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The Open Court

A MONTHLY MAGAZINE

Devoted to the Science of Religion, the Religion of Science, and the Extension of the Religious Parliament Idea

Founded by EDWARD C. HEGELER.

ANCIENT GREEK BOAT SHOWING AN EYE ON THE PROW.
(See pages 549-550.)

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CONTENTS:

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Frontispiece to The Open Court.

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JOHN NAPIER AND THE TERCENTENARY OF THE INVENTION OF LOGARITHMS.

BY PHILIP E. B. JOURDAIN.

THREE hundred years ago—in 1614—was published at Edinburgh John Napier's "Description of the Wonderful Canon of Logarithms": *Mirifici Logarithmorum Canonis Descriptio, ejusque usus in utraque Trigonometria; ut etiam in omni Logistica Mathematica, amplissimi, facillimi, et expedissimi explicatio.* Napier was described on the title-page as "Authore ac Inventore, Joanne Nepero, Barone Merchistonii, etc. Scoto"; and this has given rise to the notions, which are met more particularly in French books, that Napier was that kind of peer known as a "Baron," and that his name should be spelt "Neper." However, John Napier was not a member of the peerage: he was a Scotch "laird"—an unofficial title which corresponds to the English "lord of the manor" —of an ancient and respected family. His eldest son Archibald was the first Lord Napier properly speaking, for Archibald was raised to the peerage in 1627.

With regard to the name, it seems that "Alexander Napare," the first of Merchiston, acquired that estate before the year 1438 from James I, was provost of Edinburgh in 1437, and was otherwise distinguished in that reign. His eldest son, also Alexander, became in his father's lifetime comptroller to James II, and "ran a splendid career under successive monarchs." The origin of these ancestors of John Napier is very uncertain. In the thirteenth and fourteenth centuries, persons of the name of Napier were not uncommon. The Merchiston family cherished a tradition that their name was changed from Lennox to Napier by command of a king of the Scots who wished to do honor to one of their ancestors, Donald, a son of an Earl of Lennox. This Donald, it is said, had turned the tide of battle when flowing strongly against the king, and had fought so valiantly that the king declared before all the troops that he had "Na Peer." The name is probably of a more domestic origin, and commemorates virtues that are not usually associated with a warrior. On one occasion, John Napier was described, quite seriously it would seem, as "un Gentilhomme Ecassais nommé Non-pareil"; and one of the commendatory odes prefixed to the *Canon Mirificus* of 1614 ends with the lines:

"Nomine sic Nepar Parili fit et omine Non Par,
Quum non hac habeat Nepar in arte Parem."

It is perhaps of more importance that we do not know the correct spelling of Napier's name, since many forms of the word are found, such as Napeir, Nepair, Nepeir, Napare, Naper, Naipper. It seems that John Napier usually signed his name as "Jhone Neper" or "Jhone Nepair." The form now adopted by the family is comparatively modern.

John Napier was born at Merchiston Castle, near Edinburgh, in 1550, the year in which the Reformation in Scotland may be said to have begun. His father, Sir Archibald Napier, must have been not more than sixteen when he was born. In 1563, John Napier matriculated at St. Salvator's College, St. Andrews, and though his residence there seems to have been comparatively short, the influence of it on his future life was of the most far-reaching character. It was then that he received an impetus to theological studies that formed throughout his life quite as great an attraction as mathematics in any of its branches. He himself tells the story in the address "To the Godly and Christian Reader" prefixed to his first publication *A Plaine Discovery of the Whole Revelation of Saint John.* In that address is the following passage: "Although I have but of late attempted to write this so high a work, for preventing the apparent danger of Papistry arising within this Island; yet in truth it is no few yeers since first I began to precogitate the same: for in my tender yeers and barneage at Saint Androes at the Schools, having on the one part contracted a loving familiarity with a certain Gentleman, etc., a Papist; and on the other part being attentive to the Sermons of that worthy man of God, Master Christopher Goodman, teaching upon the Apocalypse, I was so moved in admiration against the blindnesse of Papists, that could not most evidently see their seven-hilled-city Rome, painted out there so lively by Saint John, as the maker of all Spiritual Whoredom, that not only burst I out in continuall reasoning against my said familiar, but also from henceforth I determined with myself (by the assistance of God's spirit) to employ my studie and diligence to search out the remanent mysteries of that holy Book; as to this hour (praised be the Lord) I have been doing, at all such times as I might have occasion."

