MATHEMATICAL LITERACY: A CASE STUDY OF PRE-SERVICE TEACHERS

by

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MAJOR PROFESSOR: Dr. Grant Miller

This study addresses the question of whether or not pre-service teachers are ready and prepared to use and teach the highly-specialized language of each discipline. The disciplinary languages present teaching and learning challenges due to their lack of parallels in daily the language (Shanahan & Shanahan, 2008). Additionally, the languages of the disciplines are rarely taught and are commonly acquired through an isolated representation of words without a situated meaning within the theory (Gee, 2002). The knowledge of the particular ways of reading, writing, listening to, and talking in the content areas provides opportunities for students' apprenticeship within the disciplines required for success in higher education contexts (Dobbs, Ippolito, and Charner, 2017). Moreover, this study addresses the question of how future teachers develop disciplinary knowledge and skills.

The purpose of this case study was to investigate how mathematical literacy is shaped and defined by the experiences, language, and disciplinary practices of pre-service teachers and experts in mathematics. This overall aim was unfolded by three guiding research questions: 1) What do the Experiences of Pre-Service Teachers and Experts in Mathematics Reveal about their Understanding of Mathematical Literacy? 2) How do pre-service teachers and experts in mathematics use language when solving mathematical problems? and 3) What literacy practices do pre-service teachers and experts in mathematics utilize when presented with modules that require mathematics problem-solving? To structure the elements of analysis for the participants'

responses, I adopted the theoretical support from the emerging disciplinary literacy framework, the novice-expert paradigm, and the tenets of M. K. Halliday's functional linguistic theory (i.e., Systemic Functional Linguistics; [SFL]).

Four faculty in the Department of Mathematics and four pre-service teachers in the Department of Curriculum and Instruction at a large Midwest university agreed to participate in this case study. For the data collection, I asked the participants to participate in two sessions. In the first sessions, the participants responded to a semi-structured interview. Afterward, in a second session, the participants solved modules of mathematical problems following three protocols: a think-aloud, a silent-solving, and an oral-explanatory.

The results of the participants' responses to the semi-structured interview and the three protocols indicated that their experiences as learners and teachers of mathematics are tied to their definitions of literacy and disciplinary literacy. The SFL analysis showed that for the experts of mathematics, mathematical problem-solving is a more abstract and cognitive practice. The pre-service teachers' registers indicated that mathematical problem-solving is experienced as more concrete and real practice. The unique literacy practices that these participants displayed showed the strong connection between language, literacy, and mathematical thought.

The implications of this study results of this study are discussed in terms of the importance of language and disciplinary literacy in preparation for future teachers as they progress in their course of study within their teaching education programs.

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DEDICATION

To Daniel. You are my rock, my soul, my best friend, my love. I could not have concluded this journey without you. Thank you for helping me to achieve my dreams.

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CHAPTER 1

INTRODUCTION

In this globalized era where fast technological, socioeconomic, and communication changes mark the norm in the daily life of young adults it seems that a college degree is more important than ever to reach the economic, intellectual, and personal growth that these changes demands. A 2013 report from College Board highlights the importance of higher education in terms of: a) better income sources and long-term employment benefits, b) healthier personal and social lifestyles, c) reduction of socioeconomic gaps, and d) return of economic investment at the state level (Baum, Ma, & Payea, 2013).

These benefits have contributed to a 14.8% increase of the number of 18- to 24-year-old students enrolled in postsecondary institutions during the last 45 years, especially among females and students from minority populations (National Center for Education Statistics, 2016). It is estimated that a bachelor's degree holder could potentially earn \$ 2.8 million, 57% more than a holder of a high school diploma (\$1.3 million on average; Carnevale, Rose, & Cheah, 2011). In addition, higher education provides young adults with a structured environment in which they can learn the necessary skills for economic independence (Hershner & Chervin, 2014).

Although a higher education degree projects better long-term benefits for its holder, it demands that students make important changes in their academic lives. When students are admitted into a college, they are expected to bring knowledge and skills that would support the demands of instruction within these institutions. Thus, the question of whether young adults are ready and prepared to succeed at a higher education institution points to policymakers, practitioners, and educators to take action to support secondary students to achieve this goal.

In the United States, the issue of college and career readiness has been the focal point of discussion of the role of secondary education to provide students the learning and academic tools to succeed in higher education institutions (Finkelstein et al., 2013; Henry & Stahl, 2017; Holschuh, 2014). Moreover, the discussion conveys concerns related to the United States educational system's ability to compete economically and technologically in an international arena. Thus, in 2009, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) launched the Common Core State Standards Initiative (CCSS), which, according to Rothman (2011), aims to: a) prepare secondary students for the first years of college; b) provide standards which are internationally benchmarked; c) educate parents, students, and teachers about what it is important to learn at each grade level; and d) represent a consensus among states about the knowledge and skills students should develop during the school years regarding the place they live.

In its inception, the CCSS initiative was limited to English language and mathematics for students K-12 grades and did not provide guidance for post-secondary instruction (Rothman, 2011). However, extensions of the CCSS initiative have resulted in the adoption of standards for history, social studies, sciences, and technical subjects, for which students are expected to read, write, and effectively use language in a variety of content areas (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The adoption of the CCSS implies the application of particular ways of reading and writing unique to each one of the content areas (Cervetti & David Pearson, 2012; Loveland, 2014; Manderino & Wickens, 2014; Shanahan & Shanahan, 2014), and unveils the necessity of a disciplinary framework to support the learning of knowledge and skills that are required for secondary students to prepare for college (Hynd-Shanahan, 2013; Shanahan & Shanahan, 2017; Zygouris-Coe, 2012). It is precisely through the engagement toward literacy practices, unique of the content areas that the CCSS aim to address its purpose to prepare secondary students for college and career life. The CCSS recognize that students have different literacy practices across the content areas, and it is the gain of knowledge and skills specific to the areas, which prepares students to face multiple literacy practices required by the different disciplines in higher education institutions (Kendall, 2011).

The Problem of Study

Although the adoption of the CCSS supposes to open a bridge between secondary and higher education, there are concerning data showing low levels of predicted academic success among high school students. For instance, the American College Testing (ACT) 2018 results show that only 38% of the high school graduates who took the ACT met at least 3 of the benchmarks for career readiness and 35% of these graduates did not meet any of the benchmarks (English, Reading, Math, and Science, American College Testing Inc., 2018). The Scholastic Aptitude Test (SAT) 2018 reports that only 49% of the high school graduates met the benchmark for career readiness (College Board, 2018).

These reports are accompanied by concerning rates of academic failure in classes such as college algebra, in which no more than 50% of college students pass with a grade higher than a C (Ganter & Haver, 2011). This low rate of academic success in college algebra and the disappointing indicators for career readiness displayed by the ACT and SAT scores challenge educators and researchers and raise questions related to whether high school students are prepared to develop the knowledge and practices that they need to be successful in a higher education institution. Moreover, these results challenge the CCSS aim to prepare high school students with specific ways of knowing found in academic areas in higher education institutions (Kendall, 2011). Additionally, these results questions about how prepared are teachers to guide students to meet the aim of the CCSS to develop disciplinary practices (Saavedra & Steele, 2012) and whether teacher preparation programs are equipping the new generation of teachers to situate their practices under the context of the CCSS (Kober & Rentner, 2012; Liebtag, 2013; Rothman, 2012; Wilhoit, 2012).

A disciplinary literacy framework allows educators, researchers, and practitioners to observe the secondary students' development of knowledge and skills that would allow them to be successful in higher education institutions (Manderino & Wickens, 2014; Shanahan & Shanahan, 2008, 2012, Zygouris-Coe, 2012). Additionally, a disciplinary literacy framework supports the interpretation and understanding of the specific knowledge and skills that college students develop in their discipline(s); for example, in areas such as mathematics, students need to identify the particularities of mathematical texts (e.g., formulas, equations, graphs) to interpret their meaning and practical applications (Siebert & Draper, 2012). Furthermore, a disciplinary literacy framework highlights the unique literacy practices that teachers bring to their classrooms and that are unique of the discipline they teach (Bain, 2012; Fang, 2014; Love, 2009; Temple & Doerr, 2018; Zhang & Chan, 2017).

As I will present in the forthcoming sections, for this study the academic areas are understood as communities of practice (Lave & Wenger, 1991) with shared unique communicative practices (Airey, 2011). The communicative essence of the academic areas (Hillman, 2014) requires of detailed study of the role that language plays in the development of particular ways to read and write in the disciplines (Fang, 2012; Feez & Quinn, 2017; Rezat & Rezat, 2017; Snow, 2010; Townsend, 2015). In areas such as mathematics, despite the ample study of the role that language plays in mathematical teaching and learning (e.g., Bartolini Bussi & Mariotti, 2008; Boero, Douek, & Ferrari, 2008; Cocking & Mestre, 1988; Moschkovich, 2010b; Radford & Barwell, 2016a), there are only a few studies that have observed the relationship among language, disciplinary literacy, and mathematics; and these studies have focused mainly on secondary students (e.g., Kleve & Penne, 2016; Mongillo, 2017; Yore, Pimm, & Tuan, 2007). There is a research gap in the study of language and disciplinary mathematical literacy in college students.

Nevertheless, there is a rising interest in the development of disciplinary literacy in a particular group of college students, the pre-service teachers. This interest comes from a common understanding among disciplinary literacy researchers of the pre-service teachers' need to be prepared to guide a new generation of students to develop specific literacy practices and be college and career ready (Colwell & Enderson, 2016; Colwell & Gregory, 2016; Feez & Quinn, 2017; Lenski & Thieman, 2013).

Research about disciplinary literacy in pre-service teachers has addressed issues of pre-service teachers' ability to apply content area literacy strategies in their classrooms (e.g., Feez & Quinn, 2017; Lenski & Thieman, 2013; Orr & Kukner, 2015); understandings of, attitudes toward, and beliefs about disciplinary literacy (e.g., Colwell & Enderson, 2016; Colwell & Gregory, 2016; Gritter, 2011; Masuda & Ebersole, 2013); and teaching preparation and disciplinary literacy (e.g., Colwell, 2012; Ingram, Bumstead, & Wilson, 2016). However, most of this body of research comes from research on the development of disciplinary literacy in areas such as geography (Bauch & Sheldon, 2014), humanities (Cisco, 2016), and science (Ruzycki, 2015). Mathematics has not been observed as meticulously as the rest of the content areas in this regard (Shanahan, Shanahan, & Misischia, 2011).

The aforementioned research body has identified gaps in the study of disciplinary literacy in college students as well. These gaps are more notorious in areas such as mathematics, in which despite the ample study of its relationship with language, there is scarce evidence of how college students develop the language related to mathematics as a discipline. In pre-service teachers, the lack of the mathematical language could be seen as problematic as they are the ones that would use their discursive resources to make sense of the sophisticate mathematical concepts within their classrooms to facilitate students' learning (Street, 2005).

Since the areas of concern of a disciplinary literacy framework are deeply in contact with language (Shanahan & Shanahan, 2018), it seems appropriate to analyze the development of the disciplinary language in mathematics from a linguistic perspective. In this regard, disciplinary literacy researchers have found in the tenets of Halliday's functional linguistic theory a valid methodological approach to analyze language development within the disciplines (Ebbelind & Segerby, 2015; Huang, Berg, Siegrist, & Damsri, 2017; Moore & Schleppegrell, 2014; Schleppegrell & Achugar, 2003).

Along with the study of disciplinary literacy from a functional linguistic perspective, a great area of concern of disciplinary literacy research is the study of the experts' disciplinary literacy practices (Shanahan & Shanahan, 2018). The novice-expert paradigm as a methodological approach distinguishes the fundamental differences that occur among the disciplines (Shanahan & Shanahan, 2012), the disciplinary practices that are categorized as exemplary within each discipline (Shanahan et al., 2011), and the specific content knowledge that is build within each discipline (e.g., Wineburg, 1991).

Thus, this study aims to investigate and analyze how language mediates the use of heuristics that pre-service teachers and mathematical experts display when defining mathematical literacy and solving mathematical problems. I abandon linear models of the expert-novice paradigm; rather, it is my belief that the mathematical experts' heuristics and linguistic repertoires are points of reference to trace the unique practices of mathematics as a discipline; which addresses the CCSS aim of preparing secondary students with the specifics ways of knowing found in the academic disciplines (Kendall, 2011).

Researcher's Rationale

When I started designing this project, I experienced a sense of insecurity. I consider myself an emerging scholar in the area of language, particularly how languages are learned and acquired; I have been trained as an applied linguist and most of my professional experience is related to teaching English to speakers of other languages (TESOL). However, my encounters with mathematics have been mere as a college and doctoral student. I did not feel capable of developing this study. I brought this concern to my doctoral committee members, from whom I received two essential pieces of advice: develop a pilot study with the overall goal of gain the confidence I needed to move forward in the design of this study, and state your positionality and acknowledge your strengths and weakness to develop this project. In the forthcoming sections, I will start with the latter and describe my position as a researcher. Then, I will present the design and preliminary results of the pilot study.

Researcher's Positionality

I have always thought of mathematics as an elusive subject. As a college student, I took the required courses of mathematics, but I avoided majors in which I had to invest myself in learning this subject beyond what a 'mere mortal' would do. I would not say that I had a traumatic experience as a learner to avoid studying mathematics; however, I was never encouraged to think about sciences or technology as a possible professional path.

I had to think in a career 'appropriate for a woman'. I repeated to myself - engineering, architecture, or computer sciences are majors for men; you are not good at math, after all, you studied in a high school administered by nuns, they only teach how to do crafts and become a good servant of God; you have no chance in those fields. It was my belief that I was not good at mathematics. Deep inside of me, there was a person curious for the sciences but suppressed by my own beliefs of what I was capable of doing. However, overtime, my relationship with mathematics has evolved. I started enjoying the required calculations I needed to perform in my introductory statistics course as a master student. Then, as a Ph.D. student, I was deeply invested in my advanced statistics courses, which made me to take more advanced statistics classes, which were based on the mathematical processes behind the statistics computations.

Language(s) have always fascinated me. I started to learn English as a foreign language as a teenager. I studied French for two years in college. I took a year of Kichwa, one of the native languages of my country, before starting a bachelor's in applied linguistics. I own a double master's degree in applied linguistics and TESOL. I have more than 10 years of experience of teaching future teachers of English as a foreign language.

As a doctoral student, I have been exposed to myriad of theories of learning, language and literacy acquisition, and language and education. During my doctoral program, I have been able to explore myself as a learner and discover how to conjugate my area of expertise with current trends in education, especially with the ones situated under a socio-cultural perspective. My interest in the topic of this study emerges in light of the notion of language as a cultural and a learning tool (Vygotsky, 1978; Vygotsky & Luria, 1978), its essential role in shaping the Discourses (Gee, 2008) found in education contexts, and how language mediates the learning and acquisition of literacy (Gee, 2006).

In this study, I am a mathematics outsider. I acknowledge that my lack of expertise in mathematics could lead to possible misinterpretations and ignore underlying patterns (Berger, 2015); however, if I approach mathematics with different lenses, I would be able to focus myself in the language that is used to make meaning of mathematics rather than in the processes to solve a math problem. As an outsider, I could trace patterns that could be underestimated by experts in the field (Chavez, 2008) and observe mathematics and its intrinsic and unique relationship with language (O'Halloran, 2005).

The Pilot Study

With the overall goal to experience as a researcher, I developed a pilot study during the Spring and Summer 2019 semesters. Although my initial goal to develop this pilot was to become a stronger researcher, it also guided three important goals for the design of this project: a) to test the instrument and its future application , b) to develop a set of strategies to support the linguistic analysis, and c) to identify any potential threat to the rigor and validity of my instrument.

This pilot study followed the same methodology that I intend for this project. I invited pre-service teachers and experienced professors of mathematics to participate. I was granted permission by the instructor of CI 220: Mathematics Content and Methods for the Elementary School to visit this class and invite students to participate in this pilot. I invited professors from the Department of Mathematics to participate.

Two students taking CI 220 accepted to participate in the pilot. Cesar (all the names are pseudonyms), a student majoring in Special Education, and Sophie, who is majoring in Elementary Education, agreed to participate. From the pool of faculty of the Department of Mathematics, only one instructor, Dr. Acosta, agreed to participate. As my initial intention was to interview two instructors of mathematics, I invited to participate instructors from other departments as well. One instructor teaching MATH 101: Contemporary Mathematics, Susan, agreed to participate.

I conducted two data collection sections. In the first session, the participants responded to a semi-structured interview, which aimed to explore their experiences as learners of mathematics, their definitions of literacy and mathematical literacy, and their vision of mathematics as a discipline. During the second session, I asked the participants to follow a think-aloud protocol when solving nine mathematical problems, solve silently a new set of nine problems, and follow an oral-explanatory protocol, in which the participants would explain to me how to solve a new set of nine mathematical problems.

The pilot helped me to frame certain theoretical considerations for the development of this project:

1) The conceptual framework needs to provide a working definition of expert and expertise. I consider both of the instructors who participated in the pilot to be experts. However, the level of expertise that both displayed was considerably different. Having a working definition of an expert would allow me to redefine the recruitment of the experts for this study, as well as, to draw a more detailed participant's profile.
2) After the pilot, I questioned the development of the Review Literature of this study. The pilot showed that certain procedures of data collection need to be aligned with current research. More specifically, I questioned the importance of the oral-explanatory protocol for this study, and whether current literature in the filed supports the inclusion of this protocol.
3) The pilot revealed that I need more theoretical support for the development of the items in the semi-structured interview.

Additionally, the pilot raised questions regarding the methodology of this project. The methodological issues found in the pilot will be considered in Chapter 3: Methodology.

Conceptual Framework

To structure the elements of analysis and understand the phenomena proposed by this inquiry, this study draws in multiple, cohesive, and interrelated theories, which would interlink concepts that support one another to provide a comprehensive understanding of the phenomena to be observed (Jabareen, 2009) and a broader view of how participants' thought is shaped by diverse contexts within educational research (Agee, 2002).

Thus, this Conceptual Framework will be guided by the concepts emerging from disciplinary literacy theory, the novice-expert paradigm, and Halliday's functional grammar theory of language.

Disciplinary Literacy Theory

Perhaps, the current interest in disciplinary literacy theories is one of the results of the adoption of the CCSS initiative by more than 41 states in the United States. The CCSS define the skills and knowledge that students should develop from K-12 to succeed at the postsecondary educational settings (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and provide guidelines for a reflective implementation of the literacy practices associated with each discipline (Zygouris-Coe, 2012). These guidelines aim for a connected transition from secondary to postsecondary settings (Holschuh, 2014b), which would secure academic success when students face more specialized literacy practices found in higher education contexts. However, the CCSS does not provide an overt definition of disciplinary literacy nor draw a disciplinary literacy framework; rather, the definition of disciplinary literacy is constructed by current research conducted by educators and researchers interested in this area of study.

In their seminal article, Shanahan and Shanahan (2008) define disciplinary literacy as "advanced literacy embedded in the content areas" (p.40), which follows a developmental pattern from the use of basic literacy conventions to the use of specialized advanced language and literacy practices. According to Shanahan and Shanahan, the more specialized literacy practices are, the less generalizable they become; and concernedly, these specialized practices are rarely taught.

The social and communicative nature of the disciplinary literacies has been noticed by other researchers as well. For instance, Airey (2011) defines disciplinary literacy as "the ability to appropriately participate in the communicative practices of a discipline" (p. 3). Dobbs, Ippolito, and Charner-Laird (2017) describe it as "the study of how both experts and novice students read, write, and communicate with specific disciplinary fields" (p. 1). Disciplinary literacy is not just limited to convey the expected register for reading and writing but also includes the necessary reasoning, investigating, and speaking required to learn and construct complexed advanced knowledge (McConachie, 2010), which carries the meaning that is required to be accepted and used among the practitioners within each discipline (Langer, 2011).

These definitions, however, need to be understood at their underlying levels as well. The two main concepts that disciplinary literacy entitles evoke Gee's (2006) claims that literacy is more than the simple ability to read and write. It is a controlling force that critiques the use of secondary languages. Gee posits that literacy is a secondary discourse (e.g., schooling discourses) that can survey primary discourses (e.g., family discourses) or other dominant discourses (e.g., discipline discourses). According to Gee, the development of secondary discourses could be explained by a process of acquisition and learning, as described by Krashen (1982) when language acquisition resembles the unconscious development of the children's first language. In contrast, language learning is the conscious development of a second language (e.g., academic, or foreign language). With this understanding of language acquisition and learning, Gee claims that literacy is not learned but acquired, and its acquisition requires of modeling practices in "natural, meaningful, and functional settings" (p. 261). In addition, Gee (2008) does not only limit the scope of literacy as a set of secondary discursive practices but also recognize its political and social implications through its controlling and critical nature.

The term discipline has also multiple layers and concomitant meanings (Krishnan, 2009). For the purposes and scope of this study, the term discipline will be used interchangeably with academic discipline, as it is related to the context of higher education. Academic disciplines are shaped by individuals with shared knowledge, beliefs, and practices (Lave, 1998); and they could be understood as what Lave and Wenger (1991) define as communities of practice:

> An intrinsic condition for the existence of knowledge, not least because it provides the interpretative support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning (p.98).

Lave and Werner's concept of communities of practices frames the disciplines as learning communities, in which their members share not only knowledge, but also norms of conduct, beliefs, customary traditions, symbols, language, and other symbolic representations of communication (Becher & Trowler, 2001). This set of cultural artifacts are mediators of the construction of the academic practices unique to each discipline and mediate the development of specific discursive practices, in which language becomes a cultural tool (Vygotsky & Luria, 1978), with a fundamental role in shaping the identity of a particular disciplinary community.

Mathematical Literacy

The CCSS standards for mathematical practice includes the need of students to engage with mathematics as a discipline while they grow in their understanding and developing of the procedural skills required to process mathematical tasks from elementary to high school (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Concurrently, the CCSS echo the standards proposed by National Council of Teachers of Mathematics (NCTM) and the National Research Council (2001) report 'Adding it Up', in which the emphasis in instruction should be oriented toward students' development of mathematical practices such as reasoning and proof, communication, representation, and connections (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The recognition by the CCSS of the distinctive features of mathematics as a discipline provides a start point toward a definition of mathematical literacy. However, it deems important to clarify certain terms that could be interpreted as synonyms, but they do not capture essentially the aim of the CCSS to understand mathematical teaching and learning from a disciplinary perspective.

The term quantitative literacy is sometimes assumed a synonym of mathematical literacy. However, quantitative literacy, also termed as numeracy or quantitative reasoning, refers to the individual's ability to reason and solve every-day quantitative problems (Madison, 2015). The following example, taken from Ramirez (2006) illustrates the accounts of quantitative literacy:

> In the 2002 Presidential election in Ecuador, six candidates obtained voting percentages between 11.9% and 20.6 % in the first round […The two first candidates in the first round were Gutierrez and Noboa; next, in the second round Gutierrez beat Noboa with

54.8% of the votes. Edwing Gutierrez was the President of Ecuador from 2002 to 2005.

We put forth some questions:

- Would Gutierrez have beaten Roldos, Borja or Neira, etc., in a one-on one competition?
- Was E. Gutierrez the most desired candidate in the election of 2002? (pp. 190 -191).

This example shows an every-day problem that requires of mathematical knowledge to interpret and apply the data in an every-day situation. The Association of American Colleges and Universities (2009) defines quantitative literacy as:

> a "habit of mind," competency, and comfort in working with numerical data. Individuals with strong QL [quantitative literacy] skills possess the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate) (p.1).

This concept contrast with proposed definitions of mathematical literacy, such as the one provided by the Expert Group for Mathematics of the Programme for International Student Assessment (PISA) of the Organization for Economic Cooperation Development (OECD), which defines mathematical literacy as:

> an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2017, p. 67).

Even though these definitions seem to be similar in their surface, each of them deal with different mathematical abilities. On one hand, quantitative literacy focuses on the individual's ability to apply quantitative principles to daily-life situations, while in the other hand, mathematical literacy emphasizes the learning and acquisition of mathematical knowledge through a developmental process occurring mainly within school contexts (Madison, 2015), and it notices the importance of mathematics for other disciplines by its ability to explain different surrounding phenomena (Lange, 2003).

The OECD definition of mathematical literacy falls short to recognize the importance of the ample social practices that mathematics creates and promotes (Jablonka, 2003); and concernedly, it is strongly based on Western tradition (Eivers, 2010; Stacey, 2010). The term mathematical literacy shall not only include the individual's ability to solve math problems; it shall also embrace the importance of contextualizing mathematical learning and practice, positioning it within a socio-cultural perspective (Colwell & Enderson, 2016; Jablonka, 2003, 2015), and acknowledging the metacognitive processes that are required to translate mathematics into real-world situations (Lester, 2013; Pugalee, 2004; Schneider & Artelt, 2010).

The National Research Council (2001) report 'Adding It Up' adopted the term "mathematical proficiency" (p. 106) as an umbrella for the terms: mathematical literacy, mathematical competence, numeracy, and mastery of mathematics. However, Kilpatrick (2001) claims that even though ' Adding it Up' did not adopt the term mathematical literacy, it reflects the strands of mathematical proficiency:

> (a) *conceptual understanding*, which refers to the student's comprehension of mathematical concepts, operations, and relations; (b) *procedural fluency*, or the student's skill in carrying out mathematical procedures flexibly, accurately, efficiently, and

> appropriately; (c) *strategic competence*, the student's ability to formulate, represent, and solve mathematical problems; (d) *adaptive reasoning*, the capacity for logical thought and for reflection on, explanation of, and justification of mathematical arguments; and (e) *productive disposition*, which includes the student's habitual inclination to see mathematics as a sensible, useful, and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one's own efficacy as a doer of mathematics (p. 107).

Although these strands recognize as important the cognitive processes unique of mathematics, they neglect to include the socio-cultural and discursive practices that mathematical practices generate and promote (Moschkovich, 2015), which are the standpoint for the development of literacy in mathematics (Gee, 2006; Lea & Street, 2017; Moje, 2008; Street, 2005). The disciplinary discourses shaped by mathematical learning and acquisition have a fundamental role in the relationship between mathematical practices and the context where these practices occur (Yore et al., 2007).

The Novice – Expert Paradigm and Disciplinary Literacy

One of the areas of interest in disciplinary literacy research is to observe how experts make meaning, communicate, and approach literacy within their area of expertise (Shanahan & Shanahan, 2018). The study of experts in disciplinary literacy could illuminate how the academic disciplines develop and construct their unique literacy practices (Shanahan & Shanahan, 2008), how these literacy practices differ across disciplines (Shanahan et al., 2011), and how novice practitioners differ in their approach to literacy compared with experts in their fields (Shanahan, & Shanahan, 2012).

However, there is a voice of caution when studying novices and experts. Jacoby and

Gonzales (1991) claim that perspectives in which novice and experts are defined as a fixed dichotomy, where the former lacks of the features of the latter, fail in capturing the complexity and fluidity of the development of knowledge within the disciplines. Rather, Jacob and Gonzales propose to understand novice and experts under their unique baggage of ways of knowing; acknowledging that both will react and perform distinctively. Besides, Scardamalia and Bereiter (1991) argue against to define expertise as a static term, as it would change from discipline to discipline and from individual to individual. Instead, expertise is constructed under a continuum of experiences (Daley, 1999; Petcovic & Libarkin, 2007; Warren, 2011); where an individual could display a high level of knowledge in a specific area and perform as a novice in others depending on their degree of disciplinary involvement (Mieg, 2009; Rouet, Favart, Britt, & Perfetti, 1997; Alan H Schoenfeld & Herrmann, 1982)

This disciplinary involvement is defined by Mieg (2009) as "expertise as professionalism" (p.93), in which expertise is defined as "professional engagement of distinguished individuals in support of their fields" (p.93). This definition of expertise positions the individual's level of commitment with their fields and their socio-cognitive relationships as critical to developing excellence in their profession (Mieg & Evetts, 2018). In areas such as mathematics, expertise could be developed under a variety of social activities, in which the experienced mathematician develop a unique point of view and competence to make meaning of the mathematical structures (Schoenfeld, 2016).

Thus, an expert in mathematics is an individual who does not only display a conceptual understanding of the necessary procedures to solve mathematical problems but also who can translate mathematical principles into their professional practice. An expert in mathematics would be able to communicate with fellow experts (Bryce & Blown, 2012), exchange practices and knowledge with professionals from other disciplines (Collins, 2011), and set standards for excellence within their own disciplines (Shanteau, 1988).

Functional Theory of Language

The functional theory of language follows M. A. K. Halliday and collaborators' work, in which language is viewed from a standpoint of its properties to create and express meaning, and studies language not through its constituents; but through its function (Halliday & Matthiessen, 2014). Halliday & Hasan (1989) view language as a “system of meanings” (p. 4) that are constructed not in isolation, but in the context where they are socially adapted. Halliday's functional perspective of language, also called as systematic-functional theory of grammar or systemic-functional linguistics (SFL), divorces itself from traditional views of grammar, in which grammar is understood as a rule-governed system conformed from fixed structures (c.f., Chomsky, 1957); instead, SFL proposes to understand GRAMMAR (SFL conventions uses small capitalization for the lexicogrammatical and discourse systems) as a meaning-making resource within its discursive context (Matthiessen & Halliday, 2009), which in Halliday’s terms is “the powerhouse where meanings are created” (Halliday, 1994, p. 15). It is the SFL understanding of language as a contextualized resource that allows this perspective to unveil how language uses selectively different means under specific social situations (Schleppegrell, 2012; Young, 2011).

Researches in disciplinary literacy (e.g., Fang & Schleppegrell, 2010; Gebhard, 2010; Huang et al., 2017; Schleppegrell, Achugar, & Oteíza, 2004; Shanahan, C. & Shanahan, 2018; Shanahan, T. & Shanahan, 2012) have found in the tenets of Halliday’s functional theory of langue the theoretical support to understand the systematic linguistic choices that occur within the context of disciplinary texts (Ebbelind & Segerby, 2015; Gebhard, 2010; Shanahan & Shanahan, 2018).

According to Young (2011), SFL is based on four tenets, which view language as a network of relationships, a system constructed by sub-systems, a functional entity, and a structure that forms from function. These tenets are developed through Halliday's notion of register, which is defined as "a configuration of meanings that are typically associated with a particular situational configuration of *field, mode, and tenor* [emphasis added] " (Halliday & Hasan, 1989, pp. 38-39).

Thompson (2004) describes the field, mode, and tenor as the metafunctions of the langue, also referred as experiential (field), interpersonal (tenor), and textual (mode) which are defined by Eggins (1994) as:

- Field: what the language is being used to talk about;
- Tenor: the role relationship between the interactants.
- Mode: the role language is playing in the interactions.

At a glimpse, these definitions portrait a transparent representation of the scope of the metafunctions of the language; and therefore, they could explain how language varies under social contexts. However, the field, mode, and tenor are constructs of the different conditions under which social-situated registers emerge under a continuum of social interactions (Eggins, 1994). Through this study, I will use the terms experiential, interpersonal, and textual to refer to the metafunctions of the language.

Figure 1 summarizes the continuing metafunctions that shape the resulting register under a particular social situation.

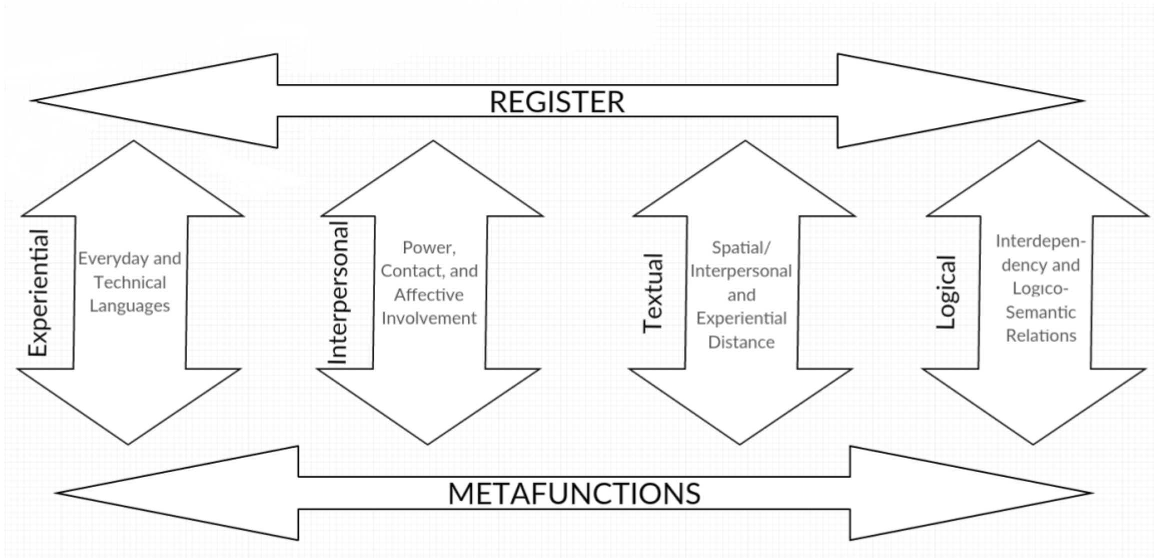


Figure 1

Metafunctions of the Language and their Components as Described by Halliday (1978) and Eggins (1994)

Figure 1 displays a fourth metafunction: the logical metafunction, in which the text is not analyzed at the clause level, but at the complex configuration level given by joining two or more clauses (Thompson, 2004). The conjoined clauses are termed by Halliday (1994) as the "clause complex" (p.216), which displays the logical relationships between clauses; therefore, the logical structure of the language.

As Colombi & Schleppegrell (2002) have noted, SFL is a theory which focuses on the social context in which the registers are produced and how they are actively constructed. Moreover, the SFL methodological tools for the analysis of how the systematic linguistic choices contribute to the formation of social contexts, how language contributes to the meaning-making, for example, of abstract concepts, and how language contributes to the development of specialized language found especially within the academic disciplines (Fang, 2005).

