Distributed Detection With Multiple Sensors: Part I—Fundamentals

Ramanarayanan Viswanathan
Southern Illinois University Carbondale, viswa@engr.siu.edu

Pramod K. Varshney
Syracuse University

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Distributed Detection With Multiple Sensors: Part I—Fundamentals

RAMANARAYANAN VISWANATHAN AND PRAMOD K. VARSHNEY, SENIOR MEMBER, IEEE

Invited Paper

In this paper, basic results on distributed detection are reviewed. In particular, we consider the parallel and the serial architectures in some detail and discuss the decision rules obtained from their optimization based on the Neyman–Pearson (NP) criterion and the Bayes formulation. For conditionally independent sensor observations, the optimality of the likelihood ratio test (LRT) at the sensors is established. General comments on several important issues are made including the computational complexity of obtaining the optimal solutions, the design of detection networks with more general topologies, and applications to different areas.

I. INTRODUCTION

In recent years, signal processing with distributed sensors has been gaining importance. The relatively low cost of sensors, the inherent redundancy possible with multiple sensors, the availability of high speed communication networks, and increased computational capability have spurred great research interest in this topic [1]. Distributed sensor systems were originally motivated by their applications in military surveillance with respect to command, control, and communications [2], [3] but are now being employed in a wide variety of applications.

Interest in decentralized detection and estimation has surfaced with anticipated applications in multiple-target detection and estimation using multiple sensors, which may be geographically dispersed [2]. In classical multisensor detection and estimation, it is assumed that all the local sensors (such as radar, sonar, infrared) communicate all their data to a central processor that performs optimal detection and tracking of targets based on conventional statistical techniques. In decentralized processing, some preliminary processing of data (often lossy compression) is carried out at each sensor and condensed information is sent from each sensor to other sensors and ultimately to the central processor which is often known as the fusion center. In the terminology of distributed sensor networks, we say that the network has intelligence at each node [4]. The centralized scheme may be too restrictive and in some practical cases even unwise. For example, when coverage areas of sensors do not completely overlap, it is possible that a signal return may be received at some but not all the sensors. In such cases, an optimum scheme would have to be based on decentralized processing of the observations at the sensors. Some of the advantages of distributed signal processing schemes are reduced communication bandwidth requirement, increased reliability, and reduced cost. In addition, a distributed system architecture may yield a better response to rapid changes in background scenario. Unlike the central processor in centralized systems, the fusion center of a decentralized system has only partial information as communicated by the sensors. This results in a loss of performance in decentralized systems as compared to centralized systems. However, the performance loss can be made small by optimally processing the information at the sensors [5]. The objective of most studies in the field is to develop computationally efficient algorithms at the sensors and at the fusion center. In general, a distributed sensor network has to address the issues of choice of topology, ability to reconfigure the structure in the case of sensor/link failures, existence of communication between sensors and feedback communication between the fusion center and the sensors, and robustness of signal processing algorithms with respect to probability models, jammers, and other external threats.

Following the broad classification of statistical inferencing into hypothesis testing and estimation, one can identify two areas of distributed signal processing, namely distributed detection and distributed estimation. Here we concern ourselves primarily with the fundamentals of the distributed detection problem. Some advanced topics that involve locally optimal detection, sequential detection, non-parametric methods, robust procedures, and CFAR tech-
Fig. 1. Parallel topology with fusion center.

Fig. 2. Parallel topology without fusion center.

Fig. 1—4 we show three major topologies used for distributed signal processing [1]. These are called parallel (Figs. 1 and 2), serial or tandem (Fig. 3), and tree (Fig. 4) configurations. Some notations that describe the observations at various points in these configurations as well as the notations that describe some pertinent probabilities are given at the bottom of the page.

II. NEYMAN–PEARSON FORMULATION

In this section, we present several interesting and fundamental results from the Neyman–Pearson (NP) formulation of the distributed detection problem. Both parallel and serial configurations are considered. We assume a binary hypothesis testing problem in which the observations at all the sensors either correspond to the presence of a signal (hypothesis $H_1$) or to the absence of a signal (hypothesis $H_0$).

A. Parallel Configuration

First, let us consider the parallel configuration of $N$ sensors that is shown in Fig. 1. We assume that the sensors do not communicate with each other and that there is no feedback from the fusion center to any sensor. Let $y_i$ denote either a single observation that is available at the $i$th sensor, or, in the case of multiple observations, a sufficient statistic that might exist for the given binary hypothesis testing problem [7]. The $i$th sensor employs the mapping rule $u_i = \gamma_i(y_i)$ and passes the quantized information $u_i$ to the fusion center. Based on the received
information \( \mathbf{u} = (u_1, u_2, \ldots, u_N) \), the fusion center arrives at the global decision \( u_0 = \gamma_0(\mathbf{u}) \) that favors either \( H_1 \) (say \( u_0 = 1 \)) or \( H_0 \) (say \( u_0 = 0 \)). The NP formulation of the distributed detection problem can now be stated as follows: for a prescribed bound on the global probability of false alarm, \( P_F \), find (optimum) local and global decision rules \( \Gamma = (\gamma_0, \gamma_1, \ldots, \gamma_N) \) that minimize the global probability of miss \( P_M \). Variations of this formulation include the optimization of only the fusion rule for a given set of local decision rules and the optimization of only the local decision rules for a given fusion rule. Also, the solution to the problem depends on whether the sensor observations are conditionally independent (conditioned on the hypothesis) or not.