Napier has been instanced by Mach as one of those who believed that philosophy and science must be founded on theology. "Napier," says Mach,[1] "applied himself to some extremely curious speculations. He wrote an exegetical commentary on the Book of Revelation, with propositions and mathematical demonstrations. Proposition XXVI, for example, maintains that the pope is the Antichrist; proposition XXXVI declares that the locusts are the Turks and Mohammedans; and so forth."

Various references in Napier's mathematical works can only be explained on the assumption that he could not divert his attention from theological studies sufficiently long to enable him to carry out cherished mathematical investigations. Whatever we may think of the ascendancy that James VI acquired over the church in Scotland, Professor Gibson[2] is inclined to believe that it was James's victory over the Presbyterian party, to which Napier belonged, that compelled Napier to withdraw from the ecclesiastical field and devote himself to his mathematical studies.

A second edition of the *Plaine Discovery,* revised and enlarged, was published in 1611, and the book continued to be republished for several years. It was also translated into many foreign languages.

There were traditions that Napier was in league with the devil, and these traditions might be met with about Edinburgh up to within not very many years ago. Among these traditions is one of a jet-black cock which was his constant companion, and was supposed to be a familiar spirit bound to him in that shape. Mark Napier, in his *Memoirs of John Napier of Merchiston, his Lineage, Life, and Times, with a History of the Invention of Logarithms,* which was published at Edinburgh and London in 1834, took the story of the cock so seriously that he tried to rationalize the tradi-

[1] *The Science of Mechanics,* 3d ed., Chicago and London: The Open Court Publishing Co., 1907, p. 447.

[2] George A. Gibson, "Napier and the Invention of Logarithms," *Proc. Roy. Phil. Soc. of Glasgow,* 1914, p. 8. To this paper (pp. 3-24), and to the biography, by Dr. J. W. L. Glaisher, of Napier in the *Encyclopædia Britannica* (11th ed. Vol. XIX, pp. 171-175) this article is very largely indebted.

tion by suggesting that Napier played upon the belief in his witchcraft to frighten his servants into confession of misdemeanors.

From the parish of Killearn come other traditions. Adjoining the mill of Gartness are the remains of an old house in which John Napier resided a great part of his time when he was making his calculations. It is reported that the constant noise of the cascade never gave him uneasiness, but that the clack of the mill, which was only occasional, greatly disturbed his thoughts. Therefore, when in deep study, he was sometimes under the necessity of desiring the miller to stop the mill that the train of his ideas might not be interrupted. He used frequently to walk out in his nightgown and cap. This, with some things which to the vulgar appeared rather odd, fixed on him the character of a "*warlock*." There is evidence that even Napier himself, like other eminent men of that time, was not free from a belief in magic.

After the publication of the *Plaine Discovery*, Napier seems to have occupied himself with the invention of secret instruments of war. These consisted of (1) a mirror for burning the enemies' ships at any distance, (2) a piece of artillery destroying everything round an arc of a circle, and (3) a round metal chariot so constructed that its occupants could move it rapidly and easily, while firing out through small holes in it. Besides this, Napier as the owner of large estates turned his attention to the improvement of agriculture.

But Napier's chief claim to remembrance is the invention of logarithms. It is a remarkable thing that, with one possible exception, there has been no rival claimant to the discovery of logarithms. Let us first consider the few hints that mathematicians had given before Napier's time.

A Frenchman, Nicolas Chuquet, in his work *Le Triparty en la science des nombres* of 1484, seems to have been the first to consider an arithmetical progression 1, 2, 3, 4, and so on, side by side with a geometrical progression which we would now write a, a^2, a^3, a^4, and so on; and to remark that the product of any two numbers of the geometrical progression is a term of the same progression, whose rank is the sum of the ranks of the two factors.[3] The same idea also appeared with the German "cossists" and with Michael Stifel in 1544.[4]

The exception referred to above is Joost or Jobst Bürgi (1552-

[3] M. Cantor, *Vorlesungen über Geschichte der Mathematik,* Vol. II, 2d ed., Leipsic, 1900, pp. 350-351.

