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Enhancing Small Cell Transmission Opportunity Through Passive Receiver Detection in Two-Tier Heterogeneous Networks

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Enhancing Small Cell Transmission Opportunity through Passive Receiver Detection in Two-Tier Heterogeneous Networks

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Abstract—In this paper, we investigate how to embed small cells into a macro cell to enhance the performance of two-tier heterogeneous networks. Conventionally, overlay and underlay schemes are adopted by small cells, where the former enables a small cell to use the macro cell idle bands with transmitter detection, while the latter enables a small cell to use the macro cell busy bands with a certain access probability. To enhance the performance of small cells, we propose to exploit more busy band opportunities for small cells through passive receiver detection, i.e., identifying the location of an active macro user that occupies a certain band. Then, a small cell may access the busy band when the active macro user is far away from the small cell. In our method, the small cell uses the energy of the received signal from the macro cell as the test statistic. We obtain the closed-form distribution of the test statistic and design two detectors with one and two thresholds, respectively. Our results demonstrate that the proposed detectors achieve about 100% to 300% more transmission opportunities than the conventional energy detector in typical two-tier heterogeneous networks without requiring any prior information of the macro user signal.

Index Terms—Heterogeneous networks, receiver detection, small cells, transmission opportunity.

I. INTRODUCTION

The two-tier heterogeneous network (HetNet) is a competitive candidate for future wireless communication systems since it is capable of providing high data rate services in dense areas [11]–[7]. In a two-tier HetNet, a number of small cells are overlaid with a macro cell. To avoid co-channel interference, small cells may access the idle bands of the macro cell, called overlay, which works well in low and medium load scenarios [8] [9], i.e., the macro cell has enough idle bands to accommodate the small cells. However, when the macro cell is with high load, it may have few idle bands. This reduces the transmission opportunities of small cells and impedes their development in dense areas. To deal with the issue, people allow small cells to access the busy bands, called underlay, but need to control the interference to active macro cell user equipments (M-UEs). Most of the existing contributions [10]–[13] determine an access probability of small cells to guarantee the outage probability of active M-UEs based on Poisson point process (PPP) model. However, small cells can only obtain limited transmission opportunities since the access probability relies on the density of M-UEs.

An effective approach to improve the transmission opportunities is to allow small cells to access the busy bands according to the location of active M-UEs: if an active M-UE is outside a small cell’s coverage, the small cell may access the M-UE’s busy band; if the active M-UE is inside the small cell’s coverage, the small cell can keep silence to avoid interference. In other words, small cells are required to identify the location of active M-UEs. In practice, it is very difficult to detect whether an active M-UE is inside the coverage of a small cell. This is because most of the existing detection methods [14]–[16] belong to transmitter detection. In downlink transmission, since the active M-UE does not transmit signals, the small cell can not identify the active M-UE based on the signal from the active M-UE. In uplink transmission, since the small cell does not know the transmission power of the active M-UE, it still can not identify the active M-UE. Therefore, an alternative solution to identify the active M-UE is to enable the small cell to conduct receiver detection in downlink transmission.

Recently, a new category of receiver detection methods, called proactive detection, has been proposed in cognitive radio systems to identify an active primary receiver [17]–[24]. In these methods, a cognitive user first transmits some jamming signals to artificially trigger the closed-loop power control (CLPC) between primary transceivers. Then, the primary transmitter adjusts the transmission power accordingly. This power adjustment indicates that the primary active receiver is inside the coverage of the cognitive user and the jamming signals cause interference to the active primary receiver. Thus, by observing the power adjustment of the primary signal, the cognitive user can detect the nearby active primary receiver. However, such proactive methods are not suitable for HetNets because the jamming signals may cause severe interference if the active M-UE is inside the coverage of the small cell.

In this paper, we propose a passive receiver detection method to identify the active M-UE in HetNets, where no jamming signal is used. By measuring the energy of the received signal from a macro cell base station (M-BS), a small cell base station (S-BS) can identify the location of the active M-UE and find the transmission opportunity of the busy band, i.e., the...
band is being occupied by the active M-UE, but the active M-UE is outside the coverage of the small cell. We further design two detectors with one and two thresholds, respectively, which allow the small cell and the macro cell to simultaneously access the same band. With the proposed detectors, the S-BS can obtain significant transmission opportunities in an “opportunistic” way, i.e., identifying the active M-UE that is outside the coverage of the S-BS and accessing the busy band without interfering with the active M-UE. This is different from overlay methods [14]–[16] that work in an “on and off” way, i.e., turning on and off the S-BS when a band is idle and busy, respectively. This is also different from underlay methods [10]–[13] that work in a “blind” way, i.e., accessing the busy band with a certain probability to avoid interference to active M-UEs. We notice that conventional detectors [25] have been applied to detecting active receivers in [26] and [27]. But the performance is poor since the decision is made by observing the oscillator leakage from an active receiver with extremely low signal-to-noise ratio (SNR). We also notice that the double-threshold detector in [28] has been used in spectrum sensing, but it cannot detect the active receiver since it belongs to transmitter detection technique.

The main contributions of this paper are summarized as follows:

1) We propose a passive receiver detection method for S-BS to identify the location of an active M-UE, which uses the energy of the received signal from M-BS as the test statistic;
2) We obtain the closed-form distribution of the test statistic to design the thresholds and evaluate the performance;
3) We design two detectors with one and two thresholds, respectively, which enable a small cell to coexist with a macro cell in an “opportunistic” way.

The proposed receiver detection can embed small cells into a macro cell, which raises a new framework of HetNet, named tier aggregation. The tier aggregation is a spectrum sharing paradigm that allows the small cell to concurrently access the busy band of the macro cell if the M-UE is outside the interference range of the small cell. It requires the small cell to identify the location of the M-UE to avoid the interference. This is different from conventional overlay spectrum sharing, where the small cell may access the busy band of the macro cell if the M-UE is out of the S-BS’s coverage. This is also different from conventional underlay spectrum sharing, where the small cell can access the idle band of the macro cell. This is also different from conventional underlay spectrum sharing, where the small cell may access the busy band, but with limited transmission opportunities. Therefore, the tier aggregation is expected to significantly increase the area spectrum efficiency of two-tier HetNets.