SLF makes extensive use of labels, which purpose is to show how the grammar of clauses and texts are attached to their meanings (Martin, Matthiessen, & Painter, 1997). In the forthcoming sections, I will provide an account of the SLF labels and definitions of each of the metafunctions of the language. Additionally, I will follow Halliday and Matthiessen (2014) conventions of capitalization, bolding, and italicization of the labels used for SLF analysis.

The Experiential Metafunction: Clause as Representation.

The experiential metafunction, also termed field, describes how language enables speakers to act, experience, or relate to each other. The experiences that speakers undergo while they interact with the world generate patterns of processes, which tell about the events that occur during these interactions (Halliday, 1994). To describe the systems of processes that occur when speakers experience the word, Halliday uses the term TRANSITIVITY[1], which refers to "the type of process designated in the clause and the consequences of this for the types of participants that can occur in the clause" (Hart, 2014, p. 22). Halliday and Matthiessen (2014) identify 6 types of processes in the English language, which include each particular domain where the experience takes place. Additionally, each of the process types relates to different Participants[2] and Circumstances.

The first type of process is the material, the process of doing and happening. In this type of process, the clause displays change in events through the input of some energy (Halliday & Matthiessen, 2014). In the material processes, Halliday labels the participant as Actor and the circumstance as Goal. As Halliday and Matthiessen define, the Actor is the one who produces the change while the Goal is the result of the intentional production of change. The material process can be of two types Creative or Transformative (done to). In Creative processes, the Actor brings to existence the Goal. Figure 2 describes the functional analysis of a creative clause.

[1]According to Halliday and Matthiessen's (2014) conventions in SFL, small capitals are used for the name of a system.
[2] Initial capital is used for structural function names.

I	made	some cookies	for the potluck
Actor	**Process: material, creative**	**Goal**	**Circumstance**

Figure 2
Functional Analysis of a Material Creative Clause

Figure 3 shows the functional analysis of a transformative clause, in which the Actor transforms the Goal.

The car	crashed	into that house
Actor	**Process: Material**	**Goal**

Figure 3
Functional Analysis of a Material Creative Clause

The second type of process is the mental, referred by Halliday as processes of sensing. The mental processes are a semantic category that deals with mental activities such as thinking, reasoning, feeling, imagining, and so forth (Thompson, 2004). As in the material processes, the mental processes require a different label for their participants; in this case, it is the Senser, "who feels, thinks or perceives, must be human or an anthropomorphized non-human. It must be conscious being" (Eggins, 1994, p. 242). As the core of the mental processes focuses on senses and feelings, they differ from the material process in failing to explain 'who does what' within the clause. For this reason, Thompson advises giving the mental process the following more specific categories: emotion (processes of feelings), cognition (processes of knowing), perception (processes of the senses), and desideration (processes of wanting).

Differently than the material, the mental processes allow a second entity to fill the place of the Senser. Halliday labels this entity as the Phenomenon, which is what the Senser feels thinks, perceives, or desires (Eggins, 1994). The Phenomenon occupies a less restricted position compared with the Goal position within the material processes; it can be the object of the mental experience, or it can be the metaphorically realized as a nominal group (Halliday & Matthiessen, 2014). Figure 4 shows the functional analysis of the Phenomenon occupying the Goal position.

She	wanted	more than just cookies
Senser	**Process: mental, desideration**	**Phenomenon**

Figure 4
Functional Analysis of a Mental Clause - Desideration Clause

Figure 5 displays a functional analysis, in which the Phenomenon is realized metaphorically as a nominal group.

The following calculations	seem to daunt	her students
Phenomenon	**Process: mental, perception**	**Senser**

Figure 5
Functional Analysis of a Mental, Perception Clause with Phenomenon as a Nominal Group

The Relational processes are the third type of processes; Halliday refers to them as processes of being and having. The primary function of the relational processes is to define and portrait (Halliday & Matthiessen, 2014). In the relational clause, there is not a process in the sense of something being produced; instead, the relational processes describe the relationships that the participants are experiencing and the conditions in which these relationships occur (Thompson, 2004). As the essence of the relational processes is to describe relationships, the labels that Halliday uses for the material and mental processes do not capture the kind of relations that occur within the relational clauses (Thompson, 2004).

Halliday labels the relational processes depending on the type of relationship that occurs within the clause. Halliday and Matthiessen organize and label the relational clause system according to the type of relation (intensive, possessive, and circumstantial), which can follow two models of being: attributive and identifying. The attribute relational processes assign a quality, characteristic, or classification to a Carrier, which is a noun or nominal group in the relational clause (Eggins, 1994). Figure 6 displays an example of the functional analysis of a relational attributive clause.

The turkey	was	superb
Carrier	**Process: rel, attrib**	**Attribute**

Figure 6
Functional Analysis of a Relational, Attributive Clause

The identifying relational processes define the identity of a Token (identity given by form) by assigning a Value (identity given by function) (Halliday, 1994). One of the distinctions between attributive and identifying relational clauses is that the processes that occur in the latter are interchangeable; however, the attributive clauses are not interchangeable. Figure 7 and 8 display the differences between attributive and identifying relational clauses.

The turkey	was	superb
Carrier	**Process: rel, attrib**	**Attribute**
* Superb	was	the turkey

Figure 7
Interchange between Attribute and Carrier in a Relational, Attributive Clause

As Figure 7 displays, the interchange of the Attribute and Carrier in a relational, attributive clause results in an ungrammatical clause in the English language. The relational identifying clause allows the speaker to choose what they value in it, being the Value of a higher content value than the Token (Halliday & Matthiessen, 2014, p. 279). Figure 8 displays the interchange between Token and Value.

The result of this problem	is	five
Token	**Process: rel, ident**	**Value**
Five	is	the result of this problem
Value	**Process: rel, ident**	**Token**

Figure 8
Interchange of Token and Value in a Relational, Identifying Clause

The fourth type of process in the transitivity system is the Verbal processes, which encompass the verbs that reflect the transference of messages through language (Thompson, 2004). In the verbal clause, the Sayer is the participant who sends the Verbiage (message) to the Receiver. However, not all verbal clauses are structured linearly. According to Thomson, in some verbal clauses, the message is expressed in a separated reported clause, which is called projection. Figure 9 shows a linear verbal clause, while Figure 10 displays a separated projected clause.

They	ordered	me	to pick him up
Sayer	**Process: verbal**	**Receiver**	**Verbiage**

Figure 9
Functional Analysis of a Verbal Clause

In a separated projected clause, the participants and processes are analyzed separately to distinguish between the processes of the verbal clause and the ones used in the reported clause.

He	told	me	that you were at home
Sayer	**Process: verbal**	**Receiver**	
Projecting			**Projected**

Figure 10
Functional Analysis of a Separate Projected Clause

The last two types of processes are the behavioral and the existential**.** Halliday and Matthiessen (2014) refer to the behavioral to the typically human processes of physiological or psychological representations (e.g., laugh, cough, or swallow). The behavioral processes seem to overlap the semantic connotations of the mental processes; however, it is in the physiological representation that they differ. On the one hand, the mental processes are internal processes of the mind, while on the other hand, the behavioral processes display signs of physiological functions (Thompson, 2004). In the behavioral clause, the participant is labeled as Behaver.

Figure 11 shows the functional analysis of a behavioral clause.

The young man	chews	tobacco	without knowing its effect
Behaver	**Process: behavioral**	**Range**	**Circumstance**

Figure 11
Functional Analysis of a Behavioral Clause

According to Thompson, the Range is one of the possible participants of the behavioral clause as it provides additional information about the domain of the process.

The existential processes refer to the existence of an entity within the clause. The use of *there* recognizes these processes. In the existential clause, there occupies the place of the subject as it is not the process of the clause. These types of clauses have only one participant: the Existent (Thompson, 2004). Additionally, the existential clause requires a circumstantial element of time or location (Halliday & Matthiessen, 2014). Figure 12 captures the functional analysis of an existential clause.

There	‘s	bread and milk	in the fridge
	Process: existential	**Existent**	**Circumstance**

Figure 12
Functional Analysis of an Existential Clause

To conclude this section, it deems important to summarize the system of labels of the experiential metafunction. The system of labels for the experiential metafunction is summarized in Figure 13.

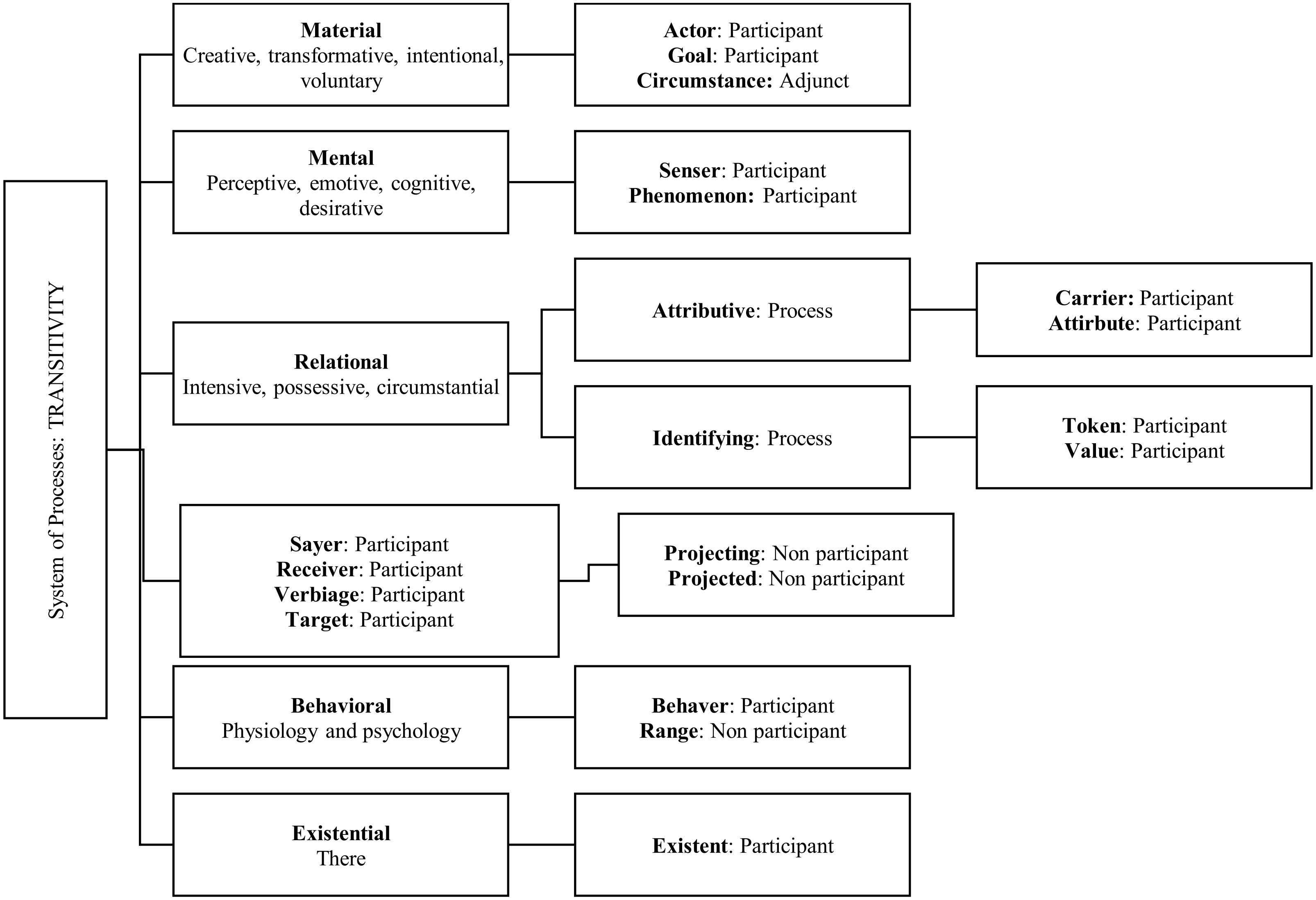


Figure 13
Labels of the Experiential Metafunction – Transitivity System

The Interpersonal Metafunction: Clause as Exchange.

The interpersonal metafunction deals with the interpersonal meanings that occur within a clause. Eggins (1994) refers to this metafunction as tenor: the relationships that occur among interactants. These relationships are framed by the nature of the language to exchange goods and services or information. These exchanges are based on four main functions of the language: offer, command, statement, and question**,** which will be matched with an expected or unexpected response (Halliday, 1994). These functions as well as the responses produced among interactants are associate with particular grammatical structures, which Halliday labels as the MOOD (with capital M): a system that provides to the interactants the linguistic resources to exchange goods and services or information using particular grammatical structures depending of the context of the interaction (Halliday & Matthiessen, 2014).

Two main elements constitute the structure of the MOOD: the Subject (a nominal group) and the Finite (operator), which is part of a verbal group. Thompson (2004) explains that in order to identify the subject and the finite within a clause, it is necessary to add a tag question to the clause. Figure 14 illustrates how a tag question is used to identify the subject and finite.

Working on this proof	was	a real challenge	wasn't	it?
Subject	**Finite**		**F**	**S**

Figure 14
Tag Question to Identify the Subject and Finite

The Subject, traditionally identified as a nominal group, provides information about the entity that is "responsible for the functioning of the clause as an interactive element" (Halliday & Matthiessen, 2014, p. 146). The Finite has the property to promote the interactions between interactants in positive, negative, time, or modality terms (Martin et al., 1997). It is through this property that the clause can be arguable (Eggins, 1994) and the interaction between interactants develops.

All the elements that are not the Subject nor the Finite are called the Residue, which consists of Predicator, Complement, and Adjunct. In the English clauses, there can be one predicator, one or two complements, and, in principle, up to seven adjuncts (Halliday & Matthiessen, 2014).

Each of the constituents of the residue realizes specific functions within the clause. Table 1 summarizes the functions of the elements of the residue and provides examples of the grammatical structures of these elements as described in Halliday and Matthiessen (2014), Thompson (2004), and Eggins (1994).

Table 1

Elements of the Residue

Element	Description	Function	Example
Predicator	Verbal group that does not include the Finite	Describe the processes that affect the Subject, the secondary tenses, and the voice of the clause.	*Peter has been **working** so hard*
Complement	A nominal group that has the potential to be the Subject of the clause	In cases where the Complement cannot become the subject of the clause, it offers an attribute to the subject.	*Daniel is working with **his colleagues***
Adjunct	Elements that do not have the potential to be the Subject of the clause.	Provide additional information about when, how, where, or why the event in the clause happened.	*Has Anne provided any response **yet**?*

The Adjunct(s) can be modal, conjunctive, circumstantial, or comment depending on the additional meaning that they provide to the experiential clause. The modal adjuncts provide interpersonal meaning to the clause, which implies that the interactants would have additional information to maintain the interaction (Eggins, 1994). The conjunctive modals are elements that signal how the clause fits as a whole in the preceding text (Thompson, 2004). According to Eggins, the circumstantial adjuncts add experiential meaning related to the processes of the clause, while comment adjuncts assess the clause as a whole. Figure 15 exemplifies the structure of the Mood and Residue.

We	weren't	prepared	enough	to endure	that journey
Subject	**Finite**	**Predicator**	**Adjunct: circumstantial**	**Adjunct: comment**	**Complement**
Mood		**Residue**			

Figure 15
Structure of the Mood and Residue

Thompson claims that the structure of the MOOD provides the elements to create a system of choices, which allows interactants to exchange information in declarative and interrogative terms; however, the MOOD system is more complex and includes as WH-interrogatives, explanative, and imperative choices as well. These choices command the structure of the experiential clause and the type of exchange that occur among interactants. Thus, the MOOD provides the information to the interactants to continue or conclude the exchange based upon the grammatical structure of the experiential clause.

The Textual Metafunction: Clause as Message.

The textual metafunction or mode is represented by what Halliday calls Theme, which is concerned about the organization of individual clauses and their role in the organization of the whole text (Martin, Matthiesssen, & Painter, 1997). The Theme is the starting point of the message and directs the interactants to locate the clause within its context and enables them to process the message (Halliday & Matthiessen, 2014). The second element of the textual clause is the Rheme, the element that provides additional information to the message. In simple terms, the Rheme is the remaining elements that are not part of the Theme (Eggins, 1994).

The Theme can be unmarked or marked. The unmarked Theme is the one found, for instance, in declarative clauses, in which the Subject and the Theme are the same and are commonly a nominal group (Thompson, 2004). In certain clauses, the Theme conflates with other elements of the MOOD system, such as adjuncts; Eggins explains that in this case the Theme is marked. Figures 16 and 17 exemplify unmarked and marked Themes.

Raphael	was	delighted	to see the mountains
Subject	**Finite**	**Predicate**	**Adjunct: circumstantial**
Mood		**Residue**	
Theme	**Rheme**		

Figure 16
Unmarked Theme - Theme as the Subject

If I were you,	I	wouldn't	show	her	that picture
Adjunct: conjunctive	**Subject**	**Finite**	**Predicate**	**Adjunct: modal**	**Complement**
	Mood		**Residue**		
Theme	**Rheme**				

Figure 17
Marked Theme – Adjunct as Theme

As Figure 17 illustrates, interpersonal and experiential elements can be part of the Theme as well. Depending on the metaelements, it is possible to distinguish different types of Themes depending on the elements that are part of them. According to Eggins (1994), Topical Themes are the ones, in which a Transitivity element occupies the initial position in the clause. Interpersonal Themes display a mood element as the Theme of the Clause. Finally, the Textual

Themes do not display an experiential or interpersonal meaning; however, they provide important cohesive structure to relate the clause to its context. Table 2 summarizes the different types of Themes as defined by Eggins and Halliday and Matthiessen (2014).

Table 2

Summary and Examples of the Types of Themes

Type of Theme	Description	Example
Topical	Contains one experiential element	***Whether*** *you like it or not*
Interpersonal	Contains a mood element	***Would*** *you be okay with it?*
Textual	Contains a textual element	***Finally,*** *my package has arrived*

The word Theme is the label used to describe the THEMATIC system, which according to Fries (1995) has four main functions within a clause: 1) Signals the maintenance or progress of the purpose of the message, 2) specifies or changes the framework to interpret the upcoming clauses, 3) signals the boundaries of the text, and 4) shows what the speakers intents to mark as the starting point of the message.

To summarize, Halliday's labeling system indicates the class and function of each one of the elements of a clause. Additionally, labeling tells about the different functions and meanings that simultaneously occur within a clause (Eggins, 1994). However, Halliday (1994) cautions and advises not to assume that there is a transparent correspondence between function and labeling, the same element of a clause can function differently depending on the context where the clause occurs. The system of labeling indicates the structure of a given clause and how its elements behave within its boundaries.

Guiding Research Questions

The main aim of this project is investigating how pre-service teachers develop their disciplinary heuristics, and how language mediates their learning and acquisition of mathematical literacy. In order to achieve this aim, the following research questions are proposed:

1. What do the experiences of pre-service teachers and experts in mathematics reveal about their understanding of mathematical literacy?

This question accounts for the importance of the pre-service teachers' experiences as learners in shaping their set beliefs and attitudes that could be translated into their teaching and literacy disciplinary practices. The mathematics experts' experiences are worthy of study because of their influence in their practice as educators.

2. How do pre-service teachers and experts in mathematics use language when solving mathematical problems?

The second question addresses the role that language plays in the learning and acquisition of mathematics as a discipline and how language mediates the development of the unique literacy practices found in mathematics. By looking at the linguistic registers that experts in mathematics utter while solving mathematical problems, I aim to trace patters of language development in pre-service teachers.

3. What literacy practices do pre-service teachers and experts in mathematics utilize when presented with modules that require mathematics problem-solving?

In the last question, I intend to observe the literacy practices that are particular to mathematics. How pre-service teachers read and write when solving mathematical problems could illuminate the practices that are particular to mathematics as a discipline. Similarly, the way by which mathematical experts read and write could provide clues of the literacy practices that pre-service teachers might apply in their professional practice.

Statement of the Problem

This study addresses the question of whether or not pre-service teachers are ready and prepared to use and teach the highly specialized language of each discipline. The disciplinary languages present teaching and learning challenges due to their lacks of parallels in daily language (Shanahan & Shanahan, 2008). Additionally, the languages of the disciplines are rarely taught and are commonly acquired through an isolated representation of words without a situated meaning within the theory (Gee, 2002). The knowledge of the particular ways of reading, writing, listening to and talking in the content areas provide opportunities for students' apprentice within the disciplines required for success in higher education contexts (Dobbs, Ippolito, & Charner 2017).

Moreover, this study addresses the question of how future teachers develop disciplinary knowledge and skills. It is expected from teachers, especially at the secondary levels, to be experts in the disciplines they teach; however, it is concerning that is not uncommon to find that teaching education programs do not focus instruction in the disciplinary ways of knowing and talking (Dobbs et al., 2017). Thus, the design of teaching education programs should include explicit instruction of the disciplinary ways of speaking, listening, reading, and writing in addition to the general abilities that every pre-service teacher would develop during their academic program, but also on the specific language and literacy practices unique to the content area of their specialty.

This study enlightens how language mediates the acquisition of skills related to a specific area of knowledge. Therefore, it would contribute to the study of academic language acquisition with a focus on mathematics. It provides a general understanding of how college students (pre-service teachers) develops disciplinary literacy. The analysis of how college students develop specific ways of talks, read, and write would draw a possible path for secondary teachers, policymakers, and administrators on what kind of knowledge and skills high school students should develop to be successful at higher education institutions and as Langer (2011) claims, the importance of academic literacy relies not only on the pedagogical content related to the disciplines, but also on the linguistic, cognitive, and sociocultural practices proper of each discipline.

CHAPTER 2

REVIEW OF LITERATURE

This review of literature is guided by the research questions that I posit for this study. The first section of this review aims to draw the relationship between learning experiences and mathematical literacy practices. Then, I will delve into current research about how pre-service teachers develop their mathematical literacy. In its final section, this review entails research about how differently experts and pre-service teachers use language to display their disciplinary literacy.

Pre-Service Teachers' and Experts' Disciplinary Experiences as Learners of Mathematics

Pre-service teachers' experiences as learners are important to study because they play an important role in shaping their goals, intentions, and beliefs (Towers, Hall, Rapke, Martin, & Andrews, 2017) about themselves as future teachers and for their future students as well. Research in pre-service teachers' experiences as learners, in areas such as mathematics, shows how they face their own and their students' learning (Ball, Lubienski, & Mewborn, 2001; Llinares & Krainer, 2006), their understanding of the purpose of mathematics, (Stuart & Thurlow, 2000), and their establishment of their systems of knowledge, beliefs, and attitudes related to mathematics education (Ernest, 1989; Stuart & Thurlow, 2000). Additionally, pre-service teachers' of mathematics relate their mathematical-communicative encounters within classrooms with their experiences as learners (Brendefur & Frykholm, 2000).

Pre-service teachers' experiences have also important implications in shaping the cultural norms and practices that they bring to their discipline (Blömeke & Kaiser, 2015; Moje, 2008; Pajares, 1992) and in understanding the ways of reading, writing, and communicating that are unique to their area of expertise (Botha, 2011; Colwell & Enderson, 2016; Colwell & Gregory, 2016; Guillaume & Kirtman, 2010). In mathematics, for example, pre-service teachers' beliefs and experiences seem to be related to their understanding of the importance of developing disciplinary knowledge, language, and ways of communicating for the individual's ability to apply mathematics into real-word situations (Guillaume & Kirtman, 2010)

Although, pre-service teachers' experiences as learners are important in shaping their future classroom practices, these experiences can also lead to emotional challenges in pre-service teachers, especially in mathematics (Philipp, 2007). For instance, Bekdemir (2010) found that pre-service teachers' negative experiences, when learning mathematics, are related to the development of mathematical anxiety (see Tobias [1980] for a detailed definition of this construct), which has an impact on teaching confidence (Brady & Bowd, 2005; Bursal & Paznokas, 2006), sense of efficacy (Gresham, 2008; Swars, Daane, & Giesen, 2006), and avoidance to teach it (Kelly & Tomhave, 1985).

Similarly, in the case of the experts (professionals engaged with teaching mathematics), learning experiences account for their development of teaching beliefs as well (Cross, 2009; Kane, Sandretto, & Heath, 2002). However, experts' learning experiences shape their teaching practices and beliefs differently. It seems that experts use their learning experiences as reflective processes about the nature of learning (Boston, 2013). Additionally, experts' experiences are reflective of the domains that construct their area of expertise (Eicher & Erens, 2015; Kagan, 1992; Neumann, 2001; Oleson & Hora, 2014) and are conjoined with their professional experiences within their disciplines (Alexander & Dochy, 1995; Hativa, Barak, & Simhi, 2001; Oleson & Hora, 2014). The union of learning and professional experiences contribute to the development of specialized teaching practices displayed by more experienced teachers, especially in higher education contexts (Oleson & Hora, 2014).

Pre-Service Teachers' Development of Disciplinary Literacy

Research about pre-service teachers' development of disciplinary literacy could illuminate the disciplinary practices that teachers would apply later in professional settings (Carlson, 2015; Johnson, Watson, Delahunty, McSwiggen, & Smith, 2011; O'Brien & Stewart, 1992; Olson & Truxaw, 2009; Pytash, 2012; Short, 1995); therefore, the forthcoming section will discuss research about how preservice teachers develop their understanding of disciplinary literacy. Especial attention will be given to research about the development of mathematical literacy in pre-service teachers; however, findings in other disciplines (e.g., Language Arts, Geography, Science) will be also considered for this section.

Early in their programs, pre-service teachers are able to distinguish practices that are unique to their disciplines (Masuda, 2014), and this ability continues growing through their undergraduate programs, including in those who pursue a master's degree (Park, 2013) and moves from a traditional understanding of literacy, as decoding of printing reading and writing, towards a broad interpretations of the multiple forms of literacy within academic contexts (Masuda & Ebersole, 2013). According to Masuda (2014), the emergence of pre-service teachers' disciplinary literacy includes an understanding of discipline-related habits of thinking, reading and writing practices and demands, use of disciplinary language and vocabulary, and application of discipline-related instructional tools. These understandings come across the use and application of content area literacy strategies to support students' learning; and although pre-service teachers recognize the importance of strong reading and writing skills for disciplinary instruction, they did not display an overt acknowledgment of disciplinary-related practices to support students who are struggling with academic reading and writing (Masuda, 2014).

Similarly, Pytash (2012) designed a study in which pre-service teachers from different areas build their understanding of disciplinary literacy by creating an assignment in which they recognize the literacy practices unique to their disciplines.

In Pytash's study, the participant pre-service teachers had to ask a professional in their area of expertise, about the types of writing they perform during their daily professional practice. Then, within their content areas, these pre-service teachers reflected upon the disciplinary practices they consider prevalent and unique. Pytash found that the pre-service teachers recognized the different narrative genres that are unique to each discipline, examined the narratives genres and content of these narratives, and analyzed the kind of language, specific vocabulary, and disciplinary slang that the authors used to convey meaning.

In areas such as mathematics, pre-service teachers do to only have a complex understanding of the importance of foundational writing and reading skills; but also an understanding that these skills should function in ways that allow students to interpret, recognize, reasoning, and solve mathematical problems (Masuda, 2014). Mathematical literacy in pre-service teachers describes their foundational knowledge of content-area literacy, the importance of disciplinary language and communication for mathematical instruction, and the application of mathematical principles for a variety of real-life situations (Colwell & Enderson, 2016).

In summary, the development of disciplinary literacy in pre-service teachers follows a progressive path. From a basic stance, in which literacy is defined under traditional lenses, toward a more complex understanding of the role that disciplinary practices play in constructing a variety of literacy forms in the content area classrooms. Among these disciplinary practices, researchers (e.g., Adams, 2003; Colwell & Enderson, 2016; Kaiser, 2005; Lenski & Thieman, 2013; Masuda, 2014; Park, 2013) have identified language as one of the most disciplinary features in the content areas; thus, the forthcoming sections will discuss the relationship between language and mathematics with special attention about how language mediates mathematical problem-solving.

Mathematical Problem-Solving Strategies by Pre-Service Teachers and Experts in Mathematics

Mathematical problem-solving has been largely studied for its importance in educational, cognitive, and didactic features of mathematical education. As Halmos (1980) claims, problem-solving is central for mathematics, and "the mathematician's main reason for existence is to solve problems, and that, therefore, what mathematics *really* consists of is problems and solutions" (p. 519, emphasis in original). Moreover, the study of mathematical problem-solving provides clues to understand other concepts related to mathematics, such as an individual's beliefs about mathematics as discipline (Weber & Leikin, 2016).

Much of the research on mathematical problem solving followed A. H. Schoenfeld's study of the relationship between students' abilities to solve problems and whether or not these abilities can be taught (Weber & Leikin, 2016). Schoenfeld (1985) proposes that the mathematical problem-solving performance is mediated by: 1) the individual's mathematical knowledge required to solve a problem (resources), 2) a set of strategies and techniques that are either effective to solve a problem, or applied to solve unfamiliar problems (heuristics), 3) metacognitive decisions to determine and apply the required strategies to solve a problem (control), and 4) the way in which an individual approaches mathematics, which is negotiated by the individual's beliefs about mathematics (i.e., belief systems).

Schoenfeld identified resources, heuristics, control, and belief systems as the kind of knowledge and behaviors required to solve mathematical problems by studying the differences between the novices' and experts' performances when solving problems. Schoenfeld &

Herrmann (1982) found that novice mathematicians approach mathematical problems by perceiving them from a surface structure, in which, although the problem is perceived with certain elements of mathematical knowledge, it is not approached from a more abstract stance found in more experienced practitioners (Chi, Feltovich, & Glaser, 1981; Hardiman, Dufresne, & Mestre, 1989).

On the other hand, experts perceive problems from a deep structure, which implies the experts' ability to apply unique mathematical principles to solve problems (Schoenfeld & Herrmann, 1982a). Similar results were found in Hardiman, Dufrense, and Mestre (1989) when studying differences between novice students and expert physicists. However, in the Hardiman et al (1989) study, another point of discussion of these differences rely on the experts' ability to use principles, concepts, and definitions to provide an explanation of their understanding of both the problem and how to solve it. Hardiman et al. note that less experienced students base their explanations of their previous experiences with similar problems, and these explanations were less elaborated than their counterpart.

The Hardiman et al. 's findings seem to be related to the 'know-how' that Chick and Stacey (2013) define as intrinsic to mathematical problem-solving. Chick and Stacey found that experienced mathematicians' approach problem-solving by not just activating the necessary cognitive processes but also applying their knowledge of problem-solving into situations outside the mathematical domain. This ability seems to intersect with the teacher's pedagogical knowledge, which allows teachers to expand their explanation of a problem in terms of its applicability in daily routines, and support students' learning with issues that emerge when solving a problem (Borko & Livingston, 1989).

The teachers' ability to explain a problem in the disciplines is constructed and constrained by his/her language (Leinhardt, 2010). Therefore, the forthcoming section will discuss how language enables learners to solve mathematical problems from a disciplinary perspective and using as the framework of reference Halliday's functional grammar.

Mathematics and Language

Regardless of the perception about how mathematical problems are solved, the different views about problem-solving intersect in a common point: how language mediates the individual's ability to solve a mathematical problem. Mestre (1988) found that language proficiency could influence problem-solving performance. This finding ties to the claims that language is fundamental to express mathematical ideas (Moschkovich, 2010; Radford & Barwell, 2016b; Schleppegrell, 2007; Winsløw, 2000). For instance, Winsløw (2000) claims that language is used to interpret the abstracts elements of mathematical thinking and to enlight the nature of mathematical knowledge.

The relationship between language and mathematics has been tested under multiple perspectives. One of them, for example, has analyzed the relationship between the discrete features of the language and the development of mathematical teaching and learning in areas such as vocabulary (Capraro & Joffrion, 2006; Nagy, Townsend, Lesaux, & Schmitt, 2012; Riccomini, Smith, Hughes, & Fries, 2015), nominalization (Bueno, 2012), or modality (Hodges, 2013). These areas of research, even though important to shape an empirical understanding of how mathematics and language are connected, do not address the wide range of functions that language performs, especially as a social and communicative endeavor. As Duval, Ferrari, Høines, and Morgan (2005) argue regarding the importance of language for problem-solving:

> Significant steps are being made in describing forms of language that are appropriate for expressing mathematical ideas or for engaging in mathematical 'forms of life' [emphasis

in original] and that function effectively for learners engaged in mathematical problem-solving. These descriptions involve not only identification at the lexical level of vocabulary, notational or graphical elements but also the choice, combination, and manipulation of these in texts that are functional in producing and/or communicating mathematics. (p. 797)

In other words, the relationship between mathematics and language goes beyond the discrete features of the language and how they are applied under mathematical contexts. This relationship is innate to the functions that language plays under the social and semiotic nature of the teaching and learning of mathematics (Morgan, 2006). In addition, the nature of mathematics is not represented by only one set of objects (i.e., numbers); it encompasses letters and words, symbols, graphics, diagrams, and so on. The multiple sets of objects that are essential to structure the mathematical thought shape its multisemiotic nature (O'Halloran 2000, 2005, 2015), in which the functions of the language create Discourse (Gee, 2008) to represent the mathematical thought through its symbolism and the graphic representation (O'Halloran, 2015).

Moreover, mathematical Discourses are dependent on the social context where they are systematically produced and reproduced (Ongstad, 2006). For instance, Ernest (2006) found that the development of the mathematical semiotic systems involves two social roles: the listener or reader, and the speaker or writer; while, these social roles are constrained by semiotic bundles such as gestures and gazes (Arzarello, 2006). These findings depict the multiple sources of semiotic representations unique to the mathematical Discourses, which are mediated by the social contexts where they are systematically acquired.

The mediation of the social contexts in the acquisition and development of the mathematical Discourses evoke Halliday's approach to understand the relationship between language and mathematics. For Halliday, the mathematical Discourse, defined as register, emerges in a developmental fashion, not because it progressively incorporates technical vocabulary, but because it is intrinsically related to the nature of mathematics as a discipline, which depends on the social activities unique to it (Halliday, 1978). It is Halliday's Systemic Functional Linguistic (SFL) framework that explores "the nature of interpersonal, experiential, logical and textual meanings afforded by symbolism, and the strategies through which these meanings are encoded" (O'Halloran, 2005, p. 96). In order to continue exploring current research on the relationships among language, mathematics, and disciplinary literacy, the forthcoming sections will emphasize SFL as a framework of choice to understand these relationships.

