THE STATICS OF LEONARDO DA VINCI*

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Introductory Note on the Manuscripts

Leonardo da Vinci lived in the fifteenth and early sixteenth centuries (1452-1519). At the age of fourteen he was apprenticed to Andrea Verrocchio, a famous artist of those days. Verrocchio's tastes, and as a consequence his circle of acquaintances, were wide, and from all these Leonardo imbibed and developed a passion for scientific inquiry side by side with his development as an artist. At Florence, in his early days, he came under the influence of such men of science as Benedetto del'Abbaco, Giovanni Agiropulo, L. P. Alberti and Toscanelli.

In 1483 Leonardo migrated to Milan, where he took service under Ludovico Sforza in the capacity of consulting engineer, architect and sculptor, and he was busily employed in all these capacities. His chief scientific friendship during this period was with Fra Luca Pacioli, the famous mathematician.

Leonardo's stay in Milan ended in 1499 with the collapse of the power of Ludovico Sforza, and for some years we find da Vinci back again in Florence. In 1506, however, he accepted an invitation from Louis XII of France to return to Milan. He remained there till 1512, and later, in 1515, Francis I of France, Louis XII's successor, invited him to take up his residence in the Castle of St. Cloud, near Amboise. Here he spent the remainder of his days. He died on May 2, 1519.

The reading of Leonardo da Vinci's manuscript has been a task of enormous difficulty with which is honorably associated the names of a small band of enthusiastic students, chief among whom may be mentioned J. Paul-Richter, G. Piumati, and C. Ravaisson-Mollien.

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Leonardo, from the time he was twenty years of age onwards, invariably wrote in a manner apparently calculated to confound his would-be readers. We may summarize the characteristics of his script under four heads.

(1) He wrote from right to left after the fashion of the Semitic group of languages; (2) his handwriting was of the kind known as "mirrored," i. e., reversed in a manner such as would be produced by looking at a normal script through a mirror; (3) he employed an elaborate scheme of abbreviations, and (4) he omitted the use of punctuation. It is accordingly much to the credit of the patient workers to whom reference has been made above that in spite of these difficulties the writings of this great genius of the Italian Renaissance have been rendered available to the world of science and letters generally. The chief of Leonardo's manuscripts are collected together and housed as follows: (1) the Codex Atlanticus, a huge miscellaneous collection at Milan, in Italy; (2) a number of note-books, lettered consecutively A, B, C, D, etc., housed at the Institute of France, Paris; various volumes in England at the British Museum, the South Hensington Museum, at Molkha Hall, Leicester, and at Windsor Castle.

PART I

IT it proposed in this paper to consider the various contributions by Leonardo da Vinci in the field of statics, a study to which he referred as the "paradise of the mathematical sciences."[1] His notes on this subject are found scattered throughout his writings with a frequency and a persistence which show clearly the importance which he attached to it. They have at various times been considered by a number of students—notably by Grothe,[2] and Schuster[3] (and more superficially by Seailles)[4] in the realms of theoretical mechanics, and by Feldhaus[5] in the study of Leonardo's applications of mechanical principles to mechanisms. At the outset it may frankly be stated that Leonardo suffered badly from the want of a precise and an accurate scientific vocabulary. All the modern ideas of force, motion, mass, inertia, work, moment, etc., are constantly to be found amongst the note-books of our philosopher; but they are cloaked in a phraseology which is rarely precise, which is frequently puzzling, and which is seldom rigid. Yet condemnation for this would surely be most unjust. The notion of rigidity in scientific thought had no place in the fifteenth century. Looseness of expression is frequently the result of ignorance. A wealth of such looseness is not an unknown device as a cloak of assumed wisdom—a pseudo-learnedness, as it were. No one, however, can accuse Leonardo da Vinci of belonging to this class of writer. There is, however, the other side of the picture—the looseness of expression due to a sheer inadequacy of words to convey new ideas such as may occur to a writer who lives ahead of his times. It is to this side of the picture that Leonardo belongs. Many of his ideas, lost with the dispersal of his manuscripts, had to be rediscovered by others long after him. That these later philosophers had the benefit of a later and a more complete terminology with which to state their discoveries is a fact

[1] Ms. E., Fol. 8v.

