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A DERIVATION OF THE PERCENTILE BASED TUKEY DISTRIBUTIONS AND A COMPARISON OF MONOTONIC VERSUS NONMONOTONIC AND RANK TRANSFORMATIONS

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by

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A Dissertation
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DISSERTATION APPROVAL

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TITLE: A DERIVATION OF THE PERCENTILE BASED TUKEY DISTRIBUTIONS AND A COMPARISON OF MONOTONIC VERSUS NONMONOTONIC AND RANK TRANSFORMATIONS

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The Method of Moments (MOM) has been extensively used in statistics for obtaining conventional moment-based estimators of various parameters. However, the disadvantage of this method is that the estimates "can be substantially biased, have high variance, or can be influenced by outliers" (Headrick & Pant, 2012). The Method of Percentiles (MOP) provides a useful alternative to the MOM when the distributions are non-normal, specifically being more computationally efficient in terms of estimating population parameters. Examples include the generalized lambda distribution (Karian & Dudewicz, 1999), third order power method (Koran, Headrick & Kuo, 2015) and fifth order power method (Kuo & Headrick, 2017). Further, the HH, HR and HQ distributions, as extensions of the Tukey g-h (GH) family, are of interest for investigation using the MOP in this dissertation. More specifically, closed form solutions are obtained for left-right tail-weight ratio (a skew function) and tail-weight factor (a kurtosis function). A Monte Carlo simulation study which includes the comparison of monotonic and nonmonotonic transformation scenarios is also performed. The effect on Type 1 error and power rates under severely nonmonotonic scenarios are of special interest in the study. Dissimilarities of not strictly monotonic scenarios are discussed. The empirical confirmation that Rank Transform (RT) is appropriate for 2x2 designs is obtained.

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CHAPTER 1

INTRODUCTION

In the early 1960's Tukey proposed the family of symmetric H distributions (Tukey, 1960) for the purpose of creating nonnormal random deviates. These distributions are based on a monotone transformation of standard normal random deviates (Z) using the following quantile function:

$$q(h) = Z * \exp(0.5hZ^2) \qquad \text{where } h > 0. \tag{1.1a}$$

Equation (1a) models symmetric distributions that have heavier tails than the normal probability density function (PDF). Over the years, the family of H distributions became more popular in terms of applied research with examples of modeling stock returns on the New York Stock exchange (Badrinath & Chatterjee, 1988, 1991), financial times stock exchange index returns (Mills, 1995), returns of aluminum and zinc (Fischer, Horn & Klein, 2006), solar flare data (Goerg, 2011), extreme oceanic wind speeds (Dupuis & Field, 2004) and operational risk (Guegan & Hassani, 2009).

Subsequently, associated with the topic of H distributions (Hoaglin, 1985; Tukey, 1977), the quantile functions of the g and g-h families were developed and are as follows:

$$q(g) = (\exp(gZ) - 1)/g \qquad h=0 \text{ (lognormal)} \tag{1.1b}$$

$$q(gh) = ((\exp(gZ) - 1)/g\ ) \ * \ \exp(0.5hZ^2) \qquad g\neq 0,\ h>0 \tag{1.1c}$$

Unlike the Pearson (1895, 1901, 1916) system of distributions, the family of g-h monotonic distributions does not cover the entire set of values in the skew and kurtosis plane (lower boundary for kurtosis is -2). In 2000, the extension to the Tukey family (Morgenthaler &

Tukey, 2000) was derived with additional families denoted as HH, HR, HQ, and HHH distributions.

The HH distributions are an asymmetric generalization of the family of H distributions. Instead of considering the one parameter of h, a pair of parameters (h_L and h_R – for transforming left and right tail separately) is considered as follows:

$$\mathrm{q(h)} = \begin{cases} Z * \exp(0.5h_L Z^2) & Z \leq 0 \\ Z * \exp(0.5h_R Z^2) & Z \geq 0 \end{cases} \qquad h_L \neq h_R \tag{1.2}$$

for $h_R \geq 0$ and $h_L \geq 0$.

The HQ family of distributions was introduced for increasing the tail elongation, so the term $qz^4/4$ was added to the exponent for this purpose. The formula for the quantile function of the HQ distribution is:

$$q(h, q) = Z * \exp(0.5hZ^2 + 0.25qZ^4) \tag{1.3}$$

for $q \geq 0, h \geq 0$ or $h < 0, q \geq h^2/4$.

Further, the HR family of distributions also has heavy tails with shape affected. In this case, the formula for the quantile function is given as follows:

$$q(h, r) = Z * \exp(hZ^2/(2 + rZ^2)) \tag{1.4}$$

for $r \geq 0$ and $h > -2r$.

1.1. Statement of the Problem

The Tukey family of distributions could be based either on the Method of Moments (MOM, Kowalchuk & Headrick, 2010) estimates, Method of L-moments (MOL, Headrick &

Pant, 2012), or the Method of Percentiles (MOP, Kuo & Headrick, 2014). Estimates for α_1 (median), α_2 (interdecile range) (Karian & Dudewicz, 2000), left-right tail-weight ratio α_3 (a skew function) and tail-weight factor α_4 (a kurtosis function) of the Tukey g-h distribution was introduced recently in the literature (Kuo & Headrick, 2014). However, estimates for HH, HQ and HR distributions using the MOP remain to be derived.

There is also a need to compare and contrast the influence of monotonic transformations versus nonmonotonic (i.e., $h<0$) on Type 1 error and power rates, because there are many transformations that may produce nonnormal variables (e.g., power method, Pearson, GLD, Burr). This is done in the context of Monte Carlo simulation methods.

1.2. Purposes of the Study

One purpose of this study is to derive the percentile-based shape parameters α_3 and α_4 for the HH, HQ, and HR families of distributions. Comparisons are made with the MOM juxtaposed with the MOP (e.g., Koran, Headrick & Kuo, 2015).

Further, to assess the effect of monotonic and nonmonotonic transformation, a 2x2 ANOVA design is used (Akritas, 1990; Blair, Sawilowsky & Higgins, 1987; Thompson, 1991). Evaluation in terms of the effect on type 1 error and power rates is provided for fifth order polynomial power method transformations (Headrick, 2002, 2010).

We expect type 1 error and power to give similar rates for different monotonic transformations, but for nonmonotonic transformations the outcomes could be considerably different. We use weak, moderate, and strong nonmonotonicity, and expect large differences in the case of strong nonmonotonicity between the Tukey *g*-and-*h* family and power method.

1.3. Research Questions.

This study investigates the following research questions:

1. Would we get closed form solutions for MOP-based HH, HQ, and HR cumulants to obviate the need for numerical equations solving and thus having properties of existence and uniqueness?

2. What are the effects on type 1 error and power properties of monotonic versus nonmonotonic transformations? The results of the Tukey GH (Hoaglin, 1985) family of distributions are compared and contrasted to the fifth order polynomial power method (Headrick, 2002, 2010).

3. Comparisons in terms of type 1 error and power method are made in the context of parametric versus rank transformation scenarios for both power method and GH.

1.4. Definition of Terms

<u>Quantile Function of a Continuous Random Variable</u>. The quantile function or percentile function of a random variable Y, denoted as $q(y)$, is defined as the inverse function of the cdf of X. The quantile function of Y gives the value of x such that $F(x)=y$, for each value between 0 and 1. (Karian & Dudewicz, 2011, p. 8)

<u>Monotonic Transformation</u> is a transformation between ordered sets that preserves the provided order. Johnson (1949) suggested that translation systems should have monotonicity.

<u>Non-monotonic Transformation</u> is a transformation between ordered sets that does not preserve the provided order.

Power method is a technique based on polynomial transformation that proceeds by taking the sum of a linear combination of a standard normal random variable, its square, cube, 4th and 5^{th} degrees (Headrick, 2002, 2010).

Monte Carlo methods - a wide group of computational algorithms which are based on repeated random sampling to obtain numerical results. The central idea of Monte Carlo methods is using pseudo-random deviates to evaluate problems and make statistical decisions.

Rank Transformation - transformation, where we "replace the data with their ranks, then apply the usual parametric *t* test, *F* test, and so forth, to the ranks "(Conover & Iman, 1981).

1.5. Significance of the Research

The proposed advantages of this methodology are:

1. Derivation of HH, HQ, and HR provides a broader range of nonnormal distributions for the Tukey GH family in the context of MOP.

2. It is proposed that the advantage of strictly monotonic transformations yields similar results for the Tukey GH and power transformations.

3. Findings of nonmonotonic transformation yields dissimilar results in terms of Type 1 error and power.

4. The 2x2 transformation is appropriate for RT design.

1.6. Limitations of the Study

1. Most strictly monotonic transformations do not span the entire space of α_3 and α_4 plane comparative to other transformations.

2. The basis for comparison of the Tukey family in this study is the power method family of 5^{th} order transformations. There are no comparisons to other transformation methods (e.g., Burr family of distributions, Pearson system, or Generalized Lambda family of distributions).

3. The observations in the simulation study are independent and uncorrelated.

4. In this Monte Carlo study, the results are limited to the parameters introduced into the study (e.g., sample size, effect size, etc.).

1.7. Overview of the Subsequent Chapters

The organization of the following chapters is as follows. In chapter 2 the literature on Tukey *g*-and-*h*, HH, HQ, and HR distributions as well as the method of percentiles is reviewed. In chapter 3 the methodology is introduced, the derivation of MOP based location, scale, as well as shape parameters and Monte Carlo simulation are described. In chapter 4 the results of the simulation study are reported. In chapter 5 the results of chapter 4 are discussed.

CHAPTER 2

LITERATURE REVIEW

In this chapter we describe several types of transformations. Specifically, the transformations based on the: (i) Pearson system, (ii) Burr distribution, (iii) Power method, and (iv) Generalized lambda distribution are described. As it was noted in Devroye (1986, p.685) "These families of distributions are usually designed for matching up to four moments". Later the Power method was extended from third (Fleishman, 1978) to fifth order polynomial transformation setting, in Headrick (2002, 2007, 2010), pdf and cdf derived. Thus, the topic remains in the focus of current research.

2.1 Pearson system.

The Pearson system was introduced by Karl Pearson near the beginning of 20th century (Pearson, 1895, 1901, 1916). It consists of twelve member distributions. The Pearson densities are presented in Table 2.1 (adopted from Devroye, 1986).

Table 2.1 Pearson system of distributions

Name	$f(x)$	Parameters	Support
Pearson I	$C(1+x/a)^b(1-x/c)^d$	$b,d>-1;\ a,c>0$	$[-a, c]$
Pearson II	$C(1-(x/a)^2)^b$	$b>-1;\ a>0$	$[-a, a]$
Pearson III	$C(1+x/a)^{ba}e^{-bx}$	$ba>-1;\ b>0$	$[-a, \infty)$
Pearson IV	$C(1+(x/a)^2)^{-b}e^{-c(\arctan(x/a))}$	$a>0;\ b>0.5$	
Pearson V	$Cx^{-b}e^{-c/x}$	$b>1;\ c>0$	$[0, \infty)$
Pearson VI	$C(x-a)^b x^{-c}$	$c>b+1>0;\ a>0$	$[a, \infty)$
Pearson VII	$C(1+(x/a)^2)^{-b}$	$b>0.5;\ a>0$	

Pearson VIII	$C(1+x/a)^{-b}$	$1 \geq b \geq 0;\ \ a>0$	$[-a, 0]$
Pearson IX	$C(1+x/a)^{b}$	$b>0;\ \ a>0$	$[-a, 0]$
Pearson X	$(1/a)e^{-x/a}$	$a>0$	$[0, \infty)$
Pearson XI	$C(a/x)^{b}$	$b>1;\ \ a>0$	$[a, \infty)$
Pearson XII	$C((a+x)/(b-x))^{c}$	$0<b<a\ ;\ 0 \leq c<1$	$[-a, b]$

According to McGrath and Irvine (1973), random variates for all members of the Pearson family except Pearson IV could be generated using one or two gamma or beta random variates. Johnson, Kotz and Balakrishnan (1995) described the Pearson type VI and type VII distributions in detail, providing several important facts and links to other families of distributions. For example, the well-known *F* distribution is a special case of the Pearson type VI distribution, and Student t distribution is a special case of Pearson type VII distribution. It should also be noted that Type I are the general form of beta distributions and Type III are gamma distributions. The Pearson densities have the shape parameters *a, b, c, d* and the normalization constant *C*.

2.2. Burr distributions

The Burr family of densities was introduced in a series of papers by Burr (1942, 1968 1973). There are 3 positive real parameters for this family *r, k,* and *c*. All Burr distributions are related not only to each other but also to the uniform distribution through the probability integral transform (Devroye, 1986). The list of cumulative distribution functions is provided in Table 2.2.

Table 2.2 Burr Family of Distributions

Name	$F(x)$	Range for x
Burr I	x	$[0,1]$
Burr II	$(1+ e^{-x})^{-r}$	$(-\infty,\infty)$
Burr III	$(1+ x^{-k})^{-r}$	$[0,\infty)$
Burr IV	$(1+((c-x)/x)^{1/c})^{-r}$	$[0,c)$
Burr V	$(1+ke^{-tan(x)})^{-r}$	$[-\pi/2,\pi/2]$
Burr VI	$(1+ke^{-sinh(x)})^{-r}$	$(-\infty,\infty)$
Burr VII	$2^{-r}(1+tanh(x))^{r}$	$(-\infty,\infty)$
Burr VIII	$((2/\pi)arctan(e^{x}))^{r}$	$(-\infty,\infty)$
Burr IX	$1-2/(2+k((1+ e^{x})^{r}-1))$	$(-\infty,\infty)$
Burr X	$(1+exp(-x^{2}))^{r}$	$[0,\infty)$
Burr XI	$(x-(1/2\pi)sin(2\pi x))^{r}$	$[0,1]$
Burr XII	$1-(1+x^{c})^{-k}$	$[0,\infty)$

As noted in Headrick, Pant, and Sheng (2010), the Burr distributions have a number of applications, with examples in terms of life testing (Wingo, 1983; 1993), operational risk (Chernobai, Fabozzi, & Rachev, 2007), forestry (Lindsay, Wood, & Woollons, 1996; Gove, Ducey, Leak, & Zhang, 2008), fracture roughness (Nadarajah & Kotz, 2006), option market price distributions (Sherrick, Garcia, & Tirupattur, 1996), meteorology (Mielke, 1973), modeling crop prices (Tejeda & Goodwin, 2008), and reliability (Mokhlis, 2005).

The most popular in the Burr family is the type XII distribution. It was further investigated in the studies of Hatke (1949), Ord (1972), Rodriguez (1977), Berkovits, Hancock, and Nevitt (2000), Headrick, Sheng, and Hodis (2007), Headrick, Pant, and Sheng (2010), Pant (2011) and Headrick (2011).

2.3. The Power Method

The power method was introduced by Fleishman (1978). This method has been studied for several decades, which evolves to a recent monograph on this topic (Headrick, 2010). Initially the technique was based on sum of standard normal variate, its square and its cube. In Headrick (2010), it is extended to the (i) standard uniform, (ii) standard logistic, (iii) triangular (sum of 2 uniform deviates) and double logistic (sum of 2 independent standard logistics deviates). The corresponding values for kurtosis in these distributions are: -1.2 for uniform distribution, -0.6 for triangular distribution, 0 for standard normal, 0.6 for double logistic and 1.2 for logistic.

Initially the power method was developed as a technique for matching 4 moments, and it has been extended to 6 moments in Headrick (2002) and for multivariate distributions (e.g., Headrick, 2002, Headrick & Sawilowski, 1999; Vale & Maurelli, 1983).