1) Conditional Independence: Conditional independence of sensor observations implies that the joint density of the observations obeys

\[
p(y_1, \ldots, y_N \mid H_1) = \prod_{i=1}^{N} p(y_i \mid H_1), \quad \text{for } l = 0, 1, \quad (1)
\]

For the above mentioned NP problem under the conditional independence assumption, the mapping rules at the sensors as well as the decision rule at the fusion center are threshold rules based on the appropriate likelihood ratios \([8], [9]\). Here, we sketch the proof for the situation in which the \( \{y_i, i = 1, \ldots, N\} \) are binary-valued. That is, \( u_i = 0 \) or \( 1 \) which implies that the \( i \)th sensor decides on hypothesis \( H_0 \) or \( H_1 \). Since the observation at the fusion center is the vector \( \mathbf{u} \), according to the standard NP lemma, the optimal fusion center test is given by \([10]\) (see (2) at the bottom of the page) where the threshold \( \lambda_0 \) and the randomization constant \( \varepsilon \) are chosen to achieve a desired \( P_F = \alpha \). Thus the optimal fusion center test is a likelihood ratio test (LRT). In order to show that the optimal local decision rules are also LRT’s, we first look at the following lemma \([9]\).

**Lemma 1:** Let the local decision rules be such that \( P_{D_i} \geq P_{D_i} \), for all \( i \). Then, for a given vector \( \mathbf{u}^* \) such that \( \Lambda(\mathbf{u}^*) > \lambda_0 \) and any other vector \( \mathbf{u}^+ \) such that \( u_i^+ \geq u_i^* \) for all \( i \), \( \Lambda(\mathbf{u}^+) \geq \lambda_0 \).

The conditional independence assumption implies that

\[
\Lambda(\mathbf{u}) = \frac{P(\mathbf{u} \mid H_1)}{P(\mathbf{u} \mid H_0)} = \prod_{i=1}^{N} \frac{P(u_i \mid H_1)}{P(u_i \mid H_0)}, \quad (3)
\]

The lemma becomes apparent when one uses the relation \( P_{D_i} \geq P_{D_i} \) in (3).

Since the decision variables \( u_i \) are binary-valued, an LRT of the form (2) is equivalent to the fusion center decision \( u_0 = \gamma_0(\mathbf{u}) \) being a Boolean function. Since \( \mathbf{u} \) can assume \( 2^N \) possible values, the number of possible Boolean functions are \( 2^{2^N} \). However, for an optimal solution of the NP problem, the fusion rule has to satisfy Lemma 1. A Boolean function that satisfies the monotonicity property given in Lemma 1 is called a positive unate function. The number of positive unate functions for various integer values of \( N \) are provided in \([11]\). Even though this number is significantly smaller than \( 2^{2^N} \), it still grows exponentially with \( N \). For example, the number of positive unate functions for \( N = 4 \) is 168 and is 7581 for \( N = 5 \).

The optimality of a LRT at a sensor can now be established. Let \( u_0 = \gamma_0^*(\mathbf{u}) \), where \( \gamma_0^*(\cdot) \) is a positive unate function. Such a function, when expressed in a sum of product form, has no complemented variables. Hence

\[
\frac{\partial P_D}{\partial P_{D_i}} \geq 0, \quad i = 1, \ldots, N. \quad (4)
\]

Consider a set of decision rules \( \Gamma^* = (\gamma_0^*, \gamma_1^*, \ldots, \gamma_N^*) \) that achieves the desired \( P_F = \alpha \) and at the same time achieves the detection probability \( P_{D_i}^* \). For this test, assume that the corresponding local false alarm and detection probabilities are \( P_{D_i}^* \) and \( P_{D_i}^* \), respectively. Consider an alternate set of decision rules \( \Gamma = (\gamma_0, \gamma_1, \ldots, \gamma_N) \) in which \( \gamma_0 = \gamma_0^* \), with the same corresponding local false alarm probabilities \( P_{F_i}^* \) but different detection probabilities \( P_{D_i} \). Accordingly to the NP lemma, for a given false alarm probability, a LRT achieves the largest possible detection probability. Therefore, if \( \Gamma \) is such that each local decision rule is a LRT, then each \( P_{D_i} \geq P_{D_i}^* \). Since the optimal fusion rule \( \gamma_0^* \) has to be a monotone rule, (4) implies that \( P_D \geq P_D^* \). Thus an optimal solution to the NP distributed detection problem should employ LRT’s at the local sensors.

The above result is valid even if \( u_i, i = 1, \ldots, N, \) are multivalued. For a proof, see \([5]\) and \([12]\).