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we first present the basic idea of the proposed detection method, and then derive the distribution of our test statistic. Consequently, we design two detectors with one and two thresholds, respectively. In Section IV, we provide both theoretical and simulation results to demonstrate the advantages of our methods. Finally, Section V concludes the paper.
data rate, the M-BS adjusts its transmission power to meet a target SNR for the specific M-UE, defined by \( \gamma_T \). This assumption is reasonable since maintaining SNR is widely applied in practical systems, e.g., the M-BS usually adjusts the transmission power through CLPC or power allocation to provide the required data rate. In (2), we replace \( \gamma_1 \) by \( \gamma_T \), and then obtain the transmission power of the M-BS by

\[
p = \frac{\gamma_T \sigma_1^2}{h_1^2 g_1}.
\]

Furthermore, we consider both large-scale path-loss and small-scale fading in wireless channels. According to [29], the path-loss follows the model

\[
g_1 = C \left( \frac{\lambda}{4\pi l} \right)^\beta,
\]

where \( C \) is a constant, \( \lambda \) is the wavelength, \( l \) is the distance between the M-UE and M-BS, and \( \beta \) is the path-loss factor \((\beta = 2 \sim 6)\). The small-scale fading follows Rayleigh distribution with unit power, and the probability density function (PDF) is

\[
f_h(z) = 2ze^{-z^2}, \quad z \geq 0.
\]

### B. M-BS to S-BS Signal

Similarly, denote \( g_2 \) as the large-scale path-loss from the M-BS to the S-BS, and \( h_2 \) as the small-scale fading from the M-BS to the S-BS, then the received signal at the S-BS can be expressed as

\[
y_2(k) = h_2 \sqrt{g_2} \gamma_T x(k) + n_2(k), \quad 1 \leq k \leq K,
\]

where \( n_2(k) \) is the AWGN at the S-BS with zero mean and variance \( \sigma_2^2 \). We adopt the same path-loss and small-scale fading models as in (4) and (5), i.e.,

\[
g_2 = C \left( \frac{\lambda}{4\pi d} \right)^\beta
\]

and

\[
f_h(z) = 2ze^{-z^2}, \quad z \geq 0,
\]

where \( d \) is the distance between the S-BS and M-BS. Substituting (3) into (6), we have

\[
y_2(k) = \frac{h_2}{h_1} \frac{\gamma_T \sigma_1^2 g_1^2}{g_2} x(k) + n_2(k), \quad 1 \leq k \leq K.
\]

Let

\[
\Phi = \frac{g_2}{g_1}
\]

and

\[
\Omega = \frac{h_2^2}{h_1^2 T},
\]

and assume \( \sigma_1^2 = 1 \), the received signal at the S-BS in (9) can be simplified as

\[
y_2(k) = \sqrt{\Phi \Omega} \gamma_T x(k) + n_2(k).
\]

### III. Passive Receiver Detection

In this section, we first introduce the basic principle of the proposed detection method to obtain our test statistic, and then derive its closed-form distribution. Consequently, we design two detectors with one and two thresholds, respectively.

#### A. Basic Principle

The proposed detection is based on the energy of the received signal from the M-BS, which carries the location information of the M-UEs. In Fig. 1, the whole coverage of the M-BS can be divided into three regions according to the location of the S-BS. In particular, the S-BS is in Region II and the M-UEs are randomly located in one of the three regions\(^4\). If an M-UE is located in Region I or III, the S-BS does not interfere with the M-UE. Otherwise, if an M-UE is in Region II, the S-BS may cause interference to the M-UE. Then, we define the two cases as hypotheses \( \mathbb{H}_0 \) and \( \mathbb{H}_1 \), respectively, i.e.,

\[
\begin{align*}
\mathbb{H}_0 : & \text{ M-UE } \notin \text{ Regions I or III,} \\
\mathbb{H}_1 : & \text{ M-UE } \in \text{ Region II.}
\end{align*}
\]

For a guaranteed wireless service to the M-UE with a target SNR requirement, the transmission power of the M-BS is mainly determined by the location of the M-UE. If the M-UE is in Region I, which is the center of the M-BS’s coverage, the M-BS can meet the M-UE’s target SNR with low power; if the M-UE is in Region II, the M-BS needs medium power to satisfy the target SNR; if the M-UE is in Region III, which is the edge of the M-BS’s coverage, the M-BS has to use high power for the target SNR. In other words, it establishes the above corresponding relationship between the M-UE’s located region and the M-BS’s power. Based on the corresponding relationship, the S-BS can distinguish the two hypotheses by measuring the energy of the received signal from the M-BS: if the measured energy is very small or very large, the M-UE is probably in Region I or III, respectively; if the measured energy is medium, the M-UE is probably in Region II. Therefore, the S-BS can use the energy of the received signal from the M-BS as a test statistic to identify the location of the M-UE.

When we further consider the shadowing and multi-path fading effects in wireless channel, they actually introduce some uncertainty to the corresponding relationship and lead to detection errors. This is because the shadowing and fading coefficients are independent of the M-UE’s located region and affect the M-BS’s power randomly. To analyze the uncertainty and achieve good detection performance, we need to first obtain the distribution of the received signal from the M-BS: if the measured energy is very small or very large, the M-UE is probably in Region I or III, respectively; if the measured energy is medium, the M-UE is probably in Region II. Therefore, the S-BS can use the energy of the received signal from the M-BS as a test statistic to identify the location of the M-UE.

In the following three subsections, we will first derive the distribution of the test statistic. Then, we design a one-

\(^4\)In this paper, our method is to detect the active M-UE, which enables the S-BS to access the busy band that is being used by the M-BS and M-UE. In other words, the M-BS is always on and serves the M-UE inside its coverage. Thus the active M-UE is in one of the three regions. Furthermore, we use the term “M-UE” to represent “active M-UE” in the rest of this paper for simplicity.
threshold detector to maximize the small cell transmission opportunity subject to an interference constraint. Furthermore, we design a double-threshold detector which can achieve better performance.