The Power method has been the most widely applied application in terms of the general linear model and its special cases. Examples of research include Beasley (2002), Headrick (1997), Headrick and Rotou (2001), Headrick and Sawilowsky (1999), Headrick and Vineyard (2001), Kowalchuk, Keselman and Algina (2003), Rasch and Guiard (2004), Serlin and Harwell (2004) and Stein (1993). In addition to the field of education measurement, applications of the power method were extensively used by researchers with examples in Markov Chain Monte Carlo estimation (Hendrix & Habing, 2009), item response theory (Stone, 2003), and computer adaptive testing (Zhu, Yu, & Liu, 2002).

Other topics of study using the power method include logistic regression (Hess, Olejnik, & Huberty, 2001), hierarchical linear models (Shieh, 2000), multiple imputation (Demirtas & Hedeker, 2008), microarray analysis (Powell, Anderson, Cheng, & Alvord, 2002), and structural equation modeling (Hipp & Bollen, 2003; Reinartz, Echambadi, & Chin, 2002).

2.4. Generalized Lambda Distribution

In the early 1970's the class of Generalized Lambda Distributions (GLDs) was introduced by Ramberg and Schmeiser (1972, 1974). Initially the authors considered the 3-parameter case of the GLD with the following inverse distribution function formula (Ramberg & Schmeiser, 1972, p. 988):

$$x = R(p) = \lambda_1 + (p^{\lambda_3} - (1-p)^{\lambda_3}) / \lambda_2 \qquad (0 \le p \le 1) \tag{2.1a}$$

where p has Uniform(0,1) distribution. Also, its probability density function is defined as

$$f(R(p)) = \lambda_3 (p^{\lambda_3 - 1} - (1-p)^{\lambda_3 - 1}) / \lambda_2 \qquad , \tag{2.1b}$$

The parameter λ_1 serves as a center of symmetry for the generalized lambda pdf, and for negative values of λ_2 and λ_3 the density function is positive over the real line. This function can be used for approximating both medium-tailed distributions (e.g., normal) and heavy-tailed distributions (e.g., Cauchy).

Later, the GLD was generalized to the 4 parameter case to include asymmetric unimodal distributions with the following inverse distribution function (Ramberg & Schmeiser, 1974, p. 78):

$$x = R(p) = \lambda_1 + (p^{\lambda_3} - (1-p)^{\lambda_4}) / \lambda_2 \qquad (0 \le p \le 1) \qquad , \tag{2.2}$$

skewness and elongation are represented by λ_3 and λ_4, and the variance is represented by λ_2 given λ_3 and λ_4. The choice of λ_1 determines the mean, and for the case $\lambda_3 \neq \lambda_4$, the distribution is asymmetric, so the expected value is not equal to λ_1.

Another extension of the GLD was proposed by Karian, Dudewicz and McDonald (1996) by combining GLD and the generalized beta distribution. Further, the univariate GLD was extended to the multivariate case (Headrick & Mugdadi, 2006).

Applications of the GLD include areas of data mining (Dudewicz & Karian, 1999), independent component analysis (Karvanen, 2003; Mutihac & Van Hulle, 2003), microarray research (Beasley et al.,2004), operations research (Ganeshan, 2001), option pricing (Corrado,2001), psychometrics (Bradley,1993; Bradley & Fleisher,1994; Delaney & Vargha,2000), and structural equation modeling (Reinartz, Echambadi, & Chin, 2002).

It also should be noted that during 1970's it was hard to compute the parameters, therefore the need for creating tables existed. As a result, the tables for GLD distributions were introduced by Ramberg, Dudewicz, Tadikamalla, and Mykytka (1979).

2.5 Tukey *g-h* family

Continuing the discussion from chapter 1 about the quantile function of Tukey *g-h* distribution, let us introduce the features of *g* and *h* parameters. As it was noted in Headrick, Kowalchuk, and Sheng (2008), "…parameters $g, h \in R$ subject to the conditions that $g \neq 0$ and $h > 0$. The parameter $\pm g$ controls the skew of a distribution in terms of both direction and magnitude. The parameter *h* controls the tail weight or elongation of a distribution and is positively related with kurtosis".

Also, we could extrapolate the chapter 1 discussion on the moments of g-h distribution, as well as the formulae for skew and kurtosis. As further suggested by Headrick et. al. (2008), the formulae for first four moments of Tukey *g-h* are as follows:

$$E[q_{g,h}(z)] = (\exp\{g^2/(2 - 2h)\} - 1)/(g(1 - h)^{1/2}) \quad (2.3)$$

$$E[q_{g,h}(z)^2] = (1 - 2\exp\{g^2/(2 - 4h)\} + \exp\{2g^2/(1 - 2h)\})/(g^2(1 - 2h)^{1/2}) \quad (2.4)$$

$E[q_{g,h}(z)^3] = (3\exp\{g^2/(2-6h)\}+\exp\{9g^2/(2-6h)\}-3\exp\{2g^2/(1-3h)\}-1)/(g^3(1-3h)^{1/2})$ (2.5)

$E[q_{g,h}(z)^4] = (\exp\{8g^2/(1-4h)\}(1+6\exp\{6g^2/(4h-1)\}+\exp\{8g^2/(4h-1)\}-4\exp\{7g^2/(8h-2)\}-4\exp\{15g^2/(8h-2)\}))/(g^2(1-4h)^{1/2})$ (2.6)

Besides, the formula for skew is:

$\alpha_1(g, h) = [(3\exp\{g^2/(2-6h)\} + \exp\{9g^2/(2-6h)\} - 3\exp\{2g^2/(1-3h)\} - 1)/(1-3h)^{1/2} - 3(1-2\exp\{g^2/(2-4h)\} + \exp\{2g^2/(1-2h)\})(\exp\{g^2/(2-2h)\} - 1)/((1-2h)^{1/2}(1-h)^{1/2}) + 2(\exp\{g^2/(2-2h)\} - 1)^3/(1-h)^{3/2}]/[g^3(((1-2\exp\{g^2/(2-4h)\} + \exp\{2g^2/(1-2h)\})/(1-2h)^{1/2} + (\exp\{g^2/(2-2h)\} - 1)^2/(h-1))/g^2)^{3/2}]$ (2.7)

and the formula for kurtosis is:

$\alpha_2(g, h) = [\exp\{8g^2/(1-4h)\}(1 + 6\exp\{6g^2/(4h-1)\}+\exp\{8g^2/(4h-1)\}-4\exp\{7g^2/(8h-2)\} - 4\exp\{15g^2/(8h-2)\})/(1-4h)^{1/2} - 4(3\exp\{g^2/(2-6h)\}+\exp\{9g^2/(2-6h)\}-3\exp\{2g^2/(1-3h)\} - 1)(\exp\{g^2/(2-2h)\} - 1)/((1-3h)^{1/2}(1-h)^{1/2}) - 6(\exp\{g^2/(2-2h)\}-1)^4/(h-1)^2 - 12(1-2\exp\{g^2/(2-4h)\} + \exp\{2g^2/(1-2h)\})(\exp\{g^2/(2-2h)\}-1)^2/((1-2h)^{1/2}(h-1))+3(1-2\exp\{g^2/(2-4h)\} + \exp\{2g^2/(1-2h)\})^2/(2h-1)]/[(1-2\exp\{g^2/(2-4h)\} + \exp\{2g^2/(1-2h)\})/(1-2h)^{1/2}+(\exp\{g^2/(2-2h)\}-1)^2/(h-1)]^2$ (2.8)

Further, the first four moments, skew and kurtosis for *g* distributions are as follows:

$E[q_{g,0}(z)] = (\exp\{g^2/2\} - 1)/g$ (2.9)

$E[q_{g,0}(z)^2] = (1 - 2\exp\{g^2/2\} + \exp\{2g^2\})/g^2$ (2.10)

$E[q_{g,0}(z)^3] = (3\exp\{g^2/2\} + \exp\{9g^2/2\} - 3\exp\{2g^2\} - 1)/g^3$ (2.11)

$E[q_{g,0}(z)^4] = (1 - 4\exp\{g^2/2\} + 6\exp\{2g^2\} - 4\exp\{9g^2/2\} + \exp\{8g^2\})/g^4$ (2.12)

$\alpha_1(g) = (3\exp\{2g^2\} + \exp\{3g^2\} - 4)^{1/2}$ (2.13)

$\alpha_2(g) = 3\exp\{2g^2\} + 2\exp\{3g^2\} + \exp\{4g^2\} - 6$ (2.14)

And the first four moments, skew and kurtosis for *h* distributions are as follows:

$$E[q_{0,h}(z)] = 0 \quad (2.15)$$

$$E[q_{0,h}(z)^2] = 1/(1 - 2h)^{3/2} \quad (2.16)$$

$$E[q_{0,h}(z)^3] = 0 \quad (2.17)$$

$$E[q_{0,h}(z)^4] = 3/(1 - 4h)^{5/2} \quad (2.18)$$

$$\alpha_1(h) = 0 \quad (2.19)$$

$$\alpha_2(h) = 3(1 - 2h)^3(1/(1 - 4h)^{5/2} + 1/(2h - 1)^3) \quad (2.20)$$

2.6 Johnson system

The Johnson system, which is characterized by the densities of properly transformed normal variates N, was introduced by Johnson (1949). Under this system both the generation of random variates and parameters fitting are simple (Devroye, 1986). The Johnson system consists of the S_L (lognormal) densities of e^N, of S_B densities of $e^N/(1+ e^N)$, and the S_U densities of $sinh(N)=0.5(e^N - e^{-N})$.

Importantly, the requirements of a translation system were clearly stated (Johnson, 1949, p.152-153). Namely, that function *f(y)*, which serves as a basement for the system of frequency curves should have the following properties:

1. *f(y)* should be a monotonic function of *y*.
2. *f(y)* should be not only simple in form but also easy to calculate
3. The range of *f(y)* should be from -∞ to +∞.
4. The system of distributions of y should include distributions of most of the types found in the data.

In the 1980's the Johnson system was extended to theTadikamalla-Johnson system (Johnson & Tadikamalla, 1982). By extending the S_L, S_B and S_U transformations to the logistic distribution, systems L_L, L_B and L_U were derived. It should be noted that L_L distribution is also called the log logistic distribution and is a special case of Burr type XII.

CHAPTER 3

METHODOLOGY

This chapter includes the derivations of Method of Percentiles (MOP) parameters for the HH, HQ, and HR distributions. Further, there is a discussion on the Monte Carlo study – nonnormal distributions, sample sizes under investigation, pseudo random number generation, the structural model, data generation techniques for the model, and treatment effects, and calculating the *F* ratios in main effects and interaction associated with the 2x2 ANOVA model.

3.1 The Tukey HR, HQ and HH distributions.

In this section the MOP parameters for the Tukey HR, HQ, and HH distributions are derived. The derivation of MOP parameters for HR family is as follows:

$$\gamma 3 = \frac{q(Z_{0.5})-q(Z_{0.1})}{q(Z_{0.9})-q(Z_{0.5})} = \frac{(Z_{0.5})\exp\left(hZ_{0.5}^2/(2+rZ_{0.5}^2)\right)-\ (Z_{0.1})\exp\left(hZ_{0.1}^2/(2+rZ_{0.1}^2)\right)}{(Z_{0.9})\exp\left(hZ_{0.9}^2/(2+rZ_{0.9}^2)\right)-\ (Z_{0.5})\exp\left(hZ_{0.5}^2/(2+rZ_{0.5}^2)\right)} =$$

$$\frac{-\ (Z_{0.1})\exp\left(hZ_{0.1}^2/(2+rZ_{0.1}^2)\right)}{(Z_{0.9})\exp\left(hZ_{0.9}^2/(2+rZ_{0.9}^2)\right)} = 1 \qquad (3.1)$$

$$\gamma 4 = \frac{q(Z_{0.75})-q(Z_{0.25})}{q(Z_{0.9})-q(Z_{0.1})} = \frac{(Z_{0.75})\exp\left(hZ_{0.75}^2/(2+rZ_{0.75}^2)\right)-\ (Z_{0.25})\exp\left(hZ_{0.25}^2/(2+rZ_{0.25}^2)\right)}{(Z_{0.9})\exp\left(hZ_{0.9}^2/(2+rZ_{0.9}^2)\right)-\ (Z_{0.1})\exp\left(hZ_{0.1}^2/(2+rZ_{0.1}^2)\right)} =$$

$$\frac{2(Z_{0.75})\exp\left(hZ_{0.75}^2/(2+rZ_{0.75}^2)\right)}{2(Z_{0.9})\exp\left(hZ_{0.9}^2/(2+rZ_{0.9}^2)\right)} = (Z_{0.75}/Z_{0.9})\exp(hZ_{0.75}^2/(2+rZ_{0.75}^2)-\ (hZ_{0.9}^2/(2+rZ_{0.9}^2)))$$

(3.2)

Based on those equations, the formulae for *h* and *r* are as follows:

$$h = \frac{\ln\left(\gamma_4 \frac{Z_{0.9}}{Z_{0.75}}\right)}{Z_{0.75}^2/(2+rZ_{0.75}^2)-Z_{0.9}^2/(2+rZ_{0.9}^2)} \qquad (3.3)$$

$$r =$$

$$=(-(\ 2Z_{0.75}^2+2Z_{0.9}^2\ ) \pm$$

$$\sqrt{(\ 2Z_{0.75}^2+2Z_{0.9}^2\ )^2-4Z_{0.75}^2Z_{0.9}^2(4-\frac{2h(Z_{0.75}^2-Z_{0.9}^2\ )}{\ln\left(\gamma_4\frac{Z_{0.9}}{Z_{0.75}}\right)})}\ )/(\ 2Z_{0.75}^2Z_{0.9}^2\ ) \qquad (3.4)$$

The derivation of MOP parameters for HQ distribution is as follows:

$$\gamma 3 = \frac{q(Z_{0.5})-q(Z_{0.1})}{q(Z_{0.9})-q(Z_{0.5})} = \frac{(Z_{0.5})\exp\left(0.5hZ_{0.5}^2+0.25qZ_{0.5}^4\right)-\ (Z_{0.1})\exp\left(0.5hZ_{0.1}^2+0.25qZ_{0.1}^4\right)}{(Z_{0.9})\exp\left(0.5hZ_{0.9}^2+0.25qZ_{0.9}^4\right)-\ (Z_{0.5})\exp\left(0.5hZ_{0.5}^2+0.25qZ_{0.5}^4\right)} =$$

$$\frac{-\ (Z_{0.1})\exp\left(0.5hZ_{0.1}^2+0.25qZ_{0.1}^4\right)}{(Z_{0.9})\exp\left(0.5hZ_{0.9}^2+0.25qZ_{0.9}^4\right)} = 1 \quad (3.5)$$

$$\gamma 4 = \frac{q(Z_{0.75})-q(Z_{0.25})}{q(Z_{0.9})-q(Z_{0.1})} = \frac{(Z_{0.75})\exp\left(\frac{hZ_{0.75}^2}{2}+\frac{qZ_{0.75}^4}{4}\right)-\ (Z_{0.25})\exp\left(\frac{hZ_{0.25}^2}{2}+\frac{qZ_{0.25}^4}{4}\right)}{(Z_{0.9})\exp\left(\frac{hZ_{0.9}^2}{2}+\frac{qZ_{0.9}^4}{4}\right)-\ (Z_{0.1})\exp\left(\frac{hZ_{0.1}^2}{2}+qZ_{0.1}^4/4\right)} =$$

$$= (Z_{0.75}/Z_{0.9})\exp\left(\left(\frac{h}{2}\right)(Z_{0.75}^2 - Z_{0.9}^2\right)-\ \left(\frac{q}{4}\right)(Z_{0.75}^4 - Z_{0.9}^4\ ))$$

(3.6)

Based on those equations, the formulae for *h* and *q* are as follows:

$$h = 2(\frac{\ln\left(\gamma_4\frac{Z_{0.9}}{Z_{0.75}}\right)}{Z_{0.75}^2-Z_{0.9}^2} - \frac{q}{4}\,(Z_{0.75}^2 + Z_{0.9}^2)) \quad (3.7)$$

$$q = 4(\frac{\ln\left(\gamma_4\frac{Z_{0.9}}{Z_{0.75}}\right)}{Z_{0.75}^2-Z_{0.9}^2} -\ \frac{h}{2})/\ (Z_{0.75}^2 + Z_{0.9}^2) \quad (3.8)$$

It should be noted that parameter *q* affects elongation in HQ distribution. As a result, HQ distribution becomes useful for approximating heavy tails.