Systemic Functional Linguistic Studies in Mathematical Encounters

As discussed in the abovementioned sections, SFL provides a framework for an understanding of the multiple semiotic resources that are applied to make sense of the mathematical activity (O'Halloran, 2003). Studies under the SFL framework have mainly addressed two spheres: the social and discursive semiotic nature of the mathematical register.

In the social sphere, SFL has provided analytic tools to understand, for example, how mathematical register differs from everyday language (de Freitas & Zolkower, 2011). One of the results of the application of these analytical tools is the emergence of social semiotics, "the context in which he [a child] himself will learn to mean, and in which all this subsequent meaning will take place" (Halliday, 1975, p. 125). Social semiotics has been explored to address questions related to the ways in which language and other semiotic resources convey in mathematical communications and personal relations to construct meanings, beliefs, and attitudes (Morgan, 2006); to the analysis of cultural influences in mathematical education

(Ongstad, 2006), or to the emergence of linguistic devices (e.g., grammatical metaphor) that influence the interpretation of mathematical abstractions (Torr & Simpson, 2003).

The discursive sphere has focused its attention on the differences between the mathematical and daily register in classroom contexts (Schleppegrell, 2004), the multisemiotic nature of the mathematical discourse (O'Halloran, 2000), the linguistic challenges that English language learners face when acquiring the mathematical register (Olivera & Cheng, 2011).

Summary

This review of literature focuses on three main areas: a) pre-service teachers' and mathematical experts' experiences as learners of mathematics, b) the development of disciplinary literacy in experts and pre-service teachers, and c) the relationship between mathematics and language. The pre-service teachers' experiences as learners have a deep impact on their beliefs about teaching and learning, which in turn, have an impact on the way pre-service teachers perceive mathematics and mathematics education (Ernest, 1989). The experts' experiences as learners impact them differently. Experts use their experiences as reflective processes (Boston, 2013), which includes not only their experiences as learners but also their experiences as professionals, which contributes to their development of specialized literacy practices.

The development of literacy practices in pre-service teachers follows a developmental process, which starts with traditional definitions of literacy to a later wider understanding of multiple forms of literacy within academic contexts. In areas such as mathematics, pre-service teachers, early in their programs, develop notions of the importance of language for mathematical instruction. Research in the development of disciplinary literacy has identified as language as one of the most salient characteristics of the academic areas.

In mathematics, the role that language plays in its learning and acquisition has been widely studied. One of the focus of the relationship between mathematics and language is the relationship of mathematics and discrete features of language (e.g., lexicon); however, this focus of research neglects to address the importance of the social and communicative purposes of language. It is through other focuses, such as the one proposed by systemic functional linguistics (SFL) researchers that the social and communicative features of the language are better framed. SFL research has addressed the social and discursive spheres of the mathematical register; however, there is a gap in the research of the relationship between language and mathematical literacy development analyzed by the SFL lenses.

CHAPTER 3

METHODOLOGY

Three research questions guide this study: 1) What do pre-service teachers' and mathematics experts' experiences as mathematical learners reveal their understanding of mathematical literacy? 2) how do pre-service teachers and experts in mathematics use language when solving mathematical problems? and 3) what literacy practices do pre-service teachers and experts in mathematics engage with when presenting modules that require mathematics problem-solving? These research questions inquiry about the particular ways in which the participants respond to the development of their literacy practices. My understanding that each one of the participants has unique linguistic repertoires, ways to construct learning and knowledge, and responses to disciplinary literacy position the importance of the participants' unique characteristics as the foci of this study. The aforementioned characteristics of this inquiry lead to the selection of a collective case study (Creswell & Poth, 2018) as the approach of choice for this qualitative study. As Yin (2006) points, "the case study method is best applied when research addressed descriptive or oral-explanatory questions and aims to produce a firsthand understanding of people and events" (p. 112). The case study research methodology implies an in-depth study of a current phenomenon under its natural context, especially when the boundaries between context and phenomenon are not distinguishable (Yin, 2018). Additional considerations of the methodological design account my experiences during the development of the pilot study. I recorded my research in fieldwork notes taken during the pilot, and I addressed these experiences throughout each section of this manuscript.

Research Methodology

For the methodological design of this collective case study, I was guided by the Creswell and Poth's (2018) definition of case study research:

> Case study research is defined as a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving ***multiple sources of information*** [emphasis in the original] (e.g., observations, interviews, audiovisual material, and documents and reports), and reports a ***case description*** [emphasis in the original] and ***case themes*** [emphasis in the original]. The unit of analysis in the case study might be multiple cases (a multisite study) or a single case (a ***within-site*** [emphasis in the original] study)" (p. 96).

This guiding definition provides the structure of the methodological design towards a data collection from multiple sources, which provided an in-depth description of each one of the participants (Hancock & Algozzine, 2011), and drew the themes and subthemes that emerged during its development.

Data Collection Methods

Participants

I invited pre-service teachers and instructors of mathematics at a large, public-research Midwestern university in the United States. Four pre-service teachers and four instructors of mathematics accepted my invitation to participate in this study. The pre-service teachers were students at different years of their programs. The instructors of mathematics were from diverse areas of expertise and had different years of professional practice. Through this document, I refer to the instructors of mathematics as either experts in mathematics or mathematicians as both terms imply expertise. Table 3 describes the instructors of mathematics.

Table 3

Description of the Instructors of Mathematics

Pseudonym	Gender	Title at Institution	Years of Experience	Linguistic Background
Dr. Arnold	Male	Professor	25 years	Native speaker of English Intermediate speaker of German Beginner speaker of Russian
Ms. Briggs	Female	Lecturer	2 years	Trilingual speaker of Mandarin, English, and Malay
Dr. Dunn	Male	Professor	12 years	Bilingual Speaker of Mandarin and English Beginner speaker of French
Dr. McFarlane	Male	Professor	30 years	Monolingual speaker of English

The instructors of mathematics come from a variety of areas of expertise. However, to maintain the identity of the experts anonymous, I decided not to disclose their area of expertise as it could reveal their identity. Additionally, I assigned a randomly generated pseudonym to each of the participants. Table 4 describes the pre-service teachers.

Table 4

Description of the Pre-Service Teachers

Pseudonym	Gender	Program	Year	Linguistic Background
Cesar	Male	Special Education	Junior	Native speaker of English Beginner speaker of Spanish
Maggie	Female	Early Childhood	Senior	Bilingual speaker of Spanish and English
Ruby	Female	Elementary Education	Senior	Native speaker of English Intermediate learner of Spanish
Sophie	Female	Elementary Education	Senior	Monolingual speaker of English

Data Gathering Procedures

I gathered multiple sources of data. For this purpose, I followed Yin (2018) guidelines to gather "six sources of evidence" (p. 113). The six sources of evidence for this study are: 1) participants' responses to a semi-structured interview (Appendix A and B); 2) a think-aloud protocol (Appendix C), which (Ericson & Smith, 1991) define as "the best-known method for assessing differences in the mediation processes as functions of the subjects' levels of expertise" (p. 20); 3) a silent problem-solving (Appendix D); 4) an oral-explanatory protocol (Appendix E); 5) researcher's fieldwork notes, and 3) researcher's reflective journal. Table 5 displays the description of the sources of data and the data collection device for each of the research questions.

Table 5

Sources of Data, Data Collection Device, and Data Collection Timeline for each Research Question

	Sources of Data	Data Collection Device
Research Question		
1) What do pre-service teachers' and mathematics experts' experiences as mathematical learners reveal their understanding of mathematical literacy?	Semi-structured interview Researcher's fieldwork notes Researcher's reflective journal	Otter Voice Recorder Mobil device
2) How do pre-service teachers and experts in mathematics use language when solving mathematical problems?	Think-aloud protocol Oral-Explanatory protocol Researcher's fieldwork notes Researcher's reflective journal	Educreations Otter Voice Recorder Mobil device
3) What literacy practices do pre-service teachers and experts in mathematics engage with when presenting modules that require mathematics problem-solving?	Oral- Explanatory Silent-Solving protocol Researcher's fieldwork notes Researcher's reflective journal	Educreations Otter Voice Recorder Mobil device

To collect the data, I used three applications designed for mobile devices: *Educreations* (Version 2.3.3), *Otter* (Version 2.1.2), and Voice Recorder (Version 45). *Educreations* is an interactive mobile whiteboard. My intention to use this application was to record participants' voices and writing simultaneously. This type of recording allowed me to track and match the process of the participants' writing when solving the mathematical problems with their language, making it possible to have another layer of data analysis and data triangulation. I used this application during the think-aloud protocol and oral explanatory protocols. *Otter* records and transcribes conversations simultaneously, which eased the transcription of the participants' responses during the semi-structured interview, think-aloud protocol, and oral-explanatory protocol. Voice Recorder was used in a second device to record the participants' responses. This recording ensured the accuracy of the transcription as well as backed up the collected data.

Data Gathering Procedures for the Pre-Service Teachers.

The collection of the pre-service teachers' data took two stages. First, I conducted a semi-structured interview in order to ask subsequent questions depending on the situation and the participant's first response (Lichtman, 2013). In this semi-structured interview, the participants answered questions related to their experiences as learners of mathematics. In a second session, I asked the participants to solve pre-selected mathematic word problems following three different protocols: 1) think-aloud, 2) silent problem-solving, and 3) oral-explanatory. In a first step, I asked the participants to solve a set of problems following a think-aloud protocol. For the second step, I asked the participants to solve a set of mathematical problems silently. Finally, I asked the participants to use a third set of problems to explain to me how to solve them. Detailed descriptions of these protocols and the mathematical problems are reported in Appendices C, D, and E.

Data Gathering Procedures for the Experts in Mathematics.

For the data collection of the professionals in the area of mathematics, I conducted a semi-structured interview (Appendix A) similar to the one applied to the pre-service teachers. However, I modify this interview to explore how these professionals perceive the importance of disciplinary literacy in their professional lives as well as its importance when developing curriculum and class instruction. I tested the items of this semi-structured interview during the pilot study. I assessed these items according to the participants' responses during the pilot.

In a second session, I asked the professionals to solve the same sets of mathematical as the pre-service teachers did. The experts in mathematics followed the same protocols than the pre-service teachers: 1) think-aloud, 2) silent problem-solving, and 3) oral-explanatory. A detailed description of the semi-structured interview is reported in Appendix A. The three protocols and the mathematical problems are reported in Appendices C, D, and E.

Instruments

I applied four instruments to collect the data. First, I applied a guide for a semi-structured interview. The items of this guide aimed to gather information about the participants' experiences learners of mathematics, participants' concepts of literacy and mathematical literacy, and participants' concepts of mathematics as a discipline. Additionally, I included items related to the participants' linguistic background for considering particularly important for a clear understanding of their linguistic choices during the three protocols aforementioned. During the pilot study, I realized that I needed to adapt the guide for the semi-structured interview for the pre-service teachers. Thus, I designed two versions of the guide for the semi-structured interview, one for the experts and one for the pre-service teachers. These adaptations are reported in Appendices A and B.

The second instrument is a set of mathematical problems for both groups to solve during the think-aloud protocol. This set of mathematical problems is reported in Appendix C. The third instrument is a different set of mathematical problems for the silent-solving protocol. This instrument is reported in Appendix D. Appendix E reports the final instrument, which is a new set of mathematical problems that the participants need to explain to me how to solve them (oral-explanatory). The selection of the mathematical problems was performed under the supervision of an expert in mathematics. These instruments were applied to both groups of participants.

Data Analysis

The data analysis encompassed three approaches. The first approach analyzed the data obtained during the semi-structured interview. I transcribed the participants' answers to the semi-structured interview using *Otter* as the first layer of transcription. Then, in a second layer, I reviewed manually the transcription for accuracy. I performed an in-depth reading of the transcribed text and *In-Vivo* coded (Saldaña, 2010) the participants' responses. The resulting codes were inductively analyzed, which developed in overarching categories (Dey, 2005) that comprise of the emergent themes found in the participants' responses.

The second approach analyzed the data from the think-aloud and oral-explanatory protocols. I transcribed the data from these protocols using *Otter* as well. Similarly, I inspected the first transcription for accuracy. In the second approach, I applied the tenets of the Systemic Functional Linguistics (SFL) analysis. To perform this analysis, I divided the target text into clauses[3] as they are the unit of analysis in SFL (Halliday, 1994).

[3] Matthiessen, Teruya, and Lam (2010) define clause as "grammatical unit of the highest rank on the lexicogrammatical rank scale." (p.71)

In the case of this study, the resulting texts resembled monologues rather than conversations, which made the identification of clauses somewhat fuzzy. For this reason, I adopted one of the Martin, Matthiessen, and Painter's (1997) guidelines to divide the text into clauses. Martin et al. indicated that if the resulting text is a monologue, as it is the case of these participants' responses, its clauses can be identified as statements that the reader can argue with. Thus, I classified the resulting statements as clauses if they contained information that promoted an argument. One of the strategies that Halliday (1994) identified as useful to argue with a text is by inserting tags[4] at the end of the statements.

The following excerpt, from one of the participants' responses during the think-aloud protocol, is an example of how I identified the clauses within the resulting texts. I inserted tags at the end of each clause to indicate their boundaries:

> Please explain why you cannot perform **[can you?]**. So, write it out **[don't you?]**. Two over three plus three over two equals two plus three **[doesn't it?]**. Three plus two so, to solve the first half of each side **[what to do?]**. So, two plus three is six **[isn't it?]**. No, that's five **[isn't it?]**. And then three plus two is five **[isn't it?]**. So, that side equals one **[doesn't it?]**. And then for this side, you have to find a common denominator **[don't you?]**. So, that'd be six **[wouldn't it?]**. So, you have to multiply two times three, two times two, and three times two to get four over six **[don't you?]**. And then three times over two times three is six and then nine over six **[isn't it?]**. And then four over six plus nine over nine, you get thirteen over six **[don't you?]**. So, they are equal **[aren't' they?]**

[4] A tag is defined as an interrogative structure attached to the end of a declarative statement to project a positive or negative connotation of the main declarative statement (Crystal, 2008).

that's what I'm interpreting from it **[am not I?]**. So, it's just not equal to each other **[is it?]**.

The tags helped me to visualize the participants' responses; thus, be able to argue or interact with the resulting text, which made it possible to distinguish the clauses within the transcribed text. I analyzed the each of the resulting clauses under Halliday's three strands of meaning or metafunctions of the language as I discussed in Chapter 1: 1) Clause as a message (Textual metafunction), 2) Clause as an exchange (Interpersonal metafunction), and 3) Clause as a representation (Experiential metafunction).

Each of the metafunctions of the language has its own functional labels. Throughout this document, I utilize the systems of labels that Halliday and Matthiessen (2014) describe for the analysis of each of the metafunctions of the language. Table 6 exemplifies how each of the elements of the clause is labeled differently depending on the strand of meaning that is being analyzed.

Table 6

Metafunctions of the Language within Clauses

	Clause			
Metafunction	**I**	**would**	**multiply**	**both sides**
Interpersonal	Subject	Finite	Predicator	Complement
Experiential	Actor	Process		Goal
Textual	Theme	Rheme		

Additionally, I used Halliday and Matthiessen's (2014) conventions to describe each of the labels of each of the metafunctions of the language. Table 7 summarizes the conventions used throughout this study. I provided a detailed description of each of the elements of the metafunctions, the systems of labels, and the conventions used in this study in Chapter 1.

Table 7

Capitalization Labels for Systems and Resulting Clauses

Capitalization	Convention	Example
Lower case	Name of term in the system	indicative / imperative
Small capitals	Name of system	MOOD, THEME, TRANSITIVITY
Initial capital	Name of the structural function	Mood, Theme, Rheme

The last of the three approaches, I applied during this study, guided the analysis of the results for the research question 3 (RQ 3), in which I aimed to investigate what literacy practices the pre-service teachers and experts in mathematics utilize when solving modules that require mathematics problem-solving. With this goal in mind and acknowledging that this inquiry is a collective case study (Creswell & Poth, 2018), I analyzed the participants' written responses to the mathematical problems they solved during the silent- solving and oral-explanatory protocols by selecting relevant information of the practices that each of these participants displayed when solving these problems (Gillham, 2000). Additionally, I applied a holistic analysis of the participants' responses that included an in-depth and iterative reading of the data, field notes, and my reflective journal (Simons, 2009) to make sense of the participants' written responses.

Rigor and Trustworthiness

One of the most accepted criteria to ensure rigor and trustworthiness in qualitative research is the Lincoln and Guba's (1985) model (Amankwaa, 2016; Cope, 2014; Houghton, Casey, Shaw, & Murphy, 2013). In the case of this study, I embraced this model and included criteria to approach the credibility, dependability, transferability, and confirmability of the findings and of the research itself.

Credibility

The credibility of this study is achieved through multiple sources of information that attempt to observe the same linguistic choices and literacy practices in the two groups of participants. *Triangulation* is a technique to approach credibility, provide different aspects of the same reality, and use different methods of data collection that maximized the understanding of the findings (Krefting, 1991; Patton, 1999).

In this study, triangulation was drawn by the different modalities. I asked the participants to solve mathematical problems. When the participants silently read and solved the mathematical problems, think-aloud about how to solve them, and explain possible ways to solve a set of problems, they displayed linguistic and literacy strategies proper of the discipline. In other words, I triangulated the participants' responses that come from the silently, oral, and oral-explanatory modalities.

Dependability

The second criterion in the Lincoln and Guba model address issues of dependability, the ability of a researcher to replicate the research design without necessarily achieving the same results (Shenton, 2004). Houghton et al. have identified two main strategies to achieve dependability. The first one is an audit trail, in which the researcher should generate documents that track the decision-making process, advances in the process of the research, and changes and adjustments made to the research design (Creswell & Miller, 2000). The second of these strategies is reflexivity, which could be understood as a continuous practice of self-awareness that illuminates how the research process was crafted, how knowledge was constructed, and how accurate the analysis was during the research process (Pillow, 2003).

For this inquiry, the researcher journal and field notes are the documents that support the audit trail. In the researcher journal, I kept a detailed record of how I conducted the research process (Amankwaa, 2016), what decisions I made (Connelly, 2016), and how I edited and revised the final manuscript (Cope, 2014). My filed notes kept track of every review meeting with each of the committee members of this dissertation and documented their feedback and suggestions for improvement (Creswell & Miller, 2000).

Reflexivity, "the process of reflecting critically on the self as researcher" (Lincoln & Guba, 1994, p. 183), was approached by two stances: personal and epistemological (Willig, 2013). In a personal reflexivity stance, as Willig describes, I mirrored how my values, interests, linguistic background, pedagogical experiences, beliefs, and interest that shaped this research. I reflected upon questions regarding the knowledge that was constructed, the possible implications and assumptions of the findings, and the applicability of the methodical design of this inquiry. My personal and epistemological reflections were recorded in a reflective journal (Lincoln & Guba, 1985), that consists of reflective daily entries during the process of this inquiry, and that are organized in two separate sections to identify personal from epistemological reflections.

Transferability

The ability to generalize the findings of case studies has been frequently an issue of criticism toward case study research; however, this is a myth (Flyvbjerg, 2011). As Flybjerg claims, case studies can provide in-depth observations, descriptions, and analysis of the investigated phenomena. In case studies, the researcher relies on analytical generalization to position the findings within the scope of an overarching theory (R. K. Yin, 2009).

For this study, I followed Lincoln and Guba's (1985) model to ensure transferability. I provide a thick description of the natural environment where this inquiry took place.

Additionally, for this study, I adopted Ponterorro's (2006) working definition of thick description. Drawing from Ryle (1971), Geertz (1973), Denzin (1989), Holloway (1997), and Schwandt (2001); Ponterroto posits that thick description includes an accurate and detailed description of the context of the investigation, the social relationships generated during the investigation within its natural context, participants' emotions and feelings as a result of these social interactions, and a thick interpretation of the research findings. A thick description of this study would allow readers to position themselves within the context of the research and have a sense of the credibility of the results.

With this definition in mind, I collected data that support a vivid description of every stage of this research. I captured this data in the form of pictures, audio and video recordings, writing materials generated by the participants, and detailed transcriptions of the participants' interviews. The transcriptions do not only reproduce participants' utterances; but also, include conventions to denote participants' emotions during the interview.

Confirmability

The last criteria in the Lincoln and Guba's model to achieve rigor and trustworthiness in qualitative research is confirmability, in which the presentations of the findings should derive transparently from the data, not from the researcher's inventions or preferences (Shenton, 2004; Tobin & Begley, 2004). Shenton (2004) proposes five strategies to ensure confirmability: 1) triangulation, 2) researcher's acknowledgment of her own beliefs and assumptions, 3) identification of the limitations of the study, 4) in-depth methodological description, and 5) the use of diagrams to demonstrate an audit trail (p.73).

As I described in the criteria above, triangulation, audit trail, and thick descriptions are part of the strategies that I utilized to achieve rigor and trustworthiness. However, to address the specific issue of confirmability, I overtly stated my set of assumptions, beliefs, and limitations as a researcher, a professional, and as a person under the Researcher's Positionality section discussed in Chapter 1.

CHAPTER 4

RESULTS

As I stated in Chapter 1, the main aim of this study is to investigate how pre-service teachers develop their disciplinary heuristics and how language mediates their learning and acquisition of mathematical literacy. To achieve this aim, I proposed three research questions (RQ):

1. What do the experiences of pre-service teachers and mathematicians reveal about their understanding of mathematical literacy?
2. What literacy practices do pre-service teachers and experts in mathematics utilize when presented with modules that require mathematics problem-solving?
3. How do pre-service teachers and experts in mathematics use language when solving mathematical problems?

Through this chapter, I will present the findings of a semi-structured interview and the participants' responses when solving mathematical problems using three protocols: 1) think aloud, 2) silent problem-solving, and 3) oral-explanatory. The semi-structured interview helped me explore RQ 1 while the protocols for problem-solving helped me explore RQs 2 and 3.

I organized the semi-structured interview in three sections. The first section includes participants' experiences as learners of mathematics. In the second section, I present participants' linguistic background, which is necessary when analyzing the data for RQs 2 and 3. In the third section of the semi-structured interview, I surveyed participants' understandings about literacy and mathematical literacy.

I obtained my participants' responses for RQ 2 and 3 through the think-aloud, silent solving, and oral explanatory protocols that I explained in detail in Chapter 3. Later in this chapter, I will present examples of participants' written responses to illustrate the findings for RQ 2. To explore RQ 3, I conducted systemic functional linguistics (SFL) analysis of participants' responses when solving the three sets of mathematical problems, as described in Chapter 3. The organization of this chapter follows the order of the research questions that guided this study.

RQ 1. What do the Experiences of Pre-Service Teachers and Experts in Mathematics Reveal about their Understanding of Mathematical Literacy?

To explore RQ 1, I conducted a semi-structured interview (Appendices A and B), which I structured in three sections. The first section aimed to explore participants' experiences as learners of mathematics. The second section explored the participants' linguistic background, which I presented in Chapter 3. Participants' linguistic background is deemed necessary when analyzing the results for RQ 3. The last section of the semi-structured interview focused on participants' definitions of literacy and mathematical literacy. Additionally, in the third section, I asked questions related to participants' understanding of mathematics as a discipline and the communicative strategies that they use in the mathematical classroom. I performed an in-depth reading of the transcribed text and *In-Vivo* coded (Saldaña, 2010) the participants' responses. The resulting codes were inductively analyzed, which developed in themes. Concomitantly, I organized the emerging themes into the following overarching categories (Dey, 2005): 1) Mathematical Engagement, 2) Mathematical Pedagogy, 3) Literacy, and 5) Mathematical Literacy.

Mathematical Engagement

The category of mathematical engagement encompasses the participants' description of how they learned mathematics. The responses I obtained from the participants were unique and rich in details. They reflected on their role as learners of mathematics and on the processes that underwent their mathematical acquisition and learning to eventually engage with mathematics up to the point of adopting it as the core of their professional practice. Moreover, the participants' experiences comprise their reflection on the support that they received from family and teachers to learn mathematics and the multiple opportunities they had to explore mathematics under various, socially situated contexts. Thus, the overarching category of mathematical engagement is organized around three themes: 1) acquisition and learning, 2) nurturing, 3) exploration, 4) and visualization

Acquisition and learning of mathematics.

While I was reding the participants' responses to my question of how did you learn mathematics? Krashen's (1985) Input Theory came to my mind. In his hypothesis, Krashen argues that in developing a second language, the speaker experiences two process. A first process called *acquisition*, in which the second language speaker experiences unconsciously the emergence of the second language typically experienced in natural settings (e.g., at home). A second process, called *learning*, in which the speaker consciously gains knowledge of the target language, which generally occurs in schooling settings.

I applied Krashen's hypothesis to the participants' responses and noticed that for some of them, mathematics was acquired through a subconscious natural process, similar to the one experienced by a child when learning to speak. For instance, Dr. McFarlane, an experienced professor of mathematics, explained that learning mathematics was a natural process for him, as he enlightened, "it was natural for me. I could do it on my own." Ms. Briggs, a young lecturer of mathematics and a trilingual speaker of Mandarin, Malay, and English, described a similar experience as Dr. McFarlane's one, as she pointed, "I am naturally better with numbers since [I was] young."

Other participants experienced mathematics more like a learning process. For example, Dr. Dunn, a Mandarin/English bilingual professor of mathematics, indicated that his initial mathematical learning occurred in a traditional fashion; however, as Dr. Dunn was more involved with his mathematical learning, it changed toward a process that resembles acquisition of mathematics, as he explains:

> I think I learned math in both a traditional and also in a reformed-minded way. But, you know, we all went to school, and we learned a key to the math, but with the years, I think I was able to balance both. I mean, I like playing with the math. You know if you give me a math problem with this in the textbook or elsewhere, I would play with it in multiple ways.

Similar to Dr. Dunn's experience "playing with math," Ruby, a senior majoring in elementary education, described her experience as a natural and playful way of learning mathematics:

> Elementary-wise, math was super fun, easy. It wasn't really a hard topic for me, and I didn't really know anyone else who struggled with math. Like, [in] third grade when we were learning multiplication, they taught us songs to remember it. And that's still what I used to remember multiplication facts […] And then fourth grade it was still reinforcing that it was lots of repetition, but it wasn't in a super harsh way. It was still in a fun way […] Middle school, still a lot of repetition, but lots of hands-on. Fun teachers who taught us songs and that type of stuff. Then, [in] high school, it was more boring, more bland. It was more like the classroom had nothing in it. It was just take notes the entire time, then you have 50 homework problems, then you come back to class. He [the math instructor] checks the homework, new lesson in like just a very strict schedule, which is fine, I did

learn a lot from it, but it wasn't very engaging necessarily, just lots of notetaking.

The “playfulness” of these learning experiences was not exactly the same for all the participants. Some of them, at an early stage, learning mathematics was not an engaging experience. For instance, Dr. Arnold described his early experience with mathematics as non-enjoyable; however, once he experienced mathematics as an acquisition processes, his appreciation of this subject shifted toward a perspective where he related mathematics to mathematical thinking, as he explains:

> It was a highly non-enjoyable experience up to a certain point. Somewhere around middle grades, I would say, seventh or eighth grade, I discovered algebra, I discovered mathematical thinking and then began to enjoy it quite a lot more. To the point of that in high school, I was reading a lot of mathematics independently and learning mathematics.

It is through experience mathematics as mathematical thinking – the ability to be flexible, efficient, and resourceful when dealing with new mathematical problems (Schoenfeld, 1985) that Dr. Arnold acquired mathematics. His mathematical acquisition made Dr. Arnold engaged with the subject, which is expressed in his inner desire to experience mathematics in a more independent fashion.

Similar to Dr. Arnold, Cesar, a junior in the Special Education Program, had a more positive engagement toward mathematics once he acquired mathematical foundational concepts, as Cesar said:

> [I]n my elementary years of learning mathematics, I had a relationship that was either good or bad with mathematics [but] not really having a deeper connection to mathematics. But, when I got into high school and learning about mathematics, I had more of a negative relationship with mathematics […]. I used to say; *I don't want to go*

> *further and pursue anything in mathematics* [emphasis from the participant] because, you know, *it just isn't for me* [emphasis from the participant]. I automatically wrote down on papers […], but as I moved into college, I got a lot better with mathematics […] I learned how I feel about math. [In college], I break down the concepts I've learned even in elementary school, I break down those concepts in college.

Cesar mathematical learning led him to disengage with the subject. It was once he acquired mathematical foundational concepts that he was able to develop a deeper appreciation and connection toward mathematics, which resulted in his ability to understand mathematical processes independently.

Nurture.

For some of these participants, mathematical engagement was not only the result of acquisition and learning processes, but also the result of support and guidance that these participants received from family and teachers. For instance, Sophie, a senior in the Elementary Education Program, described positive experiences when learning mathematics in the first years of schooling, as she explained, "I never had a like a horrible time learning math. It was usually pretty enjoyable." I can argue that Sophie's experience is related to the positive relationship that she developed with her teachers, as she narrated "[S]o I love my first-grade teacher. And from her model way from her being a role model." Much of Sophie's engagement with mathematics was related to a positive relationship with her teacher of mathematics.

For Cesar, the support he received from his instructors in college is one of the key elements for learning mathematics better, as he explains:

> I think what helped me to learn mathematics better was realizing; I think more elementary [or] high school teachers. Maybe they're learning more and more today, but

> when I was in elementary school, there was more of like, you had a gap in your information. So, then you were just behind and they kind of left you behind. But more teachers in college, you're like, *whoa* [emphasis from the participant]. I feel like more and more of these teachers are considering that you don't know this, so they are having these specific topics like, *oh* [emphasis from the participant], I assume gaps even coming into college about fractions, basic fraction operations. And teachers would, you know, college teachers that work for big jobs and work for NASA and big mathematical jobs and things like that sat down and explained those concepts to me that I had gaps in, and I think that's what really pushed me forward, and that cleared those gaps for me.

Cesar explained that the support that he has received from his instructors in college facilitated his mathematical learning; therefore, narrowing the gaps he brought from high school. Moreover, Cesar's admiration for his instructors' content knowledge and professional activity motivate him to seek by himself different ways to narrow his assumed gaps and move forward in his mathematical learning in college.

The influence of others in these participants' mathematical engagement is not only related to the impact of instructors and schools; families are crucial support for the learners of mathematics as well. Maggie, a Spanish/English bilingual, senior student in the Early Childhood Program, tied her mathematical engagement to the support her family provided. She indicated that her experiences when learning mathematics were mostly enjoyable, and "I credit that to my parents because they would challenge me at home with mathematical problems that were advanced [for age], and they made it into kind of like a game." Again, the "playfulness" of learning mathematics emerged as one of the indicators of the natural acquisition of these participants' mathematical thinking. Additionally, in Maggie's case, her family created a natural environment, which facilitated the development of her mathematical thinking.

Maggie elaborated more about her experiences as a learner of mathematics and indicated that her family encouraged her to face the challenges of being a learner of mathematics, as she explained:

> I think I was like five years old when he [my father] started teaching me a long division. And he would always say like, *I know that it's that it can be hard, but as long as you put effort in it you will get it* [emphasis added]. And I've kept that attitude like throughout my educational career, I always think back to that specific memory, and I'm like, *I can get it, I know it can be hard, but I can get it* [emphasis from the participant].

Maggie's experiences indicate that the support of her family was crucial for her mathematical learning and the development of her mathematical thinking. The encouragement that her family provided made possible for her to construct a system of beliefs that promoted a conducive environment for the acquisition of mathematics and a natural and positive emergence of her mathematical thinking and learning.

Exploration.

Maggie's experience evidences the role of teachers and families in the success of mathematical learning. She highlights the importance of her instructor's openness to explore different approaches to solve mathematical problems. By exploring other ways to solve mathematical problems, in Maggie's case, resulted in a deeper connection and engagement with mathematics, as she explained:

> I think teachers providing more than one way to do it because sometimes I would get it in one way, and I would try the same way in a different problem, or I would experiment with a different approach what that one wouldn't work. And then, the teachers were kind

> of guiding me to like, *oh well this is what you did on this one* [emphasis from the participant]. *How about you try it on this one* [emphasis from the participant]. So then, they helped me to make sure there's more than one way or more than one answer, [it] is not always straightforward. [A]nd I think to have like an open mind to the differences, kind of helped too.

Maggie indicated that for better mathematical learning, it was important that her instructors provided different approaches to experience the same problems and to apply the same approach to different problems. In other words, Maggie believes that what helped her to learn mathematics was first the opportunity to explore the mathematical problems in different perspectives, and second the opportunity to apply her mathematical educated guesses across different problems. Additionally, Maggie highlighted the importance of her instructor's guidance for her mathematical learning to orient her mathematical choices to solve problems, which resulted in a deeper engagement with mathematics as a subject.

In a similar fashion, Dr. Dunn indicated that part of his engagement with mathematics was the result of his instructor's guidance to connect mathematics with daily-life experiences, as he explained:

> I think that the teachers that I had, they all like math, and they do understand the math. The teachers I had, they all had a good understanding of the mathematics, they were able to make the connections. I can't recall any moments that I was really upset with the mathematics or my teachers. It was not always easy, but I had a good time just making progress or solving math problems. I think I enjoyed the outcome of mathematical thinking. To solve problems has been in ways that make sense, I think that sense-making has been at the core of my experience.

As Dr. Dunn's depiction of his experience as a learner of mathematics shows, it was not only the positive attitudes that his instructors displayed toward mathematics that supported his mathematical learning and engagement, it was also their ability to provide their students opportunities to make connections between the abstract concepts learned in class with more concrete situations, which made Dr. Dunn fully engaged with mathematics. The opportunities that Dr. Dunn had to connect mathematics with the real word created a meaning making environment, which facilitated the emergence of his mathematical thinking, and enabled his ability to solve a wider variety of mathematical problems, leading him to a higher level of engagement with mathematics.