B. The Distribution of the Test Statistic

When the S-BS uses $K$ samples to calculate the energy of the received signal from the M-BS, we have

$$E = \sum_{k=1}^{K} y^2(k).$$

(14)

Substituting (12) into (14), and simplifying the expression, we obtain

$$E = \sum_{k=1}^{K} \Omega \Phi \gamma_T x^2(k) + \sum_{k=1}^{K} 2\sqrt{\Omega \Phi} \gamma_T x(k) n_2(k) + \sum_{k=1}^{K} n_2^2(k).$$

(15)

In the above expression, $\Omega$, $x(k)$, $\Phi$, and $n_2(k)$ represent four random variables: $\Omega$ is determined by the ratio of the power of the two small-scale fadings $h_1$ and $h_2$; $\Phi$ is determined by the ratio of two path-loss values $g_1$ and $g_2$; $x(k)$ is the signal from the M-BS with normalized power; $n_2(k)$ is the AWGN.

It is very difficult to obtain the distribution of the test statistic in (15), since it is a combination of the four random variables. To obtain a closed-form expression, we ignore the noise and make the following approximation,

$$E \approx \sum_{k=1}^{K} \Omega \Phi \gamma_T x^2(k) = K \Omega \Phi \gamma_T.$$  

(16)

The assumption that allows us to make the approximation in (16) is that the performance loss is negligible. Specifically, the detection error mainly occurs when the M-UE is around the two boundaries of Region II. In this case, the M-BS is similar to the M-UE, which makes the S-BS experience similar SNR with the M-UE. Since the M-UE’s target SNR is usually a medium or high value, the S-BS observes the medium or high SNR accordingly. Therefore, when we calculate the energy of the received signal from the M-BS in (15), we can ignore the noise and make the approximation in (16).

We notice that the approximation gap between (15) and (16) becomes large when the S-BS experiences a low SNR (i.e., the signal strength from the M-BS is low). This case only occurs if the M-UE is in Region I and (or) if the S-BS is in the cell edge. In this case, since the SNR difference between the M-UE and the S-BS is significant, it results in a large margin for the approximation gap. As a result, the approximation gap in the low SNR case causes negligible effects on the detection performance.

Now, the energy of the received signal in (16) is determined by the two random variables $\Omega$ and $\Phi$. Next, we will first derive the probability density functions (PDFs) of $\Omega$ and $\Phi$, respectively, and then obtain the cumulative distribution function (CDF) of the test statistic $E$.

1) The PDF of $\Omega$: Since the small-scale fadings follow independent Rayleigh distribution, the power of them follows exponential distribution, i.e., $h_1^2 \sim e^{-u}$ and $h_2^2 \sim e^{-u}$.

According to [30], the CDF of $\Omega = h_2^2/h_1^2$ can be calculated by

$$F_{\Omega}(\omega) = \int_{0}^{\infty} y f_{\Omega}(y, \omega)dy = \frac{\omega}{1 + \omega},$$

and the PDF of $\Omega$ can be obtained by

$$f_{\Omega}(\omega) = \frac{1}{(1 + \omega)^2},$$

which follows F-distribution.

2) The PDF of $\Phi$: Since the M-UE is uniformly distributed in the coverage of the M-BS, the PDF of the distance between the M-UE and M-BS can be obtained by

$$f_1(l) = \left\{ \begin{array}{ll}
\frac{2l}{\pi r^2}, & \varepsilon \leq l \leq d-r \\
\frac{2l}{\pi r^2}, & d-r < l < d+r
\end{array} \right. \quad (\mathbb{H}_0),$$

where $\varepsilon$ is the minimum distance between the M-UE and M-BS. When the path-loss factor is $\beta = 2$, we substitute (4) and (7) into (10), and then obtain

$$\Phi = \frac{g_2}{g_1} = \frac{l^2}{d^2},$$

(20)

Given a distance between the S-BS and M-BS, i.e., $d$, we first obtain the CDF of $\Phi$,

$$F_{\Phi}(\phi) = P\left(\frac{l^2}{d^2} \leq \phi\right) = \left\{ \begin{array}{ll}
\frac{\phi d^2}{\pi \varepsilon r^2}, & \varepsilon \leq l \leq d-r \\
\frac{\phi d^2}{\pi r^2}, & d-r < l < d+r
\end{array} \right. \quad (\mathbb{H}_0),$$

(21)

Then, the PDF of $\Phi$ becomes

$$f_{\Phi}(\phi) = \left\{ \begin{array}{ll}
\frac{\phi^2 d^4}{\pi^2 \varepsilon^2 r^2}, & \mathbb{H}_0, \\
\frac{\phi^2 d^2}{\pi r^2}, & \mathbb{H}_1.
\end{array} \right.$$  

(22)

3) The CDF of $E$: As indicated in (16), i.e., $E = K \Omega \Phi \gamma_T$, our test statistic $E$ has the same distribution as $\Omega$ for a given value of $\Phi$ ($K$ and $\gamma_T$ are constants). Then, we obtain the conditional PDF of $E$ from (18) as follows,

$$f_E(\xi|\Phi) = \frac{K \Phi \gamma_T}{(K \Phi \gamma_T + \xi)^2}.$$  

(23)

Then, the closed-form CDF of $E$ in both $\mathbb{H}_0$ and $\mathbb{H}_1$ can be obtained by (24) and (25) at the top of the next page, respectively.

Fig. 2 plots the theoretical CDF curves and the corresponding scenarios of the test statistic $E$ based on (24) and (25), where different distances between the S-BS and M-BS are considered. We also provide simulation curves for comparison, where the system parameters are the same as that in Section IV. From Fig. 2(a), when the S-BS is close to the M-BS, i.e., $d = 100$ m, the CDF curves of $\mathbb{H}_0$ are on the right side of the CDF curves of $\mathbb{H}_1$. However, when the S-BS is far away from the M-BS, i.e., $d = 300$ m and $d = 400$ m in Fig. 2(c) and Fig. 2(e), the CDF curves of $\mathbb{H}_0$ dramatically shift to the left side of the curves under $\mathbb{H}_1$. On the other hand, the CDF curves of $\mathbb{H}_1$ slightly shift to the left as the S-BS moves away from the M-BS and the shift is too small to be observed.
This shift is reasonable since the whole coverage of the M-BS is divided into three regions according to the location of the S-BS. When the S-BS is close to the M-BS, Region I is small and Region III is large, and the M-UE is more likely to appear in Region III, resulting in that the M-BS uses lower power to serve the M-UE. Thus, the CDF curves of $\mathbb{H}_1$ shift to the left as the S-BS moves away from the M-BS. However, the shift of the CDF curves of $\mathbb{H}_0$ is almost indistinctive. This is because under $\mathbb{H}_1$, the M-UE and S-BS are both in Region II and experience similar path-losses and receive similar energies.

