THE SCIENTIFIC ACHIEVEMENTS OF BLAISE PASCAL.

BY T. J. McCORMACK.

M. LÉVY-BRUHL, whose article in the present *Open Court* on the philosophy of Pascal will have won the attention of our readers for one of the most remarkable geniuses of modern times, well remarks that by a strange irony of fate the very weapons which Pascal forged for the corroboration of his faith were by his successors turned into one of the deadliest instruments for its destruction. Science pursues its irresistible march through and in spite of the personalities in which it is incarnate, and amid all their aberrations, works out unfailingly its own salvation. Where it is impeded it sweeps to ruin, with puissant hand, the obstacles that are set in its path, and by virtue of that immanent and mighty power in the world which makes for truth, accomplishes eternally its own aims. "There are two souls," sayeth the poet, "inhabitant in every breast"; the one destined to immortality, the other to decay; the one clad in the eternal raiment of truth, the other in the muddy vestures of clay. And so it was with Blaise Pascal. Even that which was perishable in him had more, perhaps, of immortality in it than have the perishable parts of most men; but as compared with the standard of creation which he himself had set, it was foreordained to annihilation; and the World-Spirit, mocking and girding, as it were, at his worser half, and thinking forsooth that it was merry sport—

> "To have the engineer
> Hoist with his own petard,"

made his better part the saviour of the whole, and has handed him down to us, as an admirer has characterised him, "one of the sublimest spirits of the world."

Apart from Pascal's merits as a shaper of French literary style, and as an ethical essayist of the first order, it is his achievements in mathematics and physics, and not his pietistic and theological maunderings, however much they have been lauded, that constitute his real contribution to civilisation. We shall supplement, therefore, the fine exposition which Professor Lévy-Bruhl has given of his philosophy, by a popular reference to his work in science; but shall first give from another pen a brief sketch of his life.

"Blaise Pascal was born at Clermont in Auvergne, France, June 9, 1623. He was the only son of Etienne Pascal, President of the Court of Aids in that province, himself a learned and respectable man and able mathematician, who, when his boy had reached his eighth year, resigned his office and removed to Paris, for the purpose of watching over his education. From his childhood, Blaise displayed abilities far above the common order, and evinced so inquiring a spirit that, as his sister has recorded, he would not rest without knowing the reason of everything. The bent of his infantile genius was decidedly mathematical; but his father who was his only preceptor, and who was anxious that his attention should not be distracted from the study of the dead languages, resolved to exclude every notion of geometry from his mind, removed all books which treated of that science, and even abstained in the child's presence from any conversation on mathematical subjects with his friends. Notwithstanding these precautions, however, young Blaise, when only in his twelfth year, without the aid of books or oral instruction, began to draw figures with charcoal on the floor of his room, and had, without any assistance, made some progress in geometry before his father surprised him in these researches.

"After this discovery, he was thwarted no more in the pursuit of mathematical investigations; and at sixteen years of age he produced a treatise on the conic sections, of such excellence as to provoke the incredulity and wonder of Descartes, who would not believe that so extraordinary a performance was the work of a mere youth. In his nineteenth year he invented an ingenious machine for making arithmetical calculations, which excited the admiration of his times; and, afterwards, at the age of twenty-four years, the conjecture of Torricelli that the atmosphere had weight, and that this quality might account for effects before ascribed to the horror of a vacuum, led him to institute many able and successful experiments on this subject, which confirmed the truth of Torricelli's idea, and established his own scientific reputation. The results of these labors were collected into two essays, which appeared after his death, *On the Equilibrium of Liquids*, and *On the Weight of the Atmosphere*.

"From these researches, made before he had completed his twenty-fifth year, the great mind of Pascal was diverted entirely to objects of religious contemplation; and thenceforward he abandoned almost entirely the pursuits of science. He practised the most rigid abstinence from all worldly enjoyments, and wore next to his skin a cincture of iron studded with points, which he struck with his elbow into his flesh, as a punishment to himself whenever any sinful thought obtruded itself into his mind.

"It is a curious exemplification of the anomalous conditions of the human mind, that, while Pascal was immersed in these superstitious observances, he published his famous *Provincial Letters*, in which, under the name of Louis de Montalto, he assailed the morality of the Jesuits with equal wit and argumentative acumen. He was induced to write this work by his adoption of the opinions of the Jansenists, which he warmly espoused, and which involved him in the religious disputes of his age and country. Among the fruits of his devotional exercises may also be named his *Pensées*, which were collected and published after his death; and in which he has beautifully availed himself of an idea of one of the ancient fathers, that he who believes in the existence of a God gains eternally if he be right, and loses nothing if wrong; while the atheist gains nothing if right, and renders himself miserable eternally if he be wrong. The weakly frame of Pascal was reduced to premature old age by infirmities, which were aggravated by his ascetic habits, but which he bore with exemplary patience; and he died in Paris, August 19, 1662, aged thirty-nine years. His life was written elaborately by his sister, Madame Perier." (*Quoted from the English Cyclopædia.*)

PASCAL'S ARITHMETICAL MACHINE.