Although, not all the participants had encouraging support as learners. Dr. McFarlane's experience portraits rather a discouraging experience, as he narrated:

> Second grade, we were doing subtraction, and they did five minus two is three, and I said, okay, what's two minus five; and the teacher said *you can't do that* [emphasis from the participant], and I said, *are there numbers below zero*? [emphasis added], and they said no. Then in fourth grade, I found out there were, and I was very mad. *Someone stole my idea* [emphasis from the participant]. But that was I was seeing things that most adults don't see.

Dr. McFarlane's innate mathematical curiosity was discouraged in his classroom; however, his inclination toward finding the sense of mathematical phenomenon emboldened him to go further in the study of the sciences and realized that he has a different way to perceive mathematics:

> I knew it [mathematics] was my thing from sixth grade. I mean, I really knew this is what I'm going to do, not knowing what a mathematician was, but knowing I wanted to be

> involved with this [mathematics]. So, math, science, physics was there from the very beginning. In college, I actually started majoring in physics and knew I wanted to be a scientist of some sort. I ended up basically making things they don't always work. In the lab, I'm not the best, but the math on paper, I'm really good. So that kind of pushed me to [mathematics]. My natural talent was there.

Dr. McFarlane's response shows that his innate mathematical curiosity and inclination toward sciences fueled his mathematical learning. Dr. McFarlane's determination to make sense of mathematical phenomena outlasted the discouraging discourses that he faced during his early schooling years. His case shows that mathematical learning is not constrained to someone else's guidance and support; rather, it was Dr. McFarlane' strong agency that encouraged his exploration of mathematics, which made possible for him to make sense of mathematical concepts. For Dr. McFarlane, mathematical learning is strongly tied to an innate curiosity that leads the learner to find paths to make sense of the mathematical phenomena.

Visualization.

Seeking for a better understanding of these participant's experiences as learners of mathematics, I asked what helped them to learn mathematics better. Their responses highlighted the visual representations of mathematics and its applications in different contexts, situations, and subjects helped them to learn it. The perception that visualization helped these participants to learn mathematics is pointed by Ms. Briggs as:

> I think it's visualization, you have to be able to imagine what's going on. And it's really better when you have a situation to explain why you're doing that math thing. Instead of just giving you the formula and say go *ahead* [emphasis from the participant]. I can [have a visualization] as well. But then, when it gets to higher levels, you really want to know

> why you need this [information or concept]. Then it makes it more fun to go through the process of learning.

Ms. Biggs's description of visualization does not imply the graphic representation of a mathematical formula in a single plane; rather, it refers to her ability to transform the abstract concepts behind the mathematical symbolism into mental representations. Ms. Briggs's ability to 'imagine' mathematical concepts helps her to apply such concepts into different contexts and situations, and even though this process requires high-level of thinking and reasoning.

Similar to Ms. Briggs's experience, Dr. Dunn indicated that visual elements helped him to learn mathematics, as he explained:

> Drawing pictures. Doing puzzles. Even telling stories. And I remember very clearly when I was in elementary school [or] in middle school, I like to read and reread puzzle books. Like so many puzzles. Sometimes, sometimes like number puzzles, and sometimes they're just like geometrical shapes.

Dr. Dunn provides explicit examples of how he transforms the mathematical concepts into visual, tangible elements to express meaningful mathematical ideas. Dr. Dunn's visualization does not only imply traditional ways to perceive figures, he also uses verbal elements (stories) as tools to visualize the abstract mathematical concepts.

Furthermore, visualization for these participants implies to experience mathematics across different contexts and subjects. Ruby, a senior in the Elementary Education Program, indicated what helped her with her mathematical learning to see its applications across different subjects, as she explains:

> I think seeing it in different subjects helped too. Just if I saw it across the board like when you're in high school. I was in physics, or I think it was statistics, and just kind of having

> like math seen in different ways, because physics has a lot of math out here. So, I think just seeing it applied across subjects helped a lot. Just to see its relevancy.

Some of these participants explained that for a better learning of mathematics, it is necessary to visualize it across subjects and under different situations. It is their ability to imagine mathematics under different contexts what enables them to make sense of the abstract mathematical concepts and transform them into a real representation of these participants' mathematical thinking.

Mathematical Pedagogy

As part of understanding what these participants' experiences reveal about their understanding of mathematical literacy, I was curious about their experiences when teaching mathematics. I found that these participants' experiences shape an overarching category that I called Mathematical Pedagogy, which is structured by three themes: 1) content knowledge, 2) pedagogical knowledge, and 3) affective filter.

Content Knowledge.

When I asked these participants about the challenges they experience when teaching mathematics, the pre-service teachers expressed to have trouble when needed to 'unpack' the mathematical knowledge they gained in high school to develop a broader understanding of the concepts behind this knowledge to be prepared to teach mathematics. For instance, Sophie explains that she struggles to bring foundational mathematical concepts to her current math content course, as she explained:

> [S]o like, in Dr. Walters's [pseudonym] class [Math Content for Elementary School course] right now. One of the things so we've been talking about is functions. I have noticed that I've been having some difficulty with that. And really, I think that's because I

don't have a lot of history with learning that. We didn't really talk about that a lot [in class]. And also, this is kind of like a broad term [functions], but like algebra in general, I do see myself, like forgetting the things that I've previously learned. Because it's been so long since I've done it. So that's probably one thing to miss. That is a challenge for me.

In her response, Sophie points her struggle with mathematics to a sense of insecurity that makes her hesitant of her mathematical knowledge, which inhibits her ability to find different resources to effectively solve mathematical problems.

Maggie describes content knowledge as one of her challenges when teaching mathematics as well. Maggie, a bilingual speaker of Spanish and English, struggled not just to develop her content knowledge, but also to develop it in a second language. When I asked about the challenges she experienced while learning mathematics, her response was:

> *Language* [emphasis from the participant] That's *why I laughed* [emphasis from the participant] When you said that you were going to talk about language. I was like *oh I relate* [emphasis from the participant]. It was just like, my parents taught me in Spanish, and I learned it [mathematics] in Spanish until I was 16 [years old]. And so, I didn't transition like from one group to the next, I actually transitioned in February of 10th grade. So, it was in the middle of content, and they had been learning that content throughout the year, and it was right before test time. I'm pretty sure it was like a standardized state test that they were about to take, and be reading the questions I was like, *what is this asking me* [emphasis from the participant]. So, I think learning to interpret all of the words and the nuances between the words and math problems and like, *what information I needed to gather* [emphasis from the participant]. It tripped me up.

As Maggie explained, she ascribes her mathematical knowledge to her academic language development. Her struggle seems to be related with how she developed her cognitive academic language (CALPS; Cummins, 1979) in mathematics. As becoming a Spanish/English bilingual, Maggie experienced the struggles of learning the specific academic language demanded to perform a standardized assessment. Maggie did not find difficulties in performing the mathematical calculations, the difficulties appeared when she needed to interpret the specific language required to understand how to ponder the mathematical problems, by which her mathematical knowledge was tested.

Pedagogical Knowledge.

The second theme that emerged among the pre-service teachers' responses regarding the challenges they experience when teaching mathematics is related to their pedagogical knowledge. The pre-service teachers are concerned about gaining the required pedagogical knowledge to be prepared to support their students' mathematical learning. For instance, Cesar's response indicates his concern about being prepared to adapt his approach to teach mathematics depending on his students' backgrounds, as he noted:

> I feel the challenges is that number one, not every student that you're going to teach, or that I've taught learns exactly the same as you, and putting things in terms of how because you got to consider more and more because I feel like this is something new in the education field that a lot of teachers are doing. You gotta put it in terms, basic terms, first you gotta build that information, and then you could start delivering that information to a wide range of students.

Cesar is concerned about his ability to transform the mathematical abstract concepts into information, which his students would be able to relate to, and therefore transform it into mathematical knowledge.

Similar to Cesar's response, Maggie indicated that she struggles to differentiate instruction and assessment depending on her students' backgrounds, as she explained:

> For me, one of the biggest things was learning how to differentiate instruction and assessing in a way that provides you with accurate data. So, we have a lot of assessment classes. They're [the assessments] all really different. So, some are developmental; some are literacy; some are other kinds of assessments. I think learning how to when to use each assessment, what to use it for, how to interpret that data, and how to use the data to further guide your instruction has been like a lot.

Maggie's response portraits her care of adapting her instruction and assessment with information that she can extract from her students. She displays a solid understanding of the importance of students' profiles to guide instruction. However, the multiple layers of instruction and assessment overwhelm Maggie and make her hesitant of her pedagogical knowledge.

Along with the rest of the pre-service teachers, Ruby's response is aligned with her concerns about her pedagogical knowledge, as she explains:

> [S]o, because I was learning better with repetition and just consistency and hands on stuff, I struggled with geometry. I loved anything algebra related because that's just, it just was easier to learn for me because of how I learned math in elementary school. Once I got to geometry, I really struggled with the explaining part. And I think that's what made my mind turn in college when Dr. Walters [pseudonym] is focusing on the process. Like, that's how I struggled so much in geometry […]. And I think that was because it was still more of like a wider approach to everything, and you're given like a real-life problem to solve. And, I wasn't sure how to apply what I knew to a real-life problem. So, I think it kind of went along with those subjects.

Ruby's struggle started with her predisposition to apply the same approaches to deliver instruction as the ones she was exposed in early schooling years. Unfortunately, this mechanistic approach to learn mathematics was not successful to support her gain of the required pedagogical knowledge to make the mathematical concepts real in the classroom.

When I asked the experts in mathematics about the challenges they experience when teaching their courses, they distanced the challenges of teaching from themselves and related them to their students' readiness to take advanced-level mathematical classes.

For instance, Dr. Arnold noted it is a challenge when his students are not ready to face the demands of the high-level classes that he teaches. He explained, "so there is a collection of experiential challenges that the students have had, where they simply don't have the right experiences to be prepared for what I'm trying to teach them." The concern about students' readiness appeared in Dr. McFarlane's comments as well:

> There are two types of challenges here. One is […] pretty common. The big one here is we don't do a proper placement or even admissions. So, I mean, the first time I taught calculus, I had students who didn't know trigonometry. And finally found out [they have] never taken a trigonometry course. There's no way you can pass, right?

Similarly, Dr. Dunn expressed concerns about students' readiness to take his classes, as he stated, "it takes time to understand why you have so many young college students or even classroom teachers who struggle, genuinely, with mathematics in the basic ideas and the basic connections." Additionally, Dr. Dunn discussed as a challenge the affective response of his students toward mathematics:

> It has to do with the way math was taught early in their lives, but as an educator, you know […] what can I do, and how can I engage them [my students] in a way so they do

not only understand the math, but appreciate it? The rhythm of mathematics, the pleasure of doing the math, and they convey that to [their future students]. That's an ongoing battle. It's really a challenge, and we don't have a quick solution. And to me, that's the big challenge. It is sometimes very frustrating when you have, you know, college students who, I mean, just want to pass the class, to get a grade. There's nothing wrong with that, to get a grade, but on top of that, since they are going to the teachers, we [as faculty] would like them to know and appreciate [math]. And we truly, honestly, and genuinely appreciate the beauty of mathematics so that children will get a good mathematical experience. That's a hard problem.

Relatedly with Dr. Dunn's reference of his challenges as an educator of mathematics, Dr. Arnold indicated that his students' affective responses toward mathematics are his main challenge:

The really tough challenge is the affective challenge. Quite a lot of students—especially at the start of their study of university mathematics, but even the ones that have had a few years of it or are math majors, sometimes math education majors—should have had enough exposure to know better. A lot of times, there is some really naïve affect to the things, beliefs that you can solve the problem, or you can't with almost…there is a belief that learning is impossible.

Ms. Briggs describes the same affective response from her student as a challenge:

The hardest thing for me is actually having someone who doesn't want to learn. Yeah, because if you have someone that really wants to learn and work hard for it, as long as you have a little bit more of patience, you just keep moving.

This section dealt with participants' challenges when teaching mathematics. The experts in mathematics addressed their challenges as their students' readiness to take advanced-level classes and the affective responses that they get from their students toward mathematics. The pre-service teachers consider that it is their pedagogical and content knowledge what make them to struggle when teaching mathematics. They position themselves in a critical stance and argue that their pedagogical and content knowledge is not robust enough to face the demands of teaching.

Literacy

I continued exploring these participants' experiences and how they shaped their concepts of literacy and the subsequent definition of mathematical literacy. With this aim in mind, I asked the question *What is literacy?* The participants provided rich and complex definitions, which departed from ways of describing literacy as an act of phonetically decoding printed symbols (e.g., Flesch, 1985). Three themes emerged from the participants' definitions of literacy: 1) Communication at high levels, 2) disciplinary practice, and 3) learning tool

Communication at High Levels.

Differing from views of literacy as the ability of phonetically decoding the printed symbols (e.g., Flesch, 1995), these participants describe literacy as an essential tool to promote communication in areas of expertise.

For instance, Dr. Arnold understands that literacy is not just the mechanics of reading and writing but being able to use these skills at high levels to promote the intercommunication of ideas, as he stated:

> I normally understand it to have two components or a set of different productive components. In the receptive component, one should be able to read with understanding at high levels. One should be able to perceive ideas in his mind, what is being

> communicated and should be able to see how those ideas coordinate with one another […]. There's also a productive side of literacy, which is more or less the same thing and refers to be able to capture the interplay of ideas and to produce some document […]

This definition of literacy acknowledges the importance of literacy to communicate ideas and to connect disciplinary knowledge with the ability to read and write. Thus, literacy is fundamental to promote the exchange of ideas with others, who would react to those ideas and use them to promote a higher level of understanding. In other words, literacy is central for cognitive processes and communicative practices that occur in disciplinary environments (Airey, 2011).

Dr. Arnold's definition of literacy concurs with Dr. Dunn's response, as he defines it as, "understanding with confidence", and echoes Cesar's understanding of literacy as:

> [L]iteracy is building of angular about a certain subject to the point where it's a measurement of understanding of a certain topic and understanding [it] so much that you can explain that topic to someone else, which is very important. And basically, if you are literate in the subject that you were talking about, you're pretty much an expert on it […]

In his response, Cesar is not only discussing the cognitive processes of literacy, but also exploring the relationship between literacy and language (Gee,1996).

Expanding more in the relationship between literacy and language, Sophie's response portraits literacy as a learning tool mediated by language, as she explains: "[…] like the language that you use, while you're learning math [for instance], and like the understanding that you have with that language." In the same fashion, Ms. Briggs directed her response toward the relationship between literacy and language. For Ms. Briggs, literacy and language are synonyms, as she stated: "I'm not even sure if I know the definition. I'm thinking about it as language." Even though. Ms. Briggs was hesitant in her response, she connected literacy with language. Ms. Briggs's account of literacy as language was also displayed when she described mathematical literacy as "using numbers to tell stories."

Disciplinary practice.

For these participants, literacy is bonded with disciplinary practices and literacy as part of disciplinary expertise. As Cesar mentioned in his response, literacy is embedded in the practices of the experts. Some of these participants defined literacy as a disciplinary practice, as well. Ruby noted the differences between literacy across disciplines, as she explained:

> Literacy, I think, of reading, writing, talking listening, just overall engagement, but across the curriculum so literacy, I think, looks different in every single subject. In math, you have to have a certain set of literacy skills to be successful; you have to think in a different way, discuss it [mathematics] in a different way, how you solve your problem. Science, you're investigating more; that's kind of a form of literacy that you follow.

Similar to Ruby's response, Dr. McFarlane displayed a similar way of understanding literacy under disciplinary lenses. He described literacy as a decoding process; however, his understanding of literacy includes its significance across the disciplines, as he denoted "I mean just the literal word or just the ability to read and write in a language[…]I mean in broader terms of cultural literacy, it has its significance in a particular field, but other uses of words." Dr. McFarlane's position, as an expert in mathematics, forged a vision of literacy within the disciplines, which implies that the ability of reading and writing is reflected in how these skills are mainly constructed to apply them to specific disciplinary contexts.

Learning tool.

These participants define literacy as a learning tool in their mathematical classes. I found that these participants use literacy strategies, which they apply and adapt to their students and goals of instruction in mathematics. Dr. Dunn and Ms. Briggs indicated that they use literacy strategies in their classrooms and consider them of value for their professional practice, Dr. Dunn said:

> I think that the moment we teach as a teacher, we use language, we can't teach without language, we can't do math without language. I have, you know, emphasized the importance of word, words, and the contexts. And also, I have made it explicit connections between doing the math and the reading is not just a word. It's a discourse, which means it's a social media is a social and psychological process. And it's a process of mini construction.

The pre-service teachers indicated that they use literacy practices and strategies in their math classes as well. As reading is required for mathematical classes as well as for the other content areas, the pre-service teachers apply strategies such as close-reading, keyword finding, and checking for understanding when they plan for mathematical instruction. In this regard, Sophie mentioned that "because you have, like, when you're reading, you're looking for keywords, or you're looking for like the context to understand like, what's going on? So, you definitely have to be able to do that with the math problems."

The pre-service teachers apply literacy strategies to increase their students' comprehension of the problems that they need to solve. For instance, Maggie mentioned that reading helps to promote reflection of mathematical problems, as she explained:

> I think my teachers really always did approach math as they approach literacy in terms of the reading and the writing and the problem solving. Because with math problems, you have to like *see* [emphasis from the participant] the information interpret it and then

> answer. And that reminds me of reading too, where you read a passage, interpret it, and then provide a response and a reflection. So it's same processes, and it means writing for both which I think is a huge part of literacy, and manipulatives for both, like for reading my teacher would always have like pictures of the stuff that we needed to be focusing on or whereas for math, literacy can be the beads or the Unifix[®] cubes or anything like that.

Ruby indicated that in addition to a math journal, she uses discussion as a literacy strategy to inquiry how her students have found the solution for a problem, as she explained:

> I usually incorporate lots of discussion with math, or if they're working in pairs, they talk through every single thing that they do together, teach each other, what they did in their problem, not lots of writing other than just solving the general problem.

The literacy strategies that these participants indicated to use in their classes are specialized for mathematics as a subject. For these participants, literacy practices support their teaching of mathematics and the processes involving solving mathematical problems. Reading is considered a crucial skill that is necessary for teaching mathematics, and these participants recognize that the reflective nature of reading is applicable when they read mathematics as well. These participants use other forms of literacy, such as art and discussion, to make sense of the mathematical processes that they deliver in class. In other words, for these participants, literacy in mathematics is a meaning making process (Goodman, Fries, and Strauss, 2016) to make sense of the disciplinary texts unique to mathematics as a subject.

Mathematical Literacy

In Chapter 1, when I drew working definitions of disciplinary literacy and mathematical literacy to guide the development of this study, I acknowledged that both of these terms underlie intrinsic components. I approached the analysis of the participants' responses examining the components of their definition of mathematical literacy. Thus, for these participants, mathematical literacy is an overarching category that include the following themes: 1) foundational knowledge, 2) mathematical discourses, and 3) social dimensions of mathematics.

Foundational Knowledge.

To be considered mathematical literate, at an initial stage, a person needs to be able to comprehend and understand essential mathematical concepts (Kilpatrick, 2001). When I asked these participants to describe what is essential for a person to be considered mathematically literate, there was a consensus among them about the importance of foundational or basic knowledge to be mathematical literate. However, I found essential distinctions regarding how these participants define foundational knowledge. For instance, Dr. Dunn addressed the need to understand core mathematical concepts embedded in daily-life situations as foundational for the development of mathematical literacy, as he explains:

> A mathematical literate person. First, you should have a basic understanding and appreciation for mathematical phenomena, like when you see things in life. You know you make trees and flowers, you kind of see the geometry, even the number sense, the way you count, right? When you look at a map, you see shapes. When you see things in the stores like percentages or the price tags. You kind of understand what they mean by those a 25% off, or you know, four apples for $2, or five apples for $2, what does it mean? A mathematical person, I think, just not necessarily knows a lot of advanced math, which is great if they do, but I think it's a general appreciation of mathematics.

Dr. McFarland addressed the role of foundational knowledge in the development of mathematical literacy as well. However, he defined foundational knowledge as the core knowledge that an individual is expected to develop within the specialized domain of mathematics, as he explains: “that's the literate part of. First, we [faculty] would argue of the curriculum in the Ph.D. program [in mathematics] based on arguments like that, *what are the core things everybody in the field should know?* [emphasis added] You know, what a calculus student should know.” For Dr. Dunn, the development of mathematical literacy begins within the layperson’s basic understanding of the surrounding mathematical concepts. In contrast, Dr. McFarlane explains that mathematical literacy develops from the foundational, specialized knowledge required by a person who is in deep contact with mathematics as a discipline.

In Dr. McFarlane’s and Dr. Dunn’s cases, foundational knowledge is an overreaching concept that involves the multiple components of mathematics. However, not all of these participants define foundational knowledge in those terms. For example, Maggie pointed that the foundational knowledge in mathematics is related to the understanding of numbers and the relationships among them. She brings her background as a pre-service teacher in the Early Education program and understands that early numeracy concepts are one of the cornerstones for the further development of computational skills; as she explained: “[W]hen early mathematical education fails, it's harder to catch up in later grades. So, I think the foundations are most critical”. Her understanding of numeracy as foundational for the development of mathematical literacy is reflected in her vision of mathematical problem-solving. When I asked *what is essential for a student to know in order to solve a mathematical problem?* Maggie’s response comprised the importance of numeracy knowledge at the early stages of the student’s development of problem-solving skills, as she explained: “what numbers are and what they look like. I can attest to people in like later grades who don't necessarily know. So, it's very sad, but it happens”.

Ruby also addressed the importance of numeracy in the development of mathematical literacy and problem-solving skills; however, she displays a more holistic understanding of mathematical problem-solving, as she explains:

> I think it'd be like a core understanding of like, *what is it?* [emphasis added]. So, *is it an addition problem? is it a subtraction problem?* [emphasis added] General knowledge of that understanding of numbers 1, 2, 3, 4, 5, and then maybe different methods to solving it, like if they're [the students are] visual learners [...] But like, yeah, just their own method of solving it, whether it's a visual way of writing it out; but knowing to the core of what problem [the student] is solving.

Even though her response addressed the need to know the rules and a preference toward a mechanistic approach (Treffers, 1996) toward problem-solving, Ruby recognizes that there is not a single way to solve mathematical problems. Moreover, Ruby considers that the student's mathematical problem-solving emergent skills require multiple and interrelated abilities that would support the learner's development of mathematical literacy.

Numeracy is not the only essential concept that a person needs to construct to be mathematical literate, these participants also noticed the importance of semiotics as foundational knowledge in mathematics. When I asked Ms. Briggs about the qualities of a mathematically literate person, her response included the importance for a person to know the symbolism that is behind mathematical computations as she said:

> At least to understand what the symbols do [...] Plus, minus are easy to understand like multiplication and division [...]. You know, it's a multiplication relationship [...]. So, I think the basic idea is, you kind of understand the symbols or what they mean; then the equation doesn't seem to be overwhelming anymore.

In her response, Ms. Briggs' equates the knowledge of the mathematical symbolism with the individual's ability to comprehend a mathematical problem. Ms. Briggs' vision of problem-solving aligns her preference toward a more mechanistic approach to solve mathematical problems. When I asked her, *what is essential to know to solve mathematical problems?* Ms. Briggs displayed a traditional understanding of mathematical problem-solving, as she explained:

> It's kind of related back to practice [mathematical problem-solving]. They [students] can listen to someone, like a tutor if they don't understand the teacher […] So this is needed to solve this problem. Because honestly, sometimes, the math doesn't have a direct reason why you need to use that equation. Like the mean [statistics mean] is easy, but like standard deviation, how do you want to explain it? Because there aren't any whole numbers and then divide numbers, the story is different, you just need to know if you ask for this, then *this is the formula* [emphasis from the participant]. It is kind of like memorization. I would say like math has its rules around it, you need to understand those rules, and then use those rules to practice until you're very familiar with those rules it goes back to practice, but I added the rules inside.

Cesar acknowledged the need for semiotic knowledge to be mathematically literate, and he equates the ability to understand the mathematical symbols with the ability to read and understand mathematics, as he pointed:

> First off, reading math that has symbols and that has variables, I believe, it is once you start just like reading the book, I guess it is related in this way when you get over those symbols that in terms of everything once you start learning them are simple, but once you get over that hump of first learning them and reinforcing that learning, then the reading becomes more fluent, you're reading those problems more fluently. And then once you

have that fluency of the small symbols and the small problems, you can relate them to big problems.

Cesar’s response evokes Ms. Briggs’ point of equaling semiotic knowledge with reading fluency in mathematics, as well as with Ms. Briggs’ mechanistic view of mathematical problem-solving, as he explains: “[Y]ou kind of just have to learn it the way it is. Otherwise, you pretty much get left behind.” In the same fashion, Sophie displayed a mechanistic understanding of problem solving. Additionally, she acknowledged that “they [the students] have to be able to, like, I mentioned before, like, understand what is being asked, and know, like, how is solve what is being asked.”

The participants’ differences in their account of what foundational knowledge seems to be in deep connection with their concepts of mathematical problem-solving. As previously described, Dr. McFarlane defined foundational knowledge as core concepts in the field. Moreover, he argued that basic computational skills are required to be literate in mathematics as well; however, those skills are not developed in isolation; instead, they should be intertwined with additional mathematical abilities that contribute to the required fluency and knowledge to solve mathematical problems. In other words, Dr. McFarlane recognized that it is essential to develop foundational computational skills to solve mathematical problems, but the computational skills are developed along with other essential mathematical skills to develop mathematical literacy, as he explained:

> Skills that are basic skills that are basic computational skills, there's putting things together, analyzing decomposing a problem putting it back together, stepping back and looking at another the point of view, those kinds of problem-solving skills or higher-order thinking skills come in.

Aligned with the importance of these participants' understanding of mathematical problem-solving to identify the foundational knowledge to develop mathematical literacy, Dr. Dunn claims that for a student to know how to solve a problem, they need to experience it by firsthand, as he claims:

> You have to do it. I always say that. You know, what's the easiest way for somebody to get to know the taste of something you have never tasted before or assume is not poisonous to get to know the flavor.

Aligned with his belief that to be mathematical literate a person should develop an "appreciation of mathematics", Dr. Dunn indicated that it is essential for a person to 'taste' the mathematical problems as a first step toward an approach to solving problems. Additionally, Dr. Dunn used the metaphor of a poem to imply that when solving a mathematical problem, there should be multiple interpretations:

> You have to do it. It's an ongoing process [mathematical problem-solving]. Like [when] you are reading a poem, you have to read this, again and again. Put yourself in it [in the poem] and know your meaning. Your interpretation is just one of the many versions. You know the limitations; you've got to do it to play with it [the mathematical problem]. And using words, using pictures using actions to understand what it means to do math. There's no difference between understanding a poem and understanding a word math problem; both you have to, in both cases, you have to use your imagination.

These participants' understanding of how mathematical literacy develops accounts for the importance of foundational knowledge; however, they differ in how they define foundational knowledge. For some of them (e.g., Dr. Dunn and Dr. McFarlane), foundational knowledge is an overarching concept that involves an appreciation and understanding of mathematics as either daily-live phenomena or a disciplinary practice. For other participants (e.g., Maggie and Ms. Briggs), foundational knowledge is related to the knowledge of discrete skills such as numeracy or semiotics. Even though their definitions of foundational knowledge differ, these participants' shared view of what is essential for a learner to know if order to solve a mathematical problem reflects their understanding of the qualities of a mathematical literate person.

Mathematical Discourses.

Similar to these participant's responses to the question of *what is literacy?* their responses to *what is mathematical literacy?* describe cognitive, linguistic, and communicative processes embedded in advanced literacy practices (Shanahan & Shanahan, 2008) that develop either within a disciplinary context or applied to daily-life situations.

For instances, in the same fashion when he defined literacy, Dr. Dunn addressed the importance of language to define mathematical literacy, as he illustrated:

> Mathematical literacy is to talk about math, and you know, when we talk about math, we talk about it as a certain language, right? So, you talk about math and math relations, and show your understanding and to show your appreciation and show some confidence, so that's my informal way to define mathematical literacy. So, yes, it has to do with the language. You talk about something that's mathematically significant to use your language. I think the bottom line is you got to understand why and communicate the processes in a meaningful way, start a conversation about math using language in a natural human way by that I mean, not just a formula, not just a term, but start a conversation about the math used in language.

According to Dr. Dunn, mathematical literacy encompasses more than computational skills and knowledge of numeracy. It requires to use language as a mediation tool to perform the required computational and cognitive processes to obtain the solution to a given mathematical problem. Another point that Dr. Dunn provided in his account to define mathematical literacy is the aesthetic interpretation of being mathematical literate; he explains:

> First, you should have a basic understanding and appreciation for mathematical phenomena like when you see things in life. You know you make trees and flowers, you kind of see the geometry, even in the number sense. I think, you know, it's just not necessary to know a lot of advanced math, which is great if they do, but I think it's a general appreciation of mathematics, like the music, you know, and you like the music, and you can maybe have a tune if you're happy. That's my understanding of mathematics, a literate person in appreciating the mathematical side, the quantitative side, and also see the artistic side of life.

The aesthetic reference of mathematical literacy is also acknowledged by Ms. Briggs, who expands her understanding of reading in math as "simple, pretty, not so alarming." She explores even further her aesthetic understanding of mathematical literacy when Ms. Briggs explains that during her mathematical instruction, she utilizes art as literacy, as she narrated:

> I think mine [literacy] is arts. Because I'm a very crafty person. And I never find anything that could combine math and art hardly. I always find a way to like to draw them [mathematical concepts] in a picture that you can understand. A picture and then they [students] change from picture back to formula; they can remember what it is.

Similar to Dr. Dunn's account of mathematical literacy, Dr. Arnold uses the term 'communication' in his description of what it means to be mathematically literate: "[O]n the receptive side, it means that when they [students] receive communication about mathematics, they are able to see through the document into the ideas and how they interact with one another."

Dr. Arnold strongly connects his understanding of mathematical literacy with his definition of literacy, in which he highlights the importance of communicating ideas; for him, mathematical literacy is “doing that where the ideas are mathematical ideas.” Thus, mathematics is embedded in Discourses (Gee, 2008) – particular ways of socially engagement through language, literacy, cultural artifacts, and sets of beliefs. In the case of these participants, mathematical literacy is defined in base of the mathematical Discourses that distinguish the cognitive activity of the mathematical processes.

Social Dimensions of Mathematics.

Every definition of mathematical literacy should include the social dimensions that influence or are influenced by mathematical practices (Jablonka, 2003). One of these dimensions is related to the social contexts where the mathematical phenomena are applied. These participants discussed the importance of connecting mathematics with communities and daily-life situations. As Ruby pointed:

> We have to write [a lesson plan], so there's a connection to life and community part on every single lesson planner, right. And it's in the introduction, so for every single lesson I teach, I connected to a career, and then I told them to channel their inner. I taught a math lesson last semester, and it was on measurements, and I said: [W*]ell, why do you think this is even important to learn?* [emphasis from the participant] and all the students gave me all types of situations that they would need measurements for.

Maggie explained that for young students is crucial to know that mathematics is around them to develop their mathematical understanding, as she explained: “I think the idea that like math is all around us, whether it's counting or like, I said, problem-solving, critical thinking skills.” Sophie also related math with real life:

> I feel like it's also important to relate the topics to situations that they're used to. So, one thing that I always go back to is like when you're doing addition or subtraction. I always relate like the numbers to like possible. So, I like soccer, there's something about kids like, so you have so if you're explaining like two plus two equals four, we're going to be like: [*O]kay, you have two popsicles. And then your friend gave you two more. So now, how many do you have?* [emphasis from the participant] Like being able to put those questions into reality real-life terms.

Furthermore, the experts in mathematics connected mathematical concepts to real life. For instance, Dr. McFarlane explained that mathematics always has two components; one component is solving the problems in paper. The second component is to know how to translate a particular equation into a real-life situation. To make the mathematical concepts real, Dr. McFarlane indicated that he brings realia to explain how specific mathematical formulas apply to the objects that his students use daily, as he further explained:

> I try to relate objects, experiences. So, for example, we do this long derivation of the formula for the components of the acceleration of a curve. And there's the forward component and the normal component. And I explained to them [students], this is *why their parents have gray hair when they get teenagers* [emphasis from the participant] because they're teaching their kids to drive, and their kids are going around the curve, and they don't realize that the formula is the curvature, they think curvature, a sharper curve I better bit of slowdown. But the formula for that acceleration factor his curvature v squared where they don't realize the v square that's why the wheels squeal.

Dr. Dunn also uses real objects to explain abstract mathematical concepts that become real; thus, making his classes active environments. For example, he indicated that he uses origami to explain geometric shapes:

> All those things can be taught in action, so I will say mathematical modeling use modeling using a variety of tools from paper to words to technology by technology I mean pencil is our technology, paper is also technology right, use a variety of tools to show the multiple aspects of an idea.

Dr. Dunn favors a more socially-connected approach to understand mathematics and strongly oppose to reduce the mathematical concepts to formulas or even worst to sequences of steps that are not related to the meaning of the mathematical concept, which his students are learning and developing for a later application in the real world, as he explained:

> Many [students] come to us knowing formulas are mathematics. The formulas are not mathematics. Let me give an example. Just a few days ago, I had a student, two students, presented their research. One did her research on PEMDAS, it is the is the order of operation *Please Excuse My Dear Aunt Sally* [emphasis from the participant]. And she did the research on that and explained the limitations of that so-called math. *It's an acronym, it's a mnemonic* [emphasis from the participant]. Then, after a few days, another student presented her research on FOIL [First, Outer, Inner, Last] …. And she presented that as mathematics when neither PEMDAS nor FOIL are included in common core standards. They are not mathematics.

According to Dr. Dunn, these acronyms lead students to make mistakes by obscuring the concepts behind the mathematical problem or what real-life situations that problem is addressing. He explained further: "[I]n the same way, we eat pizza, but we do not eat the word *P I Z Z A* [spelling; emphasis from the participant]. Does that make sense? So, I think that's, that's a long answer to the question of how to teach math."

Another social dimension of mathematics is noticed when mathematics, as a discipline, is understood as a community of practice (Lave & Wenger, 1991). I asked these participants about the communicative practices they display when they interact with their peers, colleagues, students, and instructors. Noteworthily, these participants indicated that body language is one of the communication channels they prefer for subtle exchange of information with their instructors and students in the mathematical classroom. For instance, Ruby explained, “with my professors. It's never typically one on one, so when I do communicate with them when I'm listening to something they say, I always like nod my head or like, make sure that they know I'm understanding.” The pre-service teachers use body language and gestures to send the message that they are following through the concepts explained in class.