Furthermore, when we compare the theoretical curves with the simulation ones, they overlap very well except for a small gap that appears in the curves of $\mathbb{H}_0$ when $d = 400$ m and $0 \text{ dB} < E < 13 \text{ dB}$. The reason is that the approximation in (16) is inaccurate when the S-BS is far away from the M-BS and experiences low SNR. This will not affect the performance of the proposed methods, which will be shown in the next section.

### C. One-Threshold Detector (OTD)

In this subsection, we first discuss the conventional OTD in two special scenarios. Then, we introduce two access probabilities into the OTD in the general scenario. Finally, we calculate the detection threshold and the two access probabilities to maximize the transmission opportunity under an interference constraint.

1) Two Special Scenarios: In Figs. 2(a) and 2(b), when the S-BS is close to the M-BS, i.e., $0 < d \leq r$, it has only Regions II and III, which are corresponding to $\mathbb{H}_1$ and $\mathbb{H}_0$, respectively. Since the $\mathbb{H}_0$ CDF curve is on the right side of the $\mathbb{H}_1$ CDF curve, we can find a threshold $\eta^\prime$ to distinguish $\mathbb{H}_0$ and $\mathbb{H}_1$, i.e.,

$$
\text{Decision result} = \begin{cases} 
D_0, & E > \eta^\prime, \\
D_1, & E \leq \eta^\prime,
\end{cases}
$$

where $D_0$ and $D_1$ are denoted as the decisions on $\mathbb{H}_0$ and $\mathbb{H}_1$, respectively.

On the other hand, in Figs. 2(e) and 2(f), when the S-BS is far away from the M-BS, i.e., $R - r \leq d < R$, it has only Regions I and II, which are corresponding to $\mathbb{H}_0$ and $\mathbb{H}_1$, respectively. Since the $\mathbb{H}_0$ CDF curve in this scenario is on the left side of the $\mathbb{H}_1$ CDF curve, we can find a threshold $\eta^\prime\prime$ to distinguish $\mathbb{H}_0$ and $\mathbb{H}_1$, i.e.,

$$
\text{Decision result} = \begin{cases} 
D_0, & E \leq \eta^\prime\prime, \\
D_1, & E > \eta^\prime\prime.
\end{cases}
$$
2) The General Scenario: In Figs. 2(c) and 2(d), when the S-BS is in the medium range of the M-BS's coverage, i.e., \( r < d < R - r \), it has three regions. Then, the detection becomes complicated because we have only one threshold \( \eta \). On one hand, if the threshold \( \eta \) is relatively high, the case \( E > \eta \) indicates that the M-UE is inside Region III (\( H_0 \)) and the S-BS is able to access the busy band. In contrast, the case \( E \leq \eta \) indicates that the M-UE may appear in either Region I (\( H_0 \)) or II (\( H_1 \)). Then, the S-BS still has the opportunity (i.e., the probability that the M-UE is in Region I) to assess the busy band. On the other hand, if the threshold \( \eta \) is relatively low, the case \( E \leq \eta \) indicates that the M-UE is inside Region I (\( H_0 \)) and the S-BS is able to access the busy band. In contrast, the case \( E > \eta \) indicates that the M-UE may appear in either Region II (\( H_1 \)) or III (\( H_0 \)). Then, the S-BS still has the opportunity (i.e., the probability that the M-UE is in Region III) to assess the busy band. As a result, on both sides of the threshold, i.e., \( E > \eta \) and \( E \leq \eta \), the S-BS has the opportunity to access the busy band.

To maximize the small cell transmission opportunity, it is reasonable to introduce two access probabilities \( q_1 \) and \( q_2 \) for \( E > \eta \) and \( E \leq \eta \), respectively, where \( 0 \leq q_1 \leq 1 \) and \( 0 \leq q_2 \leq 1 \). Then, the S-BS can make full use of the transmission opportunities under both cases (i.e., \( E > \eta \) and \( E \leq \eta \)). In particular, with the two access probabilities, the S-BS is able to automatically adjust the decision rule between (26) and (27), which makes the OTD work in all three scenarios. For example, when \( q_1 = 1 \) and \( q_2 = 0 \), the S-BS accesses the busy band if \( E > \eta \), which is equivalent to (26); when \( q_1 = 0 \) and \( q_2 = 1 \), the S-BS accesses the busy band if \( E \leq \eta \), which is equivalent to (27).

Next, we obtain the optimal access probabilities \( q_1^* \) and \( q_2^* \) as well as the corresponding optimal threshold \( \eta^* \) so that the small cell transmission opportunity can be maximized.

3) Calculate the Threshold and Access Probabilities:

Based on the above discussion, the probability that the S-BS may access the busy band, i.e., the small cell transmission opportunity, becomes four components, i.e.,

\[
PO = q_1 \left( \Pr \{ E > \eta \mid H_0 \} \Pr [ H_0 ] + \Pr \{ E > \eta \mid H_1 \} \Pr [ H_1 ] \right) \\
+ q_2 \left( \Pr \{ E \leq \eta \mid H_0 \} \Pr [ H_0 ] + \Pr \{ E \leq \eta \mid H_1 \} \Pr [ H_1 ] \right) .
\]

(28)

Similarly, the interference probability to the M-UE has two components, i.e.,

\[
P_I = (q_1 \Pr \{ E > \eta \mid H_1 \} \Pr [ H_1 ] + q_2 \Pr \{ E \leq \eta \mid H_1 \} \Pr [ H_1 ] ) \zeta (d).
\]

(29)

where \( \zeta (d) = S_c / S_{II} \) is the area ratio between the coverage of the S-BS and Region II. The expression of \( \zeta (d) \) varies with different ranges of \( d \) and we obtain them in Appendix A.

Given an interference constraint \( I_c \), which is the maximum allowable interference probability to the M-UE, the optimal threshold as well as the optimal access probabilities can be obtained by

\[
\max \left\{ \begin{array}{l}
PO \\
\end{array} \right\} \\
\text{s.t.} \quad P_I \leq I_c, \quad 0 \leq q_1 \leq 1, \quad 0 \leq q_2 \leq 1, \quad \text{and} \quad \eta \geq 0.
\]

(30)

Substituting (28) and (29) into (30), and considering different distance ranges between the S-BS and M-BS, the expression (30) can be expanded to (31)-(34) at the top of the next page.