Natural arithmetical machines have been in use among savage and civilised nations from the earliest time. Their employment, however, from our present advanced point of view, denotes rather an inferior than a superior stage of intellectual development. The fingers, strings of beads, knots in cords, notches in sticks, etc., etc., were the means primitively employed in computation; counting was a motor act, an act of sense, and not one of the intellect; the results were the actual things added or subtracted, and not symbols representing those results. The original intellectual advance, therefore, consisted rather in the abolition of this primitive machinery and in the substitution for it of a procedure which was mainly psychical and mnemonic, involving a mechanical knowledge of the simple combinations of numbers, of the multiplication table, and of the use of pencil and paper. But with the transference of computation to the psychical domain, came the new danger of the multiplication of errors; the more delicate and sensitive a mechanism is, the more apt it is to become deranged.

The abacus of the Romans and the swanpan of the Chinese were advanced types of the natural counting machine; but their manner of recording results, though an improvement on the old, and slightly symbolical, was almost identical with the operation to be performed, and involved little saving of labor. M. Huc, in his *Travels in Tartary, Thibet, and China*, has aptly illustrated this point by his recital of the astonishment of the Chinese money-changers at the rapidity and accuracy with which he and his companion performed their calculations,—a rapidity and accuracy that were far in advance of anything that the users of the swanpan could offer. The abacus, which in its various forms continued to be used in the South of Europe until the end of the sixteenth century, and in

England until even a later period, met, therefore, the natural fate of imperfection.

Nevertheless, after the abolition of the natural machines, and after facility in calculating by the new mental machines had been thoroughly established, the want of an absolutely certain, mechanical means of saving mental labor, and of eliminating psychical error, was widely felt by philosophers, and the first weak effort in this direction, which was after all a reaction, was a little instrument known as *Napier's Bones*.

One original source of error had been that the carrying of units to tens and of tens to hundreds, etc., was performed by the computer himself; the operation was not mechanical and was consequently liable to error. Napier's Bones merely dispensed with the use of the multiplication table, and reduced multiplication to simple addition; but the carrying was performed by the operator. Pascal, therefore, who devised a contrivance in which the operation of carrying was performed mechanically, may be regarded as the inventor of the first calculating machine. He had completed its construction in the year 1642, at the age of nineteen, and in a letter to the chancellor, Pierre Séguier, pointed out the advantages which were to be derived from its use. The instrument was the product of a long period of meditation and of experimenting, more than fifty models having been constructed before tolerable results were obtained. But Pascal's theory was far in advance of the technical art of his day, and his ideas never found full realisation; the chief cause of failure on the part of the mechanicians being their inability to overcome the friction of the parts. Copies of his machine are in possession of the *Conservatoire des Arts et Métiers*. The boldness of his views, however, were appreciated, and his plan formed the groundwork upon which a long line of mechanical philosophers from Leibnitz and D'Alembert to Babbage, Roth, Scheutz, and a host of others, have reared an astonishing superstructure.

Two of the most elaborate types of the modern calculating engine were made by the Swedish inventor, Scheutz; one of them which was exhibited in Paris in 1855, and was afterward bought for the observatory at Albany, is now in the observatory of the Northwestern University at Evanston; the second of these machines, afterward constructed for the English government, is now used for facilitating the calculation of the mathematical tables of the Nautical Almanac. This machine, which is of the size of a small pianoforte, not only calculates mathematical tables, but actually stereotypes the results in a form ready for printing. It *cal-*

culates and *stereotypes* two and one-half pages of figures in the same interval of time that it would take a good compositor to set a single page of the same figures.

THE MYSTIC HEXAGRAM.