In the similar fashion, the experts in mathematics reported using body language and gestures to communicate with their students. However, the experts in mathematics use their students’ gestures as an additional language to communicate further during a given lesson, as Dr. Arnold explained:

> There is a whole lot of nonverbal stuff that takes me farther than any of my words will. Get the right facial expression, lean back when I should lean back, lean forward when I should lean forward and tore my arms on the sides of my chair or the grasp them together in front of me. A lot of the communication with students seems to turn on that sort of thing.

Dr. McFarlane uses his students’ gestures and facial expressions to recognize if his students have any struggle with the information he presents in class.

In a different way to communicate with his students, Dr. Dunn mentioned that he uses technology (i.e., videos) as a tool of communication; but he had a word of caution and explained:

> In the past few semesters, I'm really taking advantage of videos using them to explain the processes between mathematical operations. I think videos are pretty powerful ways. However, videos do not have the meaning either. We can watch a video to understand the math […]. Well, videos are helpful in the same way textbooks are helpful. If we read a book with a paragraph, read a problem, and then, at the same time, try to do the problem struggle through that we come to understand. So, videos are the same thing.

The pre-service teachers indicated that face to face communication is more effective for studying and sharing ideas with their classmates in their math classes. Among them, only Maggie mentioned social media as a preferred channel of communication with her peers to discuss their mathematical assignments, as she explained:

> We enjoy face to face interactions. But we also have a group chat where we talk about literally everything, all of our classes all of our like assignments and things like that […] So for math class, we write it out for each other. And we let each other like see the problems and are the steps that we took to help one another. Using for other classes, I feel like those are the main forms of communication that we use, but we like getting like at the library. Just talking it out because I think face to face is more beneficial for everyone.

The pre-service teachers mentioned that they do not have many opportunities to interact face to face with their instructors. Albeit, during classes, they use body language to mark whether they are following through the discussed topic. Any of the experts in mathematics indicated using social media to communicate with their students, but they use email and online documents to communicate and collaborate with their colleagues.

These participants use different ways to communicate, depending on their needs. Body language and gestures play an essential role in classroom interactions; however, one to one interaction is the preferred way to communicate among the pre-service teachers. Experts in mathematics use email and online files as their preferred ways of communicating with their colleagues.

The preferences towards channels of communication among these participants reflected in their preference toward their participation in professional organizations. On the one hand, the experts in mathematic indicated that they are members of national and regional associations related to their area of expertise. These experts indicated that these professional groups are helpful to keep them updated, work in collaboration with colleagues in other institutions, and draw guidelines for their work in the university they work for, as Dr. Dunn pointed:

> Well, on the selfish side, I think you can always get help from this group [faculty], and whenever you struggle with a math problem, I would try my best on the problem for a few days maybe sometimes a few weeks, but when I'm really stuck, I'm going to send out to a few mathematicians and ask for help. And I'm saying something like, *I've struggled this problem, can you give me a prompt a hint, which direction I should be going*? [emphasis from the participant]. And there are many fields of mathematics and, you know, I don't think I know all of that, and I know a little bit about everything, but you need the experts to help you, give you guidance, right? So, I'm saying, *get professional help* [emphasis from the participant].

On the other hand, the pre-service teachers displayed various responses regarding their professional or social affiliations. Among them, only Ruby indicated that she is part of an association for pre-service teachers. Maggie does not engage with either social or professional associations, as she said: “for like pre-service associations, I just don't get along with some of the people and then and I try to avoid those situations.” In contrary fashion, Cesar found that his social affiliation was beneficial for his career choice, as he explained:

> I was involved with the Millenarian Youth [pseudonym] […] They have worked in many programs where I got the opportunity to go to different schools and different areas, and just spend a day talking about mathematics or science. So yeah, Millenarian Youth [pseudonym] really has had built around and given me the opportunity of knowing that I am interested in being a teacher even before I was a pre-service teacher, they gave me the opportunity to sit down and speak to children, and even after school activities to help them with homework and things like that.

These participants' vision of the importance of professional affiliations is related to their experience. The experts in mathematics agreed that professional affiliations are a central part of mathematics as a discipline. It is through professional associations that the experts in mathematics discuss current concepts to either apply to their classes or to their program. The pre-service teachers do not find it useful to participate in such professional associations; however, Cesar's experience is an example of how specific social associations could mark paths of professional development for these participants.

Final Thoughts

To end this interview, I asked these participants *how different is mathematics from the other disciplines/content areas?* My question aimed to enlighten the characteristics of mathematics as a discipline. The experts in mathematics acknowledged that one of the points that make mathematics different than other subjects is that people are afraid of it, as Dr. Arnold commented, "I think it has a difference. People are scared of mathematics. And this is such a large effect that it is an important part of way more of what we do than it is at all". The same concern is shared by Dr. Dunn who said,

> There's also a culturally constructed or socially constructed image for mathematics. When you hear something like *I am not a math person. I'm not; I can't do math; I hate math* [emphasis from the participant]. So, that's a socially constructed thing, and it is rarely do we hear people say that I'm not an English person, but you have to use English anyway.

Mathematical anxiety (Tobias, 1980) was not the only characteristic that the participants mentioned to be unique to mathematics. Ms. Briggs considers that what makes mathematics unique is that it can be applied to other sciences, as she explained, "I fell that math is the only thing that I can think of combining everything…I can use math in both geospatial sciences and economy, even history actually." Other characteristics that the participants consider unique to mathematics is its use of problem-solving skills, as Ruby added, "I don't really think problem-solving with many other subjects I'd say that's kind of the big thing that sets it apart." For Sophie, mathematics is unique because it allows people to understand it in multiple ways, as she explained, "I thought math is very, like complex, and it can be understood and like a variety of ways, like the way one person sees how you can solve something may not be the same way that someone else does."

The experts in mathematics concernedly claimed that there is not the same sense of fear in other disciplines than in mathematics. These experts discussed this fear as a relevant component of the learners' ability to understand mathematics. The pre-service teachers provided a different perspective in this regard, considering that what makes mathematics different than other disciplines is its complexity, problem-solving structure, and flexibility to be understood under different perspectives.

RQ 2. How do pre-service teachers and experts in mathematics use language when solving mathematical problems?

To explore RQ 2, I asked the participants to perform two protocols. In the first protocol, called think-aloud (Appendix C) the participants said aloud every thought that came to their minds while solving a set of mathematical problems. In the second protocol, the oral-explanatory (Appendix E), the participants orally explained to me how to solve eight mathematical problems. Both groups orally solved the same nine problems for the think-aloud protocol and provided explanations for the same eight problems in the oral-explanatory protocol.

I present examples of the findings within figures that illustrate the position of the elements of the clause. These figures follow the SFL tradition to present the clause analysis of the Textual, Interpersonal, and Experiential metafunctions[5] of the language. Additionally, these figures include the clause divided into its elements in the first row and the function of each element in the second row. Following Halliday and Matthiessen's (2014) conventions, I bolded the functions of each element within its clause.

The Textual Metafunction: Clause as Message

I started the SFL analysis of these participants' clauses by observing their linguistic choices within the THEMATIC system. The THEMATIC system provides information about the starting point of the messages contained within the clause, the maintenance of the purpose of the messages (Halliday, 1994), the boundaries of the clauses, and the changes in the context to interpret the upcoming clauses (Fries, 1995). The THEMATIC system represents the textual metafunction, in which the clauses enable the negotiations between interactants; thus, as Halliday

[5] The analysis of the Logical metafunction is not part of this study

states, the clause is the message as well. In the case of this study, the textual metafunction shows the participants' linguistic choices to negotiate with the mathematical problems that I asked them to solve.

The first step for thematic SFL analysis was to identify the first element of each clause. The first element of the clause in the THEMATIC system is labeled as Theme. Themes can be Marked or Unmarked. Following Halliday's (1994) labeling guidelines, I labeled Unmarked Theme if it displayed a traditionally called Subject[6] (i.e., noun or noun phrase) as its first element of the clause. Marked clauses are those in which the first element was other than a Subject, such as MOOD elements (e.g., do/don't, should, can), adjuncts (e.g., however, so, and), or complements (e.g., prepositions, noun phrases). Figure 18 displays an example from Dr. Arnold's responses, in which the clause displays an Unmarked Theme.

You	cannot	perform	this equation
Theme: unmarked	**Rheme**		

Figure 18
Unmarked Theme from Dr. Arnold's Response

However, the identification of Marked or Unmarked Themes did not only determine whether the first element of the clause was a Subject, but also there were instances in which the clauses displayed a complex first element that made their identification challenging. For instance, the clause displayed in Figure 19, from Cesar's responses, shows a complex theme.

Each number	itself	has been	tripled	to the next number.
Theme: unmarked	**Rheme**			

Figure 19
Unmarked Complex Theme from Cesar's Response

[6] Halliday and Matthiessen (2014) conventions indicate to use initial capital for the names of structural functions (e.g., Theme, Subject, Rheme)

At a simple glance, the clause in Figure 19 seems to be a Marked Theme; however, the pronoun *each* is part of the Unmarked Theme as it refers to the following noun *number*. I classified as Marked Themes the clauses that displayed other elements than a Subject as their starting point. For example, in Figure 20, Dr. McFarlane uttered the following clause:

And so	that'	s	three minus one minus one
Adjunct: conjunctive	**Theme: marked**	**Rheme**	

Figure 20

Marked Theme from Dr. McFarlane's Response

Some of the Marked Themes showed a complex structure, as well. In the following example, displayed in Figure 21, Sophie chose to produce multiple thematic elements as the starting point of the following clause:

So	by looking at this	my first instinct	would	be	to say	that x is a zero
Adjunct: continuity	**Adjunct: prepositional phrase**	**Theme: marked**	**Rheme**			

Figure 21

Multiple Thematic Elements in a Marked Theme from Sophie's Responses

In total, in the think-aloud and oral-explanatory protocols, the experts in mathematics produced1,573 clauses and the pre-service teachers 1,373. These clauses were analyzed in the THEME system to observe these participants' choices to organize their responses when solving the different sets of mathematical problems. Table 8 summarizes the participant's choices to indicate how they contextualize their responses.

Table 8

Participants' Choices for the First Element of Each Clause

Experts in Mathematics				Pre-Service Teachers			
Marked Theme		Unmarked Theme		Marked Theme		Unmarked Theme	
n	%	n	%	n	%	n	%
889	56.51	684	43.48	888	64.67	485	35.3

As Table 8 displays, these participants show a preference toward Marked Themes, which is reflected by their choices of adjuncts (especially, *and*, *then*, and *so*) or complements as the first elements of the clause. The participants' choices reflected a preference toward a sequenced context when they solved the mathematical problems and indicated that the context of their response should follow a sequence as most of the clauses are related to the previous ones and set the context for the next clause. Rather than a fragment unit, the sequenced clauses seem to provide cohesion and continuity to the development of their processes to solve the problems.

This finding is illustrated in Ruby's solution for the Problem 6 (Table 9) *Solve for x:* $2 + 2x = \frac{4x}{2} + \frac{4}{2}$ in the oral-explanatory protocol. Ruby's frequent choice of conjunctive (*and, but*) and continuity (*so*) adjuncts as Themes exemplifies her preference toward referring and relating the previous clause to the next one, which describes her process of solving-problem as a sequence of steps. The focus of the clauses changes when she needs to include a new process (e.g., add, multiply, subtract) to compute the result of this problem.

The experts in mathematics display preferences toward Marked Themes as well. However, the frequency of this choice is smaller (n= 889; 56.51%) compared to the one displayed by the pre-service teachers (n= 888; 64.67%). In other words, The experts in mathematics solved the problems as a sequence as well; however, they tend to switch the focus of the clause more frequently, from a sequence to a context that would allow them to make sense of the immediate process they would need to perform to solve a problem. In the following example, displayed in Table 10, Dr. Dunn's solved Problem *6 If you toss two fair dices, each of which has 1, 2, 3, 4, 5, 6 on its six faces, what is the probability of getting a sum of 11?* during the think-aloud protocol.

Table 9

Ruby's Response for Problem 6 Think Aloud Protocol – Thematic Analysis

Line	Clause	THEME Marked	THEME Unmarked
1	so we write out our equation	✓	
2	and we're solving for x.	✓	
3	so, we'll first work on simplifying this side	✓	
4	so, because they have the same denominator	✓	
5	we can go ahead and add them together		✓
6	and condense it so that would go to this	✓	
7	so turn into four x plus four over two.	✓	
8	so then we actually get rid of the denominator	✓	
9	so we'll multiply each side by two to get rid of it	✓	
10	so we do that	✓	
11	because that's the opposite function of that	✓	
12	so since it's dividing.	✓	
13	we're going to multiply		✓
14	so you cancel that out	✓	
15	so multiply both sides by two	✓	
16	so that turns into two times two	✓	
17	which is four plus two times two x	✓	
18	which is four x equaling	✓	
19	and this cancels out so four x plus four.	✓	
20	and since both sides are equal.	✓	
21	we were to subtract four that gets zero		✓
22	and if we were to subtract four x,	✓	
23	we would get zero.		✓
24	it just can't work		✓
25	both sides are equal		✓
26	but there's not solution	✓	
227	because always would be zero on both sides.	✓	

As Dr. Dunn's response shows, he changed the focus of his clauses to provide a context that would make sense to the computational processes he had to perform. For instance, lines 5 through 7 transition from the Theme *that*, showing the focus of the message as the number of options available to solve the problem, to the conjunction *and*, to the Theme *we* that brings back the performer of the problem as the main focus of this clause.

Table 10

Dr. Dunn's Response for Problem 6 Think Aloud Protocol – Thematic Analysis

		THEME	
Line	Clause	Marked	Unmarked
1	we have two dice.		✓
2	each dice has six choices right		✓
3	this dice one this dice two and each has six choices one through six right one through six		✓
4	and that's all together we have 36 possibilities 111213141516, all the way to six	✓	
5	that's 36 possibilities		✓
6	and we're looking at.	✓	
7	we're looking at 11		✓
8	because there are many ways to do this	✓	
9	I'm going to try to find you		✓
10	how many ways can you get 11 to five and a six and six and five		✓
11	there are not other ways.		✓
12	four and six will be 10.		✓
13	so we have two,	✓	
14	you can see		✓
15	that there are two there	✓	
16	so it's a really two out of 36 whatever that is	✓	
17	if we need a fraction we can just use that one	✓	
18	we can see why over well over 18 or translate this into a decimal		✓
19	if you really want it,	✓	
20	but for now as I said one out of 18 or 212 3 6	✓	
21	that's the probability of getting the sum of 11		✓

Continuing with the analysis of the THEMATIC system, I analyzed the kind of structures these participants chose as the Themes of their clauses. There are three types of Themes: 1) the Topical Theme, 2) the Interpersonal Theme, and 3) the Textual Theme.

The Topical Theme contains elements of the experiential metafunction (e.g., Participants, circumstances, processes) of the language, which describes the processes by which the interactants act, experience, and interact with each other and with the context of the interaction (Halliday, 19994). The Interpersonal Theme includes a mood element (e.g., should, could, would) in the Subject position to use language to exchange goods, services, or information (Eggins, 1994). The Textual Theme that includes the elements such as Continuity (e.g., first, then, after) and Conjunctive Adjuncts (e.g., and, so, but), which indicate the structure and cohesion of the message contained within the clause (Thompson, 2004). Figures 22 and 23 display examples of the types of Themes that were found among these participants.

The clause in Figure 22 is an example of a Topical Theme clause because the first element that Ms. Briggs chose as the starting point *that* functions as the Carrier (noun or nominal group) of this attributive clause.

That'	s	too small of a decimal
Carrier	**Process: attributive**	**Attribute**
Theme: unmarked, Topical	**Rheme**	

Figure 22
Ms. Briggs' Choice of a Topical Theme

Figure 23 displays an example of Interpersonal Theme. In this clause, from Sophie's response, the first element of the clause is an infinitive structure, which is a mood element and can be analyzed in the following fashion:

Let's	say	you made x negative three
Subject	**Predicator**	**Adjunct**
Mood	**Residue**	
Theme: unmarked, Interpersonal	**Rheme**	

Figure 23
Interpersonal Theme within an Infinitive Structure

The last type of Theme found in these responses was the Textual Theme, in which the focus of the clause is an element that does not express experiential or interpersonal meaning, but it provides cohesion and continuity to the message of the clause. As I explained before, most of these participants' responses show a sequence of clauses with the majority of choices of first elements that indicate continuity. It is not surprising that most of the participants' responses were Textual Themes, as most of their Themes were Unmarked. Table 11 summarizes the types of Themes found among these participants.

The participants' preference toward the Textual Themes indicates that when solving mathematical problems, they used their linguistic repertoires to sequence the processes (e.g., add, subtract, or divide) they needed to perform to obtain the solutions to the problems. While sequencing their responses, these participants preferred structures that show cohesion and continuity; thus, integrating the context of the mathematical problems to make sense of their responses.

Table 11

Participants' Choices of Types of Themes

Experts in Mathematics						Pre-Service Teachers					
Topical		Interpersonal		Textual		Topical		Interpersonal		Textual	
n	%	n	%	n	%	n	%	n	%	n	%
451	28.7	413	26.2	709	45.1	199	14.5	343	25.0	831	60.5

The experts in mathematics rely less frequently on Textual Themes (n=709; 45.1%) than the pre-service teachers (n= 831; 60.5%), and these experts prefer Topical Themes more frequently (n=451; 28.7%) than the pre-service teachers (n=199, 14.5%). The differences in preferences when choosing the first element of the clause could be accounted for these participants' differences in what it means to solve a mathematical problem. For the pre-service teachers and experts in mathematics, mathematical problem solving is a sequenced, cohesive, and integrative endeavor. However, the experts in mathematics shift more frequently the focal point of the message toward the individual solving the problems, rather than to be focused on the sequence or continuity of the processes they need to perform to find the solution of a given problem.

The Interpersonal Metafunction: Clause as Exchange

To analyze the participants' clauses as an exchange, I observed whether each clause contained elements that are conducive to the interaction between the speaker, the message, and the interlocutor. Moreover, I analyzed whether the resulting clauses had the grammatical structures that are essential to exchange goods, services, or information when they were solving the mathematical problems, as they are the main functions of the language (Eggins, 1994). MOOD is the system that provides these grammatical structures to promote the linguistic exchange between these participants and the mathematical text to solve mathematical problems during the protocols mentioned in the previous sections.

I analyzed the participants' responses and classified them according to their choices to exchange information in declarative, interrogative (including Yes-No and WH-interrogatives), explanative, and imperative structures. Since the MOOD system provides the elements to analyze the clause as an exchange, I excluded clauses that exclusively displayed TRANSITIVE elements from this analysis, as I discuss them in the forthcoming section.

In its core, the MOOD system is structured by two elements that constitute the Mood (as an element); these elements are the Subject[7] and Finite. For the analysis of the participants' responses, I focused on the structure of the Finite as it provides information about tense (past,

[7] In SFL analysis Subject is defined as "Functional element of structure in the **interpersonal (modal)** [emphasis from the author] structure of the clause invested with the modal responsibility for the validity of the **proposition** or **proposal** [emphasis from the author] realized by the clause." (Matthiessen, Kazuhiro, Teruya, and Lam, 2010, p. 208)

present, future) and modality (probability and possibility). The structure of the Finite is essential to exchange information among the interactants. Additionally, it provides linguistic choices to exchange or argue about the messages contained within the clause. These linguistic choices determine the intention of the exchange of messages among interactants as well. Additionally, I observed whether the clauses contained polarity elements (positive or negative), as they provide additional information about the nature of the exchange.

In total, the pre-service teacher produced 593 clauses that contain MOOD elements, and the mathematicians produced 536 of this type of clauses. I analyzed their clauses in terms of tense (Temporal Operators) and modality (Modal Operators). In some instances, traditionally classified modal operators, such as *would* and *should*, displayed a tense meaning; therefore, they were classified as tense operators instead. Figure 24 exemplifies an instance where a traditionally considered modal operator indicates a future meaning.

Well	I	would	first	take	this	to make it easier for myself
Vocative	**Subject**	**Finite**	**Adjunct: circumstantial**	**Predicator**	**Complement**	**Adjunct: comment**
	Mood		**Residue**			

Figure 24
Would as a Tense Operator – Response from Cesar

The analysis of the MOOD system of these clauses shows that more than half of the pre-service teachers' clauses (n= 236, 58.2%) were Future Positive clauses. I interpret this finding as to the pre-service teachers' preference toward linguistic structures that allow them to extract information from the text. These structures act as cognitive-mediating tools that provide semantic and semiotic meanings to know how to solve the problems. Problem -solving is not an exchange that is happening at the moment; rather the exchange of information helped the pre-service teachers to know what to do in the further processes to compute the solutions of these problems. Figure 25 exemplifies the pre-service teachers' preferences toward Future Positive clauses.

And now	we'	re	going to start	putting	some of these together
Vocative	**Subject**	**Finite**	**Predicator**	**Adjunct: modal**	**Complement**
	Mood		**Residue**		

Figure 25
Future Positive Clause from Maggie's Response

In the example displayed in Figure 26, Maggie uses the information form the text to indicate the steps she would take to solve the mathematical problem. In the case of this clause, Maggie provides information about the events that she would perform shortly.

The experts in mathematics did not display a marked preference in the MOOD system. As Table 12 shows, the experts in mathematics' preferences for Temporal Operators is divided between Present Positive (n= 133; 39.1%) and Future Positive (n= 129; 37.9%). It seems that in the case of the experts in mathematics, the mathematical problems promoted an exchange of information situated in the present, which allowed these experts to argue with the given problems. This interpretation is exemplified in Figure 26 where Dr. McFarlane uses the present positive to exchange information with the text in the form of a positive WH-interrogative and argued with it about the validity of the processes he was performing while solving the mathematical problems.

So	why	is	this useful?
Vocative	**Subject / WH-**	**Finite**	**Complement**
	Mood		**Residue**

Figure 26
Present Positive WH-interrogative Clause from Dr. McFarlane Response

Even though interrogatives, like the one displayed in Figure 26, show a thematic element, it shows how Dr. McFarlane assures the validity of the processes that he performed to solve Problem 7. The Mood (*is*) indicated that the exchange of information occurred in the present.

Halliday and Matthiessen (2014) identify low, median, and high degree of modality among the Modal Operators. Low degree of modality is expressed by operators such as *can*, *may*, or *would*. *Will*, *would*, or *should* indicate median modality. High modality is expressed by modals such as *ought to*, *must*, or *have to*. Table 13 displays the results of the choices of Modal Operators for both groups of participants. I evaluated the modal operators in terms of polarity and the degree of possibility or probability.

Overall, among the clauses the participants produced while solving the mathematical problems, the pre-service teachers chose to specify whether the processes they performed were either certain or probable 31.7% of the time. The experts in mathematics' choices were somewhat similar to the pre-service teachers and chose 36.5% of their clause to express information about probability or possibility.

Among the clauses that displayed a modal operator, these participants preferred the Low Positive modality as the most common operator to express modality. However, I noticed that the experts in mathematics chose High Positive modality structures more frequently (n= 49; 9.1%) than the pre-service teachers (n=17; 2.9%). Even though the High Positive modal operators were not the participants' first choice to indicate possibility or probability, this difference could account for a somewhat higher level of certainty of the process the experts in mathematics were performing while solving the mathematical problems.

and	you	have to	make	it	equal to two	basically
Adjunct: conjunctive	**Subject**	**Finite**	**Predicate**	**Complement**	**Adjunct: circumstantial**	**Adjunct: mood**
	Mood		**Residue**			

Figure 27
High Modal Operator Analysis – Ms. Briggs' Response to Problem 7 in the Think Aloud Protocol

Among the experts, Ms. Briggs was the participant with the most responses of High Positive modal operators. Figure 27 presents the functional analysis of one of Ms. Briggs' clauses showing her use of a High Positive modal operator.

In this clause, Ms. Briggs portraits herself confident in the process she was performing. In the exchange of information with the text, Ms. Bridges shows that she was sure about the processes she was preforming; moreover, she was sure about the processes that were necessary to solve the mathematical problem.

The Experiential Metafunction: Clause as Representation

The last analysis, I performed on the participants' clauses was for the TRANSITIVITY system, which describes the experiences that the speakers are undergoing while they interact with the world (Halliday, 1994), in this case, while solving mathematical problems. These experiences involve processes that describe what the speakers are doing, sensing, saying, behaving, being, or having during these interactions (Halliday & Matthiessen, 2014). Table 14 shows each of these processes, their elements (participants), and examples from the participants' responses.

Table 12 - Processes of the Experiential Metafunction as Synthesized by Thompson (2004)

Process	Description	Participants	Example
Material	Processes of actions	Actor Goal Circumstance	If you draw the graph is actually the whole thing. (Ms. Briggs)
Mental	Processes of the mind	Senser Phenomenon	I know how to do it (Sophie)
Relational	Processes of relationships (*Be* and *Have*)	Carrier /Attribute Token/Vaue	So, the numerators are the same (Dr. Arnold) We have two dice (Dr. Dunn)
Verbal	Processes of saying	Sayer Verbiage	Let's say a multiplication problem but broken up (Ruby)
Behavioral	Processes of the human physiology	Behaver Range Circumstance	if you persisted in this plan of calling it x (Dr. Arnold)
Existential	Processes of existence of an entity	Actor Existent	How many possible numbers are there? (Dr. McFarlane)

Table 1

Participants' Choices of Tense Including Polarity Operators

Temporal Operators																							
Pre-Service Teachers												Experts in Mathematics											
Past Positive		Past Negative		Present Positive		Present negative		Future Positive		Future Negative		Past Positive		Past Negative		Present Positive		Present negative		Future Positive		Future Negative	
n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
37	9.1	2	0.5	92	22.7	34	8.3	236	58.2	4	0.1	38	11.1	7	2.0	133	39.1	31	9.1	129	37.9	2	0.6

Table 2

Participants' Choices of Modality Including Polarity Operators

Modal Operators																							
Pre-Service Teachers												Experts in Mathematics											
Low Positive		Low Negative		Median Positive		Median Negative		High Positive		High Negative		Low Positive		Low Negative		Median Positive		Median Negative		High Positive		High Negative	
n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
88	14.8	4	0.7	68	11.5	2	0.3	17	2.9	9	1.5	85	15.8	12	2.2	48	8.9	1	0.2	49	9.1	1	0.2

I observed the patterns of processes that these participants produced while they were solving the mathematical problems and classified their responses into one of the six types of processes that Halliday describes for the English language.

Out of the total number of clauses analyzed in the TRANSITIVITY system, 65.9% (n= 1,037) of the experts in mathematics' and 56.8% (n=780) of the pre-service teachers' clauses displayed TRANSITIVITY elements. Depending on the processes that these clauses exhibit, I classified each of them into the six categories of processes that Halliday distinguishes for the English language.

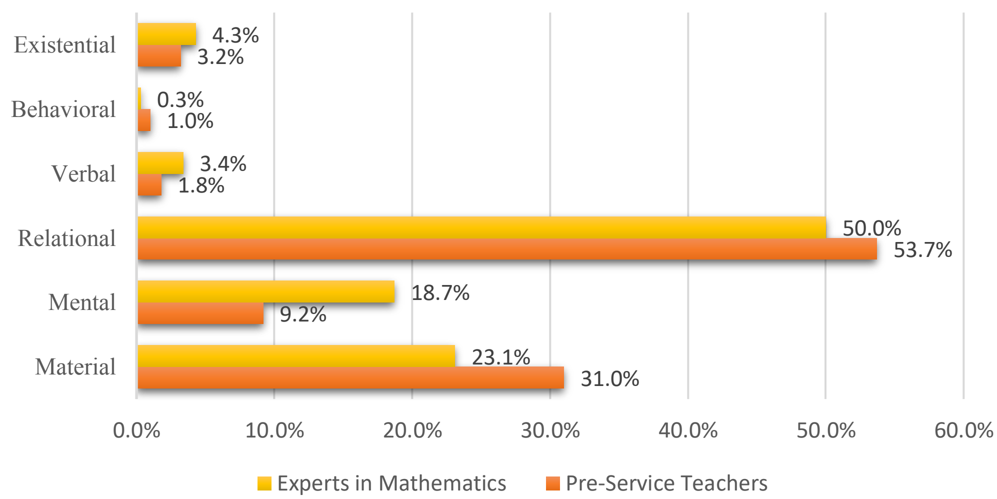


Figure 28
Participants' Choices of Processes – TRANSITIVITY System

As it is displayed in Figure 28, both groups showed a preference toward clauses that unveil relational processes. The relational processes define attributes and identities that are mostly realized by the verbs *be* and *have* (Halliday and Matthiessen, 2014). In the case of the pre-service teachers and experts of mathematics, their choice of relational processes while solving the mathematical problems indicates that they rely upon their semiotic understanding of the mathematical symbolism to assign an attribute to the participants of the clause. In other words, the pre-service teachers and experts in mathematics use their linguistic repertoires to define and characterize the mathematical symbolism embodied in the computational processes to solve the mathematical problems.

In Table 15, I present an excerpt from Ms. Briggs' response to Problem 2 *Please explain why you cannot perform 2/3+3/2=2+3/3+2* in the think-aloud protocol to exemplify how she uses her linguistic repertoire to either assign attributes or characterize the computations she needed to perform to solve Problem 2.

Table 15

Transitivity Analysis of Ms. Briggs' Responses for Problem 2 in the Think-Aloud Protocol

		TRANSITIVITY					
Line	Clause	Material	Mental	Relational	Verbal	Behavioral	Existential
1	Well, because you don't have the same denominator.			✓			
2	And then you are just suffered			✓			
3	Because I'm not supposed to ask you			✓			
4	that I would change it to the same denominator.	✓					
5	Three and two, three times two is the easiest.			✓			
6	So, two times two times two two out three times three times three to the other side.	✓					
7	And then it's four or three to six plus 3 3 9 3 to six.			✓			
8	So It's supposed to be 9 9 plus 4 13 over six.			✓			
9	That's why you cannot perform whatever you just gave me			✓			
10	because that would be five 0 five.			✓			

Ms. Briggs' response to Problem 2 reveals how she experiences the processes that are involved in solving this problem. As most of her responses are relational processes, she describes a preference toward assigning an attribute or identifying a characteristic of the processes that she is performing while solving this problem.

In Figure 29, the SFL analysis shows that Ms. Briggs understands the relationship between the Carrier and the Attribute. However, her choice of *you* as the Carrier of the process distance her from the processes she is performing. Therefore, she seems to be an observer of the processes to assess the computations that she is executing to solve the problem.

Three and two,	three times two	is	the easiest
	Token	**Process: relational, identifying**	**Value**

Figure 29
Transitivity Analysis of a Relational, Identifying Clause – Ms. Briggs' Response

Figure 29 describes the analysis of Ms. Briggs' choice of a relational, identifying clause, in which Ms. Briggs opts to value the process from an expression with a lower value (called a Token; *[three times two]*) to another with a higher content value (called Value; *[easiest]*). In this fashion, Ms. Briggs relates the processes of solving Problem 2 according to its importance for solving this problem.

The pre-service teachers and experts in mathematics selected Material as their second choice of processes. Material processes involve actions that result in a change (Eggins, 1994) and require of an Actor to perform the process to achieve a Goal. Thus, the Actor and the goal become the participants of the Material clauses (Halliday & Matthiessen, 2014). Figure 30 shows the analysis of a Material clause from Ruby's response.

then	you	divide	both sides by seven	to solve for x
	Actor	**Process: material, transformative**	**Goal**	**Circumstantial**

Figure 30
Transitivity Analysis of a Material, Transformative Clause– Ruby's Response

Relational processes, together with the Material and Mental, are the most frequent choice in the TRANSITIVITY system of the English language (Halliday and Matthiessen, 2014). The Material processes are the second choice among the pre-service teachers and experts in mathematics. Material processes involve actions that resulted in a change. As Figure 31 exemplifies, the Material processes require of an Actor, who produces the change toward obtaining a Goal.

that	we	know	Es el seis [it’s six] six squared
	Senser	**Process: mental, cognition**	**Phenomenon**

Figure 31
Transitivity Analysis of a Mental Clause – Maggie’s Response

Regarding the Mental processes, the experts in mathematics chose more than twice the number of processes that reflect mental activity (n = 194; 18.7%) than the pre-service teachers (n= 72; 9.2%). The Mental processes describe what these participants felt, sensed, knew, and desired when they were solving the mathematical problems. In Figure 31, I analyze one of Maggie’s clauses, which includes a Mental process.

In this case, Maggie relates her experience of solving this problem with her previous knowledge (*I know*), translanguaging (García & Wei, 2014) to Spanish to complete the Mental process.

These participants did not choose Verbal, Behavioral, and Existential as frequently as the other types of processes. I can explain this finding focusing on the underlying semantics of the aforementioned processes. As the participants solved the problems, these processes did not provide as the semantic resources to make sense, describe, or explain the participants' experiences when solving the problems. For instance, the Behavioral processes describe events that are mostly related to the physical response to a Mental process. Among the very few examples of using Behavioral processes, Sophie produced the clause displayed in Figure 32.

Let's	skip	that one
	Process: behavioral	**Range**

Figure 32
Transitivity Analysis of a Behavioral Clause – Sophie's Response

The Existential processes describe the presence of an entity within the clause and require the use of *there* to signal this presence. The mathematicians produced 45 Existential clauses (4.3%), while the pre-service teachers produced 25 (3.2%). Figure 33 displays an example of an Existential clause.

There	will be	a sequence of thee	multiplying by three
	Process: existential	**Existent**	**Circumstance**

Figure 33
Transitivity Analysis of a Behavioral Clause – Cesar's Response

The TRANSITIVITY analysis indicates that mostly these participants describe the events that undergo the processes for solving the problems from a relational stance. This finding implies that these groups of participants showed a preference for defining and assigning attributes to the processes they needed to perform while solving mathematical problems. The attributes that these participants assigned to the Carriers seem to be related to their semiotic knowledge of the mathematical process. This knowledge allowed these participants to describe the different processes they needed to perform and make sense of them.