In the above derivation from (30) to (31)-(34), we obtain three expressions (32)-(34) from one interference constraint (29). This is because the CDFs of \( H_1 \) in (29), i.e., \( \Pr \{ E > \eta \mid H_1 \} \) and \( \Pr \{ E \leq \eta \mid H_1 \} \), have different closed-form expressions for different ranges of \( d \). Furthermore, the objective function \( P_O \) in (31) strictly increases as \( q_1 \) and \( q_2 \) grow, respectively, but its monotonicity in terms of \( \eta \) is determined by \( q_1 \) and \( q_2 \). Thus, it is very difficult to obtain the closed-form expressions of \( q_1^* \), \( q_2^* \), and \( \eta^* \). Therefore, we will obtain them numerically.

D. Double-Threshold Detector (DTD)

In this subsection, we will design another detector using two thresholds. As shown in Fig. 2(c), when the distance between the S-BS and M-BS is \( d = 300 \) m, half of the \( H_0 \) curve is on the right side of the \( H_1 \) curve (\( E > 25 \) dB), and the other half of the \( H_0 \) curve is on the left side of the \( H_1 \) curve (\( E < 25 \) dB). This indicates that the S-BS in such a location should access the busy band if the test statistic of the S-BS is either large enough or small enough. Then, we set two thresholds \( \eta_L \) and \( \eta_H \) to distinguish \( H_0 \) and \( H_1 \), i.e.,

\[
\left\{ \begin{array}{l}
H_0 : \quad E \leq \eta_L \text{ or } E \geq \eta_H , \\
H_1 : \quad \eta_L < E < \eta_H .
\end{array} \right.
\]

(35)

This rule also works for the other two cases when the S-BS is close and far away from the M-BS, respectively, e.g., Fig. 2(a) and Fig. 2(e). Then, the small cell transmission opportunity can be expressed as

\[
P_O = \Pr \{ E \leq \eta_L \text{ or } E \geq \eta_H \mid H_0 \} \Pr [ H_0 ] \\
+ \Pr \{ E \leq \eta_L \text{ or } E \geq \eta_H \mid H_1 \} \Pr [ H_1 ] .
\]

(36)

The interference probability to the M-UE can be expressed as

\[
P_I = \Pr \{ \eta_L < E < \eta_H \mid H_1 \} \Pr [ H_1 ] \zeta (d),
\]

(37)

where \( \zeta (d) \) is also obtained from Appendix A.

Given an interference constraint \( I_c \), the optimal thresholds that maximize the small cell transmission opportunity can be obtained by

\[
\max \left\{ \begin{array}{l}
P_O \\
\end{array} \right\} \\
\text{s.t.} \quad P_I \leq I_c \text{ and } 0 < \eta_L \leq \eta_H .
\]

(38)

Substituting (36) and (37) into (38), and considering different distance ranges between the S-BS and M-BS, the expression (38) can be expanded to (39)-(42) at the top of the next page.

Since both the objective and constraint functions in (38), strictly increase with \( \eta_L \), and strictly decrease with \( \eta_H \), the maximum transmission opportunity is achieved when \( P_I = I_c \). However, it is very difficult to obtain the closed-form expression of the optimal thresholds. Therefore, we will calculate them numerically.

IV. Simulation Results

In this section, we demonstrate the advantages of the proposed detectors, called one-threshold detector (OTD) and
subject to when $\varepsilon \leq d \leq r + \varepsilon$,

$$[ (q_2 - q_1) \frac{d^2 \eta}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta}{\varepsilon^2 K \gamma_T + d^2 \eta} \right) + q_1 \left( \frac{(d+r)^2 - \varepsilon^2}{R^2} \right) ] \zeta(d) \leq I_c, \quad (32)$$

when $r + \varepsilon < d \leq R - r$,

$$[ (q_2 - q_1) \frac{d^2 \eta}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta}{(d-r)^2 K \gamma_T + d^2 \eta} \right) + q_1 \left( \frac{4dr}{R^2} \right) ] \zeta(d) \leq I_c, \quad (33)$$

when $R - r < d \leq R + r$,

$$[ (q_2 - q_1) \frac{d^2 \eta}{R^2 K \gamma_T} \ln \left( \frac{R^2 K \gamma_T + d^2 \eta}{(d-r)^2 K \gamma_T + d^2 \eta} \right) + q_1 \left( \frac{R^2 - (d-r)^2}{R^2} \right) ] \zeta(d) \leq I_c. \quad (34)$$

subject to when $\varepsilon \leq d \leq r + \varepsilon$,

$$[ \frac{(d+r)^2 - \varepsilon^2}{R^2} + \frac{d^2 \eta_L}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta_L}{\varepsilon^2 K \gamma_T + d^2 \eta_L} \right) - \frac{d^2 \eta_H}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta_H}{\varepsilon^2 K \gamma_T + d^2 \eta_H} \right) ] \zeta(d) \leq P_I, \quad (40)$$

when $r + \varepsilon < d \leq R - r$,

$$\left[ \frac{4dr}{R^2} + \frac{d^2 \eta_L}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta_L}{(d-r)^2 K \gamma_T + d^2 \eta_L} \right) - \frac{d^2 \eta_H}{R^2 K \gamma_T} \ln \left( \frac{(d+r)^2 K \gamma_T + d^2 \eta_H}{(d-r)^2 K \gamma_T + d^2 \eta_H} \right) \right] \zeta(d) \leq P_I, \quad (41)$$

when $R - r < d \leq R + r$,

$$\left[ \frac{(R^2 - (d-r)^2)}{R^2} + \frac{d^2 \eta_L}{R^2 K \gamma_T} \ln \left( \frac{R^2 K \gamma_T + d^2 \eta_L}{(d-r)^2 K \gamma_T + d^2 \eta_L} \right) - \frac{d^2 \eta_H}{R^2 K \gamma_T} \ln \left( \frac{R^2 K \gamma_T + d^2 \eta_H}{(d-r)^2 K \gamma_T + d^2 \eta_H} \right) \right] \zeta(d) \leq P_I. \quad (42)$$

double-threshold detector (DTD). To compare with the conventional transmitter detection, we also provide the performance of the energy detector (ED) [25] [31], where the threshold is calculated under Neyman Pearson criteria with 1% false alarm probability. In the simulation, we assume that the M-UE is uniformly distributed inside the macro cell, where the radius of the macro cell’s coverage and that of the small cell’s coverage are $R = 500$ m and $r = 100$ m, respectively, the target SNR at the M-UE is $\gamma_T = 20$ dB, the interference probability constraint is $I_c = 0.01$, the minimum distance between the M-UE and M-BS is $\varepsilon = 36$ m [32], the number of samples is $K = 2$, and $N = 10^4$ Monte Carlo trials are conducted for each simulation curve.