[4] *Ibid.,* pp. 397, 403, 431-432. Cf. p. 635.

1632 or 1633), an ingenious watchmaker and mechanic. But Napier's *Canon mirificus* was published six years before Bürgi's *Progress Tabulen*; Bürgi's tables are very imperfect compared with Napier's; and there is, according to Gibson, every reason for believing that Napier had formed his conception of logarithms and begun their calculation quite as early as Bürgi—probably much earlier. Moritz Cantor,[5] however, states that Bürgi's work was probably earlier than Napier's. Still Bürgi's work has not had the slightest influence, so far as can be traced; either on the theoretical or on the practical development of the theory of logarithms.

The *Canon* contains fifty-seven pages of explanatory matter and ninety pages of tables. An English translation of the first part was made by Edward Wright and published in 1616. Napier's treatment is based on the comparison of the velocities of two moving points. Suppose one point P to set out from the point A and to move along the line AX with a uniform velocity V; then suppose another point Q to set out from B on the line BY, of given length r, at the same time that P sets out from A and also with the velocity V, but to move, not uniformly, but so that its velocity at any point (D) is proportional to the distance DY from the end of the line BY. If, now, C is the point that P has reached when Q, moving in the way described, has reached D, then the number which measures AC is the "logarithm" of the number which measures DY.

Let us try to form some notion of the way in which Napier was led to the invention of these logarithms. Throughout all his life he was more or less busied with devices for making multiplications, divisions, and extractions of the square and cube roots of great numbers capable of being carried out more quickly and easily. One of the first results which he obtained was a method by which the numbers that were to be multiplied, divided, or to have their roots extracted, are replaced by other numbers called "artificial numbers" by means of which all that the numbers first mentioned can do is done far more easily. To replace the name "artificial number" Napier afterwards invented the name "logarithm," which is derived from two Greek words meaning "ratio" and "number." Indeed, he used the idea we have touched upon of the comparison of an arithmetical progression with a geometrical progression. Other more or less well-known devices for shortening calculations were published by Napier in 1617.

It is necessary to emphasize the fact that the invention of

[5] *Ibid.*, pp. 725-729.

logarithms was made long before the theory of indices began to grow up. It is not a very difficult deduction from this theory, which began with the introduction of our present very convenient notation for indices. At the present time, we say that, if a, x, and m are three numbers such as $a^x = m$, then we call a the "base" and x the "logarithm of m to the base a", $x = \log_a m$. From this we see at once that the logarithm of the product of numbers is equal to the sum of the logarithms of those numbers, the logarithm of the quotient of two numbers is equal to the logarithm of the numerator diminished by the logarithm of the denominator, the logarithm of a power of a number is equal to the logarithm of that number multiplied by the index, and the logarithm of the nth root of a number is equal to the logarithm of that number divided by n.

However, Napier himself made no explicit use of a base. What is now called the "Napierian base" is the incommensurable number e, but Napier at first implicitly used $1/e$ as base. Bürgi's base was e. The idea of integral indices was only beginning to be known in Napier's day, while those of fractional and negative indices were quite unknown then and for long after.

Napier had the needs of trigonometry primarily in view, and he usually spoke of BY (or r) as the "whole sine" and DY as a "sine." It must be remembered that at that time a sine was a line and not a ratio, as it is with us. In the table which formed the second part of Napier's book, the logarithm of sines and tangents of all angles from 0° to 90°, at intervals of one minute, were given.

Returning to the consideration of Napier's moving points; when Q is at B, the point P is at A, so that the logarithm of the whole sine BY is zero. The logarithms of numbers greater than the whole sine are negative.