Even though the local decision rules as well as the global fusion rule are LRT’s, finding the actual LRT’s is quite difficult. This is because the threshold \( \lambda_0 \) in (2) as well as the local thresholds \( t_i \) that enter the local tests

\[
\frac{p(y_i \mid H_1)}{p(y_i \mid H_0)} = \Lambda(u_i) \begin{cases} > t_i \text{ then } u_i = 1 \\ < t_i \text{ then } u_i = 0 \end{cases}
\]

need to be determined so as to maximize \( P_D \) for a given \( P_F = \alpha \) (if the likelihood ratio in (5) is a continuous random variable with no point mass, then the randomization is unnecessary and \( \varepsilon_i \) can be assumed to be zero without losing optimality). Since (2) is known to be a monotone fusion rule, one can solve for the set of optimal local thresholds \( \{t_i, i = 1, \ldots, N\} \) for a given monotone fusion rule and compute the corresponding \( P_D \). One can then successively consider other possible monotone fusion rules and obtain the corresponding detection probabilities. The final optimal solution is the one monotone fusion rule and the corresponding local decision rules (5) that provide the

\[
\Lambda(\mathbf{u}) = \frac{P(\mathbf{u} \mid H_1)}{P(\mathbf{u} \mid H_0)} > \lambda_0 \text{ decide } H_1 \text{ or set } u_0 = 1 \\
\Lambda(\mathbf{u}) < \lambda_0 \text{ decide } H_0 \text{ or set } u_0 = 0
\]

(2)
Finding the optimal solution in this fashion is possible only for very small values of N. The complexity increases with N because 1) the number of monotone rules grows exponentially with N and 2) finding the optimal \( \{t_i, i = 1, \ldots, N\} \) for a given fusion rule is an optimization problem involving an \((N - 1)\) dimensional search (it is one dimension less than \(N\) because of the constraint \(P_F = \alpha\)).

The question of randomization is thoroughly analyzed in [5]. It considers three cases: 1) no randomization in (5), yielding a deterministic strategy, 2) each sensor choosing randomization rules of the type (5) independently of all other sensors, leading to independent randomization, and 3) a strategy called dependent randomization in which a member \( \Gamma^i = \{\gamma_0^i, \ldots, \gamma_N^i\}, k = 1, 2, \ldots, K \) is chosen with some probability \(p_k\). It is shown in [5] that if the likelihood ratio in (5) has no point masses, then an optimal solution within the set of deterministic strategies is also an optimal solution within the set of independent randomization strategies.

An earlier attempt to solve the NP distributed detection problem assumed that the optimal decision rules can be obtained by maximizing the Lagrangian, \(P_D + \lambda(P_F - \alpha)\) [13]. Unfortunately, this method may not always yield the correct solution [5]. The reason for this, as explained in [5], is that if one plots all the possible receiver operating curves (ROC’s) (showing \(P_D\) versus \(P_F\)) corresponding to different sets of decision rules \(\Gamma\), they may have nonconvex regions for some probability distributions. In such situations, the optimal solution to the NP problem does not maximize the Lagrangian over the nonconvex regions and the solution resulting from the maximization of the Lagrangian is not optimum. Examples where the Lagrangian approach fails are shown in [8]. Maximization of Lagrangian may still be used for situations where the ROC is convex, e.g., derivation of local decision rules for a fixed fusion rule.

2) Conditionally Dependent Observations: The observations at the sensors are dependent when the joint density of the observations, given the hypothesis, cannot be written as the product of the marginal densities, as in (1). Such situations would arise if one detects a random signal in noise or if the sensor noise samples are correlated when detecting a deterministic signal in noise. It is shown in [14] that for the conditionally dependent case, the optimal tests at the sensors are no longer of the threshold type based solely on the likelihood ratio of the observations at the individual sensors. In general, the optimal solution is intractable. When the observations are discrete and conditionally dependent, the optimal solution is nonpolynomial complete [15]. When the joint distributions of the observations at the sensors have a certain structure, the performance of certain distributed decision rules can be easily determined [16]–[18].

B. Serial Configuration

In a serial or tandem configuration of \(N\) sensors, the \((j - 1)\)th sensor passes its quantized information to the \(j\)th sensor which generates its quantized information based on its own observation and the quantized data received from the “previous” sensor (see Fig. 3). The first sensor in the network uses only its observation to derive its quantized data for use by the next sensor. The last sensor in the network makes a decision as to which one of the two possible hypotheses the observations at the sensors correspond to. Optimal solution to the NP problem corresponding to the serial configuration is readily available when the observations at the sensors are conditionally independent [5], [19]. The problem is intractable when the conditional independence assumption is not valid. Therefore, we assume conditional independence in the sequel.