The precocity of Pascal's genius was most distinctly marked in his geometrical researches. The story that he rediscovered the principal propositions of Euclid while a mere boy may be taken *cum grano salis*, but it is certain that the famous *Essay on Conics*, which was brought into public notice by Leibnitz after Pascal's death, was written in his sixteenth year. Unfortunately, a part only of the original essay was published, and the world knows the remaining contents only from Leibnitz's report. In this essay ap-

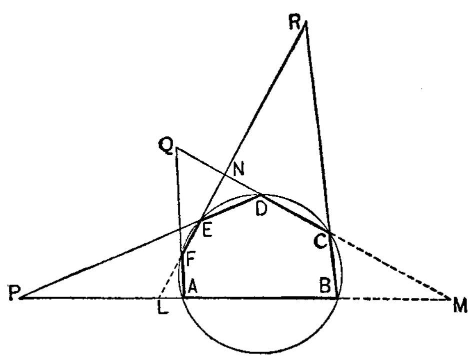

Fig. 1.
(From Beman and Smith's *Geometry*.)

pears the famous result known as Pascal's Theorem, viz., "that if a hexagon be inscribed in a conic, the points of intersection of the opposite sides will lie in a straight line." A special case of this theorem, and the one which is most popularly known, goes by the name of The Mystic Hexagram; it is that of a hexagon described in a circle. In the annexed cut (Fig. 1), if the inscribed hexagon $ABCDEF$ be such that BA and DE meet at P, CD and AF at Q, BC and FE at R, then the points P, Q, R, are collinear; that is, lie in one straight line.

The wealth of content of Pascal's treatise was no less great than the methods which he employed were ingenious. Mersenne, who must be supposed to have seen the original work, remarks that from *a single theorem* Pascal deduced four hundred corollaries, which included all the results of Apollonius.

The last geometrical, and in fact the last mathematical, work of Pascal was that on the Cycloid, or the curve traced out by a point on a moving waggon wheel. It was produced during his period of religious seclusion, and owes its existence to eight nights of sleeplessness induced by a terrific toothache; it gives a tolerably full account of the geometry of the Cycloid,—reached by methods which, according to D'Alembert, form a connecting link between the geometry of Archimedes and the infinitesimal calculus of Newton and Leibnitz,—and suggests the thought that it would have been better for the world if Pascal had suffered from the same stimulating malady during the remainder of his natural life.

It remains to be noted that Pascal also left a fragment of a work which is the first modern attempt toward a philosophy of mathematics.

THE ARITHMETICAL TRIANGLE.

Pascal's researches in arithmetic and in the theory of numbers are most beautifully illustrated by his arithmetical triangle, which forms a conspicuous example of the close, logical interconnexion of the laws of mathematics generally. Pascal's arithmetical triangle was a magical key by which he unlocked the secrets of many problems; it is represented in the annexed cut, the construction of which is as follows (Fig. 2):[1]

The Arabic numeral 1 is first repeated in a given number of squares, say ten, in a horizontal line; the second horizontal line contains one square less, namely nine, the next still one square less, namely eight, and so on. The numbers which fall in the successive squares of each row below that of the first are the sums of the numbers in all the squares which lie over and to the left of that number in the horizontal line above; thus, in the fourth row the number 20 is equal to the sum of $10+6+3+1$, which are the numbers above and to the left of it in the horizontal row just preceding.

Let us see what are some of the consequences of this simple construction. The horizontal rows form what are called *figurate numbers;* the numbers in the first line are of the first order, that is, mere natural number-signs; the numbers in the second line are numbers of the second order, or the natural numbers proper, and represent the results of the *summation* of the numbers of the first order; the numbers in the third line are of the third order, or

[1] A somewhat similar table of numbers, but without the applications afterwards discussed, appeared in Stifel's *Arithmetica Integra* (1544); the fact has, however, no bearing on Pascal's researches.

triangular numbers, because if represented by points they can be disposed in the shape of triangles; the numbers in the fourth line are of the fourth order, etc. Again, the triangle possesses as many vertical columns as horizontal columns, and any column is the same as the correspondingly numbered row; thus running upward from 20 in the fourth column, we find the same numbers that we find in running to the left of 20 in the fourth row. Taking the diagonal rows we find that their sums are in geometrical progres-

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	
1	3	6	10	15	21	28	36		
1	4	10	20	35	56	84			
1	5	15	35	70	126				
1	6	21	56	126					
1	7	28	84						
1	8	36							
1	9								
1									

Fig. 2.
(From Cantor's *Geschichte der Mathematik.*)

sion, each being twice the preceding. These rows represent the successive powers of 2.

The arithmetical triangle was made to solve problems in the theory of permutations and combinations. For this purpose, the diagonal rows are used; these diagonal rows have remarkable properties.

We take the seventh, for instance; the figures 1, 6, 15, 20, 15, 6, 1, each show in how many ways six objects can be transposed,—first, when they are all alike; second, when five are alike; third, when there are two groups, one of four alike and one of two alike;

fourth, when there are two groups, each of three alike, etc. Any row, say the fifth, gives the coefficients of the expansion of the binomial $(a+b)$ to that power which is one less than the number of the given row—in the present case one less than 5, or 4, thus:

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$

The coefficients of the expansion of $(a+b)^8$ are given in the ninth diagonal row.