[2] H. Grother, *Leonardo da Vinci als Ingenieur und Philosoph.*, Berlin.

[3] F. Schuster, *Zur Mechanik Leonardo da Vinci*, Erlangen, 1910.

[4] G. Seailles, *Leonardo de Vinci, l'Artiste et le Savant*, Paris, 1906.

[5] F. M. Feldhaus, *Leonardo da Vinci Als Techniker und Erfinder*, Jan., 1913.

that need neither detract from the credit due to Leonardo, nor blind us to the difficult path of inadequate language through which he had to grope his way. Yet one other factor requires mention in this connection. Clarity of thought in the exact sciences could not properly come into its own until the test of experiment had been recognized and practised. It is a commonplace in the history of science that little real headway was made until the advent of such pioneers of experimental science as Galileo in Italy and Gilbert in England. These preachers, by inaugurating a new era in scientific research, heralded the downfall of the dogmatists, and paved the way for such an advance in accurate scientific knowledge as to render absolutely imperative an expansion of technical vocabulary.

Leonardo, more than one hundred years before Galileo, was most definitely an experimentalist. If the great Tuscan philosopher was indeed the "Father of the Experimental Philosophy," then da Vinci was its grandfather. "Experience (i. e., experiment)," says he, "never deceives; it is only our judgment which deceives us, promising from it the things which are not in its power."[6] The appeal to experiment pervades all his writings on scientific topics. Small wonder, therefore, that his ideas and his discoveries outstripped his language. Living and thinking in terms of the late sixteenth and seventeenth century, he was yet compelled to express himself in the restricted language of the fifteenth. Vagueness of expression in the circumstances was inevitable. For these reasons, therefore, it behooves us to approach the notes on mechanics in no hypercritical spirit, but to make due allowances by assuming for ourselves rather the role of the "man in the street" to whom ideas of scientific rigidity are foreign and are replaced by what he would call "the common sense of it."

Introduction.—Da Vinci's Statics covers a very wide field. In a sense this is not surprising. Unlike dynamics, the way had been pointed by his predecessors. He had at hand the fruits of the labors of Aristotle, Archimedes, Jordanus Nemorarius and his anonymous successor, and of others; and he applied these materials in his own way with his usual vigor and independence. If the range of da Vinci's statics was wide, it was also within the possible limits of his days, thorough.

Centers of Gravity.—The fundamental contributions to Mechanics by Aristotle and the later Alexandrian school centered chiefly round the lever laws, or the laws of the balance, and the conception

[6] Codex Atlan., fol. 154r. See also fols. 3r and 119r.

of centers of gravity. Naturally, therefore, the traditional approach to the whole subject by all later writers was through these fundamentals, and in this respect Leonardo was no exception.

The law of the balance, imperfectly presented by Aristotle, received its real scientific development at the hands of the great Syracusan philosopher. Its demonstration was based on certain definitions and axioms,[7] amongst which is the statement that in every heavy body there is a definite point called *a center of gravity*, at which we may suppose the weight of the body collected.

A consideration, therefore, of Leonardo's pronouncements upon the subject of centers of gravity constitutes a suitable starting point for our study. This was a subject of peculiar importance and interest to da Vinci. It touched upon matters vital to his professional career. As an engineer he was concerned with the stability of the structures and the machines he was called upon to devise; as an artist he was interested in the balance of the human frame. References to the center of gravity are therefore plentiful.[8] Yet it is difficult to find any actual attempt at definition. This, however, need not be surprising. The conception of the center of gravity was one which had grown up literally through the ages. It had permeated all existing writings on mechanics. It was, so to speak, a commonplace of ancient and mediaeval science. Let us remember, further, that da Vinci has not left a complete treatise, but only a compendium of notes. Where he might have attempted a general definition in a text-book, one can understand its omission from a collection of notes. Nevertheless, that Leonardo regarded the conception of center of gravity as fundamental to mechanics is clear enough from his remark that, "Mechanical Science is very noble and useful beyond all others, for by its means all animated bodies which have movement perform their operations; which movement proceeds from their center of gravity. This is situated at the center, except with unequal (distribution of) weight." [9] It is clear, too, that he studied the subject experimentally. In Manuscript "B" is shown a sketch of a suspended weight with the note, "The center of all suspended weights is established under its support," [10] and in a similar sketch of a suspended artificial bird we read, "This is done to find

[7] These have been ably presented by J. M. Child in a recent paper, "Archimedes' Principle of the Balance and Some Criticisms Upon It," in C. Singer's *Studies in the History and Method of Science*, Vol. II, p. 490. Oxford, 1921.