Let us consider the case when each sensor in the serial configuration makes a binary decision, i.e., \(u_i \in \{0, 1\}\), for \(i = 1, 2, \ldots, N\). Denoting the false alarm and the detection probabilities of the \(j\)th stage(sensor) as \(P_{Fj}\) and \(P_{Dj}\), respectively, the NP problem can be stated as follows: subject to the constraint \(P_{FN} \leq \alpha\), find the decision rules at all the sensors so that these rules together achieve the maximum possible \(P_{DN}\). It is shown in [19] that the solution to the above problem yields likelihood ratio threshold rule for the \(j\)th stage, with the likelihoods being computed using the observations at the \(j\)th sensor and the decision from the \((j - 1)\)th stage. The solution is based on the following observation. From the NP fundamental lemma.
it is clear that the last \((N)\)th stage must employ a LRT to achieve the maximum possible \(P_{DN}\). The proof is then completed by showing that the probability of detection at the \(j\)th stage, for any given \(P_{Fj}\) and \(P_{F(j-1)}\), is a monotonic increasing function of \(P_{DN}\) and hence the test at the \((j-1)\)th stage must be a likelihood ratio threshold test. As in the parallel case, tests at all the sensors are LRT’s, but solving for the optimal thresholds that need to be employed in these tests is, in general, quite difficult. However, for the serial case, there exist algorithms that can obtain optimal thresholds with complexity that is linear in \(N\) [20].

In general, the serial network has serious reliability problems. Delays accumulate because each stage has to wait for results from the previous stage. This delay problem can be overcome by modifying the communication structure [19]. A more serious problem is that the performance degrades considerably if the “link” in the serial configuration is broken at an intermediate stage. Even though the performance of a serial network in the presence of failures is an important issue, we do not address it here. In the rest of this section, we investigate the question, can the serial network provide a better detection performance than the parallel network in the absence of any failures? For a two-sensor network we have the following proposition [1], [5], [19].

Proposition One: For distributed detection networks consisting of two detectors, the optimal tandem network performs at least as well as the optimal parallel network.

Proof: Consider a parallel fusion network with two local detectors and a fusion center. Let \(\gamma^* = \{\gamma^*_0, \gamma^*_1, \gamma^*_2\}\) be the set of optimal decision rules for the fusion center and the two local detectors. Decision rules \(\gamma^*_1\) and \(\gamma^*_2\) operate exclusively on their observations \(y_1\) and \(y_2\) to yield the decisions \(u_1\) and \(u_2\). The fusion rule \(\gamma^*_0\) determines the global decision \(u_0\) based on the local decisions \(u_1\) and \(u_2\).

Now consider a two-detector tandem network in which the detectors employ the decision rules \(\gamma^*_1\) and \(\gamma^*_2\). The first detector employs \(\gamma^*_1\) to operate on its observation and provides its decision to the second detector. The second detector employs \(\gamma^*_2\) to operate on its observation to come up with its preliminary decision. Then, it uses the fusion rule \(\gamma^*_0\) to combine its preliminary decision and the decision received from the first detector to yield the final decision. The tandem network designed in this ad hoc (not necessarily optimal) manner can always duplicate the performance of the optimal two-detector parallel fusion network. Thus, the optimal tandem network performs at least as well as the optimal parallel network.

Similar results on the relative performance of serial and parallel networks consisting of more than two detectors are not available. For networks in which sensors generate binary decisions, a related result is that there always exists a better serial rule than a parallel fusion rule that is implementable as a sequence of two-input and one-output Boolean rules [19]. However, it is possible that an optimal parallel fusion rule does not belong to the class of fusion rules that are implementable as a sequence of two-input and one-output rules and the parallel network might considerably outperform the serial network. In fact, asymptotically for large \(N\), as compared to the parallel scheme, the probability of a miss for a serial network goes to zero at a much slower rate [21].

In the case of tree networks of the type in Fig. 4, for conditionally independent observations, it can be shown that an optimal solution to the NP problem yields threshold tests based on the likelihood ratios [5]. Solving for the optimal thresholds is, in general, complicated.

III. BAYESIAN FORMULATION

This section presents a Bayesian formulation of the distributed detection problem in which the objective is to minimize the Bayesian risk. Assignment of costs to different courses of action and knowledge of prior probabilities are required for the solution of this problem. First, the problem is solved for parallel and serial network topologies and then several other detection network topological structures are discussed. Throughout this section we assume that the sensor observations are conditionally independent.

A. Parallel Configuration

We consider the system structure described in Section II-A. Before we deal with the general problem in Section III-A3, we consider two special cases in the next two subsections.

1) Parallel Configuration without a Fusion Center: Consider a parallel network without fusion shown in Fig. 2. All of the sensors observe a common phenomenon and make local decisions regarding it. These decisions are not combined to yield a global decision. Costs of decision making at different sensors are assumed to be coupled and a systemwide optimization based on the coupled cost assignment is performed so that the resulting sensor decision rules are coupled. For simplicity of presentation, we limit our attention to a two-sensor network here. More general results can be obtained in a similar manner. Let \(P_0\) and \(P_1\) denote the a priori probabilities for the two hypotheses \(H_0\) and \(H_1\), respectively. The costs of different sensors are not available. For networks in which sensors generate binary decisions, a related result is that there always exists a better serial rule than a parallel fusion rule that is implementable as a sequence of two-input and one-output Boolean rules [19]. However, it is possible that an optimal parallel fusion rule does not belong to the class of fusion rules that are implementable as a sequence of two-input and one-output rules and the parallel network might considerably outperform the serial network. In fact, asymptotically for large \(N\), as compared to the parallel scheme, the probability of a miss for a serial network goes to zero at a much slower rate [21].