The arithmetical triangle was also employed by Pascal for the solution of questions in the theory of probabilities, of which he and Fermat are the joint founders. The original problem was as follows:

"Two players of equal skill want to leave the table before finishing their game; their scores and the number of points which constitute the game being given, it is desired to find in what proportion they should divide the stakes."

Pascal asks first, How many games must each player win in order to gain the entire stakes? He then adds the two numbers so obtained, and seeks the corresponding base, or diagonal row, in his arithmetical triangle, in which column he adds together the numbers of as many squares as correspond in each case to the number of games to be won, and so obtains the sums representing the inverse ratio of equitable division; thus, if the first player lacks two plays to win, and the second four, the base is six; the sum of the first four numbers in this diagonal row is

$$1+5+10+10=26,$$

and the sum of the first two numbers in the same row is $1+5=6$; wherefore, the ratio required is 26 : 6.

We see in these results the unerring sense which Pascal possessed for detecting the *determinative elements* of a given mass of mathematical experience. The arithmetical triangle of Pascal is a rough, unhewn quarry-block, in which the polished mathematical statuary of Newton and the Bernoullis was potentially contained. The mere result of additions, it yet involves the consequences of complicated formulæ, and so offers many a fruitful philosophical lesson. Its study might be profitably pursued in elementary arithmetical instruction and made the basis of much inductive work. As a recreation, the study of its properties is to be preferred to the study of magic squares.

THE PHENOMENA OF LIQUID PRESSURE.

The achievements of Pascal in physics most popularly connected with his name relate to the phenomena of liquid and atmospheric pressure. The problems of hydrostatics, or the equilibrium of liquids, their transmission of pressure equally in all directions, early engaged the minds of natural philosophers, and the researches of Archimedes, Stevinus, and Galileo, virtually disposed of the question. Pascal's share in the work consisted principally in his application to the problem, of the principle of virtual displacements—the principle that the work performed by a small weight acting through a long vertical distance is equal to that of a large weight acting through a short vertical distance, if the products are equal. We shall mention first his ingenious experiments illustrating the increase of pressure with the depth of a heavy liquid.

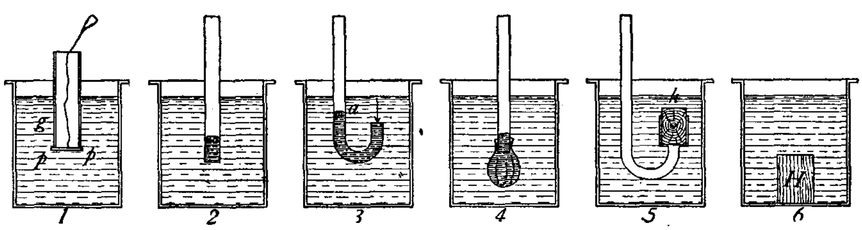

Fig. 3.

"In Fig 3, cut 1, is an empty glass tube *g* ground off at the bottom and closed by a metal disc *pp*, to which a string is attached. The whole is plunged into a vessel of water. When immersed to a sufficient depth we may let the string go, without the metal disc, which is supported by the pressure of the liquid, falling. In 2, the metal disc is replaced by a tiny column of mercury. If (3) we dip an open siphon tube filled with mercury into the water, we shall see the mercury, in consequence of the pressure at *a*, rise into the longer arm. In 4, we see a tube, at the lower extremity of which a leather bag filled with mercury is tied: continued immersion forces the mercury higher and higher into the tube. In 5, a piece of wood *h* is driven by the pressure of the water into the small arm of an empty siphon tube. In 6, a piece of wood *H* immersed in mercury adheres to the bottom of the vessel, and is pressed firmly against it for as long a time as the mercury is kept from working its way underneath it."

An ingenious illustration of a familiar mechanical principle is Pascal's Paradox. "A vessel *g* (Fig. 4), fixed to a separate support and consisting of a narrow upper and a very broad lower cylinder, is closed at the bottom by a movable piston, which, by means of a string passing through the axis of the cylinders, is independently suspended from the extremity of one arm of a balance. If *g* be filled with water, then, despite the smallness of the quantity of water used, there will have to be placed on the other scale-pan, to balance it, several weights of considerable size. *But if the liquid be frozen* and the mass loosened from the walls of the vessel, a very small weight will be sufficient to preserve equilibrium." The solution is that the *tiny* quantity of liquid lifted in any small displacement is forced *through the whole height of the narrow*

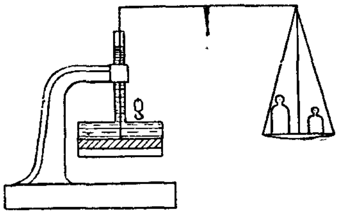

Fig. 4.

neck, while the *heavy* weights in the scale-pan move through only a *small* vertical distance.