The derivation of MOP parameters for the HH distribution is based separately on h_L and h_R ($h_L \neq h_R$) and as follows:

$$\gamma 3^L = \frac{q(Z_{0.5})-q(Z_{0.1(L)})}{q(Z_{0.9(R)})-q(Z_{0.5})} = \frac{(Z_{0.5})\exp\left(0.5h_LZ_{0.5}^2\right)-\ (Z_{0.1(L)})\exp\left(0.5h_LZ_{0.1(L)}^2\right)}{(Z_{0.9(R)})\exp\left(0.5h_RZ_{0.9(R)}^2\right)-\ (Z_{0.5})\exp\left(0.5h_RZ_{0.5}^2\right)} =$$

$$\frac{-\ (Z_{0.1(L)})\exp\left(0.5h_LZ_{0.1(L)}^2\right)}{(Z_{0.9(R)})\exp\left(0.5h_RZ_{0.9(R)}^2\right)} \quad (3.9)$$

$$\gamma 3^R = \frac{q(Z_{0.5})-q(Z_{0.1})}{q(Z_{0.9})-q(Z_{0.5})} = \frac{(Z_{0.5})\exp\left(0.5h_RZ_{0.5}^2\right)-\ (Z_{0.1(R)})\exp\left(0.5h_RZ_{0.1(R)}^2\right)}{(Z_{0.9(L)})\exp\left(0.5h_LZ_{0.9(L)}^2\right)-\ (Z_{0.5})\exp\left(0.5h_LZ_{0.5}^2\right)} =$$

$$\frac{-\ (Z_{0.1(R)})\exp\left(0.5h_RZ_{0.1(R)}^2\right)}{(Z_{0.9(L)})\exp\left(0.5h_LZ_{0.9(L)}^2\right)} \quad (3.10)$$

$$\gamma 4^{L} = \frac{q(Z_{0.75}) - q(Z_{0.25})}{q(Z_{0.9}) - q(Z_{0.1})} = \frac{(Z_{0.75(R)}) \exp\left(\frac{h_R Z_{0.75(R)}^2}{2}\right) - (Z_{0.25(L)}) \exp\left(\frac{h_L Z_{0.25(L)}^2}{2}\right)}{(Z_{0.9(R)}) \exp\left(\frac{h_R Z_{0.9(R)}^2}{2}\right) - (Z_{0.1(L)}) \exp\left(\frac{h_L Z_{0.1(L)}^2}{2}\right)} \tag{3.11}$$

$$\gamma 4^{R} = \frac{q(Z_{0.75}) - q(Z_{0.25})}{q(Z_{0.9}) - q(Z_{0.1})} = \frac{(Z_{0.75(L)}) \exp\left(\frac{h_L Z_{0.75(L)}^2}{2}\right) - (Z_{0.25(R)}) \exp\left(\frac{h_R Z_{0.25(R)}^2}{2}\right)}{(Z_{0.9(L)}) \exp\left(\frac{h_L Z_{0.9(L)}^2}{2}\right) - (Z_{0.1(R)}) \exp\left(\frac{h_R Z_{0.1(R)}^2}{2}\right)} \tag{3.12}$$

In case $h_L = h_R$, HH is symmetric. Consequently, the third moment is equal to 1.

3.2 The Monte Carlo procedure

Historically, von Newman and Ulam coined the term “Monte Carlo” during the 1940’s. However, the solutions connected to the Monte Carlo method existed earlier, for example “Buffon’s needle” case (1733) or estimating the correlation coefficient in *t*-distribution by Student (1908).

As some mathematical functions could not be integrated, researchers resorted to numerical methods (e.g., estimating flow of neutrons through a lead wall in nuclear reactor in Scheid (1988)). Currently, the method of Monte Carlo is the most effective and is widely used for solving complicated problems (Rubinstein & Kroese, 2008).

In the context of this study, the Monte Carlo algorithm is developed for a completely randomized 2x2 balanced ANOVA. Degree of monotonicity (monotonic/nonmonotonic) and type of setting (parametric/rank transform) are the corresponding factors. The program for Monte Carlo simulation study is based on the aforementioned algorithm and is written in FORTRAN language.

3.3 Probability distribution functions in the study

The distributions used in the study are as follows:

1. For fifth order power method transformation

- monotonic scenario (standard normal, skewness=0, kurtosis=0)

-weak nonmonotonic skewed scenario (Beta (4, 1.5), skewness= -0.693889, kurtosis= -0.068627)

-moderate nonmonotonic light-skewed scenario (Beta (4, 2), skewness= -0.467707, kurtosis= -0.375)

-moderate nonmonotonic symmetric scenario (Triangular, skewness= 0, kurtosis= -0.6)

-heavily nonmonotonic symmetric light-tailed scenario (Uniform, skewness=0, kurtosis=-1.2)

- severely nonmonotonic symmetric light-tailed scenario (Beta - like, skewness=0, kurtosis= -1.383826)

2. For Tukey *g*-*h*

- monotonic scenario (kurtosis=0, *g* distribution)

- weak nonmonotonic skewed scenario (kurtosis= -0.068627)

- moderate nonmonotonic light-skewed scenario (kurtosis= -0.375)

-moderate nonmonotonic symmetric scenario (*g*=0, kurtosis= -0.6)

-heavily nonmonotonic symmetric light-tailed scenario (*g*=0, kurtosis=-1.2)

-severely nonmonotonic symmetric light-tailed scenario (*g*=0, kurtosis=-1.383826)

The values of the parameters *g* and *h* in the Tukey distribution are based on the corresponding constants (*c*'s) of the power method. (First, MOP skewness and MOP kurtosis parameters were calculated based on formula 23 and formula 24 in Kuo and Headrick (2017). Then, from formula 15 and formula 16 in Kuo and Headrick (2014) *g* and *h* parameters are calculated.)

It should be noted that although in the literature there is a consensus on how to define the skewness (e.g., "degree of asymmetry'), there was a debate for a certain time on how to interpret kurtosis. Numerous sources relate kurtosis to the "degree of peakedness", and only recently the discussion is finished with Westfall (2014) article, unambiguously relating the kurtosis to the tail extremity, specifically stating "classical kurtosis measure and peakedness are unrelated" (p. 3).

3.4 Sample sizes used for the study

Three different sample sizes are considered for the Monte Carlo simulations. These are 10, 25 and 50 observations per cell. Different sample sizes are known to improve the generalizability of the study (Headrick, 1997).

3.5. Pseudo-random numbers

It should be noted that "Most of today's random number generators are not based on physical devices, but on simple algorithms that can be easily implemented on a computer. They are fast, require little storage space, and can readily reproduce a given sequence of random numbers. Importantly, a good random number generator captures all the important statistical properties of true random sequences, even though the sequence is generated by a deterministic algorithm. For this reason, these generators are sometimes called pseudorandom" (Rubinstein & Kroese, 2008, p.50).

For reproducibility of the results and for the purpose of bias reduction, seed numbers are included in the program. By using the seed numbers it would be easy to reveal all the succeeding numbers in the sequence.

3.6 The structural model

The ANOVA model used in the study is as follows:

$Y_{ijk}=\mu+\alpha_i+\tau_j+(\alpha\tau)_{ij}+\varepsilon_{ijk}$

where, *i*=1..2, *j*=1..2, *k*=1..10, or 1..25, or 1..50.

The notation is as follows:

Y_{ijk} – variate's observed value for k_{th} observation within the i_{th} level of factor α and j_{th} level of factor τ.

μ – the overall grand mean

α_i – the effect due to the i_{th} level of factor α subject to $\alpha_1 + \alpha_2 = 0$

τ_j – the effect due to the j_{th} level of factor τ subject to $\tau_1 + \tau_2 = 0$

$(\alpha\tau)_{ij}$ – the effect due to the interaction of the i_{th} level of factor α and j_{th} level of factor τ subject to $(\alpha\tau)_{1j}+(\alpha\tau)_{2j}=0$ and $(\alpha\tau)_{i1}+(\alpha\tau)_{i2}=0$

ε_{ijk} - stochastic disturbance term that follows different conditional distributions discussed above.

Proposed coefficients for treatment patterns (TP) are shown below (c denotes the effect size):

1 main effect, TP=1

	(τ_1)	(τ_2)
(α_1)	c	c
(α_2)	0	0

1 main effect, TP=2

	(τ_1)	(τ_2)
(α_1)	c	c
(α_2)	-c	-c

2 main effects, TP=3

	(τ_1)	(τ_2)
(α_1)	-2c	0
(α_2)	0	2c

2 main effects, TP=4

	(τ_1)	(τ_2)
(α_1)	0	-2c
(α_2)	2c	0

Interaction only, TP=5

	(τ_1)	(τ_2)
(α_1)	c	-c
(α_2)	-c	c

Interaction only, TP=6

	(τ_1)	(τ_2)
(α_1)	-c	c
(α_2)	c	-c

1 main effect + Interaction, TP=7

	(τ_1)	(τ_2)
(α_1)	c	0
(α_2)	-c	0

1 main effect + Interaction, TP=8

	(τ_1)	(τ_2)
(α_1)	0	c
(α_2)	0	-c

2 main effects + Interaction, TP=9

	(τ_1)	(τ_2)
(α_1)	c	0
(α_2)	0	0

2 main effects + Interaction, TP=10

	(τ_1)	(τ_2)
(α_1)	c	-c
(α_2)	c	c

3.7 Data generation

The procedure for data generation follows the fifth-order power methods proposed by Headrick (2002). The equation for the dependent variable (*Y*) in this study is as follows:

$Y = c_1 + c_2W + c_3W^2 + c_4W^3 + c_5W^4 + c_6W^5$

where *W* is the standard normal deviate; $c_1 - c_6$ are the constants for power function.

Tukey *g-h* transformations with monotonic, slightly nonmonotonic, moderately nonmonotonic as well as severely nonmonotonic setting are included in the list of distributions under study. They are investigated in terms of robustness of Type 1 error and power properties, specifically, how the degree of nonmonotonicity affects the properties under parametric and nonparametric scenarios. Comparison for MOM versus MOP estimates is also performed.

3.8 Treatment effects model

The parameters for the study are as follows:

1. Treatment effect sizes are 0, 0.25, 0.5, 0.75 and 1.
2. Treatment effect patterns: 1 main effect, 2 main effects, 1 main effect with interaction, 2 main effects with interaction, interaction only.

3.9 Calculating *F* Ratios

F statistics for interaction and main effects were computed on the raw scores and their ranks for the 12 (types of distributions) x 3 (sample sizes) x 5 (treatment patterns) x 5 (effect sizes) situations. This results in 900 scenarios for calculating *F* statistics.

CHAPTER 4

RESULTS

Chapter 4 has the following organization: First, the results for the standard normal distribution are presented. The results are followed by the approximation of Beta (4, 1.5), Beta (4, 2), Triangular, Uniform, and Beta (0.667, 0.667) distributions.

It should be noted that 2x2 Rank Transform is proven to be proper as was theoretically established in Blair, Sawilowsky, and Higgins (1987), Headrick and Sawilowsky (2000), Akritas (1990), and Thompson (1991).

4.1. Standard normal distribution

Type I error rates for the parametric ANOVA (*F*) as well as nonparametric rank transform (RT) tests for the case of zero size effect are shown in the Table 4.1.1.

Table 4.1.1 Simulated Type I error rates for 0 size effect standard normal, pattern "All effects null".

Statistic	Sample size		
	10	25	50
F-column effect	0.05164	0.05108	0.05016
F-row effect	0.05208	0.05056	0.05176
F-interaction effect	0.05260	0.05340	0.05160
F-GH- column effect	0.05344	0.05152	0.05092
F-GH- row effect	0.05356	0.05228	0.05256
F-GH- interaction effect	0.05220	0.04804	0.05076
Frt- column effect	0.05160	0.05076	0.05116
Frt- row effect	0.05228	0.05260	0.05124

Frt-interaction effect	0.05308	0.05356	0.05076
Frt-GH-column effect	0.04552	0.04872	0.04924
Frt-GH-row effect	0.04676	0.04904	0.05128
Frt-GH- interaction effect	0.04504	0.04616	0.04816

As expected, the rates are close to 0.05. Besides, with the increase of the sample size, the estimates become closer to 0.05 level. Now, we need to look at treatment pattern by the effect size.

Table 4.1.2 Simulated Power rates for 0.25 size effect standard normal, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.12992	0.23612	0.42448
F-row effect	0.05334	0.05284	0.05008
F-interaction effect	0.05488	0.05080	0.04948
F-GH- column effect	0.12464	0.24188	0.42060
F-GH- row effect	0.05512	0.05220	0.04964
F-GH- interaction effect	0.05056	0.05132	0.05108
Frt- column effect	0.12592	0.22860	0.40804
Frt- row effect	0.05392	0.05280	0.04964
Frt-interaction effect	0.05676	0.05160	0.05096
Frt-GH-column effect	0.11084	0.22956	0.40584
Frt-GH-row effect	0.04716	0.04972	0.04816
Frt-GH- interaction effect	0.04440	0.04660	0.04828

The column effect is present, and raises the Power to the levels 2.5 times higher than alpha for 10 observations, 4.5 times higher for 25, and 8-8.5 times higher for 50 observations. It should be noted that, the RT values for column effect are slightly lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 61.54% to 88.21% and for GH are from 59.67% to 87.75%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 60.32%-87.74%, and 54.54%-87.68% for GH scenario, being close for 50 observations.

Table 4.1.3 Simulated Power rates for 0.5 size effect standard normal, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.34248	0.69972	0.93852
F-row effect	0.05484	0.05152	0.05148
F-interaction effect	0.05464	0.05096	0.05064
F-GH- column effect	0.34332	0.69732	0.93948
F-GH- row effect	0.05524	0.05048	0.05216
F-GH- interaction effect	0.05448	0.05292	0.04996
Frt- column effect	0.33080	0.67992	0.92772
Frt- row effect	0.05468	0.05224	0.05104
Frt-interaction effect	0.05380	0.05040	0.05032
Frt-GH-column effect	0.32200	0.68572	0.93484
Frt-GH-row effect	0.04848	0.04672	0.04924

Frt-GH- interaction effect	0.04788	0.04816	0.04848

The column effect is present, and raises the Power to the levels 6.5-7 times higher than alpha for 10 observations, 13.5-14 times higher for 25, and 18.5-19 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is high. It should be noted that, the RT values for column and other effects are slightly lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 85.38% to 94.67% and for GH are from 85.42% to 94.68%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.9%-94.6%, and 84.47%-94.65% for GH scenario, being close for 50 observations.