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\[
R = \sum_{i,j,k} \int_{y_1, y_2} P(u_1, u_2, y_1, y_2, H_k) C_{i,j,k} = \sum_{i,j,k} \int_{y_1, y_2} P_k C_{i,j,k} P(u_1, u_2 | y_1, y_2, H_k) p(y_1, y_2 | H_k)
\]

(6)

\[
t_1 = \frac{P_0 \int_{y_2} p(y_2 | H_0) [(C_{110} - C_{010}) + P(u_2 = 0 | y_2)] + \int_{y_2} p(y_2 | H_1) [(C_{011} - C_{111}) + P(u_2 = 0 | y_2)]}{P_1 \int_{y_2} p(y_2 | H_1) [(C_{001} - C_{101}) + \int_{y_2} p(y_2 | H_2) [(C_{010} - C_{100}) + \int_{y_2} p(y_2 | H_3) [(C_{011} - C_{111}) + P(u_2 = 0 | y_2)]]} \]

(7)
courses of action are denoted by \( C_{ijk} \), where \( C_{ijk} \) represents the cost of detector one deciding \( H_j \), detector two deciding \( H_k \) when \( H_k \) is present. The goal is to obtain decision rules at the two detectors that jointly minimize the Bayesian risk \( R \) given by (6), as shown at the bottom of the previous page. It is shown in [1, Sec. 3.2], [2] that the resulting decision rules are LRT’s of the form (5). The threshold at the first sensor is given by (7), shown at the bottom of the previous page. Note that \( t_1 \) is a function of \( P(y_2 = 0 \mid y_2) \) which is determined by the decision rule at the second sensor. Thus \( t_1 \) is a function of \( t_2 \). A similar expression for \( t_2 \) can be obtained as a function of \( t_1 \). These expressions represent the necessary conditions that \( t_1 \) and \( t_2 \) must satisfy. Solution of these coupled equations yields a locally optimum solution. When there are multiple local minima, each must be examined to find the best solution.

2) Optimal Fusion of Local Decisions: Next, we consider the fusion of sensor decisions \( y_i \), \( i = 1, \ldots, N \), in a Bayesian framework. Each \( y_i \) is a binary random variable characterized by the associated \( P_{F_i} \) and \( P_{M_i} \). The goal is to determine the fusion rule that minimizes the Bayes risk. Once again the result for this binary hypothesis testing problem is an LRT given by [1, Sec. 3.3], [3]

\[
P(y_i \mid H_1) \begin{cases} \frac{P_{M_i}}{P_{F_i}} & i = 1, \ldots, N \in H_0 \\ \frac{P_{F_i}}{P_{M_i}} & i = 1, \ldots, N \in H_1 \end{cases} \quad \eta
\]

(8)

where \( C_{ij} \) denotes the cost of global decision being \( H_i \) when \( H_j \) is present. This LRT can be expressed in the following form [1, Sec. 3.3], [3]

\[
\sum_{i=1}^{N} \left[ u_i \log \frac{1 - P_{M_i}}{P_{F_i}} + (1 - u_i) \log \frac{P_{M_i}}{1 - P_{F_i}} \right] \begin{cases} \frac{P_{M_i}}{P_{F_i}} & i = 1, \ldots, N \in H_0 \\ \frac{P_{F_i}}{P_{M_i}} & i = 1, \ldots, N \in H_1 \end{cases} \quad \eta
\]

(9)

Thus a weighted sum of sensor decisions is formed and is compared with a threshold. The weights are functions of the probabilities of false alarm and miss of individual sensor decisions and are, therefore, functions of the quality of sensor decisions.

3) Global Optimization: Finally, we consider the system shown in Fig. 1 that consists of a number of sensors connected in parallel and with a fusion center. The goal in the Bayesian formulation is to obtain the set of decision rules \( \Gamma = \{\gamma_0, \ldots, \gamma_N\} \) that minimizes the average cost of the overall system operation. As in [1, Sec. 3.4], [22], the Bayes risk can be expressed as

\[
\mathcal{R} = C + C_F \sum_u P(u_0 = 1 \mid u)P(u \mid H_0) - C_D \sum_u P(u_0 = 1 \mid u)P(u \mid H_1)
\]

