ROGER BACON, LOGICIAN AND MATHEMATICIAN.

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ALTHOUGH Bacon was preeminently a physicist he was never tired of praising mathematics. Theologians, he said, ought "to abound in the power of numbering."[1] Then again, "divine mathematics alone can purge the intellect and fit the student for the acquirement of all knowledge." He showed himself much more wide-minded than his more famous namesake, Francis Bacon: for not only did he state as a fundamental principle that the study of natural science must rest on experiment, but he also explained how astronomy and the physical sciences rest ultimately on mathematics, and progress only when their fundamental principles are expressed in a mathematical form. Mathematics, he said, should be regarded as the alphabet of all philosophy.

Roger Bacon strove unsuccessfully to replace logic in the curriculum of the University of Oxford by mathematical and linguistic studies. In fact, he had a low opinion of the utility of logic, because reasoning seemed to him to be innate. We can form some idea of how far Bacon was in this case in advance of his times, when we reflect that even at the present day Oxford University still cultivates the Aristotelian logic with its errors and limitations, and its learned professors regard modern and more profound logical work with contempt, for no other apparent reason than that it is a product of the last sixty years. The fact is that Roger Bacon and all the really scientific objectors to scholastic logic, including Kant, were quite right: the Aristotelian or merely syllogistic logic of classes and propositions is quite insufficient for the purposes of even elementary arithmetic and geometry. For any scien-

[1] This and the following quotations are taken from W. W. Rouse Ball, *A Short Account of the History of Mathematics,* 4th ed., New York and London, 1908, pp. 169, 175, 176.

tific purpose, we must leave scholastic logic in the school. In the present problems of science we can no more effect anything with it than a regiment armed with bows and arrows could take Gibraltar. Logic has now recognized that it must, if it is to be of any real use in the world, make use of a symbolism more or less analogous to the ingeniously thought-out and economical (in Mach's sense) symbolism of algebra. Modern logicians see that using algebraical signs like "$a+b$" does not necessarily imply that "a" and "b" stand for numbers, any more than French people, when they speak of a "*chou*", mean a shoe.

II.

Let us now turn to Bacon's mathematical work. The most important part is his work on "perspective."[2] But we will here fix our attention on his discussion of "continuity." Nowadays it is the usual opinion among those who have studied the subject that it was Zeno the Eleatic who first incontrovertibly showed the untenability of the Pythagorean doctrine that lines, surfaces, and solids are composed of points. I refer more especially to the third and fourth of Zeno's famous arguments about motion, preserved—though probably in a mangled state—by Aristotle. Zeno's first two arguments about motion, which are far better known and are —unlike the others—readily refutable at the present day, may conceivably be directed against the opposite view that spaces are divisible to infinity. Aristotle was a supporter of the doctrine of infinite divisibility and an opponent of Zeno, and devoted much space in his *Physics* to the discussion of "continuity," which he expressed by the Greek word συνεχής.

Now, in the thirty-ninth chapter of his *Opus tertium*,[3] Bacon discussed continuous spatial magnitudes, and emphasized the impossibility of generating such magnitudes from single point-elements. His proof of this was as follows. If a square were formed of points—for example, suppose that a side contained 5 points and a whole square was formed out of five columns or five rows of five points each—then the diagonal would also be formed of five points. The diagonal, then, would be equal to a side, and this is geometrically impossible. Kurd Lasswitz[4] showed that this proof of Bacon's occurred previously with the Arabian mathematician

[2] Cf. Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. II, pp. 97-99. 2d ed., Leipsic, 1900.

[3] *Opera quaedam hactenus inedita* (ed. J. S. Brewer), Vol. I, 1859, p. 132. Cf. M. Cantor, *op. cit.*, p. 97.

[4] *Geschichte der Atomistik vom Mittelalter bis Newton.* Hamburg and Leipsic, Vol. I, pp. 194, 149.

Mutakallimun. It may be remarked that Bacon, especially in his work on "perspective," made great use of the writings of Arabian authors.

The Greek language was known, as Bacon himself indicated, by many of the learned of the thirteenth century, but there was for the most part a lack of Greek works.[5] It can hardly be doubted that Bacon was not familiar with the fourth and last of Zeno's arguments about motion, which was against the composition of space out of indivisibles, especially as Aristotle in his *Physics* had in all probability mangled this argument out of all similarity to its original form. And indeed, outwardly, Zeno's argument is very different from that of Bacon. But both have this in common: They can only be satisfactorily answered by one who knows that the modern theory of infinity and continuity has resolved all the contradictions that were formerly thought to subsist in these notions. In Bacon's case, we now know that the diagonal of a square may, in Georg Cantor's terminology, contain the same cardinal number of points as the side, although they are of different lengths. It is not, then, impossible to hold that a continuous line is composed of points. Both Zeno and Bacon seem to have proved that we cannot do this if the points are finite in number.

Finally, it may be remarked that Aristotle's conception seems to have made its first explicit appearance in the West in a definition[6] of Thomas Bradwardine, who was probably born a few years before Roger Bacon died.

[5] Cantor, *op, cit.*, p. 99.

[6] *Ibid.*, p. 119.