THE PHENOMENA OF ATMOSPHERIC PRESSURE.

The action of pumps in raising water, the phenomena of suction generally, were attributed by the ancient philosophers to nature's abhorrence of a vacuum—to the *horror vacui;* and to this repugnance on the part of nature to emptiness in her domain there was supposed to be no limit. Imagine Galileo's surprise, therefore, on hearing of a newly-constructed pump, accidentally supplied with a very long suction-pipe, which was unable to raise water to a height of more than thirty-two feet. Galileo's immediate thought was that the *horror vacui* possessed *a measurable power.* His pupil, Torricelli, thereupon conceived the idea of measuring the resistance to a vacuum by a column of mercury. If the pressure which forced liquids into a vacuum was *definite*, then the mercury, which was fourteen times as heavy as water, would be raised to a column-height of only one-fourteenth of that of the water-column.

The prediction was confirmed by the experiment of Viviani.

A glass tube sealed at one end, filled with mercury, and stopped at the open end with the finger, was inverted in a dish of mercury so as to stand vertically, and the finger removed; the mercury stood stationary at a height of about twenty-eight inches.

Torricelli knew from Galileo's experiment that the air had weight, and he jumped to the conclusion that the column of mercury was balanced by the columnal mass of atmosphere superincumbent upon the free surface of the mercury in the dish, just as a weight of great specific gravity on one pan of a scales may be balanced by a bulky body of light specific gravity on the other.

Pascal, hearing a rumor of the experiment, reflected on it independently, and repeated it with the most beautiful variations—with wine, with oil, with inclined columns, with siphons, etc. But his chief merit was his establishing of the complete analogy between liquid and atmospheric pressure. He conceived the earth

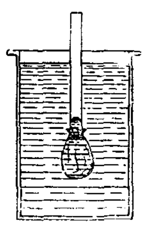

Fig. 5.

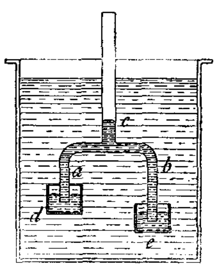

Fig. 6.

to be surrounded by an ocean of air, as the land is encompassed by an ocean of water. Every phenomenon which the new theory attributed to atmospheric pressure, he showed to have its analogy in liquid pressure. Into a deep vessel of water, a glass tube open at the top end to the air and having at its lower end a bag of mercury, is sunk (Fig. 5); as the tube descends the mercury rises by the pressure; as the tube rises the mercury falls. Why will not the same phenomenon happen in the ocean of air, if the distances of ascent be taken great enough to allow for the differences of density? Pascal requested his brother-in-law to perform the Torricellian experiment on the summit of the Puy de Dôme, a mountain in Auvergne. The mercury sank; the uses of the modern barometer were established.

"The invention of the barometer," says an eloquent writer, "is one of the most curious in the history of philosophy. No new discoveries, not even those first substantiated by the use of the telescope, ever knocked so hard at the door of a received system, or in a manner which so imperiously demanded admission, as this one." And to Pascal, more than to any other, was due the merit of having overcome the prejudices of his contemporaries.

To clinch the new theory, Pascal mimicked the flow produced in a siphon by atmospheric pressure, by the use of water. "The two open unequal arms *a* and *b* of a three-armed tube *a b c* (Fig. 6) are dipped into the vessels of mercury *c* and *d*. If the whole arrangement then be immersed in a deep vessel of water, yet so that the long *open* branch shall always project above the upper surface, the mercury will gradually rise in the branches *a* and *b*, the columns finally unite, and a stream begin to flow from the vessel *d* to the vessel *e* through the siphon-tube open above to the air."[1]

* * *

Such, in the main, were the scientific achievements of Blaise Pascal. His wit, the graces of his style, his theology and philosophy, have entered literature, and may be found in its dusty tomes, in all their pristine brilliancy, by those who have the desire to seek them. But his scientific achievements have entered life, the life of all nations, and remain there,—silent and unfelt, but none the less puissant in their eternal presence.

[1] Figures 3, 4, 5, 6 and the quotations accompanying them are from Mach's *Mechanics*.