(10)

where

\[
C_F = P_0(C_{10} - C_{00})
\]

\[
C_D = (1 - P_0)(C_{01} - C_{11})
\]

\[
C = C_{10} (1 - P_0) + C_{00} P_0
\]

and \( \sum_u \) indicates summation over all possible values of \( u \). A person-by-person optimization (PBPO) methodology is adopted for system optimization. In this methodology, while optimizing any one decision rule, it is assumed that all the other decision rules remain fixed. System design equations resulting from this PBPO procedure represent necessary but not, in general, sufficient conditions to determine the globally optimum solution. This set of equations is solved simultaneously to obtain the desired PBPO solution. It is shown in [1, Sec. 3.4], [22] that the decision rules and the fusion rule are LRT’s given by

\[
p(y_k \mid H_j)^{u_k=1} \geq \frac{P(y_0 \mid H_0)^{u_0=0} \Pi_{i=1, i \neq k}^N P(y_i \mid H_0)^{u_i=0} \Pi_{i=1, i \neq k}^N P(y_i \mid H_1)^{u_i=1}}{P(y_0 \mid H_0)^{u_0=0} \Pi_{i=1, i \neq k}^N P(y_i \mid H_0)^{u_i=0} \Pi_{i=1, i \neq k}^N P(y_i \mid H_1)^{u_i=1}}
\]

(11)

\[
\sum_{i=1}^{N} P(u_i \mid H_1)^{u_i=1} \leq \frac{C_F}{C_D}
\]

(12)

where

\[
y^k = (u_1, \ldots, u_{k-1}, u_{k+1}, \ldots, u_N)^T
\]

\[
A(u^k) = P(u_0 = 1 \mid u^k) - P(u_0 = 1 \mid u^0)
\]

and

\[
y^{ij} = (u_1, \ldots, u_{k-1}, u_j = 0, u_{k+1}, \ldots, u_N)^T
\]

A simultaneous solution of the above \((N + 2^N)\) coupled nonlinear equations yields the PBPO solution. As indicated in Section II, determination of decision rules for distributed detection networks is quite complicated. Several computational algorithms based on a variety of approaches such as Gauss–Seidel cyclic coordinate descent and gradient based techniques have been proposed in the literature [23]–[25].

Consider a special case in which the observations at the sensors are identically distributed. In this situation, it would appear that the decision rules at the sensors should be identical. But examples have been found in which nonidentical decision rules are optimal [26]–[28]. It has, however, been shown that the solution with an identical decision rule constraint is asymptotically optimal and that the identical decision rule assumption often results in little or no loss of optimality [5], [29]. Therefore, identical local decision rules are frequently assumed in many situations. Also, with identical local decision rules and identical distribution at all the sensors, the optimal fusion rule reduces to a \( K \)-out-of-\( N \) form [1, Sec. 3.4], i.e., the global decision \( u_0 = 1 \) if \( K \) or more sensor decisions are one. This structure of the fusion rule reduces the computational complexity considerably.

4) Entropy-Based Cost Functions: Thus far, the cost formulation involved assignment of a fixed cost to each possible course of action. In signal detection applications where these costs are not available or where we are interested in the amount of information we are able to transfer, entropy-based cost functions have been found to be quite useful [30]. Design of the system shown in Fig. 1 based on the following logarithmic cost function has been considered in [1, Sec. 7.2], [31]

\[
C_{ij} = \log \left\{ \frac{P(u_i, H_j)}{P(u_0)P(H_j)} \right\}, \quad i, j = 0, 1
\]

(13)
where \( u_0 \) represents \( u = i \). Based on the PBPO methodology, the fusion rule and the sensor decision rules that maximize the mutual information \( I(H; u_0) \) can be obtained. It is shown in [1, Sec. 7.2], [31] that the fusion rule in terms of a specific incoming decision vector \( u^* \) is

\[
\frac{P(u^* = u | H_1)}{P(u^* = u | H_0)} = \frac{P_0(C_00 - C_0y)}{P_1(C_11 - C_1y)},
\]

Sensor decision rules are LRT’s in which the thresholds are given by

\[
t_k = \frac{P_0(C_{10} - C_{00}, \theta_{p_k})}{P_1(C_{01} - C_{11}, \theta_{p_k})}, \quad k = 1, \ldots, N.
\]

The PBPO solution is obtained by solving (14) and (15) simultaneously.

**B. Serial Configuration**

In this section we consider the Bayesian hypothesis testing problem for a serial network (Fig. 3). For simplicity, we limit our attention to a two-detector serial network here. More general results are available in [1, Sec. 4.2]. In this network, the first detector makes the decision \( u_1 = i \) based on \( y_1 \). The second detector makes the final decision \( u_2 \) based on \( u_1 \) and \( y_2 \). Let \( C_{jk} \) denote the cost of deciding \( u_2 = j \) when \( H_k \) is present. The goal is to derive decision rules at both detectors so that the average cost of making decision \( u_2 \) is minimized. The Bayes risk in this case can be expressed as (see (16) at the bottom of the page).