[8] Eg. Codex Atlan., fol. 86ra; Ms. A, 5r; Ms. G, 78v; Ms. H, 105r; Ms. M, 37r.

[9] *Sul Volo degli Uccelli*, fol. 3r.

[10] Ms. B, fol. 18v.

the center of gravity of the bird."[11] A number of da Vinci's notes on this subject occur as incidental to his studies on the poise of the human figure. They bring out his appreciation of the need for a due distribution of weight about the "axis" (i. e., the vertical line through the center of gravity) under such varying circumstances as standing, sitting, kneeling, walking up and down hill, mounting stairs and ladders, and so on.

The following is quoted as a typical example: "A sitting man," we read, "cannot raise himself if that part of his body which is in front of his axis does not weigh more than that which is behind that axis without using his arms. A man who is mounting any slope finds that he must involuntary throw more weight forward—that is, in front of the axis and not behind it. Hence a man will always involuntarily throw the greater weight towards any point whither he desires to move than in any other direction. A man who runs down hill throws the axis onto his heels, and one who runs uphill throws it onto the points of his feet; and a man running on level ground throws it first on his heels and then on the points of his feet."[12] It is worthy of note that Leonardo was aware of the possibility of the center of gravity of a body being actually outside itself. Thus in the course of a long note in the *Codex Atlanticus*, attached to a sketch, we read, "Occasionally the center of gravity is to be found outside of the body, that is to say not within the weight of the matter, that is to say in the air."[13]

So far we may say that apart possibly from the experimental aspect we see here little that is really an advance on what had been done before. There is one aspect of the whole question of centers of gravity, however, for which claims of pioneer work may justly be made on behalf of Leonardo, namely, in the finding of centers of gravity of solid figures. Archimedes had made a thorough study of the centers of gravity of *plane* surfaces in his "*Treatise on the Equilibrium of Planes and of their Centers of Gravity*, deducing his results on Euclidean lines. Up to the time of da Vinci, however, none appears to have considered mathematically the problem of the center of gravity of the solids. It is of peculiar interest, therefore, to find in Manuscript F the following note accompanied by a sketch of two figures from which it is clear that Leonardo certainly considered the case of the tetrahedron: "The center of gravity of a pyramid is in the fourth of its axis, towards the base; and if you

[11] *Sul Volo degli Uccelli*, fol. 16 (15)v.
[12] Ms. A, fol. 28v.
[13] *Codex Atlan.*, fol. 153v.b.

divide the axis in four equal parts, and as you cut between two of the axis of this pyramid, one such intersection comes out at the above mentioned quarter."[14] Apart from this note on the two figures we have no further guide as to how da Vinci arrived at his result. The treatment appears to be modern enough, and it embodies a result which formal history had hitherto attributed to Commandin & Maurolycus in the middle of the Sixteenth Century.[15]

Leonardo's work on this subject was not undertaken in vain. Duhem has shown in a discussion on da Vinci's influence on his successors[16] that the sixteenth century philosopher, Jean Baptiste Villalpand (1552-1608) took up the study of centers of gravity, and clearly borrowed freely, though without acknowledgement, from Leonardo's writings. The importance of this lies in the fact that Villalpand was freely quoted in this, as in other subjects, by the well-known sixteenth century commentator in Mathematics, Father Mersenne in his widely read *Mechanicorum Libri*. Similar remarks apply equally to Bernardino Baldi.[17]

The Principle of the Lever

We turn next to the second of the two chief legacies of antiquity to the Statics of the Middle Ages, namely, the Principle of the Lever. We find this stated by Leonardo in its simplest form in manuscript "A". He speaks of the long arm as the lever, and the short arm as the counter lever, and his note reads: "The weight attached to the extremes of the lever made of any material whatever will lift up at the extremity of a counter lever a weight superior to itself by the same proportion as is the counter lever to the lever." Nothing could be simpler than this. The fact that the principle is more clumsily expressed, as for example in the *Codex Atlanticus*, need not seriously concern us. Schuster stresses the fact that in this collection Leonardo expresses the relationship mathematically in an unnecessarily complicated form,[18] but inasmuch as he also expressed it as above in its simplest form, all cause for doubt as to the clear mindedness of our philosopher is removed.