Table 4.1.4 Simulated Power rates for 0.75 size effect standard normal, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.63132	0.95868	0.99976
F-row effect	0.05492	0.04908	0.04980
F-interaction effect	0.05436	0.04844	0.05428
F-GH- column effect	0.64188	0.95968	0.99964
F-GH- row effect	0.05204	0.05164	0.05036
F-GH- interaction effect	0.05352	0.05124	0.05064
Frt- column effect	0.61208	0.95100	0.99940
Frt- row effect	0.05544	0.05152	0.05068
Frt-interaction effect	0.05424	0.05080	0.05376
Frt-GH-column effect	0.63812	0.95996	0.99976

Frt-GH-row effect	0.04644	0.04900	0.05104
Frt-GH- interaction effect	0.04668	0.04916	0.04996

The column effect is present, and raises the Power to the levels 12-13 times higher than alpha for 10 observations, 19 times higher for 25, and 19.9-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is very high. It should be noted that, the RT values for column and other effects are not necessarily lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 92.08% to 94.99% and for GH are from 92.2% to 94.99%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.83%-94.99%, and 92.2%-94.99% for GH scenario, being close for 50 observations.

Table 4.1.5 Simulated Power rates for 1.0 size effect standard normal, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.85964	0.99864	1.00000
F-row effect	0.05468	0.04876	0.04976
F-interaction effect	0.05448	0.05288	0.04964
F-GH- column effect	0.86428	0.99824	1.00000
F-GH- row effect	0.05304	0.05276	0.04968
F-GH- interaction effect	0.05504	0.04984	0.05084
Frt- column effect	0.84468	0.99752	1.00000
Frt- row effect	0.05548	0.04868	0.05196
Frt-interaction effect	0.05460	0.05236	0.05088

Frt-GH-column effect	0.87964	0.99872	1.00000
Frt-GH-row effect	0.04864	0.05080	0.04904
Frt-GH- interaction effect	0.05072	0.04736	0.05068

The column effect is present, and raises the Power to the levels 16.8-17.6 times higher than alpha for 10 observations, 19.8-20 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is exactly 1.0, which is the highest possible. It should be noted that, the RT values for column and other effects are not necessarily lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95% and for GH are from 94.2% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.07%-95%, and 94.3%-95% for GH scenario, being the same for 50 observations.

Table 4.1.6 Simulated Power rates for 0.25 size effect standard normal, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33920	0.69900	0.93896
F-row effect	0.34316	0.69676	0.93876
F-interaction effect	0.05488	0.05080	0.04948
F-GH- column effect	0.34004	0.69256	0.93964
F-GH- row effect	0.33884	0.68964	0.94200
F-GH- interaction effect	0.05056	0.05132	0.05108
Frt- column effect	0.32556	0.68004	0.92816

Frt- row effect	0.32540	0.67736	0.92892
Frt-interaction effect	0.05444	0.05200	0.04832
Frt-GH-column effect	0.32212	0.67440	0.93424
Frt-GH-row effect	0.32196	0.67860	0.93504
Frt-GH- interaction effect	0.04484	0.04652	0.04780

Two main effects are present, and raise the Power to the levels 6.4-7 times higher than alpha for 10 observations, 13.4-14 times higher for 25, and 18.4-18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is the high. It should be noted that, the RT values for column and row effects are lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 85.25% to 94.69% and for GH are from 85.24% to 94.62%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.62%-94.62%, and 84.47%-94.65% for GH scenario, being close for 50 observations.

Table 4.1.7 Simulated Power rates for 0.5 size effect standard normal, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86612	0.99868	1.00000
F-row effect	0.86544	0.99844	1.00000
F-interaction effect	0.05220	0.05188	0.05220
F-GH- column effect	0.86700	0.99872	1.00000
F-GH- row effect	0.86420	0.99864	1.00000
F-GH- interaction effect	0.05416	0.05188	0.05044

Frt- column effect	0.84576	0.99796	1.00000
Frt- row effect	0.84604	0.99760	1.00000
Frt-interaction effect	0.04364	0.04260	0.04496
Frt-GH-column effect	0.86832	0.99856	1.00000
Frt-GH-row effect	0.86676	0.99860	1.00000
Frt-GH- interaction effect	0.04520	0.04480	0.04100

Two main effects are present, and raise the Power to the levels 16.8-17.4 times higher than alpha for 10 observations, 19.9-20 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.00, which is the highest. It should be noted that, the RT values for column and row effects are slightly lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.2% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.09%-95%, and 94.24%-95% for GH scenario, being the same for 50 observations.

Table 4.1.8 Simulated Power rates for 0.75 size effect standard normal, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.99548	1.00000	1.00000
F-row effect	0.99532	1.00000	1.00000
F-interaction effect	0.05256	0.05140	0.05212
F-GH- column effect	0.99580	1.00000	1.00000
F-GH- row effect	0.99576	1.00000	1.00000

F-GH- interaction effect	0.05240	0.05016	0.05008
Frt- column effect	0.99256	1.00000	1.00000
Frt- row effect	0.99268	1.00000	1.00000
Frt-interaction effect	0.02640	0.02572	0.02568
Frt-GH-column effect	0.99600	1.00000	1.00000
Frt-GH-row effect	0.99616	1.00000	1.00000
Frt-GH- interaction effect	0.03000	0.02812	0.02720

Two main effects are present, and raise the Power to the levels 19.8-19.93 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.00, which is the highest. It should be noted that, the RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.97% to 95% and for GH are from 94.96% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 94.98%-95% for GH scenario, being the same for 50 observations.

Table 4.1.9 Simulated Power rates for 1.0 size effect standard normal, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	1.00000	1.00000	1.00000
F-row effect	0.99996	1.00000	1.00000
F-interaction effect	0.05520	0.05448	0.05112

F-GH- column effect	0.99996	1.00000	1.00000
F-GH- row effect	0.99996	1.00000	1.00000
F-GH- interaction effect	0.05544	0.04972	0.05144
Frt- column effect	0.99988	1.00000	1.00000
Frt- row effect	0.99996	1.00000	1.00000
Frt-interaction effect	0.00632	0.00536	0.00596
Frt-GH-column effect	0.99996	1.00000	1.00000
Frt-GH-row effect	0.99996	1.00000	1.00000
Frt-GH- interaction effect	0.01336	0.01024	0.00812

Two main effects are present, and raise the Power to the levels 19.9-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.00, which is the highest. It should be noted that, the RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 94.99%-95% for GH scenario, being the same for 50 observations.

Table 4.1.10 Simulated Power rates for 0.25 size effect standard normal, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05564	0.05300	0.04840

F-row effect	0.05224	0.05096	0.05180
F-interaction effect	0.34100	0.69608	0.93832
F-GH- column effect	0.05448	0.05008	0.04936
F-GH- row effect	0.05392	0.05288	0.04988
F-GH- interaction effect	0.33900	0.69568	0.94044
Frt- column effect	0.05552	0.05488	0.04844
Frt- row effect	0.05356	0.05144	0.05172
Frt-interaction effect	0.32716	0.67948	0.92840
Frt-GH-column effect	0.04676	0.04766	0.04896
Frt-GH-row effect	0.04696	0.04992	0.04988
Frt-GH- interaction effect	0.32384	0.68500	0.93488

Interaction effect is present, and raise the Power to the levels 6.4-6.9 times higher than alpha for 10 observations, 13.4-14 times higher for 25, and 18.4- 18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is the high. It should be noted that, the RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 85.34% to 94.67% and for GH are from 85.25% to 94.68%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.7%-94.6%, and 84.57%-94.65% for GH scenario, being almost the same for 50 observations.

Table 4.1.11 Simulated Power rates for 0.5 size effect standard normal, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05156	0.05040	0.04948
F-row effect	0.05396	0.05188	0.05144
F-interaction effect	0.86992	0.99812	1.00000
F-GH- column effect	0.05396	0.05040	0.05100
F-GH- row effect	0.05364	0.04972	0.05056
F-GH- interaction effect	0.86752	0.99852	1.00000
Frt- column effect	0.05132	0.05080	0.04928
Frt- row effect	0.05116	0.05244	0.05304
Frt-interaction effect	0.85420	0.99724	1.00000
Frt-GH-column effect	0.04972	0.04784	0.05012
Frt-GH-row effect	0.05064	0.04840	0.05048
Frt-GH- interaction effect	0.88360	0.99900	1.00000

Interaction effect is present, and raises the Power to the levels 17-17.7 times higher than alpha for 10 observations, 19.9-20 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, the RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is not necessarily lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.25% to 95% and for GH are from 94.24% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.2%-95%, and 94.34%-95% for GH scenario, being the same for 50 observations.

Table 4.1.12 Simulated Power rates for 0.75 size effect standard normal, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05300	0.05124	0.05436
F-row effect	0.05432	0.05268	0.04928
F-interaction effect	0.99492	1.00000	1.00000
F-GH- column effect	0.05448	0.05112	0.05196
F-GH- row effect	0.05236	0.05164	0.05140
F-GH- interaction effect	0.99588	1.00000	1.00000
Frt- column effect	0.05384	0.05088	0.05416
Frt- row effect	0.05120	0.05160	0.04996
Frt-interaction effect	0.99328	1.00000	1.00000
Frt-GH-column effect	0.05240	0.05248	0.05296
Frt-GH-row effect	0.05084	0.05136	0.05036
Frt-GH- interaction effect	0.99788	1.00000	1.00000

Interaction effect is present, and raises the Power to the levels 19.8-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, the RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is not necessarily lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.97% to 95% and for GH are from 94.98% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 94.99%-95% for GH scenario, being the same for 50 observations.

Table 4.1.13 Simulated Power rates for size 1.0 effect standard normal, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05464	0.05232	0.04944
F-row effect	0.05276	0.05268	0.05176
F-interaction effect	0.99996	1.00000	1.00000
F-GH- column effect	0.05528	0.05192	0.05068
F-GH- row effect	0.05456	0.05244	0.04948
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05424	0.05272	0.05068
Frt- row effect	0.05384	0.05148	0.05184
Frt-interaction effect	0.99988	1.00000	1.00000
Frt-GH-column effect	0.05404	0.05240	0.04884
Frt-GH-row effect	0.05260	0.05140	0.05040
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power to the levels 19.999-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, the

RT values for column and row effects are not necessarily lower than for parametric scenarios. Besides, interaction effect is not necessarily lower (almost always equal) for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.1.14 Simulated Power rates for size 0.25 effect standard normal, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.12948	0.23316	0.41908
F-row effect	0.05224	0.05096	0.05180
F-interaction effect	0.12720	0.24168	0.41844
F-GH- column effect	0.12532	0.24196	0.42240
F-GH- row effect	0.05392	0.05288	0.04988
F-GH- interaction effect	0.12428	0.23932	0.41820
Frt- column effect	0.12348	0.22580	0.40104
Frt- row effect	0.05332	0.05160	0.05172
Frt-interaction effect	0.12456	0.23172	0.40556
Frt-GH-column effect	0.10956	0.22832	0.40656
Frt-GH-row effect	0.04636	0.05044	0.04924
Frt-GH- interaction effect	0.10828	0.22596	0.40456

Both Interaction and column effects are present, and raise the Power to the levels 2.1-2.5 times higher than alpha for 10 observations, 4.5-5 times higher for 25, and 8-8.5 times higher for 50 observations. Also, for 50 observations, the rate is above 0.4, which is not very high. It should be noted that, the RT values for column and interaction effects are lower than for parametric scenarios. In contrast, row effect is not necessarily lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 60.63% to 88.07% and for GH are from 59.68% to 88.2%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 59.52%-87.68%, and 53.7%-87.7% for GH scenario, being almost the same for 50 observations.

Table 4.1.15 Simulated Power rates for size 0.5 effect standard normal, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33760	0.69968	0.94016
F-row effect	0.05280	0.05188	0.05144
F-interaction effect	0.34268	0.69368	0.93964
F-GH- column effect	0.34096	0.69828	0.93944
F-GH- row effect	0.05364	0.04972	0.05056
F-GH- interaction effect	0.33972	0.69840	0.93904
Frt- column effect	0.32544	0.67808	0.92872
Frt- row effect	0.05124	0.05160	0.05264
Frt-interaction effect	0.33036	0.67396	0.92940

Frt-GH-column effect	0.32364	0.68496	0.93468
Frt-GH-row effect	0.04732	0.04760	0.04916
Frt-GH- interaction effect	0.32056	0.68580	0.93524

Both Interaction and column effects are present, and raise the Power to the levels 6.4-6.8 times higher than alpha for 10 observations, 13.4-14 times higher for 25, and 18.5-18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is high. It should be noted that, the RT values for column and interaction effects are lower than for parametric scenarios. In contrast, row effect is not necessarily lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 85.2% to 94.68% and for GH are from 85.3% to 94.67%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.6%-94.62%, and 84.38%-94.65% for GH scenario, being almost the same for 50 observations.

Table 4.1.16 Simulated Power rates for size 0.75 effect standard normal, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.63732	0.95976	0.99972
F-row effect	0.05432	0.05268	0.04928
F-interaction effect	0.63988	0.95912	0.99948
F-GH- column effect	0.63888	0.95860	0.99976
F-GH- row effect	0.05236	0.05164	0.05140
F-GH- interaction effect	0.63428	0.95636	0.99952

Frt- column effect	0.61044	0.95136	0.99936
Frt- row effect	0.05108	0.04792	0.04696
Frt-interaction effect	0.61596	0.94976	0.99936
Frt-GH-column effect	0.62956	0.95704	0.99936
Frt-GH-row effect	0.04536	0.04608	0.04800
Frt-GH- interaction effect	0.62868	0.95616	0.99952

Both Interaction and column effects are present, and raise the Power to the levels 12.2-12.8 times higher than alpha for 10 observations, 18.9-19.2 times higher for 25, and 19.98-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is very high. It should be noted that, the RT values for column and interaction effects are lower than for parametric scenarios (or the same). Besides, row effect is lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 92.2% to 94.99% and for GH are from 92.2% to 94.99%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.8%-94.99%, and 92.05%-94.99% for GH scenario, being almost the same for 50 observations.

Table 4.1.17 Simulated Power rates for size 1.0 effect standard normal, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86608	0.99808	1.00000
F-row effect	0.05276	0.05268	0.05176
F-interaction effect	0.86404	0.99836	1.00000

F-GH- column effect	0.86460	0.99832	1.00000
F-GH- row effect	0.05456	0.05244	0.04948
F-GH- interaction effect	0.86588	0.99856	1.00000
Frt- column effect	0.84552	0.99752	0.99996
Frt- row effect	0.04584	0.04348	0.04544
Frt-interaction effect	0.84440	0.99972	1.00000
Frt-GH-column effect	0.86428	0.99840	1.00000
Frt-GH-row effect	0.04460	0.04440	0.04252
Frt-GH- interaction effect	0.86592	0.99880	1.00000

Both Interaction and column effects are present, and raise the Power to the levels 16.8-17.4 times higher than alpha for 10 observations, 19.94-20 times higher for 25, and 19.999-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9999, which is very high. It should be noted that, the RT values for column and interaction effects are not necessarily lower than for parametric scenarios. Besides, row effect is lower for RT scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95% and for GH are from 94.2% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.08%-94.99%, and 94.2%-95% for GH scenario, being the same for 50 observations.