System optimization is carried out based on the PBPO methodology. As shown in [1, Sec. 4.2], [8], the decision rules at both detectors are LRT’s. A single threshold is used at the first detector whereas two thresholds depending upon the decision of the first detector are used at the second detector. \( t_2^* \) denotes the threshold at the second detector when \( u_1 = i \). The three thresholds are given by

\[
t_1 = C_F \frac{P_{F2}(\theta_2^*) - P_{F2}(\theta_2^*)}{C_D \frac{P_{D2}(\theta_2^*)}{\frac{P_{D2}(\theta_2^*)}{P_{D1}}} - P_{D2}(\theta_2^*)},
\]

\[
t_2 = C_F \frac{P_{F1}}{C_D \frac{P_{D1}}{\frac{P_{D1}}{P_{D1}}}}
\]

\[
t^*_2 = C_F \frac{1 - P_{F1}}{C_D \frac{1}{\frac{1}{P_{D1}}}}
\]

where \( P_{F2}(\theta_2^*) \) and \( P_{D2}(\theta_2^*) \) represent the values of probabilities of false alarm and detection at detector two based on threshold value \( \theta_2^*, i = 0, 1 \). The above coupled equations need to be solved to determine the three thresholds. For an \( N \)-detector serial network, \((2N - 1)\) coupled equations need to be solved to determine the thresholds. For large \( N \), a more convenient approach is to represent these equations recursively [1, Sec. 4.2].

An important question is whether the parallel topology or the serial topology performs better under Bayes criterion. Proposition One given earlier is independent of the criterion and is hence valid for the Bayes criterion as well. For \( N \)-detector networks \((N > 2)\), no definitive statements can be made. It has been shown that the parallel network is better than the serial network in the asymptotic sense [5], [32]–[34]. The value of \( N \) at which the parallel network becomes superior is not known.

Another interesting issue is the ordering of nonidentical detectors in serial networks. It might be tempting to put the better detectors toward the end but there exist examples that show that placing the better detectors toward the end need not always be optimal [32]. Ordering depends on many factors such as prior probabilities, costs, etc. No general results on this issue are available.

**C. More General Network Topologies**

In Sections III-A–B, we have discussed the Bayesian formulation for two basic distributed detection network topologies. The tree network can be handled in a similar manner and decision rules can be derived at all the detectors [1, ch. 4], [35], [36]. In all the configurations thus far, information flowed in only one direction namely toward the fusion center. No provision was made for extensive communication/consultation among sensors or for feedback information flowing from the fusion center toward the sensors. Introduction of this additional communication capability improves system performance. A number of such network configurations have been considered in the literature. For example, in a parallel network with feedback, observations arrive sequentially at the sensors over the observation interval. After each received observation, sensors make tentative decisions and transmit them to the fusion center. The fusion center combines these and transmits the tentative global decision back to the sensors. Sensors use this feedback information to adapt their decision rules. Design and analysis of this system is available in [1, Sec. 4.4], [37]. Another interesting notion is that of parleying where a number of sensors reach a decision collectively [38]. Each sensor transmits its tentative decision to all other sensors. Based on its original observation and the most recent set of tentative decisions, each sensor “rethinks” and makes another tentative decision. This process continues until all sensors reach a consensus. Performance of this system is characterized by the time to reach a consensus and correctness of the result. In [39], a generalized Bayesian formulation for the design of arbitrary detection network configurations and communication structures is presented. PBPO methodology is employed to determine the decision rule at any detector of the network [1, Sec. 4.5], [39].

\[
R = \sum_{i,j,k} \int_{y_1,y_2} P_h C_{jk} P(u_2 | u_1, y_2) p(y_1, y_2 | H_k) p(y_2 | H_k).
\]
IV. DISCUSSION AND CONCLUSION

We have discussed some basic issues related to detection of signals with a collection of distributed sensors. Two main sensor topologies, parallel and serial, and two main optimality criteria, NP and Bayesian risk, were considered in some detail. A fundamental result is that for conditionally statistically independent observations at the sensors, the optimal tests at the sensors and at the fusion center, if one exists, under either of the two criteria, are likelihood ratio threshold tests. Although the optimality of the LRT’s is established, finding the actual tests involves the determination of thresholds, through a set of coupled integral equations, a task which is computationally complex.

The literature on distributed detection is quite rich and continues to grow. For a more extensive set of references, see [1], [6], [40]. We briefly describe a few other basic results from the literature. All these results assume conditionally independent observations. A somewhat surprising result is the possibility that an optimal solution to a distributed detection problem may exhibit nonsymmetry with respect to sensors even though the observations at the sensors may be identical [2], [5], [29]. However, for a large number of sensors, under some mild restrictions, the optimality of identical sensor tests was established in [29]. Also, several results on asymptotic performance (with respect to a large number of sensors) reveal that the performance crucially depends on the nature of the individual sensor ROC. For example, the asymptotic error rate at the fusion center depends on the slope of the sensor ROC at the origin and/or at $P_{fi} = 1$ [33], [41]. The suboptimality of the OR and the AND rules, for a large number of sensors, is also shown in [41]. Performance of distributed detection systems in terms of (finite) number of sensors has been investigated in [42] where results are presented on the number of sensors required to attain a certain performance. In [40], the author considers a model that allows for recursive processing if the fusion center is unable to decide on a hypothesis at a given instant. He also derives recursive probability expressions that show the progress of these probabilities as a function of the number of recursion.