[14] Ms. F, fol. 51r.

[15] See Libri, *Histoire des Science Mathematique en Italie*, p. 40. Paris. See also Dahen, *Etudes Sur L. de V.*, Vol. I, pp. 35-36.

[16] Dahen, *Etudes*, etc., p. 80.

[17] Dahen, *Etudes*, etc., p. 101.

[18] Schuster, *Zur Mechanik Leonardo da Vincis*, Erlangen, 1915, p. 34.

He realizes completely the direct consequences of the principle. In Manuscript "A", to which we have already alluded, we find a sketch illustrating the practical utility of the lever with the remark, "10 lbs at the end of a lever will do the same as 20 lbs. at the mid point, and as 40 lbs. at the fourth part." Leonardo was not alone influenced by Archimedes in his treatment of the balance. Wöpcke quotes from an Arabic work of Greek origin[19] a series of four axioms embodying various conditions under which a varying loaded lever remains horizontal. Of these, the second, which gives in effect the lever principle in its simplest form, is to be found in the *Codex Atlanticus*,[20] and the fourth (which states that in a beam loaded with a number of weights to produce equilibrium, if one weight on one side of the fulcrum is moved inwards, and another on the same side is moved outwards by a suitable amount, the beam remains horizontal) occurs in both Manuscripts "A" and "E".[21] It

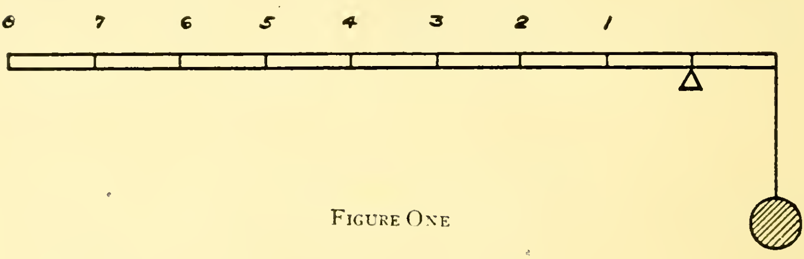

Figure One

is interesting to note that with all Leonardo's clarity of thought on this subject, he occasionally blundered over simple but not unimportant points. In a sketch[22] strongly suggestive of the steelyard,[23] (Fig. 1) we find a short arm at the extremity of which is a suspended weight, and on the other side of the fulcrum an arm eight times as long with divisions marked as shown. The lever is assumed to be heavy, and the problem Leonardo sets himself is to ascertain what weight he must suspend at the extremity of the short arm in order to counteract the effect of the heavier arm, given that each section of the balance weighs one pound. A simple calculation shows the result to be 31½ lbs. Leonardo makes it 35 lbs., and the fault of his argument lies in his unfortunate view that the

[19] Wöpcke, *Das Buch des Euklid uber die Wage*, Berlin, 1851. See also Duhem's *Etudes*, Vol. I. p. 65.

[20] *Codex Atlanticus*, fol. 154v.a. It also occurs in the *De ponderibus* of Jordanus Nemorarius.

[21] Ms. A, fol. 5r; Ms. E, fol. 143r.

[22] Ms. A, fol. 51v.

[23] It is worthy of note that the actual discussions of the Roman steelyard as such is unexpectedly absent from Leonardo's manuscripts.

weight of each section acts at the *outer end*, and not at the mid-point. On this basis, he says in effect that the weight of the short arm is cancelled out by Section 1 of the long arm, so that the weight must equal the sum of 2 lbs. (to balance the one pound of Section 2 at twice the distance) and 3 lbs. (to balance the 1 lb. of Section 3 at three times the distance), and so on to the eighth section at 7 times the distance, i. e., $2+3+4+5+6+7+8=35$ lbs. Happily, this did not satisfy our philosopher, since a later sketch is accompanied by the correcting remark that the weight must act in the middle of each portion. In this self-correction we see another reminder of the fact that the materials left to us are notes and memoranda *only*, jotted down as they occurred to the mind of the writer, so that faults were often cancelled out as occasion arose in later notes.