Table 4.1.18 Simulated Power rates for size 0.25 effect standard normal, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.07384	0.09664	0.14124

F-row effect	0.06888	0.09524	0.14140
F-interaction effect	0.07212	0.09912	0.14344
F-GH- column effect	0.07016	0.09672	0.14488
F-GH- row effect	0.07224	0.01088	0.13896
F-GH- interaction effect	0.07012	0.09904	0.14248
Frt- column effect	0.07192	0.09468	0.13924
Frt- row effect	0.06932	0.09244	0.13644
Frt-interaction effect	0.07268	0.09772	0.13892
Frt-GH-column effect	0.05976	0.09008	0.13980
Frt-GH-row effect	0.06204	0.09448	0.13068
Frt-GH- interaction effect	0.05888	0.09220	0.13472

Interaction, row and column effects are present, and raise the Power to the levels 1.2-1.5 times higher than alpha for 10 observations, 1.8-2.2 times higher for 25, and 2.6-2.9 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.1, which is not high. It should be noted that, the RT values for column, row and interaction effects are not necessarily lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 27.4% to 64.5% and for GH are from 28.57% to 65.5%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 27.87%-64.09%, and 32.07%-64.23% for GH scenario, being almost the same for 50 observations.

Table 4.1.19 Simulated Power rates for size 0.5 effect standard normal, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.12024	0.23516	0.41836
F-row effect	0.12160	0.23668	0.42028
F-interaction effect	0.12516	0.23840	0.42536
F-GH- column effect	0.12240	0.23588	0.42412
F-GH- row effect	0.12292	0.23344	0.41888
F-GH- interaction effect	0.12248	0.23908	0.41800
Frt- column effect	0.11692	0.22592	0.40020
Frt- row effect	0.11916	0.22632	0.40256
Frt-interaction effect	0.12200	0.23292	0.40612
Frt-GH-column effect	0.10804	0.22576	0.40528
Frt-GH-row effect	0.11060	0.22240	0.40452
Frt-GH- interaction effect	0.10936	0.22700	0.40156

Interaction, row and column effects are present, and raise the Power to the levels 2.16-2.5 times higher than alpha for 10 observations, 4.4-4.8 times higher for 25, and 8-8.5 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.4, which is not high. It should be noted that, the RT values for column, row and interaction effects are lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 58.33% to 88.23% and for GH are from 59.2% to 88.21%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 57.26%-87.68%, and 53.7%-87.65% for GH scenario, being almost the same for 50 observations.

Table 4.1.20 Simulated Power rates for size 0.75 effect standard normal, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.21420	0.46144	0.74500
F-row effect	0.21696	0.45852	0.74848
F-interaction effect	0.21908	0.45740	0.75396
F-GH- column effect	0.22132	0.45636	0.74952
F-GH- row effect	0.21692	0.45716	0.75332
F-GH- interaction effect	0.21652	0.45540	0.75048
Frt- column effect	0.20564	0.43984	0.72448
Frt- row effect	0.21024	0.44088	0.72416
Frt-interaction effect	0.20996	0.43892	0.73380
Frt-GH-column effect	0.20160	0.44184	0.73576
Frt-GH-row effect	0.19880	0.43484	0.73684
Frt-GH- interaction effect	0.19632	0.43692	0.73576

Interaction, row and column effects are present, and raise the Power to the levels 3.9-4.43 times higher than alpha for 10 observations, 8.7-9.2 times higher for 25, and 14.5-15.1 times higher for 50 observations. Also, for 50 observations, the rate is above 0.7, which is high. It should be noted that, the RT values for column, row and interaction effects are lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 76.64% to 93.37% and for GH are from 76.9% to 93.36%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 75.68%-93.09%, and 74.53%-93.22% for GH scenario, being almost the same for 50 observations.

Table 4.1.21 Simulated Power rates for size 1.0 effect standard normal, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.34228	0.69416	0.94028
F-row effect	0.33804	0.69436	0.94156
F-interaction effect	0.34236	0.69480	0.94160
F-GH- column effect	0.34100	0.69664	0.93840
F-GH- row effect	0.34548	0.69936	0.94040
F-GH- interaction effect	0.33944	0.69404	0.94272
Frt- column effect	0.32620	0.66632	0.92880
Frt- row effect	0.31928	0.66776	0.92928
Frt-interaction effect	0.32600	0.67156	0.92932
Frt-GH-column effect	0.31872	0.67876	0.92916
Frt-GH-row effect	0.32596	0.68112	0.93368
Frt-GH- interaction effect	0.32180	0.67780	0.93272

Both Interaction and column effects are present, and raise the Power to the levels 6.36-6.92 times higher than alpha for 10 observations, 13.32-14 times higher for 25, and 18.56-18.86 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, the RT values for column, row and interaction effects are lower than for parametric scenarios.

Overall, in parametric setting, for the power method the relative rejection rates are 85.38% to 94.69% and for GH are from 85.53% to 94.7%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.38%-94.62%, and 84.66%-94.64% for GH scenario, being almost the same for 50 observations.

4.2. Beta (4, 1.5) distribution.

Table 4.2.1 Simulated Type I error rates for size 0 effect Beta (4, 1.5) distribution, pattern “All effects null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05156	0.05136	0.05032
F-row effect	0.05236	0.05072	0.05088
F-interaction effect	0.05180	0.05364	0.05144
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.05160	0.05076	0.05116

Frt- row effect	0.05228	0.05260	0.05124
Frt-interaction effect	0.05308	0.05356	0.05760
Frt-GH-column effect	0.04580	0.04660	0.04940
Frt-GH-row effect	0.04572	0.04992	0.05044
Frt-GH- interaction effect	0.04484	0.04716	0.04744

The rates are close to 0.05 (except F-GH parametric scenarios with zeros). Besides, with the increase of the sample size, the estimates become closer to 0.05 level.

It should be also noted that F-GH nonparametric scenarios in most cases are slightly lower than 0.05, while in power method cases the values are higher than 0.05.

In general, size 0 effect rates confirm the theory. Now, we need to look at treatment pattern by the effect size for Beta (4, 1.5) distribution.

Table 4.2.2 Simulated Power rates for size 0.25 effect Beta (4, 1.5) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”

Statistic	Sample size		
	10	25	50
F-column effect	0.12756	0.23616	0.42128
F-row effect	0.05216	0.05248	0.05072
F-interaction effect	0.05616	0.05208	0.05080
F-GH- column effect	0.00080	0.13280	0.26200
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.13284	0.25116	0.44648
Frt- row effect	0.05304	0.05196	0.05156

Frt-interaction effect	0.05704	0.05192	0.05220
Frt-GH-column effect	0.66704	0.97504	0.99992
Frt-GH-row effect	0.03400	0.03508	0.03516
Frt-GH- interaction effect	0.03564	0.03416	0.03564

Column effect is present, and raises the Power for power method scenarios to the levels 2.54-2.66 times higher than alpha for 10 observations, 4.72-5 times higher for 25, and 8.42-8.94 times higher for 50 observations. Also, for 50 observations, the rate is above 0.4, which is not very high. It should be noted that, for GH scenarios the parametric values for column effects are low (0.0008-0.262), but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are high (0.66704-0.99992), but for row, and interaction are 0.034-0.03564.

Overall, in parametric setting, for the power method the relative rejection rates are 60.8% to 88.12% and for GH are from negative to 80.92%. Thus, for 50 observations GH gives close rates of relative rejection. In RT setting, the relative rejection rates for the power method are 62.4%-94.62%, and 92.5%-94.99% for GH scenario, being almost the same for 50 observations.

Table 4.2.3 Simulated Power rates for size 0.5 effect Beta (4, 1.5) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33352	0.69996	0.93956
F-row effect	0.05260	0.05156	0.05108
F-interaction effect	0.05316	0.04968	0.05224
F-GH- column effect	0. 14204	0.94796	1.00000

F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.35148	0.72244	0.94924
Frt- row effect	0.05296	0.05096	0.05300
Frt-interaction effect	0.05276	0.05164	0.05252
Frt-GH-column effect	0.99560	1.00000	1.00000
Frt-GH-row effect	0.01600	0.01396	0.01460
Frt-GH- interaction effect	0.01404	0.01396	0.01460

Column effect is present, and raises the Power for power method scenarios to the levels 6.65-7.03 times higher than alpha for 10 observations, 14-14.5 times higher for 25, and 18.79-19 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for column effects are 0.14204-1, but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are very high 0.995-1.0, but for row and interaction are 0.01396-0.016.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.68% and for GH are from 64.79% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.75%-94.73%, and 94.98%-95% for GH scenario, being almost the same for 50 observations.

Table 4.2.4 Simulated Power rates for size 0.75 effect Beta (4, 1.5) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.63676	0.95944	0.99952

F-row effect	0.05536	0.05060	0.04948
F-interaction effect	0.05236	0.04968	0.05144
F-GH- column effect	0. 84576	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.64552	0.96316	0.99972
Frt- row effect	0.05516	0.05048	0.04956
Frt-interaction effect	0.05316	0.05184	0.04964
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00252	0.00224	0.00200
Frt-GH- interaction effect	0.00296	0.00180	0.00224

Column effect is present, and raises the Power for power method scenarios to the levels 12.7-12.9 times higher than alpha for 10 observations, 19.2-19.27 times higher for 25, and 19.98-19.99 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is extremely high. It should be noted that, for GH scenarios the parametric values for column effects are 0.84-1, but for row and interaction are 0. For nonparametric scenarios values for column effects are 1.0, but for row, and interaction are 0.0018-0.00296.

Overall, in parametric setting, for the power method the relative rejection rates are 92.2% to 94.99% and for GH are from 94.09% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 92.25%-94.99%, and 95% for GH scenario, being almost the same for 50 observations.

Table 4.2.5 Simulated Power rates for size 1.0 effect Beta (4, 1.5) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.86456	0.99856	1.00000
F-row effect	0.05232	0.05248	0.05104
F-interaction effect	0.05500	0.05312	0.04940
F-GH- column effect	0. 99904	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.86336	0.99820	1.00000
Frt- row effect	0.05420	0.05104	0.05200
Frt-interaction effect	0.05524	0.05328	0.05112
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00112	0.00068	0.00072
Frt-GH- interaction effect	0.00120	0.00044	0.00048

Column effect is present, and raises the Power for power method scenarios to the levels 17.26-17.3 times higher than alpha for 10 observations, 19.96-19.97 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column effects are 0.999-1, but for row and interaction are 0. For nonparametric scenarios values for column effects are 1.0, but for row, and interaction are 0.00044-0.0012.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.2%-95%, and 95% for GH scenario, being almost the same for 50 observations.

Table 4.2.6 Simulated Power rates for size 0.25 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.34260	0.69800	0.94120
F-row effect	0.34092	0.69796	0.94076
F-interaction effect	0.05616	0.05208	0.05080
F-GH- column effect	0. 14688	0.94720	1.00000
F-GH- row effect	0. 14540	0.94736	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.34716	0.70832	0.94680
Frt- row effect	0.34780	0.70592	0.94460
Frt-interaction effect	0.05580	0.05252	0.05188
Frt-GH-column effect	0.98424	1.00000	1.00000
Frt-GH-row effect	0.98468	1.00000	1.00000
Frt-GH- interaction effect	0.00340	0.00240	0.00144

Column and row effect are present, and raise the Power for power method scenarios to the levels 6.8-7 times higher than alpha for 10 observations, 13.96-14.2 times higher for 25, and 18.8-18.94 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are 0.1454-1.0, but for interaction are 0. For nonparametric scenarios values for column and row effects are 0.98424-1.0, but for interaction are 0.00144-0.0034.

Overall, in parametric setting, for the power method the relative rejection rates are 85.29% to 94.69% and for GH are from 65.6% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.6%-94.72%, and 94.92%-95% for GH scenario, being almost the same for 50 observations.

Table 4.2.7 Simulated Power rates for size 0.5 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86796	0.99848	1.00000
F-row effect	0.86568	0.99828	1.00000
F-interaction effect	0.05316	0.04964	0.05224
F-GH- column effect	0.99908	1.00000	1.00000
F-GH- row effect	0.99908	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.85336	0.99788	1.00000
Frt- row effect	0.85196	0.99748	1.00000
Frt-interaction effect	0.04536	0.04256	0.04548
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000

Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 17-17.36 times higher than alpha for 10 observations, 19.94-19.97 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are 0.999-1.0, but for interaction are 0. For nonparametric scenarios values for column and row effects are 1.0, in contrast for interaction they are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.2%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.8 Simulated Power rates for size 0.75 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.99564	1.00000	1.00000
F-row effect	0.99516	1.00000	1.00000
F-interaction effect	0.05236	0.04968	0.05144
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.99272	1.00000	1.00000

Frt- row effect	0.99236	1.00000	1.00000
Frt-interaction effect	0.02404	0.02636	0.02808
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 19.84-19.92 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are 1.0, but for interaction are 0. For nonparametric scenarios values for column and row effects are 1.0, in contrast for interaction they are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.2%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.9 Simulated Power rates for size 1.0 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.99996	1.00000	1.00000
F-row effect	0.99996	1.00000	1.00000
F-interaction effect	0.05500	0.05312	0.04940
F-GH- column effect	1.00000	1.00000	1.00000

F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.99988	1.00000	1.00000
Frt- row effect	0.99996	1.00000	1.00000
Frt-interaction effect	0.00676	0.00764	0.00828
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 19.9992-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are 1.0, but for interaction are 0. For nonparametric scenarios values for column and row effects are 1.0, in contrast for interaction they are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.10 Simulated Power rates for size 0.25 effect Beta (4, 1.5) distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05540	0.05344	0.04876

F-row effect	0.05216	0.05248	0.05072
F-interaction effect	0.33984	0.69880	0.93900
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.14604	0.94936	1.00000
Frt- column effect	0.05540	0.05464	0.04844
Frt- row effect	0.05360	0.05148	0.05124
Frt-interaction effect	0.35424	0.71944	0.94792
Frt-GH-column effect	0.01548	0.01388	0.01404
Frt-GH-row effect	0.01472	0.01540	0.01556
Frt-GH- interaction effect	0.99496	1.00000	1.00000

Interaction effect is present, and raise the Power for power method scenarios to the levels 6.8-7.0 times higher than alpha for 10 observations, 14- 14.4 times higher for 25, and 18.8-19 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.14-1.0, but for column and row are 0. For nonparametric scenarios values for column and row effects are 0.01388-0.01556, in contrast for interaction they are 0.99496-1.0.

Overall, in parametric setting, for the power method the relative rejection rates are 85.29% to 94.68% and for GH are from 65.75% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.88%-94.97%, and 95% for GH scenario, being almost the same for 50 observations.

Table 4.2.11 Simulated Power rates for size 0.5 effect Beta (4, 1.5) distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.04988	0.05184	0.05084
F-row effect	0.05260	0.05156	0.05108
F-interaction effect	0.87144	0.99852	1.00000
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.99940	1.00000	1.00000
Frt- column effect	0.05344	0.05144	0.04944
Frt- row effect	0.05132	0.05236	0.05368
Frt-interaction effect	0.86820	0.99796	1.00000
Frt-GH-column effect	0.00092	0.00052	0.00056
Frt-GH-row effect	0.00120	0.00084	0.00056
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raise the Power for power method scenarios to the levels 17.36-17.44 times higher than alpha for 10 observations, 19.96- 19.98 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.999-1.0, but for column and row are 0. For nonparametric scenarios values for column and row effects are 0.00052-0.0012, in contrast for interaction they are 1.0.