In the following, we first compare the performance of the OTD, the DTD, and the ED. Then, we present their performance as a function of the target SNR and the radius of the S-BS. After that, we demonstrate that our methods work well even if the target SNR of the M-UE is unknown to the S-BS. Finally, we provide the performance of our methods in the case of shadowing.

Fig. 3. Transmission opportunities of the OTD, the DTD, and the ED.

A. Comparison of the OTD, the DTD, and the ED

Fig. 3 compares the transmission opportunities identified by the three detectors, the OTD, the DTD, and the ED, where both simulation and theoretical curves are provided for the OTD...
and the DTD. From the figure, the simulation and theoretical curves overlap very well, and the proposed OTD and DTD significantly outperform the conventional ED. In particular, the OTD and the DTD have an “U” shape for \( d < 500 \) m and are with the lowest transmission opportunities when \( d \approx 300 \) m. The reason is that the CDF curves of \( H_0 \) and \( H_1 \) in Fig. 2 overlap when \( d \approx 300 \) m, and then it is hard for the OTD and the DTD to distinguish the two hypotheses. When \( d > 500 \) m and the S-BS moves out of the coverage of the M-BS, the S-BS may always access the busy band without interfering with the M-BS. Thus, the transmission opportunity approaches 1 for \( d > 500 \) m. Furthermore, when comparing the OTD and the DTD, they have the same performance for most of the S-BS locations except for a small gap when \( 250 < d < 350 \) m. In this range, the DTD slightly outperforms the OTD.

Fig. 4 compares the interference probabilities of the OTD, the DTD, and the ED. From the figure, the OTD and the DTD can control their interference under the preset interference probability constraint \( I_c = 0.01 \) for \( d < 500 \) m. This is because the optimal thresholds of the OTD and the DTD are designed under the interference probability constraint. In addition, the interference probability of the ED is around 0.002 for \( d < 500 \) m, since the ED is designed under the false alarm probability constraint rather than the interference probability constraint. When \( d > 500 \) m and the S-BS moves out of the coverage of the M-BS, there is no M-UE inside the M-BS’s coverage and the S-BS causes no interference to the M-UE. Then, the interference probabilities of all three methods approach 0 when \( d > 500 \) m.

Fig. 5 and Fig. 6 provide the optimal threshold and the optimal access probabilities of the OTD, respectively, where the threshold of ED is also provided in Fig. 5 for comparison. From Fig. 5, the threshold of ED is a constant since it is determined by a fixed false alarm probability and does not change with \( d \). But the optimal threshold of the OTD varies with \( d \) since it is calculated based on an interference probability constraint. From Fig. 6, when the S-BS is close to the M-BS, i.e., \( d < 300 \) m, \( q_1^* = 1 \) and \( q_2^* = 0 \), which means that the S-BS accesses the busy band when \( E > \eta^* \); when the S-BS is far away from the M-BS, i.e., \( d > 300 \) m,
Transmission opportunity, $P_{O}$

Access probability, $q_1^*$

Access probability, $q_2^*$

$\eta$

$\text{DTD}$

$\text{OTD}$

Fig. 8. Theoretical and simulation results comparison in the OTD and the DTD.

Fig. 7 shows the optimal thresholds of the DTD as well as the threshold of the ED. From the theoretical curves of the DTD, when $d < 250$ m, the optimal high threshold $\eta_H^*$ is about 30 dB and the optimal low threshold $\eta_L^*$ is about $-80$ dB, which means that the decision mainly depends on $\eta_H^*$; when the S-BS moves away from the M-BS, i.e., $250$ m $< d < 350$ m, the decision depends on both thresholds; when the S-BS is close to the edge of the M-BS’s coverage, i.e., $350$ m $< d < 500$ m, the low threshold becomes the dominant threshold. As the S-BS keeps on moving out of the M-BS’s coverage, $d > 500$ m, the overlap between the coverages of the S-BS and the M-BS reduces to zero, and then the S-BS may always access the busy band without interfering with the M-UE. Thus, the two thresholds of the DTD become identical and can be any value, which has been omitted in Fig. 7. Furthermore, the simulation curves match the theoretical ones very well except for the lower threshold $\eta_L^*$ in the range $36$ m $< d < 250$ m and the higher threshold $\eta_H^*$ in the range $350$ m $< d < 500$ m. This is reasonable since the optimal thresholds $\eta_L^*$ and $\eta_H^*$ in these ranges approach zero and infinity, respectively. We cannot obtain the theoretical values by conducting the simulation with $10^4$ trials.

B. Transmission Opportunities versus $\gamma_T$ and $r$

Fig. 8 compares the simulation results of the OTD and the DTD with the theoretical ones in terms of the transmission opportunity. Since the largest gap between simulation and theoretical curves in Fig. 3 appears at $d \approx 300$ m, we evaluate our theoretical results under the worst case of $d = 300$ m. From the figure, the simulation and theoretical curves of the OTD do not overlap for $\gamma_T < 5$ dB, and these of the DTD do not overlap for $\gamma_T < 15$ dB. The reason is that we omit the noise in our derivation, which is not valid in low SNR regions. Fortunately, such an approximation works well in practice, which covers most target SNR values of the M-UE.

Fig. 9 gives the two optimal access probabilities of the OTD with low target SNRs. Here, we only provide the simulation results, since the theoretical results do not match the simulation ones for $\gamma_T < 5$ dB as we have discussed.

From the three sub-figures, when $d < 300$ m, $q_1^* = 1$ and $q_2^* = 0$. But when $d > 300$ m, $q_1^* = 0$ and $q_2^* = 1$. Therefore, the proposed OTD is easy to be implemented in practical systems.