RQ 3. What literacy practices do pre-service teachers and experts in mathematics utilize when presented with modules that require mathematics problem-solving?

One of the purposes of this study is to explore how this group of participants use their unique repertoires of reading, writing, talking, and communicate when they solve mathematical problems, which in turn would inform about the development of mathematical literacy in pre-service teachers. In RQ1, I observed how these participants' experiences contribute to their understanding of mathematical literacy. RQ 2 explored the participants' linguistic repertoires to make sense of mathematical problems. In the final research question, RQ3, I analyzed the participants' literacy practices while solving mathematical problems. As I belief that every individual develops unique literacy practices build upon their learning experiences, sociocultural background, and values and attitudes toward literacy (Barton & Hamilton, 2000), I present the results of this question as an individual analysis of each one of the participants' ways of reading and writing.

Dr. Arnold's Literacy Practices

Dr. Arnold started solving each problem by first reading each of them. At a simple glance, it seemed that he was reading each of the problems in a linear fashion. However, his reading practices were more complex than just using reading to decode the message and the processes needed to solve the problems. Instead, he showed a critical stance when reading these problems. Dr. Arnold was critical against the way that some of the problems were written. He stated that the language of problems 3and 5 in the think-aloud protocol was not clear enough, making the problem more challenging to solve. As a strategy to overcome this difficulty, Dr. Arnold deconstructed the text and inserted statements to clarify this problem and make sense of it. In the excerpt from Dr. Arnold's response to Problem 3, *What percent is $50 more than $20?* in the think-aloud protocol displayed in Table 16, I noticed first how he criticized the problem, and then how he created a new version of the problems to make it clearer and easier to solve.

Table 16

Dr. Arnold's Response for Problem 3 – Think-Aloud Protocol

Line	Statement
1	what percentage is $50 more than $20
2	It's **odd wording**
3	**Let's suppose the question is**
4	**if we increase from $20 to $50**
5	what percentage increases this
6	Then the question
7	it increases $30
8	which is one and a half of 20 so be a 150% increase

As Table 16 displays, Dr. Arnold assessed the problem before starting to solve it. Then, he modified it in a fashion that made sense to him to solve it in a more efficient way.

Keeping a critical stance and modifying the narrative of the problems were not the only practices that Dr. Arnold utilized when solving these problems. Additionally, Dr. Arnold omitted reading information from the text and focused his attention on key information that would help him to solve the problem. To illustrate this finding, I present an excerpt from Dr. Arnold's response to Problem 9, in the think-aloud protocol in Table 17.

Dr. Arnold's ways of reading this problem shows that he focused on key information, (bolded in Table 17) and dropped unnecessary statements as he was reading the problem. This practice provided him the information he needed to solve this problem. Additionally, Dr. Arnold used the figure to locate within it the information he extracted from the text. Figure 34 shows how Dr. Arnold used the rest of the text to obtain additional information and make meaning of the processes he needed to perform to solve this problem.

Table 17

Dr. Arnold's Response to Problem 9 in the Think-Aloud Protocol

Line	Statement
1	Number nine
2	One of these
3	**triangle ABC is equilateral**
4	**D E F are the midpoints of the sides**
5	**AC is 6'' long**
6	Measure of angle DEF
7	Alright, probably there is to do this
8	but since ABC is equilateral
9	it must be also equiangular
10	so I would mark those three angles as equal
11	and I would mark all of them as 60_o

As it is displayed in Figure 34, Dr. Arnold used writing practices to make sense of the problem as well. Even though, he omitted reading some information from the text, at the same time he was writing and drawing information that contributed to his understanding and visualization of the problem to compute the answers for questions 1 through 3 in this problem.

Dr. Arnold did not only display this kind of writing practice to solve Problem 9, but he used the same practices to solve Problem 6 in the silent-solving protocol. Figure 35 displays a sample of Dr. Arnold's writing and drawing practices.

(9) In the figure below, triangle ABC is an equilateral triangle. D, E, F are the midpoints of their respective sides. We know AC is 6 inches long.

(1) What is the measure of angle DEF?
(2) What is the length of EF?
(3) What is the distance between A and D?

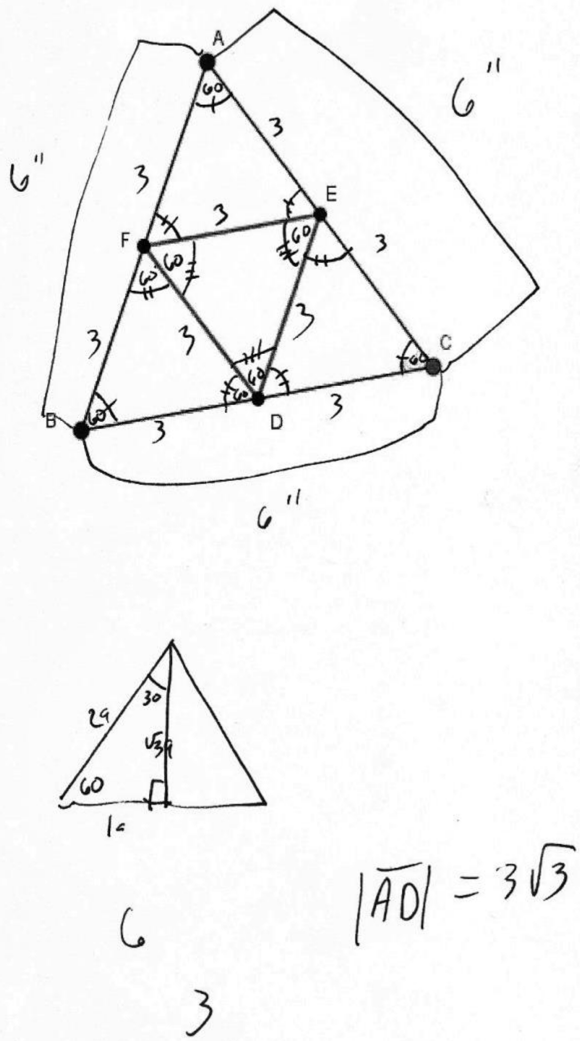


Figure 34
Dr. Arnold’s Written Sample – Problem 9 in the Think-Aloud Protocol

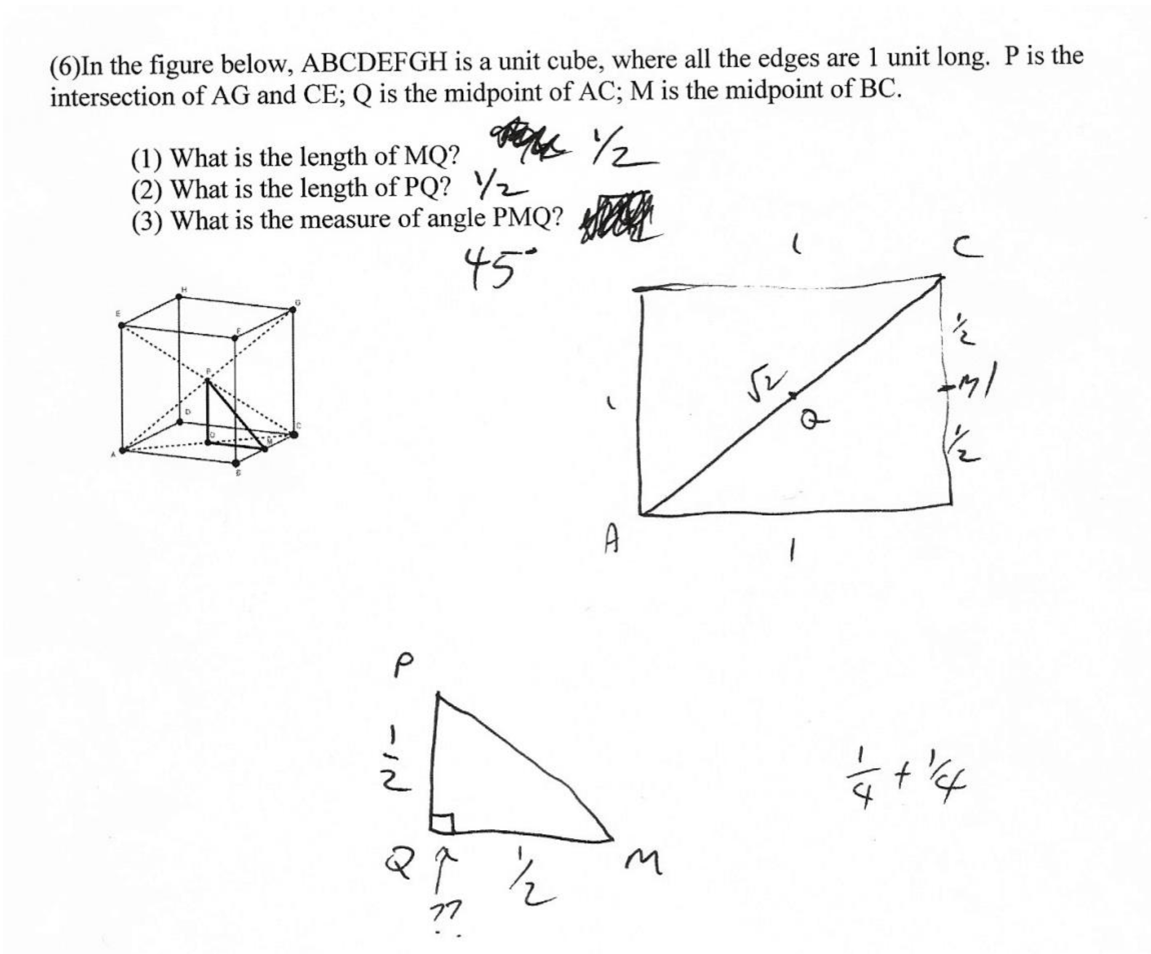


Figure 35
Dr. Arnold's Written Sample – Problem 6 in the Silent-Solving Protocol

In this case, Dr. Arnold used writing and drawing to compensate for the poor formatting of this problem. While solving this problem, Dr. Arnold was critical in the way this problem was formatted. He criticized the size of the cube, which was too small to be able to read it (as reported in Field notes 05/21). Therefore, he needed to extract the information from the graph and to construct a visual representation of the problem. The different ways of visualizing the graph allowed Dr. Arnold to make sense of this problem and solve it.

Ms. Briggs' Literacy Practices

Before starting to solve the mathematical problems, Ms. Biggs explained that she would use her native language to compute the mathematical problems. As a native speaker of Mandarin, Ms. Briggs indicated that when she is working on mathematics, Mandarin is the first language that comes to her mind. As part of this study, I asked her to use English during the think-aloud and explicatory protocols because I would have constraints to translate and analyze her responses if she would answer in a language other than English or Spanish. Ms. Briggs followed my request and talked mostly in English. However, there were moments, in which Ms. Briggs needed to use Mandarin to keep processing these problems.

Counting was the one process in which Ms. Briggs needed to translanguages[8] into Mandarin most frequently. As she stated, "and then I was doing 1 3 3 2 3 6 in Chinese [Mandarin] all the way to a three something." Additionally, Ms. Briggs used Mandarin to confirm her responses as she expressed, "So that will be [speaking in Mandarin] 3 2 6 16 double confirm." Another way in which Mandarin supported her processing of problem-solving was to make sense of the definitions she needed to apply when solving these problems. For instance, she discussed the Mandarin words for numerator and denominator to make sense of her problem-solving process, as she commented: "Funny thing is in Chinese denominator means **mando** [Mandarin word for denominator]. And then, the one on the top is numerator, numerator also write down numerator in Chinese is actually **san**

[8] Tanslanguage is defined as" the deployment of a speaker full linguistic repertoire, which does not in any way to the socially and politically defined boundaries of named languages." (Garcia & Klein, p.14)

[Mandarin word for numerator].”

However, Ms. Briggs did not perceive her English/Mandarin translanguaging as helpful for her problem-solving process; instead, she indicated that using these languages simultaneously interferers with her ability to solve these problems. For example, when Ms. Briggs was solving Problem 4 *A number is 30% more than 5, what is that number?* in the think-aloud, she stated the following:

> So, 50 of the question that I'm going back to very, very basic because I'm overcomplicating things. And eventually, I couldn't solve them. So, I should go back to. Oh, wait, I figured out where I went wrong. I don't know. I think it's because of me speaking in English, and it messed up my mind with two zero five and five zero two. In Chinese, we say 5 over 2 or 2 over 5. Okay, so I should divide it by 5. I'm pretty sure the one that I divided by four is wrong now.

In the case of Ms. Briggs, translanguaging was one of the practices that mediated her problem-solving processes. Additionally, she displayed particular literacy practices to make sense of the problems and calculate them. During the silent-solving protocol, she used writing to criticize the problems, highlight important information within each problem, focus the readers’ attention on her responses, and extract information from the problem to visualize it from a different perspective. Ms. Briggs’s responses to the silent-solving protocol are reported in Figure 36.

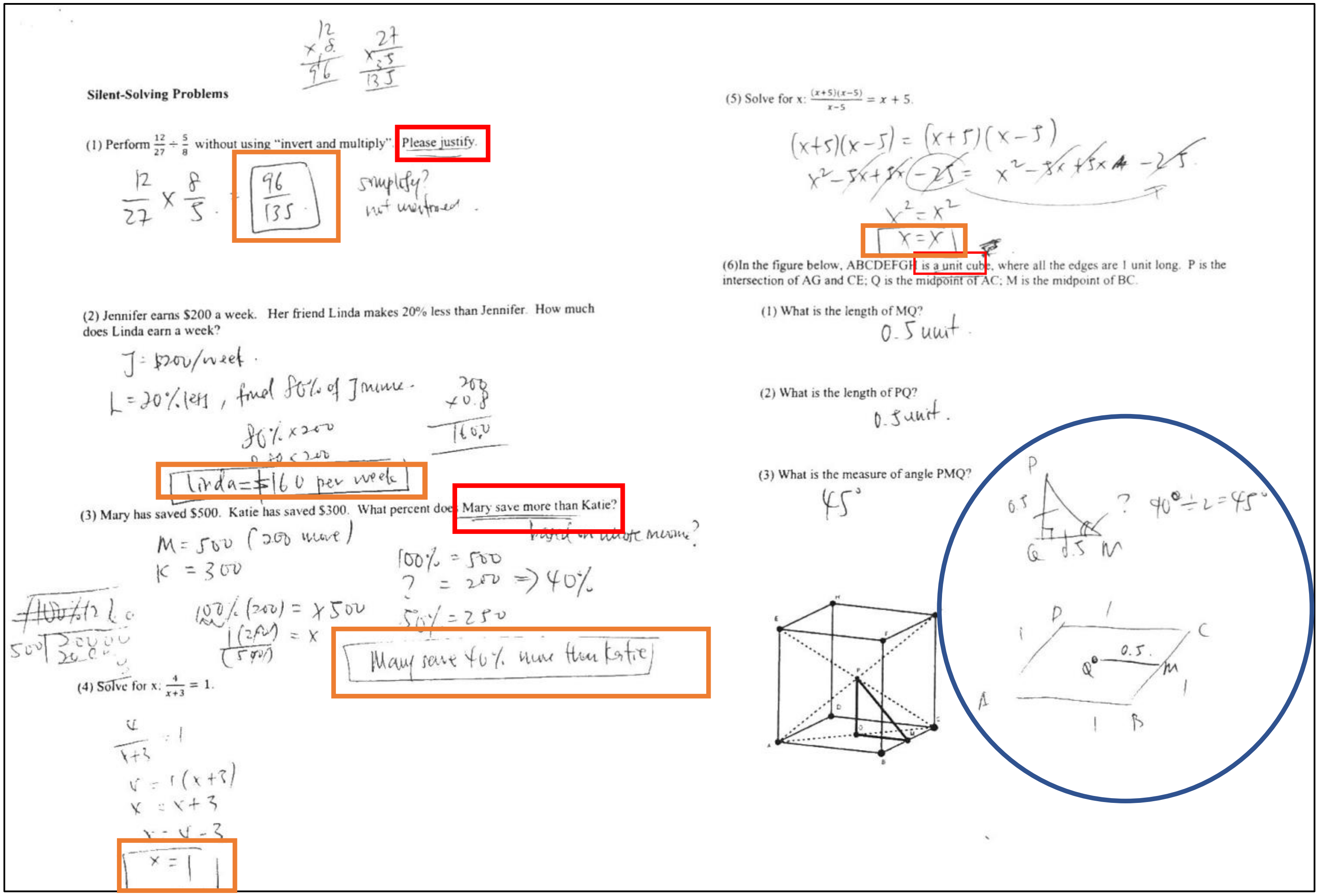


Figure 1

Mrs. Briggs' Responses to the Problems in the Silent-Solving Protocol

In Figure 36, I have marked with a red square the instances, in which Ms. Briggs underlined key terms within each problem. She highlighted her responses using a square; I repeated her same practice with an orange square that directs the attention to her practice. Ms. Biggs extracted information from the graph in Problem 6 to visualize the structures she needed information from to be able to solve the problem. I circled the figures that Ms. Briggs extracted from the graph in Problem 6.

Ms. Briggs displayed a variety of writing practices along her problem-solving. She used charts, pictures, and different ink colors to clarify the processes she performed to solve the mathematical problems. Figure 37 presents an example in which Ms. Briggs used the aforementioned writing practices.

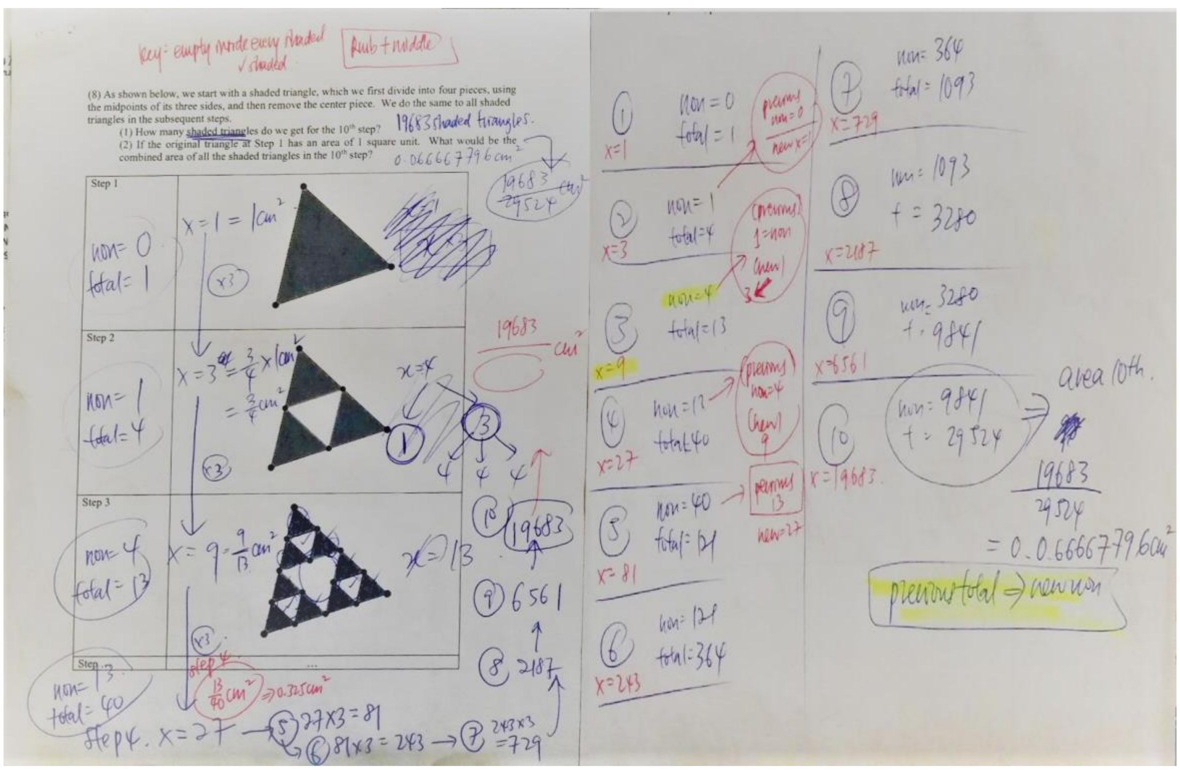


Figure 37

Ms. Briggs' Responses to Problem 8 in the Oral-Explanatory Protocol

As Figure 37 shows, Ms. Briggs used different practices to make the answer to Problem 6 less confusing. In this problem, Ms. Briggs created a chart to organize her thought process; then, she used a different color of ink (red) to point key quantities that are necessary to track to solve this problem. Additionally, she highlighted her response in the same fashion as she did for her responses in the silent-solving protocol.

Dr. McFarlane's Literacy Practices

Dr. McFarlane's reading practices distinguish a meaning-making process, in which he constructs the meaning of the text by inserting, omitting, or changing statements to create his representation of the problem. In the following example, Dr. McFarlane reconstructs Problem 9 in the think-aloud protocol by reading it in the resulting fashion: "In the figure, you have this triangle kind of skewed, equilateral D E and F are midpoints. We know the area of A C of AC, where is AC? Up here, AC is six." The original text for this problem is *In the figure below; triangle ABC is an equilateral triangle. D, E, F are the midpoints of their respective sides. We know AC is 6 inches long*. As Dr. McFarlane was reading this problem, he was creating a parallel representation of the text. This representation contains key elements that he used to calculate the responses to this problem.

Dr. McFarlane was critical toward the way the problems were stated. His criticism was related to the limitations that some of the problems seemed to offer. For instance, Problem 2 *Please explain why you cannot perform 2/3+3/2=2+3/3+2* in the think-aloud protocol implies that there is not a possible way to solve this problem. However, Dr. McFarlane indicated that it is possible to solve the problem as it is stated; he deconstructed the problem as it was presented and changed it into a way that he was able to solve it. As he explained:

> Please explain why you cannot perform this operation. This is a funny one. Why? It's incorrect, you can do it, of course, but, but you're not finding a common denominator. So it's not a valid way of adding fractions. It is something called fairy

addition; actually, it is defined. For example, if you had quizzes that were weighted differently. That's the right way to do it. But it's not like if you were talking to a student, I mean you could say you had two pies one was divided into three as well as divided into halves. Well, one of the halves says, three halves fine but anyway, so it's not a valid operation because there's not following the rules for adding fractions, so you know finding a common denominator.

In his criticism, Dr. McFarlane uses his background to modify the intention of the problem, but he understands that this problem is addressing a concept that students of mathematics should know and recognizes that the problem is testing the students' knowledge of fractions addition.

Another way in which Dr. McFarlane uses his background to make sense of the problems was his use of graphs to represent his responses. Figure 38 displays an example of Dr. McFarlane's use of graphs to illustrate his responses. Dr. McFarlane uses a graph to represent his response to Problem 7. He indicated that the graph would help to visualize the response, as he explained, "And if we want we can graph it. Okay. So why is this useful? Well, if you want to do this, so getting a feel for it so." In this fashion, Dr. McFarlane used his writing practices, not just to represent the expected response, but also to provide a more precise representation of the process that he performed.

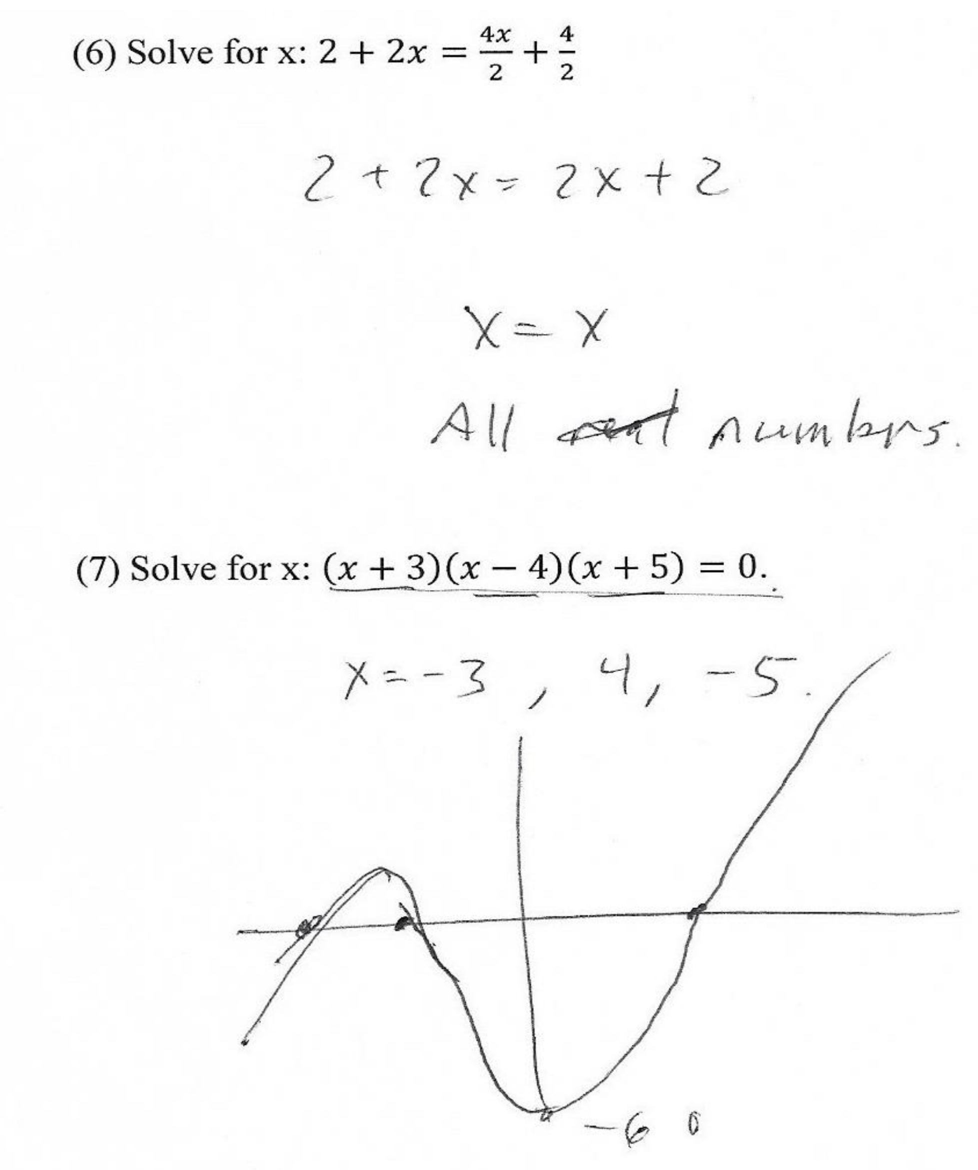


Figure 38
Dr. McFarlane's Response to Problem 7 in the Oral- Explanatory

Dr. Dunn's Literacy Practices

As the rest of the experts in mathematics display, Dr. Dunn's first literacy practice that he applies when solving the problems was reading. Dr. Dunn 's particular ways of reading these problems include re-reading. For instance, in Problem 4 in the think-aloud protocol, I can argue that Dr. Dunn needed to re-read the first statement of the problem to clarify the processes he needed to perform to solve this problem, as he said, " So a number is 30 more than five, what is that number? 30 more than 5, 30 percent more than 5, 30 percent it says what is 30 percent of five." Re-reading facilitated Dr. Dunn's identification of the information; he needed to know to solve this problem.

Another of Dr. Dunn's literacy practices, in this case involving language, was his change in intonation to signal key elements within the problem. In the think-aloud protocol Problem 5, Dr. Dunn stated the following: "The next one, Mary has saved $500. Katie has saved $300. How much **more** [change in intonation] money does Mary save than Katie." In this case, Dr. Dunn changed his intonation to mark the word **more** as a critical element to solve the problem. *More* indicated what kind of mathematical operation Dr. Dunn needed to perform to solve this problem. His change in intonation was notorious across his problem-solving. Dr. Dunn produced multiple instances in which he changed his intonation to mark the processes that he needed to perform to the mathematical problems.

The multidimensional nature of mathematics requires that Dr. Dunn used different written practices to organize the information included in each problem. Additionally, Dr. Dunn drew visual representations of the different processes involving these problems and illustrated different ways to find the solutions. As it is displayed in Figure 39, Dr. Dunn illustrated Problem 6 in the think-aloud protocol to provide a visual representation of the elements that are required to solve this problem.

Dr. Dunn used different modalities of writing when solving this problem. He used the tools available to him to write and draw, which helped him to create another representation of the text. It is crucial to notice Dr. Dunn's choice of changing ink colors to differentiate the stages required to solve this problem. As Figure 40 shows, Dr. Dunn uses color to create multiple layers towards finding the responses to these problems.

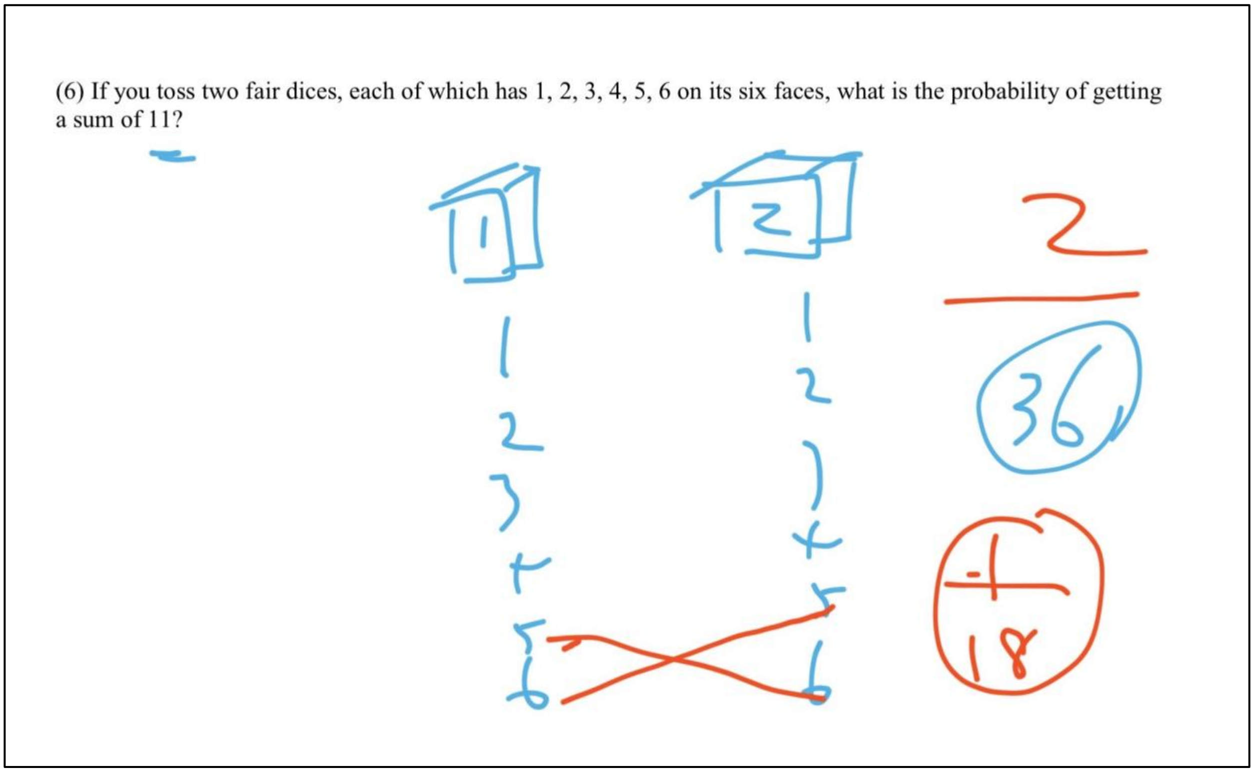


Figure 39
Dr. Dunn's Response for Problem 6 in the Think-Aloud Protocol

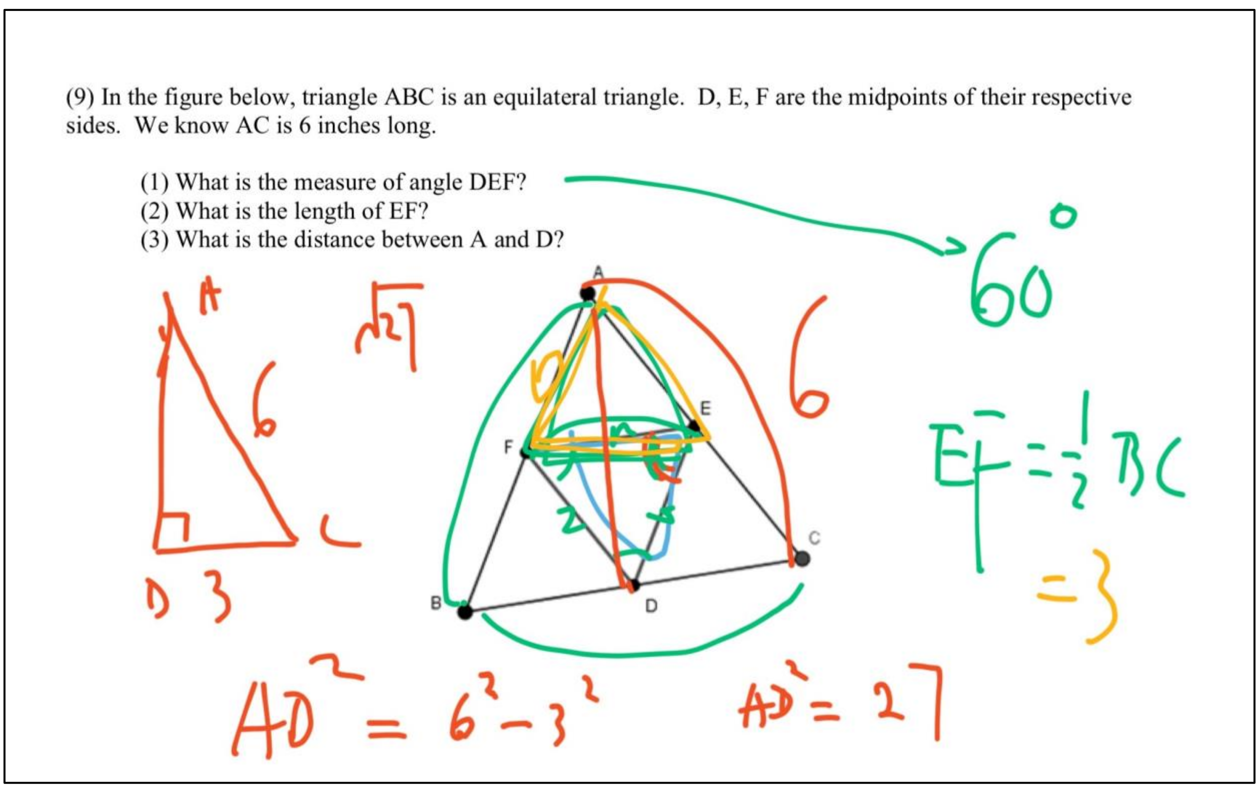
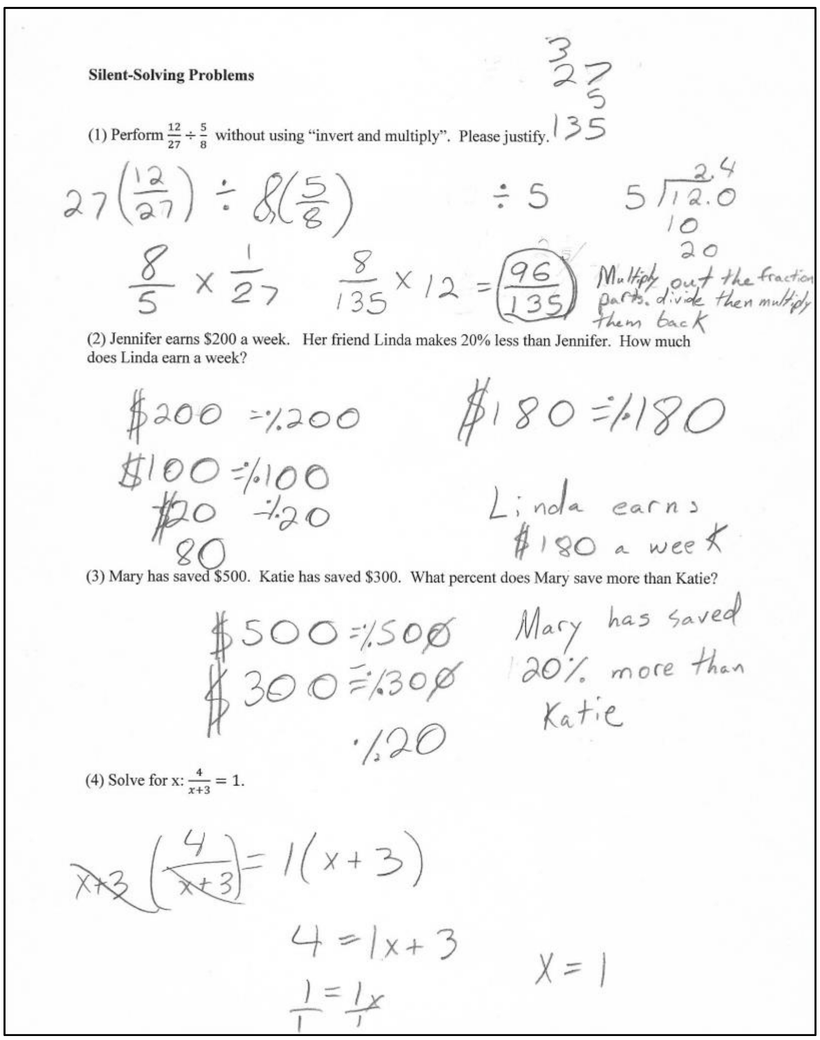


Figure 40
Dr. Dunn's Response to Problem 9 in the Think-Aloud Protocol

Dr. Dunn colored each stage of the development of this problem, indicating the information that was required to continue to the next stage. This writing practice allows Dr. Dunn to convey a more visual message of the processes that are required to solve this problem.