The Bent-Arm Lever-Conception of Potential Arm

Leonardo's next step in what we may regard as his logical scheme is of the greatest importance, since upon it hinges a number of applications which undoubtedly carried our philosopher very far. This concerns what is in effect the modern bent-arm lever. So far the lever or balance has been straight and the weights perpendicular to it. What if one arm is now bent relative to the other, so that the corresponding weight is inclined to it at some angle? The fundamental experiment upon which Leonardo bases this problem has been oft quoted, and is illustrated in Figure 2.[24] A bar *at* is pivoted at *a* and has a weight *o* suspended from *t* at *m*. A second weight is also attached to *t* by means of a horizontal cord *tn* passing over a pulley *n*. The problem is to find the ratio of the weights depending from *t* and *n* as a condition for the equilibrium of the rod *at*. Leonardo regards this as a lever problem in which the lever arm for the weight *o* is not *at*, but what he calls the *potential arm* or *potential lever ab*, and for the weight at *n* the lever arm is the potential arm *ac* He also speaks of these potential arms as *real* (i. e., in the sense of the effective arms) and the lines *ctn* and *btm* (i. e., the real lines with the cord extensions) as *semi-real*. Alternative terms also used by Leonardo are "spiritual lines" and "corporeal lines." Leonardo's conclusion therefore becomes that, as with the

[24] Ms. E, fol. 65v.

simple balance, the ratio of the two weights will be inversely as the ratio of their potential arms.

How did da Vinci arrive at this conception? The point, in view of the many problems upon which it is based, is one of importance.

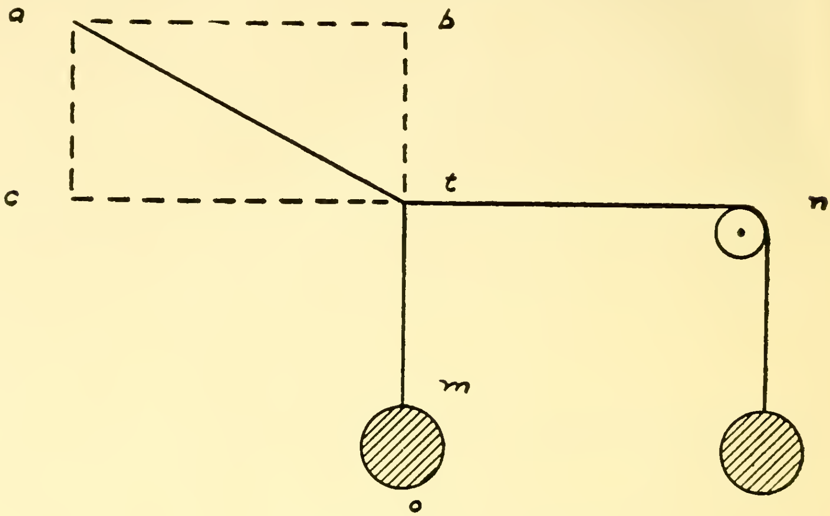

FIGURE TWO

Mach[25] has suggested the following as Leonardo's train of thought. Imagine a string laid round a pulley A (Fig. 3) and subject to equal tensions on both sides. Since BC is the portion in contact, EF will be a line of symmetry and the system will be in equilibrium. But it is to be noted that the only essential parts of the pulley are the two rigid radii, AB and AC. These suffice to determine the form of the motion of the points of application of the two strings, and the rest could be cut away without disturbing equilibrium. Hence, although the radius BA produced will cut the string at D, the lever arm for the right hand force is not AD but the potential lever AC. The view offered in this reasoning is naturally not impossible, but it seems improbable as a natural line of approach. It belongs rather to the sequence of thoughts which might be developed *after* the conception of the "potential arm" than to those which

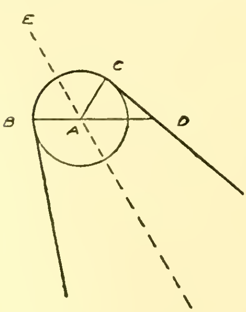

FIGURE THREE

[25] E. Mach, *The Science of Mechanics* (Eng. edition) p. 21. London, 1911.