Fig. 10 shows the transmission opportunities of the OTD and the DTD for different radii of the small cell, where $\gamma_T = 20$ dB. Here, we only provide the theoretical results for simplicity since the theoretical and simulation results match very well in high target SNR. From the figure, the transmission opportunities of the OTD and the DTD have the same trend for different distances between the S-BS and M-BS. Specifically, the transmission opportunity of all curves are equal to 1 for
In practice, the target SNR of the M-UE varies with different services and may even be unknown to the S-BS. Thus, we assume that the target SNR is uniformly distributed in a reasonable range, e.g., between 5 dB and 20 dB. Then, the CDF of the test statistic in Section III-B needs to incorporate the distribution of $\gamma_T$. Since it is very difficult to obtain the closed-form CDF in this case, we will provide simulation results instead.

Fig. 11 compares the performance of the OTD, the DTD, and the ED, where $\gamma_T$ is uniformly distributed between 5 dB and 20 dB. From the figure, the proposed the OTD and the DTD obtain almost the same performance. They achieve about 100% to 300% more transmission opportunities in average than the conventional ED. Fig. 12 provides the corresponding interference probabilities of the OTD, the DTD, and the ED. From the figure, the three methods have similar interference to the M-UE, i.e., the interference probabilities are around 0.01 for $d < 500$ m, and approach 0 for $d > 500$ m.

**D. The Case of Shadowing**

In this subsection, we provide the performance of the three detectors in the case of shadowing, i.e., path-loss, shadowing, and multi-path fading are all considered. In particular, the shadowing coefficient follows log-normal distribution with the standard variation of 12 dB\(^3\). All simulation parameters except $K = 16$ are the same as those in Figs. 11 and 12. Here, the reason that the number of samples $K$ is set to 16 is to let the ED has similar interference probability with the OTD and the DTD. This is because the OTD and the DTD are designed under the interference probability constraint. When we consider shadowing, they can automatically adjust the thresholds to reach the preset interference probability $I_c = 0.01$. However, the ED is designed under the false alarm probability. When we consider shadowing, the ED does not change the threshold, which raises the interference probability. Thus, we adjust the number of samples $K$ to raise the threshold and make the ED have similar interference probability with the OTD and the DTD. This allows us to make a comparison of different detectors in terms of the transmission opportunity.

Fig. 13 provides the transmission opportunities of the three detectors. From the figure, we observe the same trend as in Fig. 12, but with only slight performance loss at all detectors. Fig. 14 shows the corresponding interference probabilities of all three detectors. From the figure, we observe almost the same curves as in Fig. 12. Therefore, in the case of shadowing, the proposed OTD and DTD can still achieve about 100% improvement.

\(^3\)We actually provide the worst case performance since the standard variation of shadowing is usually between 4 dB and 12 dB [29].
to 300% more transmission opportunities in average than the conventional ED.

V. CONCLUSIONS

In this paper, we proposed a passive receiver detection method to enable a small cell to identify the location of an active user that is being served by a macro cell. Then, the small cell may access the busy band if the active user is outside the coverage of the small cell, and achieve more transmission opportunities. We suggested that the small cells detect the active user by using the received energy from the macro cell as the test statistic. Then we designed two detectors with one and two thresholds, called OTD and DTD, respectively. Meanwhile, as the conventional energy detector, neither the OTD nor the DTD requires any prior information of the macro cell’s signal. Our results indicated that under the same opportunities in average than the conventional energy detector.

VI. ACKNOWLEDGMENTS

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APPENDIX

A. Derivation of $\zeta(d)$

In the following, we derive the expression of $\zeta(d)$, which is the area ratio between the coverage of the S-BS and Region II, defined as $\zeta(d) = S_s/S_{II}$. Since the expressions of both areas vary with the distance $d$ between the S-BS and M-BS, we will calculate the ratio in different ranges of $d$, respectively.

1) S-BS at the center of the macro cell: Fig. 15 illustrates the system model when the S-BS is at the center of the macro cell, i.e., $\varepsilon \leq d \leq r + \varepsilon$. From the figure, the area of Region II can be obtained by

$$S_{II} = \pi \left( (d + r)^2 - \varepsilon^2 \right).$$

When $\varepsilon \leq d \leq r - \varepsilon$, the area of the S-BS’s coverage is determined by $r$ and $\varepsilon$, i.e.,

$$S_s = \pi (r^2 - \varepsilon^2).$$

When $r - \varepsilon \leq d \leq r + \varepsilon$, the area of the S-BS’s coverage is determined by $d$, $r$, and $\varepsilon$. Let $\varphi$ and $\varphi'$ be the angles shown in Fig. 15, we have $\varphi = \arccos ((d^2 + \varepsilon^2 - r^2)/2d\varepsilon)$ and $\varphi' = \arccos ((d^2 + r^2 - \varepsilon^2)/2d\varepsilon)$. Then the area of the S-BS’s coverage becomes

$$S_s = r^2(\pi - \varphi') + r^2\cos \varphi' \sin \varphi - \varepsilon^2 \varphi + \varepsilon^2 \cos \varphi \sin \varphi. \tag{45}$$

Therefore, we have the following expressions,

$$\zeta(d) = \frac{S_s}{S_{II}} = \frac{r^2 - \varepsilon^2}{(d + r)^2 - \varepsilon^2}, \quad \text{for} \quad \varepsilon \leq d \leq r - \varepsilon, \tag{46}$$

and

$$\zeta(d) = \frac{S_s}{S_{II}} = \frac{r^2(\pi - \varphi') + r^2\cos \varphi' \sin \varphi - \varepsilon^2 \varphi + \varepsilon^2 \cos \varphi \sin \varphi}{\pi \left( (d + r)^2 - \varepsilon^2 \right)}, \quad \text{for} \quad \varepsilon < d \leq r + \varepsilon. \tag{47}$$

2) S-BS in the medium range of the M-BS's coverage: Fig. 1 in Section II provides the system model of this case, i.e., $r + \varepsilon < d \leq R - r$. From the figure, the area of the region II can be obtained by

$$S_{II} = \pi (d + r)^2 - \pi (d - r)^2 = 4\pi dr. \tag{48}$$

The area of the S-BS’s coverage can be obtained by

$$S_s = \pi r^2. \tag{49}$$

Then, we have

$$\zeta(d) = \frac{S_s}{S_{II}} = \frac{r}{4d}, \quad \text{for} \quad r + \varepsilon < d \leq R - r. \tag{50}$$
3) S-BS at the edge of the macro cell: Fig. 16 gives the system model when the S-BS is at the edge of the macro cell, i.e., \( R - r < d \leq R + r \). From the figure, the area of the region II can be obtained by

\[
S_{II} = \pi R^2 - \pi (d-r)^2.
\]