Cesar's Literacy Practices

Cesar displayed a unique repertoire of literacy practices while solving the mathematical problems. One of the most salient of these practices is how he provided a narrative response to these problems. Figure 41 contains a few instances in which Cesar used a narrative to provide his response to the problem.



Silent-Solving Problems

(1) Perform $\frac{12}{27} \div \frac{5}{8}$ without using "invert and multiply". Please justify.

$27\left(\frac{12}{27}\right) \div 8\left(\frac{5}{8}\right)$ $\div 5$

$\frac{8}{5} \times \frac{1}{27}$ $\frac{8}{135} \times 12 = \frac{96}{135}$

Multiply out the fraction parts. divide then multiply them back

(2) Jennifer earns $200 a week. Her friend Linda makes 20% less than Jennifer. How much does Linda earn a week?

$200 = %200 $180 = %180

$100 = %100

$20 = %20

80

Linda earns $180 a week

(3) Mary has saved $500. Katie has saved $300. What percent does Mary save more than Katie?

$500 = %500

$300 = %300

%120

Mary has saved 120% more than Katie

(4) Solve for x: $\frac{4}{x+3} = 1$.

$x+3\left(\frac{4}{x+3}\right) = 1(x+3)$

$4 = 1x + 3$

$\frac{1}{1} = \frac{1x}{1}$

$X = 1$

Figure 41
Cesar's Responses to Silent-Solving Protocol – Problems 1 through 4

Even though Cesar provided a numerical answer for the problems illustrated in Figure 41, he re-wrote the response and provide a narrative of his response. I can argue that Cesar's way of presenting the result of this problem helped him to synthesize the processes he performed; therefore, showing his audience the product of such processes.

Another practice that Cesar is to include data into the graphs to have an additional visual representation of the problem. For example, in Problem 9 in the think-aloud protocol (Figure 42), Cesar located the measures of the sides of the triangle in the graph.

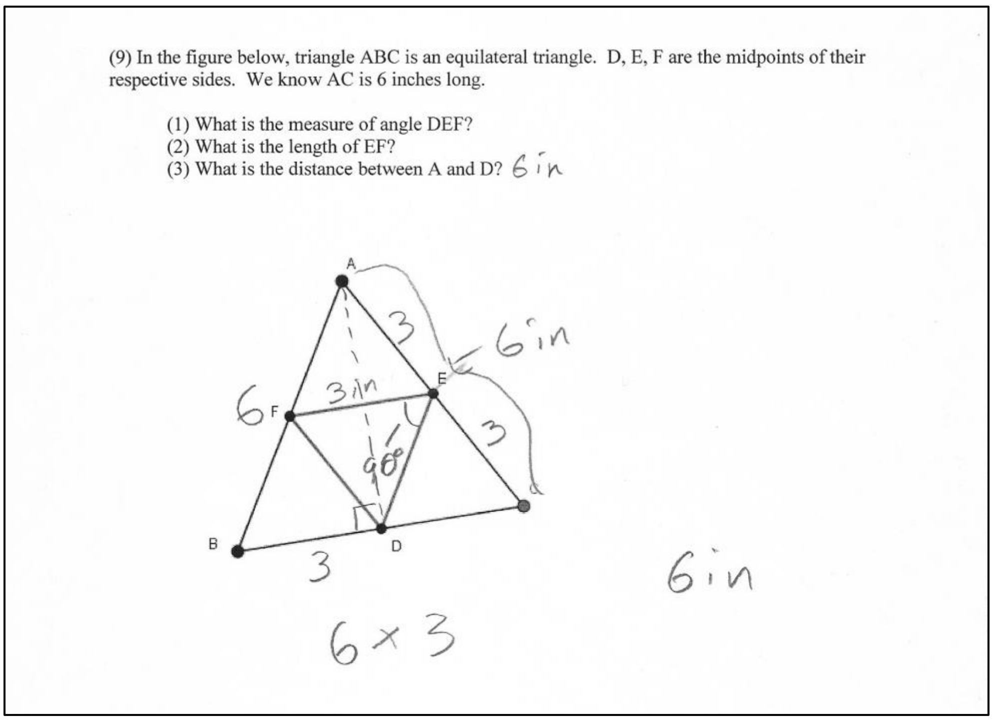
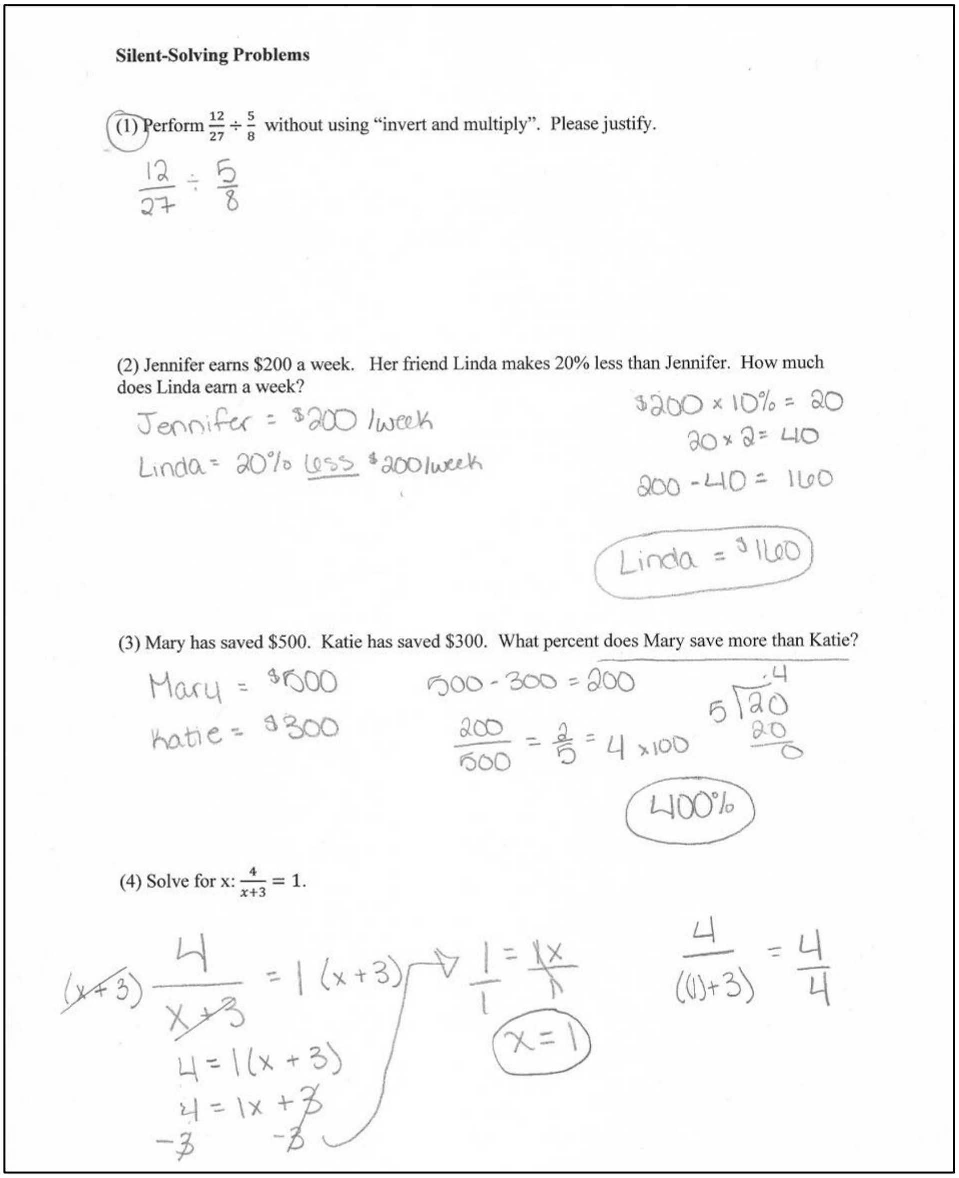


Figure 42
Cesar's Response Problem 9 in the Think-Aloud Protocol

Simultaneously, Cesar explained his mental process of solving this problem and wrote key elements, such as values and measurements that he needed to identify to be able to solve the problem. However, Cesar confused the calculations of this problem and did not provide an accurate response for it. It seems that the way Cesar interpreted the graph mislead him making not aware of the processes (Pythagorean Theorem) required to solve the problem.

Sophie's Literacy Practices

Sophie displayed unique literacy practices when solving the mathematical problems. She underlined important information from the text and marked her responses as it is displayed in Figure 45 for Sophie's responses for the silent-solving protocol.



Silent-Solving Problems

(1) Perform $\frac{12}{27} \div \frac{5}{8}$ without using "invert and multiply". Please justify.

$\frac{12}{27} \div \frac{5}{8}$

(2) Jennifer earns \$200 a week. Her friend Linda makes 20% less than Jennifer. How much does Linda earn a week?

Jennifer = \$200 /week

Linda = 20% less \$200/week

\$200 × 10% = 20

20 × 2 = 40

200 - 40 = 160

Linda = \$160

(3) Mary has saved \$500. Katie has saved \$300. What percent does Mary save more than Katie?

Mary = \$500

Katie = \$300

500 - 300 = 200

$\frac{200}{500} = \frac{2}{5} = .4 \times 100$

$5\overline{)2.0}$ = .4

400%

(4) Solve for x: $\frac{4}{x+3} = 1$.

$\cancel{(x+3)}\frac{4}{\cancel{x+3}} = 1(x+3)$

$4 = 1(x+3)$

$4 = 1x + \cancel{3}$

$-3 \quad -\cancel{3}$

$\frac{1}{1} = \frac{\cancel{1}x}{\cancel{1}}$

$x = 1$

$\frac{4}{(1)+3} = \frac{4}{4}$

Figure 43
Sophie's Responses for Problems 1 through 4 in the Silent-Solving Protocol

As Figure 43 shows, Sophie used her writing as a way to highlight important information in the text and focus the attention of the reader on her responses. Additionally, Sophie utilized a particular practice to make sense Problem 5, which is displayed in Table 18.

Table 18

Sophie's Response to Problem 5 in the Oral Explanatory Protocol

Line	Statement
1	Okay, define g of x equals three x minus one, where x is a real number.
2	So, what does g of x have an output of 26?
3	So, in this, if you're putting 26 in place of x,
4	and then you will take the equation one look like three times 26 minus one.
5	And if you use what is the word,
6	orders of operations
7	which is, I remember as **Please Excuse My Dear Aunt Sally**
8	So, you would start with this excuse,
9	you don't have parentheses
10	and you don't have an exponent
11	so, then it would go multiplication and then division
12	so, you will first start off with three times 26, which is 18 678.
13	And then, since you got rid of the multiplication and division first,
14	next would be addition and subtraction
15	So, you would take the 78 and subtract one,
16	and then that would be 77.
17	What is the value of g g zero?
18	Okay, I'm gonna assume that since this is zero
19	Everything else is zero because when you mul...no
20	because it will be a negative one?
21	I don't know how to do that one
22	Functions are a little confusing,
23	And what is G three plus four
24	So, you would be taking three plus four and putting that in place of x
25	g three plus four equals three times three plus four minus one
26	And if you're going back to orders of operations,
27	you have to do parentheses first
28	You'll do three plus four equals seven
29	And then you take seven and multiply it by three to get 21.
30	And then you'll subtract one to get twenty
31	I think

Problem 5 in the oral-explanatory protocol states the following:

5) Define g(x)=3x-1, where x is a real number.
   (a) What does g(x) have an output of 26?
   (b) What is the value of g (g (0))?
   (c) What is g (3+4)?

When solving this problem, Sophie stated that she needed to follow the orders of operations to solve the equation and additional variables. To achieve this purpose, she used a mnemonic device, as it is shown in Table 18.

In her response to Problem 5, Sophie attempted to use a mnemonic device to follow the steps she considered necessary to solve this problem. However, this strategy did not guide her into the processes she needed to perform to solve it successfully. Contrarily, it directed her attention to a prescribed sequence of steps that made her ignore the context of the variables; therefore, miscalculating her response.

Maggie's Literacy Practices

As a bilingual speaker of Spanish, Maggie's literacy practices are strongly connected to her ability to use English and Spanish simultaneously. Maggie used both languages to make sense of the problems and to perform the required processes to solve the mathematical problems successfully. Similarly to Ms. Brigg's translanguaging, Maggie used her first language (Spanish) to count. Moreover, she required to use Spanish to perform mathematical operations. In the excerpt displayed in Table 19, Maggie used both languages simultaneously to solve Problem 9 in the think-aloud protocol.

When solving this problem, Maggie required using Spanish to make sense of the processes she needed to perform. She translanguaged into Spanish to add and subtract; additionally, she used Spanish to assign attributes (Table 19; Lines 4, 11, and 14) to some elements of this problem, facilitating its processing and obtaining an accurate result. It is relevant to mention that I disclosed to Maggie that I am a bilingual speaker of Spanish as well. Interestingly, Maggie displayed multiple occasions of translanguaging in the think-aloud protocol; however, she translanguaged only for a few seconds in the oral-explanatory protocol to perform one step in a division.

Table 19

Maggie's Response to Problem 9 in the Oral Explanatory Protocol

Line	Statement
1	Okay, so then it's,
2	let's do A squared plus B squared equals C squared situation here
3	where A That's going to be the hypotenuse,
4	that we know **es el seis** [it is six] six squared
5	let me erase this
6	so, I have more space over here.
7	So then, A .
8	Let's make it
9	Let's make A the variable,
10	then that's three squared equals six squared
11	So, **va a ser** [it's going to be] A squared
12	**mas nueve mas treinta y seis** [nine plus thirty six]
13	A squared equals
14	**Treinta y seis menos nueve** [thirty-six minus nine] **diesciseis menos** [sixteen minus] nine **es** [is] seven twenty seven.
15	A equals square root of twenty seven,
16	which is, what is the square root of twenty-seven
17	but I probably did something wrong here,
18	but I'm gonna leave it.
19	That's square root twenty-seven.
20	I would probably get partial points for this one if that's wrong because the process is right

Maggie displayed unique writing practices, as well. She indicated that one of the writing practices that helps her when performing mathematical problems is to re-write the equations she needs to solve. In Problem 5) *Solve for x:* $\frac{(x+5)(x-5)}{x-5} = x + 5$ in the silent-solving protocol, Maggie experienced difficulties in solving this problem. After her unsuccessful first try, she decided to start over. On both occasions, Maggie started working on this problem by first re-writing it. Figure 44 shows Maggie's sequence to solve Problem 5.

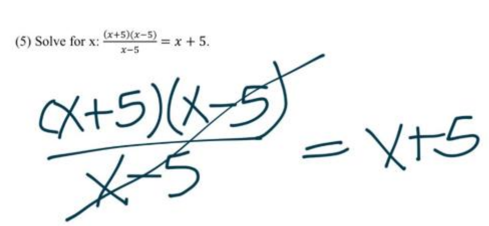
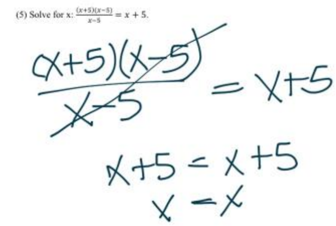
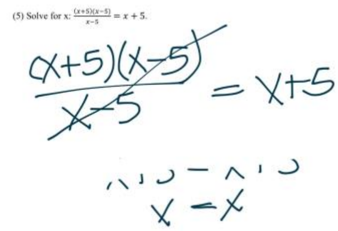
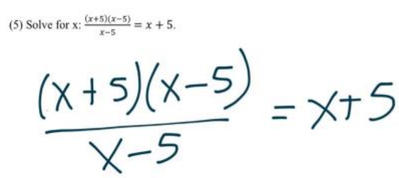
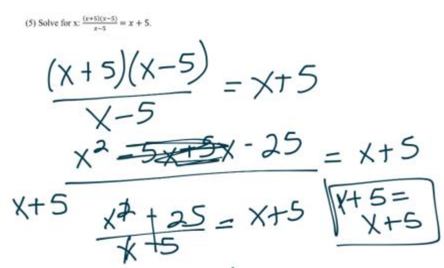
Re-Writing Problem 5 - First Try	
	
Deleting Work on Problem 5 to Start Over	
	
Re-Writing Problem 5 – Second Try	
	

Figure 44

Maggie's Sequence to Solve Problem 5 – Silent Solving Protocol

As this sequence shows, Maggie started Problem 5 by re-writing it, as she was confused with the response she calculated, she deleted all her work on Problem 5 to start working over on it. Re-writing helped her to visualize if she missed any step previously to find the answer.

Ruby's Literacy Practices

Ruby marked important information from the text of the problem and located this information in the graphs to make sense of them and to track the values and processes she needed to focus on when calculating the problems. However, she used different ways of marking key elements. Her choices of key terms are displayed in Figure 45.

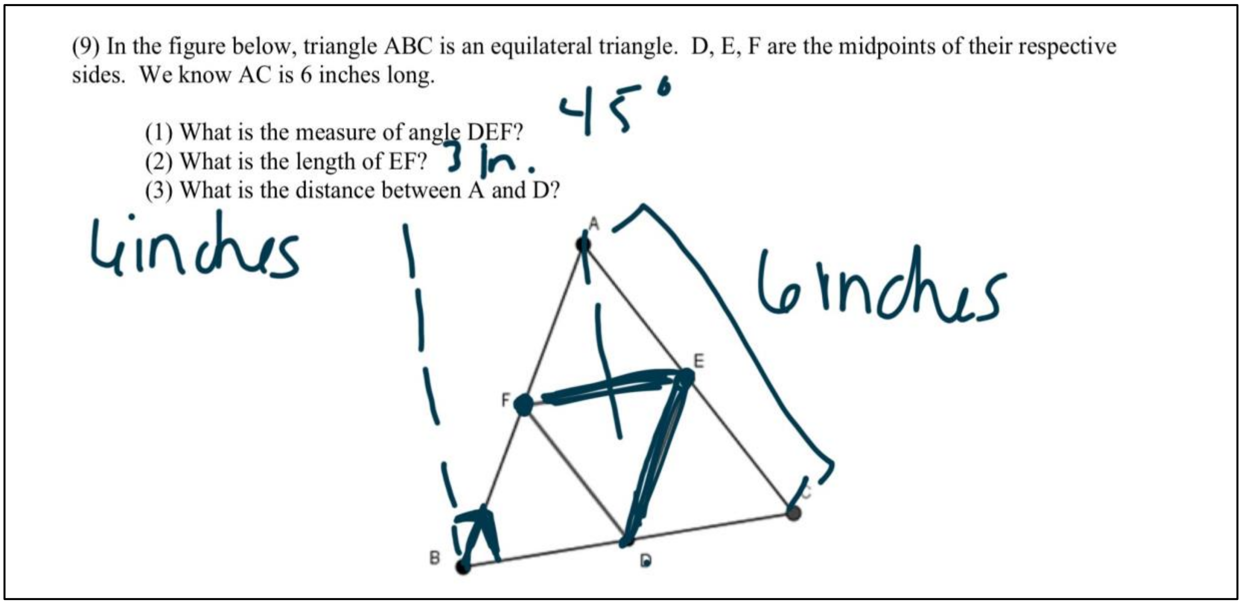


Figure 45
Ruby's Response to Problem 9 – Think-Aloud Protocol

However, it seems that Ruby had issue that Cesar did to solve this problem. She did not used the required process to solve this problem. It seems that the figure that supposed to illustrate this problem confused the participants make them calculate the solution of this problem without further consideration of the geometric shapes illustrated in Problem 9.

Ruby's unique writing practices show how she processes fractions and use graphs to make sense of them. To make sense of fraction and to solve Problem 1 *What is bigger between 5/27 and 5/17?* Please explain in the think-aloud protocol; Ruby used a graph to represent fractions and determine the response to this problem, she was the only participant that drew a graph to represent a fraction. Ruby's response to Problem 1 in the think-aloud protocol is displayed in Figure 46.

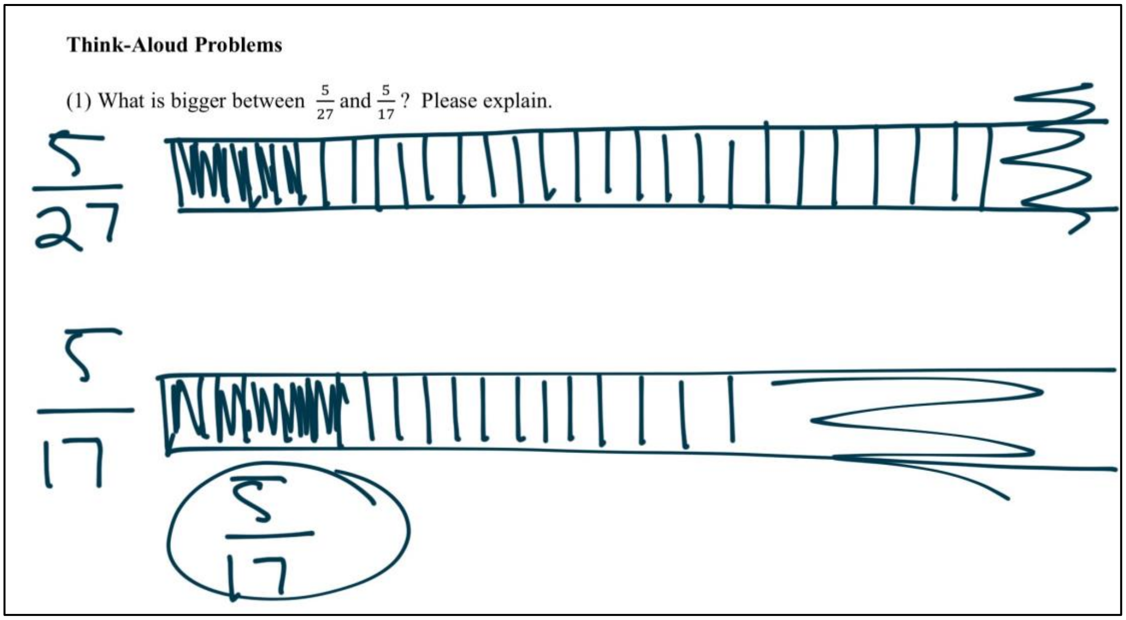


Figure 46
Ruby's Response to Problem 1 in the Think – Aloud Protocol

Another of the literacy practices that Ruby consistently displayed across the protocols was her organization of the information about key elements and values that she extracted from the text, especially in the modeling problems. Figure 47 presents how Ruby organized the information in Problems 2 and 3 in the silent-solving protocol.

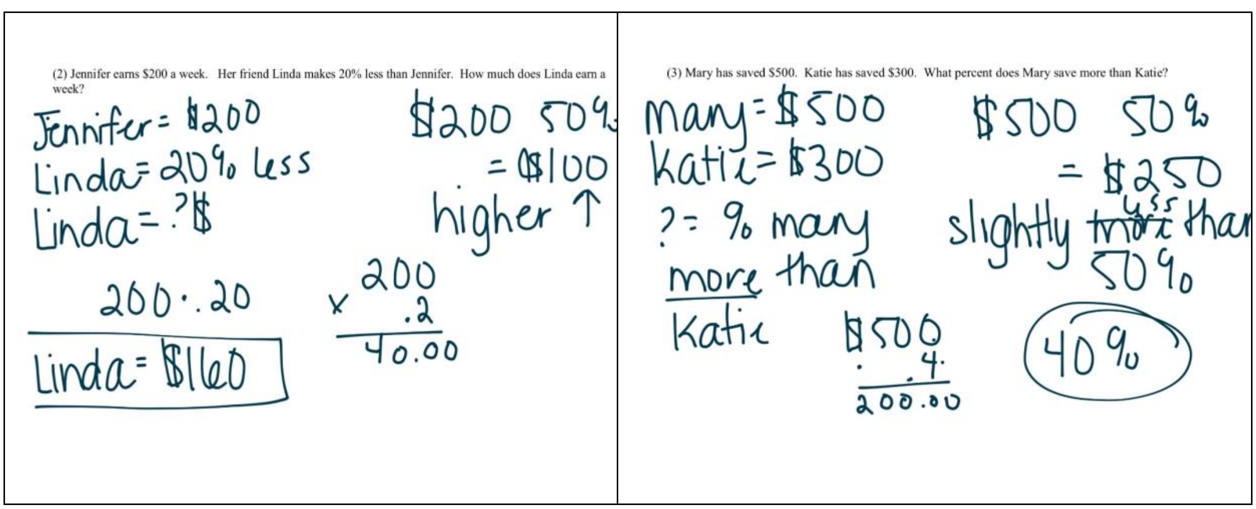


Figure 47
Ruby's Responses to Problems 2 and 3 in the Silent Solving Protocol

As Figure 47 shows, Ruby extracts information from the text by first labeling the components of the problem and assigning them the value stated in the text. She organizes this information on the left corner of the problems. On the right corner, she writes a numeric representation of her thoughts of how the problem could be interpreted. Then, Ruby performs the required calculations to find the solution of the problem. Finally, she marks the answer to the problem with a square or a circle to bring the reader's attention to her response, which was a practice found in other participants as well.

CHAPTER 5

IMPLICATIONS AND CONCLUSIONS

When a secondary student earns admission into a higher education institution, it is expected of them to bring the knowledge and skills that would support the demands of academic settings, including the specialized practices that are intrinsic to the disciplines. I designed this study to highlight the importance of college students' language and literacy practices as they become members of a discipline. Moreover, I aimed to understand how college students, who are becoming future teachers, develop literacy and linguistic practices unique to mathematics as a part of their baggage of disciplinary knowledge.

As a case study, I focused on how pre-service teachers develop mathematical literacy. I invited experts in mathematics to participate in this study as well. My intention to observe mathematicians' literacy practices and linguistic repertoires was to generate a framework to analyze and argue how pre-service teachers learn and acquire the highly specialized practices and repertoires that are found in mathematics as a discipline.

I framed this study under three theoretical considerations. First, I applied the current notions of disciplinary literacy as a conceptual framework to understand the development, learning, acquisition, and requirements of the specialized language and literacy of the disciplines in post-secondary contexts. Second, I comprised theoretical orientations to reveal the relationship between experts and novices from a non-traditional standpoint, which depicts this relationship as a continuum of experiences rather than a fixed dichotomy. Finally, I included the tenets of Halliday's (1994) functional theory of language to understand how the different functions of language shape the registers that the participants displayed when they solved mathematical problems.

The discussion of the background of this study, as well as the conceptual framework, provide theoretical elements to draw three guiding research questions: 1) What do the experiences of pre-service teachers and experts in mathematics reveal about their understanding of mathematical literacy? 2) How do pre-service teachers and experts in mathematics use language when solving mathematical problems? and 3) What literacy practices do pre-service teachers and experts in mathematics utilize wh en presented with modules that require mathematics problem-solving? These questions helped me construct a comprehensive picture of mathematical literacy in both pre-service teachers and experts in mathematics, highlighting the commonalities between these groups and illustrating the patterns of practices that they display when solving mathematical problems.

To explore these questions, I designed two data collection sessions. In the first session, I conducted a semi-structured interview with the participants, which I modified to have a more approachable instrument when I interviewed the pre-service teachers. In the second session, the participants followed three protocols to solve mathematical problems. Both groups solved the same set of problems. I recorded and transcribed the data, which I analyzed inductively. In the forthcoming sections, I discuss the results of the data analysis and posit the possible implications for literacy and language instruction as well as for teacher education in higher education contexts.

Learners' Experiences and their Understanding of Mathematical Literacy

The experiences of pre-service teachers and experts in mathematics are worthy of studying because they could help me understand their future and current teaching practices (Towers et al., 2017). Moreover, the experiences of pre-service teachers and experts in mathematics relate to their funds of knowledge that are evident in their understanding of mathematical learning.

In this study, the pre-service teachers and experts in mathematics described their experiences with mathematics as processes that resemble Krashen's (1982) hypothesis of second language development. Krashen hypothesizes that second languages are developed by two processes: 1) a process called *acquisition*, in which a second language is acquired in a similar fashion than the subconscious process experienced by a child when they learn their first language, and 2) a process called *learning*, in which second languages are consciously learned mainly through schooling.

These participants described their experiences with mathematics as acquisition and learning processes. However, these processes were not developed in a linear fashion. Instead, these participants described a dynamic path to develop their mathematical thinking (Schoenfeld, 1985). Some of the participants (e.g., Dr. McFarland and Ms. Briggs) indicated that they naturally acquired mathematics, which facilitated their engagement with the subject. For other participants (e.g., Cesar and Dr. Dunn), mathematics was learned throughout their schooling career.

These participants' experiences shifted over time, in conjunction with the kind of support and context of instruction they received. The nurturing environment that their families and teachers provided was crucial to develop a long-term engagement with mathematics to the point of adopting it as the core of their professional lives.

Although both groups described similar experiences when learning mathematics, I found contrasting beliefs about conducive practices for mathematical learning. On the one hand, the mathematical experts indicated that transforming the abstract mathematical concepts into visual artifacts is crucial for mathematical learning. Additionally, the experts believe that learning mathematics embraces the manipulation and actual use of these artifacts. On the other hand, the pre-service teachers believe that it is their former or current instructors' mathematical knowledge that provides a conducive environment for their mathematical learning.

It is not surprising to find different sets of beliefs about mathematical learning between pre-service teachers and mathematical experts. Aligned with what Boston (2013) found, experts in mathematics are more reflective about the nature of learning by recognizing the importance of visual artifacts for learning. Similar to what Hogan, Rabinowitz, and Craven III (2003) claim, novice teachers tend to rely on surface structures to understand learning. In the case of these pre-service teachers, they understand that mathematical learning is more conducive when the mathematical concepts are 'given' to them. They distance themselves from their process of learning mathematics by implying that it is somebody else's knowledge that facilitates their learning. For example, Cesar, a junior in the Especial Education program, indicated that what helped to learn mathematics better was the instruction he received from his instructors in college, as he explained:

> [C]ollege teachers that work for big jobs and work for NASA and big mathematical jobs and things like that sat down and explained those concepts to me that I had gaps in, and I think that's what really pushed me forward, and that cleared those gaps for me

Another point of contrast between these groups is their challenges when learning and teaching mathematics. As a commonality, both groups reported not having struggles when learning mathematics. This finding is related to the first result of natural and enjoyable experiences when learning mathematics. Yet, when asked about the challenges these groups experience when teaching mathematics, I obtained what appeared to be two different types of responses. The pre-service teachers perceived their background as a challenge when they are teaching, while the experts indicated that it is their students' background, and even motivation, that is the true challenge they face in their professional practice.