Napier then found the rule that, if a is to b as c is to d, then

$$\log a - \log b = \log c - \log d,$$

and hence he easily obtained all the rules required for ordinary calculations.[6]

It used to be a general opinion that there was a "metaphysical objection to the introduction of ideas of motion into geometry." This opinion seems to me to leave out of account the profound logical objections to the conception of motion which were first formulated by Zeno the Eleatic about five hundred years before our era began, and to which no satisfactory answer has been—or indeed

[6] On Napier's logarithmic work, see also M. Cantor, *op. cit.*, Vol. II, pp. 730-737.

could be—given until within the last thirty-five years. It is certainly worthy of particular remark that the notion of flowing quantities was expressed very clearly indeed in Napier's *Canon*, and Colin Maclaurin[7] remarked that "the nature and genesis of logarithms is proposed by the inventor in a method similar to that which is applied in this doctrine[fluxions] for explaining the genesis of quantities of all sorts, and is described by him almost in the same terms."

Henry Briggs (1556-1630), reader of geometry at Gresham College, London, and later Savilian professor of geometry at Oxford, welcomed Napier's book with great enthusiasm. In 1615 he wrote to Archbishop Usher: "Napper, lord of Markinston, hath set my head and hands awork with his new and admirable logarithms. I hope to see him this summer, if it please God, for I never saw book which please me better, or made me more wonder." Briggs visited Napier in 1615 and stayed with him a whole month. Indeed, Briggs, we read, was so moved that he could not rest until he had seen the inventor of logarithms. When Briggs actually saw Napier, each, it is reported, beheld the other with admiration and without a word being spoken. At last Briggs said: "My Lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help unto astronomy, viz., the logarithms; but, my Lord, being by you found out, I wonder nobody else found it out before, when, now being known, it appears to easy."

At this visit, Napier and Briggs discussed certain changes in the system of logarithms. In a letter to Napier before the visit, Briggs had suggested that it would be more convenient, while the logarithm of the whole sine was still taken as zero, to take the logarithm of the tenth part of the sine as a power of 10, and he had actually begun the calculation of tables on his proposed system. Napier agreed that a change was desirable, and stated that he had formerly wished to make a change; but that he had preferred to publish the tables already prepared, as he could not, on account of ill health and for other reasons, undertake the construction of new tables. He proposed, however, a somewhat different system from that suggested by Briggs, namely that zero should be the logarithm, not of the whole sine but of unity, while, as Briggs suggested, the logarithm of the tenth part of the sine should be a power of 10. Briggs at once admitted that Napier's method was decidedly the

[7] *Treatise of Fluxions*, Vol. I, p. 158. On Napier's idea of flowing quantities, cf. M. Cantor, *op. cit.*, Vol. II, p. 849.

better, and he set about the calculation of tables on the new system, which is essentially the system of logarithms now in use.

In 1616, Briggs again visited Napier and showed him what he had done, and would have paid him a third visit in 1617, had Napier's life been spared. In 1617 Briggs published a small book giving the logarithms of all numbers from 1 to 1000 calculated to 14 places of decimals; and these tables were very greatly extended in his *Arithmetica logarithmica* of 1624. A gap in these tables was filled up by the work of Adrian Vlacq published in 1628. Edmund Gunter published a table of some Briggian or common logarithms of trigonometrical functions in 1620. More extensive tables were published later by Vlacq and Briggs,[8] and, from that time, mathematical tables began to be very plentiful.

Napier's account of the construction of his tables of 1614 was published posthumously by his son Robert in 1619, though it seems to have been written many years before 1614.

It is pleasant to reflect that charges which have been brought against Napier of jealousy of Briggs are unfounded. Both Napier and Briggs were united by a very warm friendship for each other. Other unfounded reports are that Napier's devotion to mathematics was due to old age and the gout, and that his mathematical pursuits led him to dissipate his means.

Besides the invention of logarithms and other methods for shortening calculations, to Napier are due certain rules in spherical trigonometry and the technically important introduction of the decimal point in arithmetic.

The portrait of John Napier that is reproduced as a frontispiece to this number is from a steel engraving prefixed to Mark Napier's *Memoirs of John Napier*. This engraving is a partial copy of an authentic portrait of Napier which belongs to the College of Edinburgh. It was presented by Margaret, Baroness Napier in her own right, and there is no doubt of its genuineness. It bears the shield of arms and the initials of Napier with a date 1616 and his age. The name of the painter is unknown.

[8] On the logarithmic work of Briggs and others, see Cantor, *op. cit.*, pp. 738-748.