The area of the S-BS’s coverage is determined by \( d, r \) and \( R \). Let \( \theta \) and \( \theta' \) be the angles shown in Fig. 16, we have \( \theta = \arccos((d^2 + R^2 - r^2)/2dr) \) and \( \theta' = \arccos((d^2 + r^2 - R^2)/2dr) \). Then the area of small cell can be expressed as

\[
S_s = r^2\theta' + R^2\theta - R^2\sin\theta\cos\theta - r^2\sin\theta'\cos\theta'.
\]

Therefore, we have

\[
\zeta(d) = \frac{S_s}{S_{II}} = \frac{r^2\theta' + R^2\theta - R^2\sin\theta\cos\theta - r^2\sin\theta'\cos\theta'}{\pi R^2 - \pi (d-r)^2},
\]

for \( R - r < d \leq R + r \). (53)

In summary, we have the expression (54) at the top of the next page under different ranges of \( d \).

**B. Optimal Access Probabilities \( q_1 \) and \( q_2 \)**

In the following, we analyze the optimal access probabilities of the OTD when the target SNR is large. Let

\[
A = \Pr\{E \leq \eta|H_0\}, \quad B = \Pr\{E \leq \eta|H_1\}, \quad C = \Pr\{E \geq \eta|H_0\}, \quad D = \Pr\{E \geq \eta|H_1\},
\]

the transmission opportunity in (28) can be simplified as

\[
P_O = q_2 \left( A \Pr\{H_0\} + B \Pr\{H_1\} \right) + q_1 \left( C \Pr\{H_0\} + D \Pr\{H_1\} \right),
\]

and the interference constraint in (29) can be simplified as

\[
P_I = (q_2 B \Pr\{H_1\} + q_1 D \Pr\{H_1\}) \zeta(d).
\]

From (56), we obtain

\[
q_1 = \frac{P_I}{\zeta(d)} - q_2 B \frac{\Pr\{H_1\}}{D \Pr\{H_1\}}.
\]

and

\[
q_2 = \frac{P_I}{\zeta(d)} - q_1 D \frac{\Pr\{H_1\}}{B \Pr\{H_1\}}.
\]

Substituting (57) into (55), we have

\[
P_O = q_2 \left( A \Pr\{H_0\} + B \Pr\{H_1\} \right) + \frac{P_I}{\zeta(d)} - q_2 B \frac{\Pr\{H_1\}}{D \Pr\{H_1\}} - q_1 \left( C \Pr\{H_0\} + D \Pr\{H_1\} \right)
\]

\[
= P_I \left( 1 + \frac{C \Pr\{H_0\}}{D \Pr\{H_1\}} \right) - q_2 \left( \frac{(BC - AD) \Pr\{H_0\}}{D} \right).
\]

On the other hand, substituting (58) into (55), we have

\[
P_O = \frac{P_I}{\zeta(d)} - q_1 D \frac{\Pr\{H_1\}}{B \Pr\{H_1\}} + q_1 \left( C \Pr\{H_0\} + D \Pr\{H_1\} \right)
\]

\[
= P_I \left( 1 + \frac{A \Pr\{H_0\}}{B \Pr\{H_1\}} \right) + q_1 \left( \frac{(BC - AD) \Pr\{H_0\}}{B} \right).
\]

When the target SNR of the M-UE is large, e.g., \( \gamma_T = 20 \) dB, the CDF curves of \( H_0 \) and \( H_1 \) are different, which have been shown in Fig. 2. Specifically, if the S-BS is close to the M-BS, e.g., \( d = 100 \) m in Fig. 2(a), the CDF curve of \( \mathbb{H}_0 \) is on the right side of the CDF curve of \( \mathbb{H}_1 \), and we have \( B > A > 0 \) and \( C > D > 0 \). Since \( \Pr\{H_0\} > 0 \), \( \Pr\{H_1\} > 0 \), \( P_I > 0 \), and \( \zeta(d) > 0 \), we can obtain the maximum \( P_O \) in (60) when \( q_1 = 1 \), and the maximum \( P_O \) in (59) when \( q_2 = 0 \).

Similarly, if the S-BS is far away from the M-BS, e.g., \( d = 400 \) m in Fig. 2(e), the CDF curve of \( \mathbb{H}_0 \) is on the left side of the CDF curve of \( \mathbb{H}_1 \). Then we obtain \( A > B > 0 \) and \( D > C > 0 \). Therefore, the maximum \( P_O \) in (60) can be reached when \( q_1 = 0 \), and the maximum \( P_O \) in (59) can be reached when \( q_2 = 1 \).

In summary, when the target SNR \( \gamma_T \) is large, the optimal access probabilities of the OTD are either 0 or 1, which maximize the small cell transmission opportunity.

**REFERENCES**


\[
\zeta(d) = \begin{cases} 
\frac{r^2 - \varepsilon^2}{(d+\varepsilon)^2 - \varepsilon^2}, & \varepsilon \leq d \leq r - \varepsilon, \\
\frac{r^2 (\pi - \varphi') + r^2 \cos \varphi' \sin \varphi - \varepsilon^2 \varphi + \varepsilon^2 \cos \varphi \sin \varphi}{\pi (d+\varepsilon)^2 - \varepsilon^2}, & r - \varepsilon \leq d \leq r + \varepsilon, \\
\frac{r^2 \theta' + r^2 \varphi - R^2 \sin \theta \cos \theta' - \varepsilon^2 \sin \theta' \cos \theta'}{\pi R^2 - \pi (d-r)^2}, & r + \varepsilon < d \leq R - r, \\
R - r < d \leq R + r, & \end{cases}
\]

where

\[
\varphi = \arccos \left( \frac{(d^2 + \varepsilon^2 - r^2)}{2d \varepsilon} \right), \\
\varphi' = \arccos \left( \frac{(d^2 + r^2 - \varepsilon^2)}{2d r} \right), \\
\theta = \arccos \left( \frac{(d^2 + R^2 - r^2)}{2d R} \right), \\
\theta' = \arccos \left( \frac{(d^2 + r^2 - R^2)}{2d R} \right).
\]


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