The pre-service teachers seemed to be concerned about their mathematical knowledge and the implications that this assumed 'lack of knowledge' would have on their students' performance. This finding is related to their beliefs about mathematical learning. For these pre-service teachers, if learning is depending on the instructor's mathematical knowledge, they would assume that their background would not provide enough support for their students' learning. For instance, Ruby, a senior in the Elementary education program, explained that she learned mathematics with a different approach than the one she is currently learning to teach it. Ruby recognized the way how she learned mathematics is not helping her to apply it into real-life situations, which could be helpful to make her students connect the mathematics with daily-life problems, as she explained:

> [S]o, because I was learning better with repetition and just consistency and hands on stuff […] that's just, it just was easier to learn for me because of how I learned math in elementary school. Once I got to geometry, I really struggled with the explaining part.

In contrast, the experts in mathematics seemed to consider their students' readiness to take high-level mathematics classes as the real challenge. This finding is aligns with current research on secondary students' readiness, which indicates that in 2018 not more than 38% of high school of graduates achieve the benchmark for readiness as measured by the ACT (American College Testing Inc., 2018) and no more than 49% of these students as measured by the SAT (College Board, 2018).

The findings related to literacy and mathematical literacy show that overall, these participants evoke Gee's (2006) definition of literacy by understanding it in all its dimensions.

Even though these participants' responses seemed to indicate personal definitions of literacy, each of these responses tied literacy and Discourses. For these participants, literacy is more than the cognitive practice of decoding the printed language; literacy is essentially the exchange of ideas through communicative processes that are shaped differently depending on the context of the exchange and the disciplinary language (a Discourse) that is used in the moment of the exchange.

The participant's definition of mathematical literacy is aligned with their definition of literacy. Both groups acknowledged that mathematical literacy entails the knowledge of basic mathematical concepts and semiotic resources, which are introduced and contextualized with language (O'Halloran, 2005). Additionally, these participants appeared to include aesthetic elements to complement their definition of mathematical literacy. They also indicated that they would include aesthetic elements in their classes to make real the mathematical concepts and to expose their students to practical scenarios, in which the mathematical concepts become tangible and real. It seems that these participants' heuristics are aligned with Dewey's (1934) pragmatism regarding the value of aesthetic elements to the development of genuinely learning experiences. Dr. Dunn, a bilingual expert in mathematics with more than ten years of experience teaching mathematics in higher education institutions, indicated that to develop mathematical literacy is necessary to develop an appreciation for it as if it were music, as he explained:

> I think, you know, it's just not necessary to know a lot of advanced math, which is great if they do, but I think it's a general appreciation of mathematics, like the music, you know, and you like the music, and you can maybe have a tune if you're happy. That's my understanding of mathematics, a literate person in appreciating the mathematical side, the quantitative side, and also see the artistic side of life

For these participants, the concept of mathematical literacy is embedded to the social practices that mathematics promotes, as “it is not possible to promote a conception of mathematical literacy without at the same time – implicitly or explicitly – promoting a particular social practice” (Jablonka, 2003, p. 75). These participants are aware of the social nature of mathematics. Throughout this study, the participants remarked the practical applications of mathematics in daily life situations and the essential role of real-life contexts to make sense of the abstract mathematical concepts.

To compound a portrait of these participants’ views of literacy, mathematical literacy, and therefore disciplinary literacy, I was interested in exploring their views about their discipline as a community of practice (Lave, 1998), in which the members of this community share the social elements of interaction, such as language or tools of communication (Becher & Trowler, 2001). I explored these participants’ communicative practices with their peers and their students. Both groups described different practices to communicate with these groups. Body language is one of the tools these participants use to communicate within their classrooms with their instructors or students. The more experienced instructors explained that they could recognize their students’ frustration and struggles in their classrooms by looking at their faces.

These participants described formal ways to communicate with their colleagues. The experts in mathematics explained that they primarily use email to communicate with their colleagues, while the pre-service teachers indicated that they prefer one-to-one communication with their peers.

Another feature that I aimed to identify in my participants was their connection to different disciplinary associations. The pre-service teachers did not indicate that they belong to a professional community; instead, they are affiliated with social organizations. Contrastingly, each of the experts in mathematics named professional organizations that they are affiliated and are active members by participating in conferences and seminars.

To conclude the analysis of these participants' notions of mathematics as a discipline, I asked them about what makes mathematics different from other disciplines. The pre-service teachers indicated that problem-solving, complex structure and flexibility are the most salient features that make mathematics a unique discipline. The experts in mathematics consider that the affective domain makes mathematics different from other disciplines. The experts in mathematics concord that there is not a perceived fear in other disciplines. For the experts, it is the fear of mathematics or what Tobias (1980) defines as mathematical anxiety what shapes it as discipline.

Systematic Linguistic Analysis of Pre-Service Teachers and Experts of Mathematics when Solving Mathematical Problems

The main goal of this study is to understand how pre-service teachers develop their concepts and practices of disciplinary literacy, which refers to "the ability to engage in social, semiotic, and cognitive practices consistent with those of content experts" (Fang, 2012, p. 19). Moreover, this section aims to discuss the findings of exploring from a systematic linguistic perspective, how these participants' linguistic choices when solving mathematical problems show their engagement with mathematics as a discipline.

I applied the tenets of Halliday's functional linguistics to explore how these participants present and organized their responses (Theme analysis), exchange meaning with the text (Mood analysis) and experience the context when solving mathematical problems (Transitivity analysis). The mathematical experts' linguistic choices are seen as paths of development of disciplinary discourses and the pre-service teachers' choices as the language that is in process of learning and acquisition (Krashen, 1982).

Clause as a message, represented by the analysis of Theme was the first metafunction that I analyzed in these participants' clauses. The textual metafunction situates the context in which the clause occurs, and it provides the point of departure of where the message will go (Halliday & Matthiessen, 2014).

The Theme analysis of the participants' clauses shows that the pre-service teachers and the experts in mathematics have different choices when situating their clauses within the context and signaling the start point of the message. The pre-service teachers preferred to locate the point of departure of the message in marked textual themes, which appeared to be an attempt to sequence the messages and connect the clauses one after the other to provide a consistent message. For example, as displayed in Table 9, Ruby sequenced her response by choosing adjuncts, especially *so* and *and* as the first element of the clause.

On the other hand, the experts in mathematics preferred contextualized their messages as either an exchange of information or as an interpretation of their experiences. For example, Dr. Dunn's response when solving Problem 6 in the think aloud protocol reveals that, even though he also utilizes adjuncts to sequence his response, he uses them sporadically and chose other thematic elements (e.g., pronouns, noun phrases, conjunctions) to make sense of the processes he is performing.

> **We** have two dice. **Each dice** has six choices right. **This dice** one this dice two and each has six choices one through six, right one through six, **and** that's all together, **we** have 36 possibilities 111213141516, all the way to six. **That**'s 36 possibilities, **and** we're looking at. **We**'re looking at 11, **because** there are many ways to do this. **I**'m going to try to find you, **how** many ways can you get 11 to five and a six and six and five. **There** are not

other ways. **Four** and six will be 10. **So**, we have two, **you** can see that there are two there. **So** it's a really two out of 36 whatever that is

In other words, the mathematicians chose to either contextualize their messages as an exchange of information from the text to obtain a result, or to interpret their problem-solving process as an experience rather than a sequence of processes.

The findings in the Theme analysis could be interpreted in terms of mathematicians' in-depth understanding of the importance of the context of the problem (O'Halloran, 2005), which allows them to bring the required processes to the act of exchanging information with the text and therefore use this information to solve the problem. The mathematicians do not necessarily perceive problem-solving as a set of sequenced steps; instead, sequencing is balanced with the inclusion of other focal themes in the clause. For instance, the mathematicians included Conjunctive Adjuncts as well; however, they made this choice fewer times than the pre-service teachers. As I illustrated with Dr. Dunn's response to Problem 6 in the think-aloud protocol.

It seems that this finding does not indicate that the pre-service teachers only consider sequencing (noticeable by a more frequent use marked themes) as key for problem-solving, but their notion of problem-solving as an exchange of information and as an experience, at least at this stage, is still in development.

Mood Analysis intends to establish whether the clause is providing cues about the speech roles that the interactants assume during an interaction. The interactants can exchange messages to give or demand goods and services, or information (Halliday & Matthiessen, 2014). In conversational situations, the speech roles are somewhat transparent; thus, the interactants can respond to the message and provide what the other person's message is requesting. However, in the case of the printed text, the speech roles are opaque. In this study, the analysis of the Mood opened a window to observe how these participants exchange information with the text to make sense of the abstract mathematical concepts, the mathematical symbolism embedded in these problems, and the processes required to foreground the result of these problems.

Halliday (1989) explains that in social interactions, the lexicogrammatical structure of the clause shapes the function of the message. The lexicogrammatical structures of the MOOD are constructed in discursive contexts (Thibault, 1995), in which modality, temporality, and polarity describe the nature of the exchange (Halliday, 1994) and provide the structure for the interactants to continue or conclude the exchange.

The analysis of the Mood indicated that these participants displayed a tendency to exchange information with the text (mathematical problems) as a predicted occurrence of events, which was displayed by their preference in using Future Operators as the most commonly used structures of the MOOD. Additionally, there are differences between these two groups in how certain they describe the processes they are performing to solve the mathematical problems. According to the findings presented in Table 12, the pre-service teachers did not display the same degree of certainty than the mathematicians; instead, the pre-service teachers were more hesitant in supporting the validity of their processes by choosing more frequently Low Positive (e.g., can, could, may) and less frequently High Positive modal operators (e.g., must, have/had/has to, need to).

I analyzed the participants' clauses as a representation of their experiences when solving the mathematical problems through the TRANSITIVITY system, which describes how the interactants sense the flow of events that occur while they experience the past, present, or future reality (Halliday and Matthiessen, 2014). This flow of events is constructed by lexicogrammatical structures that unveil how the interactants do, sense, relate to, say, behave, and express existence through a system of processes that encompass a particular configuration dependent of the interactants' roles when they are experiencing the world (Eggins, 1994). This configuration fluctuates depending on the speaker's role and assigns to this speaker different configurations (i.e., participants and elements of the clause). For the sake of this study, participants and elements of the clause were not part of this analysis. In this study, The TRANSITIVITY analysis was exclusively on the processes of the clause.

After transcribing these participants' responses, I divided the resulting text into clauses. A clause is a unit of meaning that unifies the different metafunctions of the language (experiential, interpersonal, and textual; [Matthiessen, Teruya, & Lam, 2010]). Then, I classified the resulting clauses according to the process that they contain: Material, mental, relational, verbal, behavioral, or existential.

I found that there are differences between these groups in their choices of processes. Both groups show a more frequent preference towards processes that assign an attribute to the participants or relate them to the rest of the elements of the clause. However, I found different preferences for the Material and Mental processes in these groups of participants. The pre-service teachers showed a preference toward Material, while the mathematicians show a preference toward Mental processes. The material processes indicate that the pre-service teachers were experiencing mathematical problem-solving from real or tangible actions (Eggins, 1994). This difference seems to indicate that the pre-service teachers assume their role as actors that have a goal in mind and perform the processes that are required to achieve that goal. In other words, they describe their experiences of solving the problems as concrete processes, which require a canonical structure Actor, Process, and Goal. Sophie, a senior in the Elementary Education program produced the following clause when solving Problem 7 in the think-aloud protocol: “You just plug these numbers for X.” In this clause Sophie chose to use a Material process (plug) to indicate the action that she needed to perform to solve the problem.

On the other hand, the mathematicians prefer to relate their experiences of solving the mathematical problems as they were feeling, sensing, or thinking about the processes involved in finding the solution for these problems. For the mathematicians, mathematical problem-solving is experienced as a mental activity, which, I argue, shows that the experts in mathematics exhibited a more abstract thought while solving problems. For these experts, solving-problem requires a higher level of abstraction, evaluation, and conceptualization of processes required to solve a mathematical problem. For instance, Dr. Arnold produced the following clause complex: “So, I suppose the image of the natural numbers under h is going to be the even natural numbers.” In his response, Dr. Arnold chose the Mental process *suppose* to reflect the mental activity that he needed to perform to make sense of the problem that he was solving.

Literacy Practices of Pre-Service Teachers and Experts in Mathematics when Solving Mathematical Problems

In this section, I discuss the findings of the literacy practices that the pre-service teachers and experts in mathematics displayed when solving mathematical problems. I understand literacy practices as Barton and Hamilton (2000) define them:

> Literacy practices are the general cultural ways of utilizing writing language which people draw upon their lives. In the simplest sense literacy practices are what people do with literacy. However practices are not observable units of behavior since they also involve values, attitudes, feelings and social relationships. This includes people’s awareness of literacy, constructions of literacy and discourses of literacy, how people talk about and make sense of literacy (p. 7).

In Chapter 4, I presented the literacy practices that each of the participants displayed when solving the mathematical problems as I consider them a unique representation of their relationship with literacy. However, for the sake of this section, I synthesize and discuss these findings to elaborate a depiction of these participants' disciplinary literacy practices to understand how experts and novices read, write, and communicate under the specific context of their area of study (Dobbs, Ippolito, & Charner-Laird, 2016).

When presented the mathematical problems, the experts in mathematics used reading as the first way to connect with the text. However, the reading practices that they displayed were not linear. The mathematicians re-read the text, omitted or added information, transacted with the problem (Rosenblatt, 1994) to create a new text that made sense to them. In this transaction, the experts adopted a critical stance towards the problems. The mathematicians analyzed the problems and criticized the narrative of the text, its description of mathematical concepts, and whether it was an exemplary exercise to use in a mathematical classroom. As Shanahan, Shanahan, and Misischia (2011) found, for the experts in mathematics' reading is an interpretative practice, which provides additional elements of analysis. The experts in mathematics' reading practices were not oriented only to solve the problem, but to make sense of it within the context where a particular problem was presented.

Similarly, the pre-service teachers' first approach to the text was reading it. They re-read the text as well but differently from the experts. The pre-service teachers used their re-reading as a practice to extract and organize the information that the text provided. The pre-service teachers did not criticize the problems and solved them without any additional consideration other than the information presented to them.

These participants explained that visualization was one of the most efficient practices that helped them to learn mathematics better. This experience is aligned with some of the writing practices that these participants displayed. Both groups used additional resources to make sense of the text. For instance, they drew graphs and used different colors to visualize the elements of the problems that provided crucial information to solve them. Additionally, they signaled key information in the text and marked their response. I argue that the practice of marking their responses, mainly observed in the pre-service teachers, responds to their intention of showing to their audience the accurate completion of the task, as only correct answers were marked. The writing practices were commonly displayed simultaneously with reading.

The bilingual participants (Ms. Briggs and Maggie) used their languages to make sense of the problems. I used the term translanguaging to refer to the practices in which these participants used their languages. Translanguaging is a term that addresses not the existence of two separate linguistic systems in the bilingual brain but only one linguistic repertoire that has been socially constructed to appear to be two separate languages (García & Wei, 2014). Ms. Briggs and Maggie translanguaged in multiple instances while solving the mathematical problems. They counted, calculated, and defined mathematical terms in their first language. Maggie and Ms. Briggs' translanguaging can be understood as a practice to appropriate of the mathematical processes (Garza, 2017) that were required to solve these problems. By translanguaging Ms. Briggs and Maggie did not only make sense of the text, mathematical concepts and processes, but also gained ownership of the text and the processes that these participants performed to solve the problems.

To conclude this section, I summarize the main findings of this study in three points:

1. The participants' mathematical experiences are tied to their definitions of literacy, disciplinary literacy, and mathematical literacy. Moreover, these experiences are

observed in the literacy practices that these participants displayed when solving mathematical problems.

2. The experts in mathematics displayed more abstract and critical mathematical reasoning when solving mathematical problems. The registers the experts used when solving the mathematical problems reveal that for them, mathematical problem-solving is a more abstract and cognitive practice. For the pre-service teachers, mathematical problem-solving is a more concrete and real experience.
3. Mathematical symbolism and abstraction become real through language, which in turn facilitates the literacy practices that these participants displayed. The unique literacy practices that these participants displayed showed the strong connection between language, literacy, and mathematical thought.

Implications of this Study

The findings of this study show a deep connection between language, literacy, and mathematics. This connection seems to shape how pre-service teachers and experts in mathematics understand the literacy practices and linguistic repertoires that are unique to mathematics. As the primary goal of this study is to understand how pre-service teachers develop their notions of mathematical literacy, and in the following sections I present the implications of these results in terms of teacher education and future research.

Implications for Teaching Education Programs

Overall, I found that learning experiences shape the literacy practices of experts in mathematics and pre-service teachers; however, for these experiences to count as part of the pre-service teachers' funds of knowledge (Gonzales, Moll, & Amanti, 2009) and for future application in professional practice these experiences need to be transformed into practice through a reflective process. By reflecting on their experiences as learners, the pre-service teachers would recognize the practices that facilitate learning, make sense of those practices in their teaching context, and translate them into their repertoire of teaching practices.

This study informs the definitions and understanding of literacy and mathematical literacy in pre-service teachers and experts in mathematics and how these literacies are connected with language. The importance of language and the particular ways of reading and writing within academic contexts need to be explicitly discussed within teacher education programs. Future teachers need to be aware that each discipline has specific literacy practices and linguistic repertoires; and that the disciplines as communities of practice (Lave & Wenger, 1991), are constructed under common cultural norms and ways of communicating among the members of these communities.

In higher education contexts, students need to be ready and prepared to face the demands of the highly specialized language and literacy practices that the disciplines require. For pre-service teachers, these demands imply that they need to be ready to make sense of the academic texts and use them to construct their professional knowledge and that they need to translate these texts into actual classroom practices (Colwell & Reinking, 2013). The multimodal, multisemiotic, and multidimensional nature of the mathematical texts presents an additional challenge for pre-service teachers to deal with these sorts of texts. For teacher education programs, it is crucial to provide students with explicit instruction of the nature of the mathematical texts and the literacy practices that are unique to mathematics (Draper, 2008); thus, the pre-service teachers would have the tools to construct and deconstruct these texts to be more accessible to them and their future students.

Teaching education programs should enrich their curriculum with courses that explicitly address the importance of developing disciplinary literacy practices (Draper et al., 2012), and how language mediates the acquisition and learning of the disciplinary Discourses.

Implications for Future Research

The analysis of the data I presented is inconclusive because of the analysis of the data refers to each of these participants' unique experiences, linguistic repertoires, and disciplinary practices. Additionally, the SFL analysis focused only on three of the four metafunctions of the language, and it did not include the study of all the elements of the clause. Therefore, more research about the relationship among language, literacy, and disciplinary literacy is needed.

I explored how these participants' experiences shaped their understating of literacy and mathematical literacy. Yet, the issue of how sociocultural influences in shaping the pre-service teachers and experts in mathematics' notions of literacy and mathematical literacy was neglected. For instance, I did not analyze the data from Ms. Briggs and Dr. Dunn taking into consideration their sociocultural background or their early experiences of learning mathematics in a context different than the one found in educational settings within the United States. Therefore, future research should address how individuals' sociocultural norms mediate their understanding of literacy and mathematical literacy, as well.

The systematic functional linguistic analysis of the participants' registers while solving mathematical problems is brief. The three of the Halliday's Metafunctions should be explored in detail. I did not perform an analysis of the additional elements of the THEME, MOOD, and TRANSITIVITY systems. A detailed analysis of these systems would enlighten additional distinctive features of the mathematical discourses in pre-service teachers and experts in mathematics.

The translanguaging practices of bilingual speakers when solving mathematical problems is another area of interest for future researchers. Even though there is a growing body of research about the challenges that bilingual students face when they learn mathematics (see Moschkovich, 2010), the issues of translanguaging in bilingual college students is an area that requires further exploration. Moreover, the issue of the influence of translanguaging in pre-service teachers' teaching practices is another area of interest for future research.

Limitations

This study entails numerous limitations. The first of them acknowledge the small number of participants in this study. For this reason, the findings of this study are not generalizable to similar contexts or disciplines.

This case study enlightens a small area of the development field of disciplinary literacy in the case of mathematics. However, the linguistic data that I analyzed during this study does not provide evidence of the academic discourses of mathematics as a discipline. The linguistic data was collected in an artificial-created environment. Additionally, in Ms. Briggs' case, I asked her to use English exclusively to respond to the think-aloud protocol, which constrained her ability to use her linguistic repertoires freely and created an additional layer of artificial environment for her data collection. The participants' responses can vary in natural contexts (e.g., mathematical classrooms).

The characteristic of the participants does not allow me to compare these results with similar populations. These participants have unique characteristics that are shaped by their personal, professional, and disciplinary backgrounds. The linguistic and literacy practices of the experts in mathematics and pre-service teachers are not considered to be general for the discipline or generalizable across disciplines; they are limited to the context of this study.

Conclusion

The Common Core State Standards (CCSS) for Mathematics reflect a need to prepare secondary students for the rigors and academic challenges of higher education institutions. To be prepared for the academic challenges of exploring the higher education texts, the CCSS call for including the specific way of reading and writing that are unique to the disciplines (Manderino & Wickens, 2014; Shanahan & Shanahan, 2014) into each one of the content areas. This requirement of the CCSS provoked the emergence of disciplinary literacy theories, which aim to observe how secondary and college students construct their knowledge and practices unique to each of the content areas and disciplines.

This study aimed to enlighten how college students becoming future teachers develop disciplinary literacy in mathematics. Using the concepts of literacy and disciplinary literacy as a theoretical framework, the novice-expert paradigm, and Halliday's systemic functional linguistic analysis, I explored the language and literacy practices that experts in mathematics and pre-service teachers display when solving mathematical problems. The results of this study indicate that their experiences shaped these participants' literacy practices as learners of mathematics. Additionally, these groups made different linguistic choices when solving mathematical problems, which supposed to be related to the different years of experience that these participants have. These findings, even though not concluding, portray the profound relationship between language and literacy in the development of disciplinary literacy in mathematics.

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APPENDICES

APPENDIX A

GUIDE FOR THE SEMI-STRUCTURED INTERVIEW EXPERTS IN MATHEMATICS

Background questions:

1. How did you learn mathematics? Was it an enjoyable/challenging experience?
2. What did help you to learn mathematics better?
3. When did you decide to become a mathematician/ teacher of mathematics?
4. How long have you been working as a mathematics professional?
5. In what field of mathematics have you developed your professional practice?
6. What are the challenges you have experienced while learning mathematics/ becoming a teacher of mathematics? Could you describe a challenging situation/episode in your professional practice?
7. Do you speak another language(s)?
    a. What language(s) do you speak?
    b. How fluent are you in this/these language(s)?
    c. How old were you when you learned your second language?
    d. Is English the language spoken at your home?
    e. Do you know how to read and write in your additional language(s)?
    f. In what language(s) did you take your first mathematics classes? In what language did you take your professional mathematics classes?

Questions about mathematical literacy

1. How do you define literacy?
2. Could you describe mathematical literacy?
3. What does it mean to you to be mathematically literate?

4. Describe the qualities of a mathematically literate person
5. What are the specific mathematical literacy practices that you would consider necessary to be mathematically literate? Could you provide a specific example?
6. What does it mean to you to read math?
7. What is essential for a student to know in order to solve a mathematical problem?
8. Do you have any experience in which you applied general literacy strategies in your mathematics class?
9. What strategies do you apply in your class/professional practice to make mathematical concepts real/ more meaningful?
10. What heuristic have you found yourself using in your classroom/professional practice?
11. What strategies or tools of communication have you found useful to communicate with your students/ colleagues/ useful for your professional practice?
12. What community of mathematicians are you part of?
    a. What is the scope of the communities you participate as a mathematician?
    b. Do you consider important to be part of any association? Why?
13. How different is mathematics from the other content area disciplines? Could you provide some examples?
14. Anything else that you would like to say about learning and language in the field of mathematics?
15. How do you think that mathematics has shaped your understanding of the world?

APPENDIX B

GUIDE FOR THE SEMI-STRUCTURED INTERVIEW MODIFIED FOR PRE-SERVICE TEACHERS

Background questions:

1. How did you learn mathematics? Was it an enjoyable/challenging experience?
2. What did help you to learn mathematics better?
3. What are the challenges you have experienced while learning mathematics? Could you describe a challenging situation/episode as a learner of mathematics?
4. How do you like mathematics in college?
5. When did you decide to become teacher?
6. How much mathematical background do you think a teacher needs to have?
7. What are the challenges you have experienced as a pre-service teacher? Could you describe a challenging situation/episode as pre-service teacher?
8. Do you speak another language(s)?
    g. What language(s) do you speak?
    h. How fluent are you in this/these language(s)?
    i. How old were you when you learned your second language?
    j. Is English the language spoken at your home?
    k. Do you know how to read and write in your additional language(s)?
    l. In what language(s) did you take your first mathematics classes? In what language did you take your professional mathematics classes?

Questions about mathematical literacy

16. How do you define literacy?

17. Could you describe mathematical literacy?
18. What does it mean to you to be mathematically literate?
19. Describe the qualities of a mathematically literate person
20. What are the specific mathematical literacy practices that you would consider necessary to be mathematically literate? Could you provide a specific example?
21. What does it mean to you to read math?
22. What is essential for a student to know in order to solve a mathematical problem?
23. Do you have any experience in which you used general literacy strategies in your mathematics class?
24. How different is mathematics from the other content area disciplines? Could you provide some examples?
25. Anything else that you would like to say about learning and language in the field of mathematics?
26. How do you think that mathematics has shaped your understanding of the world?
27. What strategies do you apply to make mathematical concepts real/ more meaningful?
28. What tools of communication have you found useful to communicate with your fellow pre-service teachers?
29. Are you member of any RSOs on campus, what about any association of pre-service teachers?
    a. What is the scope of the communities you participate as a pre-service teacher/college student?
    b. Do you consider important to be part of any association? Why?

APPENDIX C

THINK-ALOUD PROTOCOL

In this section of your participation, I am going to ask you to solve seven mathematical problems. The first one will be considered a practice exercise. Then, I will give you, one by one, each of the problems. What I would ask you to do is to *think-aloud* while you are solving these problems. Think aloud means that you would verbalize every thought that you have while solving these problems. The idea is that you say every thought from the moment you receive the problem until you solve it. Please speak as continuously as you can. If you stop talking for 5 seconds, I will use this sign (KEEP TALKING sign) to remind you to keep talking. I will be keeping notes of participation. If you would like to feel more comfortable, I can move to the other side of the room, so my presence would not distract you from the task. Do you have any question about the procedure? Do you need I clarify the procedure?

Practice Problem

How do you find $\frac{2}{3}$ of 129? Please explain.

Think-Aloud Problems

(1) What percent is $50 more than $20?

(2) A number is 30% more than 5, what is that number?

(3) Mary has saved $500. Katie has saved $300. How much more money does Mary save than Katie?

(4) If you toss two fair dices, each of which has 1, 2, 3, 4, 5, 6 on its six faces, what is the probability of getting a sum of 11?

(5) Define h(x) = 2x, where x is a natural number.

(a) What is h(1), h(3), h(23), respectively?
(b) If h(x)=246, what is x?
(c) What is the range of the function h(x)?

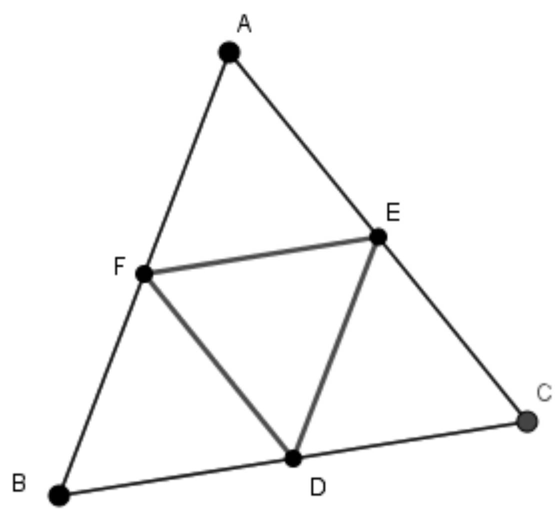
(6) Solve for x: $x^2 - 3 = 13$.

(7) In the figure below, triangle ABC is an equilateral triangle. D, E, F are the midpoints of their respective sides. We know AC is 6 inches long.

(a) What is the measure of angle DEF?

(b) What is the length of EF?

(c) What is the distance between A and D?



APPENDIX D

SILENT-SOLVING PROTOCOL

In this section, I will give you three geometry problems for you to solve. You don't need to talk anymore; instead, I would ask you to solve silently these problems. You can write and make any notes you would need. Take as much time as you need. At this time, you will not receive a practice problem. If you cannot solve the problem, you can leave it at any time you would think is good for you.

Silent-Solving Problems

(1) Perform $\frac{12}{27} \div \frac{5}{8}$ without using "invert and multiply". Please justify.

(2) Jennifer earns \$200 a week. Her friend Linda makes 20% less than Jennifer. How much does Linda earn a week?

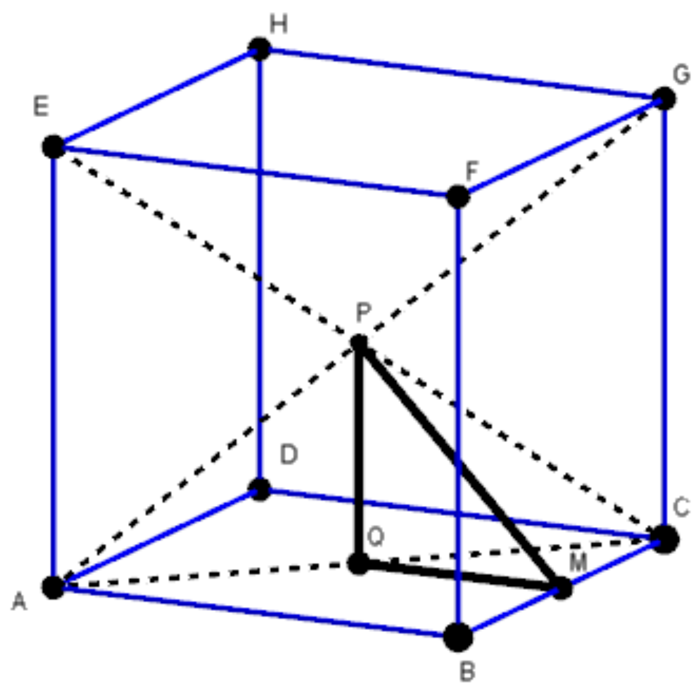
(3) Mary has saved \$500. Katie has saved \$300. What percent does Mary save more than Katie?

(4) Solve for x: $\frac{4}{x+3} = 1$.

(5) Solve for x: $\frac{(x+5)(x-5)}{x-5} = x + 5$.

(6)In the figure below, ABCDEFGH is a unit cube, where all the edges are 1 unit long. P is the intersection of AG and CE; Q is the midpoint of AC; M is the midpoint of BC.

(1) What is the length of MQ?
(2) What is the length of PQ?
(3) What is the measure of angle PMQ?



APPENDIX E

ORAL-EXPLANATORY PROTOCOL

This is the final section of your participation. Now, I would give to you three new problems. I would like you to explain to me how you would solve each of these problems. You can take a few minutes to know how to solve the problems before explaining how to solve them to me. If one of the problems gives you trouble to solve and/or explain, you can skip it and continue with the next one. Please let me know when you are ready to start.

Problems

(1) Explain why $\frac{8}{9} \times \frac{3}{4} = \frac{8\times 3}{9\times 4}$ using anything you are comfortable with.

(2) Joe got a pay raise of 25% last year. Recently, he received a 25% cut to his salary. Is he making as much as money now as he did before his pay raise?

(3) Thomas Elementary School has one seventh more girls in the third grade than Carruthers Elementary. If we know there are 96 girls in the third grade at Thomas Elementary, how many girls are there in the third grade at Carruthers Elementary?

(4) A phone number typically has ten digits, and each digit is a number from 0 to 9. If we assume all digits are fairly used in making a phone number, what is the probability that you can get guess your teacher's phone number correctly in one try?

(5) Define g(x)=3x-1, where x is a real number.

(a) What does g(x) have an output of 26?

(b) What is the value of g(g(0))?

(c) What is g(3+4)?

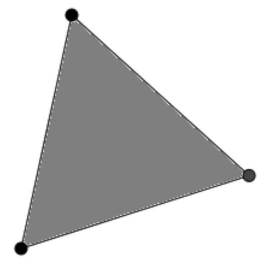
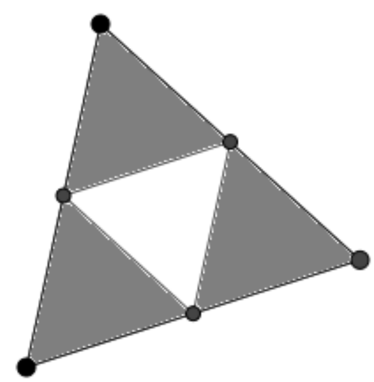
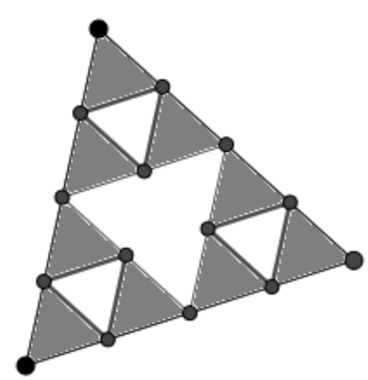
(6) Solve for x: $2 + 2x = \frac{4x}{2} + \frac{4}{2}$

(7) Solve for x: $(x + 3)(x - 4)(x + 5) = 0$.

(8) As shown below, we start with a shaded triangle, which we first divide into four pieces, using the midpoints of its three sides, and then remove the center piece. We do the same to all shaded triangles in the subsequent steps.

(1) How many shaded triangles do we get for the 10th step?

(2) If the original triangle at Step 1 has an area of 1 square unit. What would be the combined area of all the shaded triangles in the 10th step?

Step 1	
Step 2	
Step 3	
Step …	…

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