In [43] the authors describe what they call the fusion of detection probabilities. In their model each sensor, based on its observation, generates a probability that declares its confidence on the signal present hypothesis. These probabilities are then sent to a fusion center where a decision regarding the signal hypothesis is made. Even though this idea may look different, it is essentially equivalent to the sensor sending multilevel information. The statistical literature addresses a similar topic known as the combining of level of significance [44].

Since the optimal solution to the distributed detection problem is computationally complex (see [5], [15], [45] for more details on complexity issues), especially with multilevel quantization, a suboptimal procedure is to maximize a form of distance measure such as $J$-divergence. Some numerical results for a specific problem based on distance optimization are presented in [46]. Design of multilevel quantizers for distributed hypothesis testing has been further considered in [47]. An iterative procedure to cyclically improve the system performance metric based on distance measures such as the Bhattacharyya distance has been proposed. Many examples have been presented to illustrate the procedure. Several additional interesting results on the design of quantizers at local sensors are presented in [48]. Under a Bayesian formulation, [49] shows how the risk at the fusion center, corresponding to a minimum average cost receiver, can be expressed directly in terms of the detection and the false alarm probabilities of the decisions of the sensors. For identical sensors, it also brings out the explicit dependence of the risk on the sensor threshold.

Distributed detection in uncertain environments has received some attention in the literature. Applications of robust methods and nonparametric techniques to this problem are described in Part II of this paper [6]. In [50], [51], an approach based on Dempster–Shafer theory is presented. They deal with the situation when each decision maker has an unknown probability of being jammed or defective and an unknown probability to provide an incorrect decision when jammed or defective. The uncertainty is handled by discounting the degree of confidence in decisions and Dempster’s combining rule is employed for information aggregation. An alternate approach to treat uncertainty is by means of fuzzy information systems. Design of the fusion rule and decision rules at the sensors based on this approach are presented in [52] and [53]. In [52], a decision fusion scheme is derived where the uncertainty associated with local sensor error probabilities is modeled by means of fuzzy sets. A crisp decision is reached by defuzzifying the Bayesian risk based on a criterion for mapping fuzzy sets on the real line. In [53], measurement inaccuracies at sensors are modeled as fuzzy events. Bayesian decision criterion is employed in the design of decision rules at the sensors and at the fusion center.

The problem of optimization of distributed decision making structures with applications to the design of organizations has been investigated extensively [54]–[57]. A normative model to study the interactions between task structures and organizational design on the performance of hierarchical organizations has been formulated. This model has applications in a wide variety of areas such as social sciences, medical diagnosis, and military command and control [58].

Certain open problems remain to be investigated. One is the question of accuracy (performance) achievable in distributed detection. Given a complete knowledge of the probability distributions of the observations, it is possible to numerically evaluate the performance of a given distributed detection rule and compare its performance with that of a central detection rule. However, it would be nice to have a general type of accuracy bound, similar to the Cramer–Rao bound in parameter estimation, that describes the accuracy achievable by a class of distributed detection rules. Such results do exist for the asymptotic
(large number of sensors) case, but for finite sample case, evaluation of an accuracy bound remains an open problem.

Another issue is the application of distributed detection results to other areas. One such area is diversity combining in communication receivers in which independent pieces of information are available, but usually at a single site. If a complete statistical description of these samples (information) is available, then an optimal combining of these samples (the optimal central rule, in distributed terminology) can be carried out. An example of this is the maximal ratio combining [59]. When a complete statistical characterization is not possible, e.g., due to changes in link conditions, or if a simpler decision rule is desirable, then a hard limited decision of each sample can be obtained and the decisions can then be combined using a K-out-of-N rule [60]. Unlike in decentralized detection problems, where the sensors are geographically dispersed, the multiple antennas used in space diversity receivers are usually co-located. Therefore, making decisions using only the signals at each antenna and then combining these decisions using an appropriate rule is then dictated by the need for robust rules that perform reasonably well over a wide range of channel conditions. Yet another application area is the object recognition problem in computer vision, using multiple cues [61].

Many fundamental results on distributed detection have been obtained and applied to several diverse areas. Much research remains to be performed on the theory and applications of decentralized detection. Several recent results on the theoretical aspects of distributed detection are described in Part II of this paper [6].

References


Ramanayanan Viswanathan received the B.E. degree (honors) in electronic and communication engineering from the University of Madras, India, the M.E. degree (with distinction) in electrical communication engineering from the Indian Institute of Science, Bangalore, and the Ph.D. degree in electrical engineering from Southern Methodist University, Dallas, TX, in 1975, 1977, and 1983, respectively.

From 1977 to 1980 he was a deputy engineer at Bharat Electronic, Ltd., Bangalore, India. Since 1983 he has been with Southern Illinois University, Carbondale, where he is a Professor of Electrical Engineering. His research interests include detection and estimation, communication theory, and wireless communications. He co-authored a textbook, Introduction to Statistical Signal Processing with Applications (Prentice-Hall, 1996). Dr. Viswanathan is a registered Professional Engineer in the State of Illinois.

Pramod K. Varshney, for a photograph and biography, see this issue, p. 5.