Overall, in parametric setting, for the power method the relative rejection rates are 99.4% to 95% and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.24%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.12 Simulated Power rates for size 0.75 effect Beta (4, 1.5) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05336	0.05096	0.05532
F-row effect	0.05536	0.05060	0.04948
F-interaction effect	0.99492	1.00000	1.00000
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05328	0.05108	0.05340
Frt- row effect	0.05596	0.05044	0.05056
Frt-interaction effect	0.99256	1.00000	1.00000
Frt-GH-column effect	0.00324	0.00252	0.00200
Frt-GH-row effect	0.00280	0.00284	0.00188
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 19.84-19.9 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 1.0, but for column and row are 0. For nonparametric scenarios values for column and row effects are 0.00188-0.00324, in contrast for interaction they are 1.0.

Overall, in parametric setting, for the power method the relative rejection rates are 99.97% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.13 Simulated Power rates for size 1.0 effect Beta (4, 1.5) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05300	0.05260	0.05136
F-row effect	0.05232	0.05248	0.05104
F-interaction effect	0.99992	1.00000	1.00000
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05340	0.05320	0.05196
Frt- row effect	0.05232	0.05220	0.05244
Frt-interaction effect	0.99988	1.00000	1.00000
Frt-GH-column effect	0.00896	0.00848	0.00708
Frt-GH-row effect	0.01020	0.00880	0.00732
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 19.99-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 1.0, but for column and row are 0. For nonparametric scenarios values for column and row effects are 0.00708-0.0102, in contrast for interaction they are 1.0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.2.14 Simulated Power rates for size 0.25 effect Beta (4, 1.5) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0. 12756	0.23616	0.42128
F-row effect	0.05216	0.05248	0.05072
F-interaction effect	0.12708	0.24128	0.41756
F-GH- column effect	0.00080	0.01328	0.26200
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00080	0.01284	0.25880
Frt- column effect	0.13180	0.24764	0.44076
Frt- row effect	0.05256	0.05232	0.05100
Frt-interaction effect	0.13180	0.25260	0.44404

Frt-GH-column effect	0.59388	0.95828	0.99964
Frt-GH-row effect	0.02572	0.02576	0.02752
Frt-GH- interaction effect	0.59816	0.95896	0.99960

Both column and interaction effects are present, and raise the Power for power method scenarios to the levels 2.54-2.64 times higher than alpha for 10 observations, 4.72-5 times higher for 25, and 8.36-8.88 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.4, which is not very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.0008-0.2588, for column 0.0008-0.262 and for row are 0. For nonparametric scenarios values for column and interaction are 0.59-0.999, in contrast for row they are 0.025-0.027.

Overall, in parametric setting, for the power method the relative rejection rates are 60.63% to 88.2% and for GH are from negative to 80.9%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 62.06%-88.74%, and 91.58-94.99% for GH scenario, being close for 50 observations.

Table 4.2.15 Simulated Power rates for size 0.5 effect Beta (4, 1.5) distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33352	0.69996	0.93956
F-row effect	0.05260	0.05156	0.05108
F-interaction effect	0.34216	0.69420	0.93996
F-GH- column effect	0.14204	0.94796	1.00000
F-GH- row effect	0.00000	0.00000	0.00000

F-GH- interaction effect	0.14188	0.94796	1.00000
Frt- column effect	0.34344	0.70864	0.94420
Frt- row effect	0.05084	0.05040	0.05192
Frt-interaction effect	0.35000	0.70568	0.94444
Frt-GH-column effect	0.98396	1.00000	1.00000
Frt-GH-row effect	0.00356	0.00160	0.00156
Frt-GH- interaction effect	0.98436	1.00000	1.00000

Both column and interaction effects are present, and raise the Power for power method scenarios to the levels 6.8-7 times higher than alpha for 10 observations, 13.8-14.2 times higher for 25, and 18.6-19 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.14-1.0, for column 0.14-1.0 and for row are 0. For nonparametric scenarios values for column and interaction are 0.98-1.0, in contrast for row they are 0.00156-0.00356.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.68% and for GH are from 64.79 to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.44%-94.7%, and 94.92%-95% for GH scenario, being almost the same for 50 observations.

Table 4.2.16 Simulated Power rates for size 0.75 effect Beta (4, 1.5) distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.63676	0.95944	0.99952

F-row effect	0.05536	0.05060	0.04948
F-interaction effect	0.63972	0.95984	0.99944
F-GH- column effect	0.84576	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.84992	1.00000	1.00000
Frt- column effect	0.63180	0.95816	0.99936
Frt- row effect	0.05120	0.04744	0.04768
Frt-interaction effect	0.63472	0.95716	0.99952
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00004	0.00000	0.00000
Frt-GH- interaction effect	0.99996	1.00000	1.00000

Both column and interaction effects are present, and raise the Power for power method scenarios to the levels 12.6-12.8 times higher than alpha for 10 observations, 19-19.2 times higher for 25, and 19.98-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.849-1.0, for column 0.845-1.0 and for row are 0. For nonparametric scenarios values for column and interaction are 0.9999-1.0, in contrast for row they are 0-0.00004.

Overall, in parametric setting, for the power method the relative rejection rates are 92.2% to 94.99% and for GH are from 94.09 to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 92.09%-94.99%, and 94.99%-95% for GH scenario, being almost the same for 50 observations.

Table 4.2.17 Simulated Power rates for size 1.0 effect Beta (4, 1.5) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.86456	0.99856	1.00000
F-row effect	0.05232	0.05248	0.05104
F-interaction effect	0.86528	0.99876	1.00000
F-GH- column effect	0.99904	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.99904	1.00000	1.00000
Frt- column effect	0.85036	0.99776	0.99996
Frt- row effect	0.04376	0.04328	0.04424
Frt-interaction effect	0.84968	0.99824	1.00000
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00000	0.00000	0.00000
Frt-GH- interaction effect	0.99996	1.00000	1.00000

Both column and interaction effects are present, and raise the Power for power method scenarios to the levels 17.2-19.98 times higher than alpha for 10 observations, 19.98-20 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 0.999-1.0, for column 0.999-1.0 and for row are 0. For nonparametric scenarios values for column and interaction are 0.99996-1.0, in contrast for row they are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99 to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.12%-95%, and 94.99%-95% for GH scenario, being the same for 50 observations.

Table 4.2.18 Simulated Power rates for size 0.25 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.07288	0.09592	0.14364
F-row effect	0.07036	0.09432	0.14072
F-interaction effect	0.07252	0.09812	0.14256
F-GH- column effect	0.00000	0.00000	0.00012
F-GH- row effect	0.00008	0.00000	0.00020
F-GH- interaction effect	0.00004	0.00000	0.00020
Frt- column effect	0.07488	0.10048	0.15112
Frt- row effect	0.07208	0.09920	0.14808
Frt-interaction effect	0.07604	0.10388	0.15116
Frt-GH-column effect	0.19496	0.47156	0.77216
Frt-GH-row effect	0.19524	0.46664	0.77400
Frt-GH- interaction effect	0.19472	0.46500	0.77904

Column, row and interaction effects are present, and raise the Power for power method scenarios to the levels 1.4-1.52 times higher than alpha for 10 observations, 1.88-2.06 times higher for 25, and 2.8-3.3 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.1, which is low. It should be noted that, for GH scenarios the parametric values for interaction effects, column, and row are 0-0.0002. For nonparametric scenarios values for column, row and interaction are 0.19-0.77.

Overall, in parametric setting, for the power method the relative rejection rates are 28.94% to 65.2% and for GH are negative. In RT setting, the relative rejection rates for the power method are 30.63%-66.92%, and 74.32%-93.58% for GH scenario, being far from each other even for 50 observations.

Table 4.2.19 Simulated Power rates for size 0.5 effect Beta (4, 1.5) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.12036	0.23296	0.41972
F-row effect	0.12204	0.23656	0.42236
F-interaction effect	0.12372	0.23860	0.42292
F-GH- column effect	0.00072	0.01388	0.26124
F-GH- row effect	0.00084	0.01376	0.26616
F-GH- interaction effect	0.00092	0.01364	0.25860
Frt- column effect	0.12460	0.24464	0.43440
Frt- row effect	0.12644	0.24664	0.43568
Frt-interaction effect	0.12880	0.25032	0.43996
Frt-GH-column effect	0.50212	0.92956	0.99876
Frt-GH-row effect	0.51020	0.92916	0.99836
Frt-GH- interaction effect	0.49908	0.92848	0.99884

Column, row and interaction effects are present, and raise the Power for power method scenarios to the levels 2.4-2.6 times higher than alpha for 10 observations, 4.6-5 times higher for 25, and 8.2-8.8 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.4, which is low. It should be noted that, for GH scenarios the parametric values for interaction effects, column, and row are 0.0007-0.266. For nonparametric scenarios values for column, row and interaction are 0.499-0.998.

Overall, in parametric setting, for the power method the relative rejection rates are 58.46% to 88.2% and for GH are from negative to 81.21%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 59.87%-88.64%, and 89.98%-94.99% for GH scenario, being close for 50 observations.

Table 4.2.20 Simulated Power rates for size 0.75 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.21624	0.46132	0.74380
F-row effect	0.21408	0.45724	0.74852
F-interaction effect	0.21732	0.45468	0.75132
F-GH- column effect	0.01760	0.38232	0.98612
F-GH- row effect	0.01624	0.38448	0.98708
F-GH- interaction effect	0.01592	0.38344	0.98668
Frt- column effect	0.21716	0.46668	0.75168
Frt- row effect	0.22600	0.46332	0.75460
Frt-interaction effect	0.22232	0.46516	0.76028

Frt-GH-column effect	0.80480	0.99924	1.00000
Frt-GH-row effect	0.80076	0.99896	1.00000
Frt-GH- interaction effect	0.80476	0.99928	1.00000

Column, row and interaction effects are present, and raise the Power for power method scenarios to the levels 4.2-4.6 times higher than alpha for 10 observations, 9-9.4 times higher for 25, and 14.8-15.2 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.7, which is high. It should be noted that, for GH scenarios the parametric values for interaction effects, column, and row are ranging from 3 times lower to 19.6 times higher than alpha. For nonparametric scenarios values for column, row and interaction are 16-20 times higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 76.44% to 93.35% and for GH are from negative to 94.93%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 76.98%-93.42%, and 93.75%-95% for GH scenario, being close for 50 observations.

Table 4.2.21 Simulated Power rates for size 1.0 effect Beta (4, 1.5) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.34288	0.69168	0.93936
F-row effect	0.33848	0.69508	0.94012
F-interaction effect	0.34332	0.69636	0.94124
F-GH- column effect	0.14536	0.94888	1.00000
F-GH- row effect	0.14552	0.94804	1.00000

F-GH- interaction effect	0.14408	0.94724	1.00000
Frt- column effect	0.34128	0.68396	0.93572
Frt- row effect	0.33532	0.68440	0.93600
Frt-interaction effect	0.34044	0.68852	0.93704
Frt-GH-column effect	0.90132	0.99996	1.00000
Frt-GH-row effect	0.90380	1.00000	1.00000
Frt-GH- interaction effect	0.90272	0.99988	1.00000

Column, row and interaction effects are present, and raise the Power for power method scenarios to the levels 6.6-6.9 times higher than alpha for 10 observations, 13.6-14 times higher for 25, and 18.6-18.9 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects, column, and row are 2.9-20 times higher than alpha. For nonparametric scenarios values for column, row and interaction are 18-20 times higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85.23% to 94.69% and for GH are from 65.28 to 95%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 85.09%-94.66%, and 94.45%-95% for GH scenario, being close for 50 observations.

4.3. Beta (4, 2) distribution.

Table 4.3.1 Simulated Type I error rates for size 0 effect Beta (4, 2) distribution, pattern “All effects null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05140	0.05108	0.04988

F-row effect	0.05220	0.05080	0.05140
F-interaction effect	0.05200	0.05356	0.05188
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.05160	0.05076	0.05116
Frt- row effect	0.05228	0.05260	0.05124
Frt-interaction effect	0.05308	0.05356	0.05760
Frt-GH-column effect	0.04604	0.04704	0.04892
Frt-GH-row effect	0.04564	0.04988	0.05080
Frt-GH- interaction effect	0.04464	0.04756	0.04740

The rates are close to 0.05 (except F-GH parametric scenarios with zeros). Besides, with the increase of the sample size, the estimates become closer to 0.05 level.

It should be also noted that F-GH nonparametric scenarios in most cases are slightly lower than 0.05. In contrast, for power method cases the values are higher than 0.05.

In general, size 0 effect rates confirm the theory. Now, we need to look at treatment pattern by the effect size for Beta (4, 2) distribution.

Table 4.3.2 Simulated Power rates for size 0.25 effect Beta (4, 2) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.12696	0.23432	0.41960
F-row effect	0.05236	0.05204	0.05096

F-interaction effect	0.05652	0.05292	0.05116
F-GH- column effect	0.00060	0.01208	0.26064
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.12408	0.22544	0.40384
Frt- row effect	0.05352	0.05236	0.05124
Frt-interaction effect	0.05672	0.05204	0.05232
Frt-GH-column effect	0.66464	0.97452	0.99988
Frt-GH-row effect	0.03376	0.03476	0.03584
Frt-GH- interaction effect	0.03536	0.03412	0.03568

Column effect is present, and raise the Power for power method scenarios to the levels 2.48-2.54 times higher than alpha for 10 observations, 4.5-4.7 times higher for 25, and 8-8.4 times higher for 50 observations. Also, for 50 observations, the rate is above 0.4, which is not very high. It should be noted that, for GH scenarios the parametric values for column effects are low (from 80 times lower to 5.2 times higher than alpha), but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are high (13-20 times higher than alpha), but for row, and interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 60.63% to 88.08% and for GH are from negative to 80.82%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 59.68%-87.62%, and 92.48%-94.99% for GH scenario, being close for 50 observations.

Table 4.3.3 Simulated Power rates for size 0.5 effect Beta (4, 2) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33436	0.69908	0.93976
F-row effect	0.05212	0.05168	0.05260
F-interaction effect	0.05276	0.05124	0.05224
F-GH- column effect	0.13332	0.95012	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.32320	0.67828	0.92828
Frt- row effect	0.05328	0.05116	0.05260
Frt-interaction effect	0.05280	0.05172	0.05248
Frt-GH-column effect	0.99572	1.00000	1.00000
Frt-GH-row effect	0.01580	0.01372	0.01376
Frt-GH- interaction effect	0.01360	0.01316	0.01392

Column effect is present, and raises the Power for power method scenarios to the levels 6.4-6.8 times higher than alpha for 10 observations, 13.2-14 times higher for 25, and 18.4-18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for column effects are from 2.6 to 20 times higher than alpha, but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are 19.9-20 times higher than alpha, but for row, and interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85.05% to 94.68% and for GH are from 62.5 to 95%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 84.53%-94.6%, and 94.98%-95% for GH scenario, being close for 50 observations.

Table 4.3.4 Simulated Power rates for size 0.75 effect Beta (4, 2) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.63464	0.96000	0.99964
F-row effect	0.05416	0.05128	0.04996
F-interaction effect	0.05300	0.05052	0.05108
F-GH- column effect	0.85168	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.61184	0.95072	0.99932
Frt- row effect	0.05532	0.05064	0.04964
Frt-interaction effect	0.05320	0.05160	0.05000
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00244	0.00208	0.00188
Frt-GH- interaction effect	0.00260	0.00168	0.00196

Column effect is present, and raise the Power for power method scenarios to the levels 13.2-13.8 times higher than alpha for 10 observations, 19-19.2 times higher for 25, and 19.98-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is very high. It should be noted that, for GH scenarios the parametric values for column effects are from 17 to 20 times higher than alpha, but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are 20 times higher than alpha, but for row, and interaction are lower than alpha, but greater than 0.