preceded it. A more plausible origin of the conception is, however, suggested by note which appears in Manuscript "E," accompanied by a sketch (Fig. 4) which reads, "The junction of the appendices of the balances with the arms of these balances is always a potential rectangle, and is not able to be real if these same are oblique," [26] and again, "The real arms of the balance are longer than the potential arms, and as much more as they are nearer the center of the World" [27] (i. e., nearer the vertical through the fulcrum). Finally, on the next page, we read, "And always the real arms will not have in themselves the potential arms if they are not in the position of equality." [28] Here, surely, we have a more natural approach to Leonardo's important conception of the potential arms. The swinging of the arm of the balance has at all times been a familiar sight, and with da Vinci's powerful imagination, it is pertinent to believe in his quick ability to seize upon the significance of the diminishing perpendicular distance between the two suspended weights.

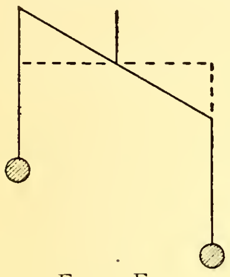

FIGURE FOUR

THE CONCEPTION OF MOMENTS

An interesting and important problem next arises. In view of the fact that the conception of the potential arm involves *both* the weight or force factor *and* the perpendicular distance on to its line of action from the point of suspension or fulcrum, did Leonardo knowingly have in his mind the idea of moments as we understand it today? Some controversy has not unnaturally developed regarding this point. Duhem,[29] and with him Mach,[30] favors the view that Leonardo understood and employed the idea of moments; Schuster[31] takes the opposite view. We are bound to express the opinion here that the balance of argument is distinctly against the statement that Leonardo conceived

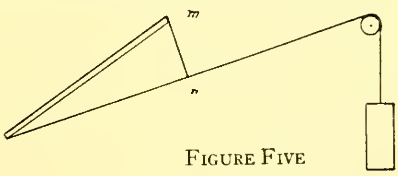

FIGURE FIVE

[26] Ms. E, fol. 64r.
[27] Ms. E, fol. 65v.
[28] Ms. E, fol. 65v.
[29] Duhem, *Etudes*, Vol. I, p. 143.
[30] E. Mach, *The Science of Mechanics*, p. 20 and Supplementary Vol., p. 7.
[31] Schuster, *Zur Mechanik*, etc., p. 43.

the idea of the moment of a force about a point. Duhem's case is an ingenious one. He cites first a passage from Manuscript "I", which, referring to a sketch (Fig. 5) of a force applied by a cord to take a mass out of the vertical, reads as follows: "*To know to each degree of movement the extent of the force of the power which moves and of the cause of the thing moved.* Make as you see in *mn* (i. e., drop a perpendicular on to the line of the force which moves) with *fh.*"[32] The point Duhem makes here is in the dropping of the perpendicular. He now quotes Leonardo's use of a tangential circle of which he speaks as the "circumvoluble."

Thus in Manuscript "M"[33] we find a diagram (Fig. 6) of a lever *fm*, having a weight of 4 lbs. suspended vertically from *m* and one of 8 lbs. at an inclined direction *fp* through the use of the pulley *p*. Leonardo clearly indicates his use of the perpendicular

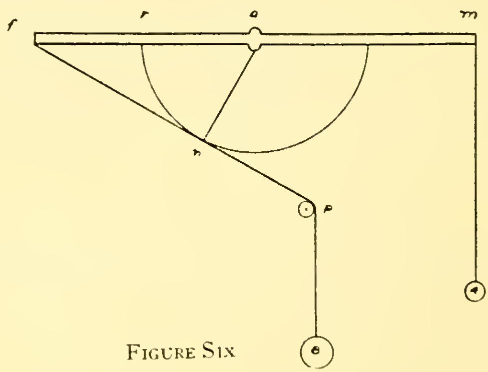

FIGURE SIX

to the line of action of the force by his employing of the "circumvoluble." It clearly emerges from the above that Leonardo da Vinci appreciated the significance of the perpendicular on to the line of action of the force. Where we cannot agree with Duhem, however, is in the contention that it follows from this that Leonardo *both* used the product of force and distance, *and* attached to this product the significance of the measure of the turning power of the force about the fulcrum. The essential alternatives are really as between the general use by Leonardo of either the factor of proportion or of the product of force and distance. The principle of the balance was to the effect that for equilibrium the *ratio* of the weights was inversely as the lengths of the arms (or of the potential arms in the

[32] Ms. I, fol. 30r.
[33] Ms. M, fol. 40r.

case of inclined forces). The principle of moments applied to the balance would express the fact that the *product* of the weight into the length of the arm was the same on both sides. These are the two alternatives, and the whole case again Leonardo's conception of the idea of moments lies in the fact that wherever possible Leonardo employs the ratio factor and not the product factor. In the light of this undoubted fact it is therefore difficult to believe that our philosopher was the originator of the conception of moments.

