of welfare criterion, Millian or utilitarian.
is how the equilibrium capital with violence $k^*_t$ (or $\tilde{k}_t$) compares with $k^{GR}$. Either there is capital overaccumulation with $k^{GR} < k^*_t < k^p_t$, or else there is underaccumulation with either $k^*_t < k^{GR} < k^p_t$ or $k^*_t < k^p_t < k^{GR}$.

In the first case, dynamic inefficiency provides a clear mandate for a policy measure that simultaneously reduces capital and violence. Of the shocks considered in the final proposition, we would want a policy that acted like shocks that decrease the effective horizon for the individual, as would an increase in impatience or the deadliness of conflict. Thus, for dynamically inefficient economies our model suggests that traditional policies like weapons control may not be welfare-enhancing. Similarly, less traditional public health policies that lead to an increase in the utility discount factor along the lines of Chakraborty (2004) might also not be desirable. Instead, other anti-violence policies may have to be considered that go beyond the scope of the final proposition and of this paper.

For dynamically efficient economies, anti-violence measures have to be neutral with respect to capital to unambiguously increase social welfare; or else they would have to also encourage capital formation to at least increase welfare in the long-run. This suggests a more nuanced approach to anti-violence policies that may require combining several measures to get the desired result. As an example of an anti-violence policy that is neutral with respect to capital, one could imagine combining a measure that acted like the shocks that decreased effective horizons with a measure that produced effects like a reduced pressure for violent appropriation. One possible solution would be to allow an increase in the deadliness of conflict and also increase property rights enforcement (through police or legal system). For capital neutrality the balance would have to be made carefully, but if capital formation were allowable or if transitional welfare had low weight, then property rights enforcement could be given a higher priority. A less traditional way of achieving the same outcome would be a policy that discouraged better health care (to reduce the utility discount factor) combined with family planning polices to reduce population growth, with greater weight given to the latter if transitional welfare effects receive little or no weight. While these policy options are not necessarily the most realistic, they do suggest the difficulty of finding socially desirable anti-violence policies when there is capital accumulation.
6 Conclusion

We extend Diamond’s (1965) overlapping generations model by introducing mortal conflict and interpreting individual survival as the outcome of a Tullock (1980) contest. Thus, we are able to capture the reality that development is accompanied by violence. We show that violence inhibits development compared to conflict-free economies. We also demonstrate that violence and capital are negatively related on the path to steady state and that the changes of the stationary levels of capital and violence can be positively or negatively related depending on the underlying shocks to economic fundamentals.

We have made strong assumptions to keep the model clean and to better focus on the capital-violence dynamics. Relaxing these assumptions would complicate but not materially affect the essence of the analysis. Though we have indicated what kinds of anti-violence measures may be most beneficial to society, our model can be thought of as providing a dynamic framework for a more detailed future analysis of the role government and social institutions in violence reduction. One important question to pursue is what types of norms and bargaining arrangements can reduce violence and also encourage capital formation, perhaps by considering arguments along the lines of Anbarci, Skaperdas and Syropoulos (2002). A related question in understanding how to reduce violence is how violent groups form in the first place and evolve in dynamic economies, which may be pursued by introducing considerations along the lines of Esteban and Ray (2006) into the dynamic framework. Finally, one may want to consider the introduction of fear, which can be an important social force as recent events have shown. If fear is interpreted as a collective force that distorts subjective probabilities, then fear can be modeled as an externality that reduces the subjective probability of survival by a factor that depends on aggregate violence. Introduction of such a fear factor can easily generate multiple equilibria and development death traps by changing the shape of the equilibrium capital accumulation relationship in ways that are familiar from the literature on poverty traps (Azariadis, 1996).
References