Overall, in parametric setting, for the power method the relative rejection rates are 92.12% to 94.99% and for GH are from 94.13 to 95%. Thus, for 50 observations GH gives very close relative rejection. In RT setting, the relative rejection rates for the power method are 91.83%-94.99%, and 95% for GH scenario, being very close for 50 observations.

Table 4.3.5 Simulated Power rates for size 1.0 effect Beta (4, 2) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.86552	0.99856	1.00000
F-row effect	0.05296	0.05280	0.05116
F-interaction effect	0.05492	0.05344	0.05004
F-GH- column effect	0.99904	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.84408	0.99744	1.00000
Frt- row effect	0.05416	0.05164	0.05168
Frt-interaction effect	0.05532	0.05360	0.05036
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00124	0.00060	0.00068

Frt-GH- interaction effect	0.00116	0.00044	0.00048

Column effect is present, and raises the Power for power method scenarios to the levels 16.8-17.3 times higher than alpha for 10 observations, 19.9-19.98 times higher for 25, and 19.98-20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column effects are from 19.98 to 20 times higher than alpha, but for row, and interaction are 0. In contrast, for nonparametric scenarios values for column effects are 20 times higher than alpha, but for row, and interaction are lower than alpha, but greater than 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99 to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.08%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.6 Simulated Power rates for size 0.25 effect Beta (4, 2) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33908	0.69880	0.94152
F-row effect	0.33880	0.69720	0.94136
F-interaction effect	0.05652	0.05292	0.05116
F-GH- column effect	0.13788	0.94964	1.00000
F-GH- row effect	0.13592	0.94980	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.32456	0.67280	0.92756

Frt- row effect	0.32496	0.67088	0.92748
Frt-interaction effect	0.05532	0.05272	0.05180
Frt-GH-column effect	0.98440	1.00000	1.00000
Frt-GH-row effect	0.98444	1.00000	1.00000
Frt-GH- interaction effect	0.00332	0.00192	0.00132

Column and row effect are present, and raise the Power for power method scenarios to the levels 6.4-6.8 times higher than alpha for 10 observations, 13.4-14 times higher for 25, and 18.4-19 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are from 2.6 to 20 times higher than alpha, but for interaction are 0. In contrast, for nonparametric scenarios values for column and row effects are 19.6-20 times higher than alpha, but for interaction are lower than alpha, but greater than 0.

Overall, in parametric setting, for the power method the relative rejection rates are 85.24% to 94.9% and for GH are from 63.21% to 95%. Thus, for 50 observations GH gives very close relative rejection. In RT setting, the relative rejection rates for the power method are 84.59%-94.6%, and 94.99%-95% for GH scenario, being very close for 50 observations.

Table 4.3.7 Simulated Power rates for size 0.5 effect Beta (4, 2) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	0.86732	0.99880	1.00000
F-row effect	0.86544	0.99820	1.00000
F-interaction effect	0.05276	0.05124	0.05224

F-GH- column effect	0.99932	1.00000	1.00000
F-GH- row effect	0.99932	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.84112	0.99748	1.00000
Frt- row effect	0.83904	0.99680	1.00000
Frt-interaction effect	0.04460	0.04284	0.04668
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 16.6-17.6 times higher than alpha for 10 observations, 19.8-19.98 times higher for 25, and 20 times higher for 50 observations. Also, for 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are from 19.98 to 20 times higher than alpha, but for interaction are 0. Besides, for nonparametric scenarios values for column and row effects are 20 times higher than alpha, but for interaction are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.22% to 95% and for GH are from 94.99 to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.04%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.8 Simulated Power rates for size 0.75 effect Beta (4, 2) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size

	10	25	50
F-column effect	0.99568	1.00000	1.00000
F-row effect	0.99564	1.00000	1.00000
F-interaction effect	0.05300	0.05052	0.05108
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.99228	1.00000	1.00000
Frt- row effect	0.99224	1.00000	1.00000
Frt-interaction effect	0.02444	0.02576	0.02692
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 19.8-19.9 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are 20 times higher than alpha, but for interaction are 0. Besides, for nonparametric scenarios values for column and row effects are 20 times higher than alpha, but for interaction are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.98% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.9 Simulated Power rates for size 1.0 effect Beta (4, 2) distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	1.00000	1.00000	1.00000
F-row effect	0.99996	1.00000	1.00000
F-interaction effect	0.05492	0.05344	0.05004
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00000	0.00000	0.00000
Frt- column effect	0.99992	1.00000	1.00000
Frt- row effect	0.99996	1.00000	1.00000
Frt-interaction effect	0.00544	0.00612	0.00612
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Column and row effect are present, and raise the Power for power method scenarios to the levels 19.98-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for column and row effects are 20 times higher than alpha, but for interaction are 0. Besides, for nonparametric scenarios values for column and row effects are 20 times higher than alpha, but for interaction are 0.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.10 Simulated Power rates for size 0.25 effect Beta (4, 2) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05616	0.05320	0.04928
F-row effect	0.05236	0.05204	0.05096
F-interaction effect	0.33764	0.69712	0.93888
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.13772	0.95160	1.00000
Frt- column effect	0.05544	0.05496	0.04872
Frt- row effect	0.05340	0.05152	0.05172
Frt-interaction effect	0.32656	0.67848	0.92820
Frt-GH-column effect	0.01496	0.01304	0.01380
Frt-GH-row effect	0.01448	0.01460	0.01556
Frt-GH- interaction effect	0.99504	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 6.4-6.8 times higher than alpha for 10 observations, 13.4-14 times higher for 25, and 18.4-18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 2.6-20 times higher than alpha, but for column and row are 0. Besides, for nonparametric scenarios values for interaction effects are 19.9-20 times higher than alpha, but for column and row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85.2% to 94.67% and for GH are from 63.69 to 95%. Thus, for 50 observations GH gives the almost same relative rejection. In RT setting, the relative rejection rates for the power method are 84.69%-94.6%, and 94.97%-95% for GH scenario, being almost the same for 50 observations.

Table 4.3.11 Simulated Power rates for size 0.5 effect Beta (4, 2) distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05088	0.05160	0.00506
F-row effect	0.05212	0.05168	0.00526
F-interaction effect	0.87156	0.99852	1.00000
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.99960	1.00000	1.00000
Frt- column effect	0.05196	0.05140	0.04880
Frt- row effect	0.05136	0.05236	0.05360

Frt-interaction effect	0.84936	0.99716	1.00000
Frt-GH-column effect	0.00084	0.00056	0.00056
Frt-GH-row effect	0.00116	0.00072	0.00052
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 16.8-17.3 times higher than alpha for 10 observations, 19.8-20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is above 0.99, which is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are 19.9-20 times higher than alpha, but for column and row are 0. Besides, for nonparametric scenarios values for interaction effects are 20 times higher than alpha, but for column and row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.26% to 95% and for GH are from 94.99 to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.11%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.12 Simulated Power rates for size 0.75 effect Beta (4, 2) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05348	0.05100	0.05468
F-row effect	0.05416	0.05128	0.04996
F-interaction effect	0.99544	1.00000	1.00000
F-GH- column effect	0.00000	0.00000	0.00000

F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05452	0.05024	0.05368
Frt- row effect	0.05484	0.05012	0.05096
Frt-interaction effect	0.99200	1.00000	1.00000
Frt-GH-column effect	0.00296	0.00220	0.00200
Frt-GH-row effect	0.00268	0.00252	0.00176
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 19.8-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 20 times higher than alpha, but for column and row are 0. Besides, for nonparametric scenarios values for interaction effects are 20 times higher than alpha, but for column and row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.98% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.13 Simulated Power rates for size 1.0 effect Beta (4, 2) distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50

F-column effect	0.05432	0.05336	0.05080
F-row effect	0.05296	0.05280	0.05116
F-interaction effect	0.99996	1.00000	1.00000
F-GH- column effect	0.00000	0.00000	0.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05308	0.05308	0.05124
Frt- row effect	0.05272	0.05240	0.05192
Frt-interaction effect	0.99988	1.00000	1.00000
Frt-GH-column effect	0.00844	0.00824	0.00728
Frt-GH-row effect	0.00972	0.00840	0.00740
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the levels 19.9-20 times higher than alpha for 10 observations, 20 times higher for 25, and 20 times higher for 50 observations. Also, for 25 and 50 observations, the rate is 1.0, which is the highest. It should be noted that, for GH scenarios the parametric values for interaction effects are 20 times higher than alpha, but for column and row are 0. Besides, for nonparametric scenarios values for interaction effects are 20 times higher than alpha, but for column and row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.3.14 Simulated Power rates for size 0.25 effect Beta (4, 2) distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.12696	0.23432	0.41960
F-row effect	0.05236	0.05204	0.05096
F-interaction effect	0.12556	0.24048	0.41712
F-GH- column effect	0.00060	0.01208	0.26064
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.05200	0.01116	0.25712
Frt- column effect	0.12240	0.22480	0.40140
Frt- row effect	0.05344	0.05216	0.05164
Frt-interaction effect	0.12412	0.23232	0.40460
Frt-GH-column effect	0.59148	0.95844	0.99964
Frt-GH-row effect	0.02548	0.00256	0.00268
Frt-GH- interaction effect	0.59504	0.95896	0.99964

Interaction and column effects are present, and raise the Power for power method scenarios to the levels 2.4-2.6 times higher than alpha for 10 observations, 4.4-4.8 times higher for 25, and 8-8.4 times higher for 50 observations. Also, for 50 observations, the rate is just above 0.4, which is not very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are up to 5.2 times higher than alpha, but for row is 0. Besides, for nonparametric scenarios values for interaction and column effects are 11.8-19.9 times higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 60.2% to 88.08% and for GH are from negative to 80.82%. Thus, for 50 observations GH gives close relative rejection. In RT setting, the relative rejection rates for the power method are 59.2%-87.64%, and from 91.55% to 94.99% for GH scenario, being close for 50 observations.

Table 4.3.15 Simulated Power rates for size 0.5 effect Beta (4, 2) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.33436	0.69908	0.93976
F-row effect	0.05212	0.05168	0.05260
F-interaction effect	0.34004	0.69452	0.93988
F-GH- column effect	0.13332	0.95012	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.13212	0.95076	1.00000
Frt- column effect	0.32028	0.67248	0.92616
Frt- row effect	0.05088	0.05096	0.05104
Frt-interaction effect	0.32724	0.66884	0.92696
Frt-GH-column effect	0.98400	1.00000	1.00000
Frt-GH-row effect	0.00344	0.00168	0.00148
Frt-GH- interaction effect	0.98412	1.00000	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the levels 6.4-6.8 times higher than alpha for 10 observations, 13.2-14 times higher for 25, and 18.5-18.8 times higher for 50 observations. Also, for 50 observations, the rate is above 0.9, which is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are 2.6-20 times higher than alpha, but for row is 0. Besides, for nonparametric scenarios values for interaction and column effects are 19.6-20 times higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 58% to 94.6% and from 62% to 95% for GH. Thus, for 10 observations GH gives slightly more rejection, whereas for 50 observations they are approximately the same. In RT setting, the relative rejection rates for GH scenario are 94.9-95%, and 84-94.6% for the power method, being almost the same for 50 observations.

Table 4.3.16 Simulated Power rates for size 0.75 effect Beta (4, 2) distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.63464	0.96000	0.99964
F-row effect	0.05416	0.05128	0.04996
F-interaction effect	0.63868	0.96056	0.99956
F-GH- column effect	0.85168	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000
F-GH- interaction effect	0.85588	1.00000	1.00000
Frt- column effect	0.60492	0.94828	0.99912
Frt- row effect	0.05132	0.04800	0.04684
Frt-interaction effect	0.60740	0.94684	0.99924
Frt-GH-column effect	0.99996	1.00000	1.00000

Frt-GH-row effect	0.00004	0.00000	0.00000
Frt-GH- interaction effect	0.99996	1.00000	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the levels 12-12.8 times higher than alpha for 10 observations, 18.8-19.2 times higher for 25, and 19.98-20 times higher for 50 observations. Also, for 50 observations, the rate is above 0.999, which is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are 17-20 times higher than alpha, but for row is 0. Besides, for nonparametric scenarios values for interaction and column effects are 19.98-20 times higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 95%, and for GH are from 94% to 95%. Thus, for 10 observations GH gives slightly more rejection, whereas for 50 observations they are the same. In RT setting, the relative rejection rates for GH scenario are 95%, and 91.6-95% for the power method, being almost the same for 50 observations.

Table 4.3.17 Simulated Power rates for size 1.0 effect Beta (4, 2) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.86552	0.99856	1.00000
F-row effect	0.05296	0.05280	0.05116
F-interaction effect	0.86432	0.99872	1.00000
F-GH- column effect	0.99904	1.00000	1.00000
F-GH- row effect	0.00000	0.00000	0.00000

F-GH- interaction effect	0.99924	1.00000	1.00000
Frt- column effect	0.83712	0.99724	0.99996
Frt- row effect	0.04368	0.04444	0.04460
Frt-interaction effect	0.83704	0.99752	1.00000
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.00000	0.00000	0.00000
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row is 0. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 94% to 95%, and for GH are 95%. Thus, for 10 observations GH gives slightly more rejection, whereas for 50 observations they are the same. In RT setting, the relative rejection rates for the power method are 94-95%, and 95% for GH scenario, being almost the same for 50 observations.

Table 4.3.18 Simulated Power rates for size 0.25 effect Beta (4, 2) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.07296	0.09564	0.14252
F-row effect	0.07028	0.09464	0.14048

F-interaction effect	0.07288	0.09856	0.14240
F-GH- column effect	0.00000	0.00000	0.00008
F-GH- row effect	0.00008	0.00000	0.00016
F-GH- interaction effect	0.00004	0.00000	0.00020
Frt- column effect	0.07236	0.09476	0.13960
Frt- row effect	0.06980	0.09316	0.13764
Frt-interaction effect	0.07356	0.09812	0.13896
Frt-GH-column effect	0.19464	0.46948	0.76972
Frt-GH-row effect	0.19448	0.46404	0.77168
Frt-GH- interaction effect	0.19332	0.46304	0.77576

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are lower than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 28% to 64%, and for GH are negative or non-interpretable. In RT setting, the relative rejection rates for the power method are 30-63%, and 74-93.5% for GH scenario.

Table 4.3.19 Simulated Power rates for size 0.5 effect Beta (4, 2) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size

	10	25	50
F-column effect	0.11956	0.23372	0.41808
F-row effect	0.12100	0.23640	0.42092
F-interaction effect	0.12276	0.23780	0.42188
F-GH- column effect	0.00052	0.01248	0.25996
F-GH- row effect	0.00084	0.01292	0.26360
F-GH- interaction effect	0.00084	0.01244	0.25736
Frt- column effect	0.11676	0.22492	0.39908
Frt- row effect	0.11884	0.22696	0.40240
Frt-interaction effect	0.12100	0.23152	0.40384
Frt-GH-column effect	0.50188	0.93032	0.99896
Frt-GH-row effect	0.50908	0.93016	0.99836
Frt-GH- interaction effect	0.49872	0.92916	0.99884

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are lower than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 58% to 88%, and for GH are from negative to 80%. Thus, for 50 observations GH gives slightly less rejection. In RT setting, the relative rejection rates for the power method are 58-87%, and 90-95% for GH scenario.