The Function of the Pulley, and the Length of the String

In connection with the general theme of the potential balance arm, we find interspersed through the Manuscripts[34] a number of variations of Fig. 6. Common to all these figures we have a graduated balance, with the arms horizontal, and with the fulcrum at the mid point. Also in every case we find at one extremity a weight hanging directly downwards, and at the center a "circumvoluble" circle. The variations occur at the other extremity. The cord comes directly from it at some angle, tangentially to a "circumvoluble," and thence by varying pulley connections to the second hanging weight. An extreme case is illustrated in Fig. 7. Here there are six pulleys, with the radius of the circumvoluble, and therefore of the potential arm one-quarter of the real arm, and the weights are as 8 to 2. As Leonardo puts it, "In the same relationship in which *mn* stands to *ms* stand inversely their powers." [35] Clearly Leonardo understood (1) that neither the length of the cord nor the number of pulleys had the slightest influence on the relationship between the powers, and (2) the arrangement of pulleeys was purely a matter of convenience.

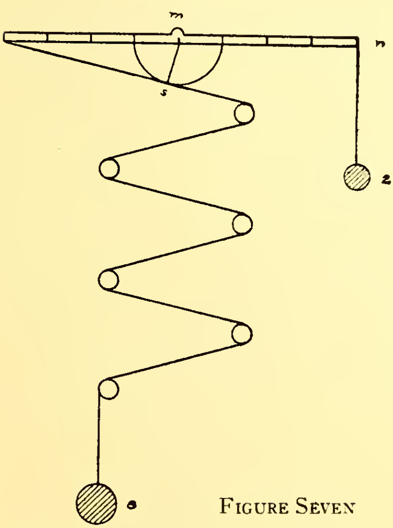

Figure Seven

[34] E. g., *Codex Atlanticus*, fol. 100r.b.
[35] Ms. M, fol. 38r.

Leonardo's Applications of the Lever Principle

We are now in a position to consider the various applications by Leonardo of the principle of the lever to mechanisms in General. As a preliminary we may consider the class of problem in which a suspended weight is deflected out of the vertical by the action of a disturbing force inclined to it at an angle. In Fig. 8,[36] we have a weight of 4 lbs. at the end of a cord deflected from the vertical *ab* into the position *am* by a tangential force *mf* applied by the weight of 1 lb. passing over the pulley *f*. The angle *amf* is therefore a right angle. Leonardo shows the position of equilibrium as such that each of the four divisions *bd*, etc., of *ab* is equal to *ac*. This is in direct accordance with the lever laws, since the potential arm for the 1 lb. weight is *am* and that for the 4 lb. weight is *ac*, so that $4 : 1 = am (= ab) : ac$.

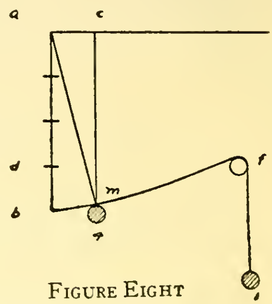

Figure Eight

Indeed, in a further sketch Leonardo traces the increase in the necessary deflecting load along *mf* from zero in the lowest division at *b* to its maximum value when *am* is horizontal. Another case of interest is that in which the deflecting force makes an obtuse angle with the suspending cord[37] (Fig. 9). The sketch is somewhat defaced in the *Codex Atlanticus*, but the meaning is, however, clear. A weight of 9 lbs. is deflected from the vertical *bm* to *bd* so that the point *c* immediately above *d* is one-third of the radius *bf* (equal to *bm*). Also the deflecting weight passes over a pulley *h* such that *bh* is a half of *bf*. The problem Leonardo sets himself is to determine the value of the deflecting weight for equilibrium in this position. The potential arm of the 9 lb. weight is *bc* equal to one-third of *bf*, and that of the

