MISCELLANEOUS.

BOOK REVIEWS AND NOTES.

THE NATURE OF MATHEMATICS. By *Philip E. B. Jourdain, M.A.* "The People's Books," London and Edinburgh: T. C. and E. C. Jack; New York: Dodge Publishing Co. 92 pages.

This is a volume of an extraordinarily inexpensive series of books on science, philosophy, religion, history, economics, and literature published in Great Britain at sixpence each. "The purpose of this little volume," says the the author (p. 7), "is not to give-like a text-book-a collection of mathematical methods and examples, but to do, firstly, what text-books do not do: to show how and why these methods grew up. All these methods are simply means, contrived with the conscious or unconscious end of economy of thought-labor, for the convenient handling of long and complicated chains of reasoning. This reasoning, when applied to foretell natural events, on the basis of the application of mathematics, as sketched in the fourth chapter, often gives striking results. But the methods, of mathematics, though often suggested by natural events, are purely logical....In this book I shall not pay very much attention to the details of the elementary arithmetic, geometry, and algebra of the many text-books, but shall be concerned with the discussion of those conceptions-such as that of negative number-which are used and not sufficiently discussed in these books. Then, too, I shall give a somewhat full account of the development of analytical methods and certain examinations of principles,"

The first five chapters are devoted to a historical exposition of the growth of mathematical methods: The growth of mathematical science in ancient times; the rise and progress of modern mathematics-algebra; the rise and progress of modern mathematics-analytical geometry and the method of indivisibles; the beginnings of the application of mathematics to natural science -the science of dynamics; the rise of modern mathematics-the infinitesimal calculus. After a chapter on modern views of limits and numbers, modern conclusions as to the nature of mathematics are dealt with in the final chapter. "In the historical part," says the author on page 8, "we shall see that the actual reasonings made by mathematicians in building up their methods have often not been in accordance with logical rules. How, then, can we say that the reasonings of mathematics are logical in their nature? The answer is that the one word 'mathematics' is habitually used in two senses, and so, as explained in the last chapter, I have distinguished 'mathematics,' the methods used to discover certain truths, and 'Mathematics' the truths discovered. When we have passed through the stage of finding out, by external evidence

or conjecture, how mathematics grew up with problems suggested by natural events, like the falling of a stone, and then how something very abstract and intangible but very real separated out of these problems, we can turn our attention to the problem of Mathematics without troubling ourselves any more as to how, historically, it gradually appeared to us quite clearly that there is such a thing at all as Mathematics-something which exists apart from its application to natural science. History has an immense value in being suggestive to the investigator, but it is, logically speaking, irrelevant. Suppose that you are a mathematician; what you eat will have an important influence on your discoveries, but you would at once see how absurd it would be to make, say, the momentous discovery that 2 added to 3 makes 5 depend on an orgy of mutton cutlets or bread and jam. The methods of work and daily life of mathematicians, the connecting threads of suggestion that run through their work, and the influence on their work of the allied work of others, all interest the investigator because these things give him examples of research and suggest new ideas to him; but these reasons are psychological and not logical." In this is shown the second object of the book.

The principal points of the book are: (1) A discussion of the question as to the "use" of mathematics; (2) The emphasis on the fact that mathematics is a living science; depends, psychologically speaking, on the natural sciences; proceeds, like them, by means of economizing thought (pp. 6, II, 12, 16, 18, 20, 28, 33, 53, 69, 71 75, 89 and what may be called *faith* (pp. 32, 34, 43, 52); (3) That the essential character of mathematics is that it deals with the notion of *any* (pp. 15, 33, 48, 69, 85, 86); (4) The nature of mathematics is *logical*, so that "all those petty questions—sometimes amusing and often tedious—of history, persons, and nations are irrelevant to Mathematics in itself" (p. 9); (5) Analytical geometry is regarded as a *picture* of algebraical processes; (6) The explanation of the infinitesimal calculus.

It may be mentioned that a part of the views here given is taken from an article in *The Monist* for 1908. ϕ