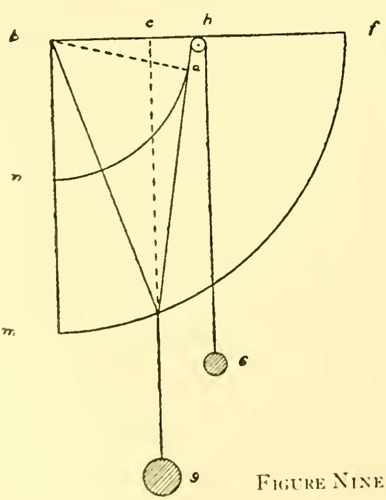

Figure Nine

[36] *Codex Atlanticus*, fol. 268v.b. See also fol. 365v.a.
[37] *Codex Atlanticus*, fol. 115r.a.

deflecting weight is *ab* the perpendicular on to *dh*, equal to one-half of *bf*. Hence the ratio of the potential arms is 2 to 3, and the ratio of the powers must therefore be 3 to 2. Leonardo thence correctly concludes that the deflecting weight must be 6 lbs.

THE INCLINED PLANE

For the next application of the principle of the lever we turn to Leonardo's treatment of the inclined plane. Stevinus,[38] as we know, has received the tribute of orthodox scientific history as having introduced the first complete treatment of the subject.[39] However, Leonardo, some 70 or 80 years before him had also tackled the problem. He shows little more than a few detached notes and sketches and his treatment is incomplete and undoubtedly lacks the brilliance of his Dutch successor. Interspersed throughout the note-books we find sketches—with and without accompanying notes—showing a double inclined plane with two weights connected by a pulley at the top. His linking up of this problem with that of the lever is clearly shown in a sketch in Manuscript "G" [40] (Fig. 10) in which we see added to the usual diagram of the double inclined plane, what we might call an equivalent simple lever, the two weights being common to the two systems. Clearly he regards the ratio of the weights for equilibrium as equal to the ratio of the basis. Again, in the *Manuscript on the Flight of Birds* we find an elaborate diagram

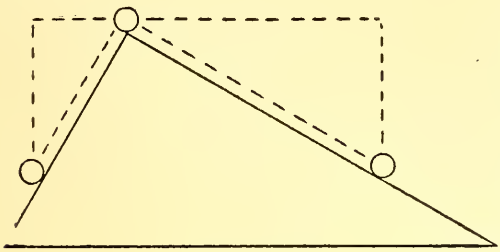

FIGURE TEN

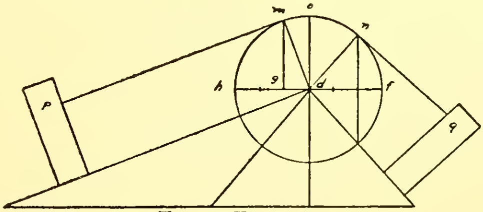

FIGURE ELEVEN

[38] Simon Stevinus of Bruges (1548-1620).

[39] Thus in Cox's *Mechanics*, Cambridge, 1904, p. 41, we read, "His discovery constitutes the second important step in the historical development of Mechanics."

[40] Ms. G, fol. 49r. See also Ms. E, fol. 1.v.

(Fig. 11) in which the values are so proportioned that the perpendicular *mg* on *hd* makes *gd* to one-third of *hd*, and *de* two-thirds of *df*. We read against this sketch as follows: "The weight *q*, because of the *right-angle n* above *df* at the point *e*, weighs two-thirds (i. e., referring to the pull on the rope *nq*) of its natural weight, which was 3 lbs.; and is a force of 2 lbs. The weight *p* which was also 3 lbs., is a force of 1 lb.; because of *m*, right angle on the line *hd* at the point *g*. Then we have here 1 lb. against 2 lbs.[41] The application of the principle of the lever is clear. With *d* as the fulcrum, the potential arm for the pull in the cord *nq* is *nd* (equal to *df*) and that for the weight *q* (considered at *n*) is *de*. Hence the pull is to the weight as 2 is to 2; and similarly for the other side of the figure.

[41] *Sul Volo degli Uccelli*, fol. 4r.

(To be continued)