Table 4.3.20 Simulated Power rates for size 0.75 effect Beta (4, 2) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.21316	0.46004	0.74280
F-row effect	0.21292	0.45600	0.74928
F-interaction effect	0.21644	0.45512	0.75272
F-GH- column effect	0.01532	0.38040	0.98648
F-GH- row effect	0.01392	0.38060	0.98712
F-GH- interaction effect	0.01384	0.37968	0.98716
Frt- column effect	0.20436	0.43696	0.72168
Frt- row effect	0.21020	0.43632	0.72012
Frt-interaction effect	0.20848	0.43544	0.72864
Frt-GH-column effect	0.80344	0.99932	1.00000
Frt-GH-row effect	0.79972	0.99900	1.00000
Frt-GH- interaction effect	0.80388	0.99928	1.00000

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is high. It should be noted that, for GH scenarios the parametric values for row and column effects are not necessarily lower than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 76% to 93%, and for GH are from negative to 94.8%. Thus, for 50 observations GH gives slightly higher rejection. In RT setting, the relative rejection rates for the power method are 80-93%, and 93-95% for GH scenario, being almost the same for 50 observations.

Table 4.3.21 Simulated Power rates for size 1.0 effect Beta (4, 2) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.33896	0.69008	0.93988
F-row effect	0.33536	0.69432	0.94112
F-interaction effect	0.34052	0.69468	0.94156
F-GH- column effect	0.13636	0.95228	1.00000
F-GH- row effect	0.13752	0.95096	1.00000
F-GH- interaction effect	0.13552	0.95044	1.00000
Frt- column effect	0.32448	0.65784	0.92488
Frt- row effect	0.31736	0.66092	0.92288
Frt-interaction effect	0.32464	0.66436	0.92416
Frt-GH-column effect	0.89884	0.99996	1.00000
Frt-GH-row effect	0.90160	1.00000	1.00000
Frt-GH- interaction effect	0.90004	0.99988	1.00000

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 84% to 94.6%, and for GH are from 62% to 95%. Thus, for 50 observations GH gives slightly higher rejection. In RT setting, the relative rejection rates for the power method are 84-94.5%, and 94.4-95% for GH scenario, being almost the same for 50 observations.

4.4. Triangular distribution.

Table 4.4.1 Simulated Type I error rates for size 0 effect Triangular distribution, pattern "All effects null".

Statistic	Sample size		
	10	25	50
F-column effect	0.05144	0.05108	0.05076
F-row effect	0.05264	0.05120	0.05212
F-interaction effect	0.05340	0.05368	0.05188
F-GH- column effect	0.03832	0.03744	0.03748
F-GH- row effect	0.03900	0.03756	0.03788
F-GH- interaction effect	0.03648	0.03408	0.03640
Frt- column effect	0.05152	0.05068	0.05108
Frt- row effect	0.05224	0.05272	0.05128
Frt-interaction effect	0.05308	0.05352	0.05076
Frt-GH-column effect	0.04548	0.04868	0.04920

Frt-GH-row effect	0.04680	0.04900	0.05128
Frt-GH- interaction effect	0.04500	0.04624	0.04816

The rates are close to 0.05, which is in line with the theory. Only the GH parametric scenario gives lower rates in all cases .

It should be also noted that F-GH nonparametric scenarios in most cases are slightly lower than 0.05. In contrast, for power method cases the values are higher than 0.05.

In general, size 0 effect rates confirm the theory. Now, we need to look at treatment pattern by the effect size for Triangular distribution.

Table 4.4.2 Simulated Power rates for size 0.25 effect Triangular distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.12728	0.23356	0.41588
F-row effect	0.05272	0.05168	0.05132
F-interaction effect	0.05728	0.05264	0.05152
F-GH- column effect	0.10752	0.22700	0.41672
F-GH- row effect	0.03860	0.03864	0.03656
F-GH- interaction effect	0.03760	0.03664	0.03632
Frt- column effect	0.11928	0.21164	0.37736
Frt- row effect	0.05360	0.05284	0.05124
Frt-interaction effect	0.05668	0.05208	0.42940
Frt-GH-column effect	0.11356	0.24216	0.48440
Frt-GH-row effect	0.04652	0.04984	0.04840

Frt-GH- interaction effect	0.04468	0.04616	0.04740

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 58% to 88%, and for GH are from 50% to 88%. Thus, for 50 observations GH gives the same rejection. In RT setting, the relative rejection rates for the power method are 58-87%, and 56-90% for GH scenario, being almost the same for 10 observations.

Table 4.4.3 Simulated Power rates for size 0.5 effect Triangular distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.33360	0.69816	0.94056
F-row effect	0.05224	0.05148	0.05256
F-interaction effect	0.05328	0.05208	0.05192
F-GH- column effect	0.32916	0.71020	0.95068
F-GH- row effect	0.04032	0.03648	0.03748
F-GH- interaction effect	0.04064	0.03736	0.03604
Frt- column effect	0.30696	0.64932	0.91060
Frt- row effect	0.05288	0.05172	0.05244

Frt-interaction effect	0.05276	0.05240	0.05284
Frt-GH-column effect	0.34196	0.71708	0.94956
Frt-GH-row effect	0.04556	0.04636	0.04808
Frt-GH- interaction effect	0.04684	0.04792	0.04700

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.7%, and for GH are from 84.8% to 94.7%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.7-94.5%, and 85.3-94.7% for GH scenario, being almost the same for 50 observations.

Table 4.4.4 Simulated Power rates for size 0.75 effect Triangular distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.63232	0.96016	0.99972
F-row effect	0.05460	0.05220	0.04988
F-interaction effect	0.05304	0.05216	0.05100
F-GH- column effect	0.64744	0.96852	0.99996
F-GH- row effect	0.03756	0.03668	0.03744

F-GH- interaction effect	0.03752	0.03688	0.03632
Frt- column effect	0.58796	0.93824	0.99888
Frt- row effect	0.05568	0.05116	0.04944
Frt-interaction effect	0.05296	0.05108	0.05036
Frt-GH-column effect	0.66812	0.97128	0.99992
Frt-GH-row effect	0.04600	0.04616	0.04864
Frt-GH- interaction effect	0.04548	0.04644	0.04760

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 95%, and for GH are from 92.2% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.5-95%, and 92.5-95% for GH scenario, being almost the same for 50 observations.

Table 4.4.5 Simulated Power rates for size 1.0 effect Triangular distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86648	0.99820	1.00000
F-row effect	0.05356	0.05236	0.05172

F-interaction effect	0.05624	0.05420	0.05080
F-GH- column effect	0.88100	0.99928	1.00000
F-GH- row effect	0.03808	0.03832	0.03640
F-GH- interaction effect	0.04168	0.03596	0.03640
Frt- column effect	0.82856	0.99612	0.99996
Frt- row effect	0.05416	0.05172	0.05160
Frt-interaction effect	0.05040	0.05368	0.05068
Frt-GH-column effect	0.90108	0.99924	1.00000
Frt-GH-row effect	0.04728	0.04720	0.04536
Frt-GH- interaction effect	0.04848	0.04388	0.04708

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95%, and for GH are from 94.3% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.9-95%, and 94.4-95% for GH scenario, being the same for 50 observations.

Table 4.4.6 Simulated Power rates for size 0.25 effect Triangular distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size

	10	25	50
F-column effect	0.33760	0.70096	0.94132
F-row effect	0.33580	0.69600	0.94100
F-interaction effect	0.05728	0.05264	0.05152
F-GH- column effect	0.32912	0.70468	0.95192
F-GH- row effect	0.32956	0.70072	0.95316
F-GH- interaction effect	0.03760	0.03664	0.03632
Frt- column effect	0.31056	0.64864	0.91276
Frt- row effect	0.31020	0.64644	0.91340
Frt-interaction effect	0.05552	0.05340	0.05120
Frt-GH-column effect	0.33872	0.70720	0.94992
Frt-GH-row effect	0.33848	0.70720	0.95000
Frt-GH- interaction effect	0.04364	0.04420	0.04580

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85.1% to 94.7%, and for GH are from 84.8% to 94.7%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.9-94.5%, and 85.2-94.7% for GH scenario, being almost the same for 50 observations.

Table 4.4.7 Simulated Power rates for size 0.5 effect Triangular distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86936	0.99884	1.0000
F-row effect	0.86664	0.99844	1.0000
F-interaction effect	0.05328	0.05208	0.05192
F-GH- column effect	0.88164	0.99924	1.0000
F-GH- row effect	0.87964	0.99936	1.0000
F-GH- interaction effect	0.04064	0.03736	0.03604
Frt- column effect	0.82988	0.99704	1.0000
Frt- row effect	0.82888	0.99664	1.0000
Frt-interaction effect	0.04408	0.04400	0.04600
Frt-GH-column effect	0.88944	0.99928	1.0000
Frt-GH-row effect	0.88760	0.99932	1.0000
Frt-GH- interaction effect	0.03980	0.03804	0.03520

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95%, and for GH are from 94.3% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94-95%, and 94.3-95% for GH scenario, being the same for 50 observations.

Table 4.4.8 Simulated Power rates for size 0.75 effect Triangular distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.99620	1.00000	1.00000
F-row effect	0.99604	1.00000	1.00000
F-interaction effect	0.05304	0.05216	0.05100
F-GH- column effect	0.99760	1.00000	1.00000
F-GH- row effect	0.99764	1.00000	1.00000
F-GH- interaction effect	0.03752	0.03688	0.03632
Frt- column effect	0.99228	0.99704	1.00000
Frt- row effect	0.99192	0.99664	1.00000
Frt-interaction effect	0.02436	0.02616	0.02708
Frt-GH-column effect	0.99804	0.99928	1.00000
Frt-GH-row effect	0.99820	0.99932	1.00000
Frt-GH- interaction effect	0.02192	0.01868	0.01760

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row

effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.9% to 95%, and for GH are from 94.9% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.9-95%, and 94.9-95% for GH scenario, being the same for 50 observations.

Table 4.4.9 Simulated Power rates for size 1.0 effect Triangular distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	1.00000	1.00000	1.00000
F-row effect	0.99996	1.00000	1.00000
F-interaction effect	0.05624	0.05420	0.05080
F-GH- column effect	0.99996	1.00000	1.00000
F-GH- row effect	0.99996	1.00000	1.00000
F-GH- interaction effect	0.04168	0.03596	0.03640
Frt- column effect	1.00000	1.00000	1.00000
Frt- row effect	0.99996	1.00000	1.00000
Frt-interaction effect	0.00564	0.00488	0.00532
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	0.99996	1.00000	1.00000
Frt-GH- interaction effect	0.00676	0.00416	0.00296

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95%, and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99-95%, and 94.99-95% for GH scenario, being the same for 50 observations.

Table 4.4.10 Simulated Power rates for size 0.25 effect Triangular distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05560	0.05296	0.04884
F-row effect	0.05272	0.05168	0.05132
F-interaction effect	0.33656	0.69752	0.93884
F-GH- column effect	0.03800	0.03564	0.03620
F-GH- row effect	0.03860	0.03864	0.03656
F-GH- interaction effect	0.32940	0.70804	0.95100
Frt- column effect	0.05524	0.05468	0.04828
Frt- row effect	0.05396	0.05204	0.05176
Frt-interaction effect	0.30880	0.64908	0.90960

Frt-GH-column effect	0.04608	0.04696	0.04808
Frt-GH-row effect	0.04616	0.04928	0.04868
Frt-GH- interaction effect	0.34256	0.71680	0.94964

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are slightly lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.7%, and for GH are from 84.8% to 94.7%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.8-94.5%, and 85.4-94.7% for GH scenario, being almost the same for 50 observations.

Table 4.4.11 Simulated Power rates for size 0.5 effect Triangular distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05104	0.05104	0.04948
F-row effect	0.05224	0.05148	0.05256
F-interaction effect	0.87040	0.99844	1.00000
F-GH- column effect	0.03904	0.03532	0.03664
F-GH- row effect	0.04032	0.03648	0.03748
F-GH- interaction effect	0.88420	0.99952	1.00000

Frt- column effect	0.05168	0.05096	0.04940
Frt- row effect	0.05088	0.05216	0.05332
Frt-interaction effect	0.83388	0.99600	1.00000
Frt-GH-column effect	0.04684	0.04384	0.04576
Frt-GH-row effect	0.04688	0.04500	0.04728
Frt-GH- interaction effect	0.90484	0.99940	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95%, and for GH are from 94.3% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94-95%, and 94.5-95% for GH scenario, being almost the same for 50 observations.

Table 4.4.12 Simulated Power rates for size 0.75 effect Triangular distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05340	0.05100	0.05452
F-row effect	0.05460	0.05220	0.04988
F-interaction effect	0.99628	1.00000	1.00000

F-GH- column effect	0.03916	0.03652	0.03816
F-GH- row effect	0.03756	0.03668	0.03744
F-GH- interaction effect	0.99768	1.00000	1.00000
Frt- column effect	0.05412	0.05104	0.05316
Frt- row effect	0.05568	0.05180	0.05092
Frt-interaction effect	0.99176	1.00000	1.00000
Frt-GH-column effect	0.04636	0.04432	0.04524
Frt-GH-row effect	0.04380	0.04416	0.04332
Frt-GH- interaction effect	0.99912	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.9% to 95%, and for GH are from 94.9% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.9-95%, and 94.9-95% for GH scenario, being the same for 50 observations.

Table 4.4.13 Simulated Power rates for size 1.0 effect Triangular distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50

F-column effect	0.05572	0.05200	0.04980
F-row effect	0.05356	0.05236	0.05172
F-interaction effect	0.99996	1.00000	1.00000
F-GH- column effect	0.04008	0.03760	0.03688
F-GH- row effect	0.03808	0.03832	0.03640
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05356	0.05292	0.05036
Frt- row effect	0.05416	0.05208	0.05204
Frt-interaction effect	0.99992	1.00000	1.00000
Frt-GH-column effect	0.04444	0.04308	0.04164
Frt-GH-row effect	0.04352	0.04216	0.04200
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95%, and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99-95%, and 94.99-95% for GH scenario, being the same for 50 observations.

Table 4.4.14 Simulated Power rates for size 0.25 effect Triangular distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.12728	0.23356	0.41588
F-row effect	0.05272	0.05168	0.05132
F-interaction effect	0.12532	0.23884	0.41924
F-GH- column effect	0.10752	0.22700	0.41672
F-GH- row effect	0.03860	0.03864	0.03656
F-GH- interaction effect	0.10724	0.22320	0.41228
Frt- column effect	0.11896	0.21196	0.37708
Frt- row effect	0.05340	0.05240	0.05160
Frt-interaction effect	0.11976	0.21968	0.37912
Frt-GH-column effect	0.11376	0.24172	0.42936
Frt-GH-row effect	0.04584	0.04948	0.04884
Frt-GH- interaction effect	0.11156	0.23824	0.42716

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 60% to 88%, and for GH are from 53% to 88%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 58-86.8%, and 55-88.3% for GH scenario, being almost the same for 50 observations.

Table 4.4.15 Simulated Power rates for size 0.5 effect Triangular distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.33360	0.69816	0.94056
F-row effect	0.05224	0.05148	0.05256
F-interaction effect	0.33868	0.69208	0.93928
F-GH- column effect	0.32916	0.71020	0.95068
F-GH- row effect	0.04032	0.03648	0.03748
F-GH- interaction effect	0.32928	0.71024	0.95200
Frt- column effect	0.30732	0.64940	0.91188
Frt- row effect	0.05172	0.05112	0.05216
Frt-interaction effect	0.31224	0.64388	0.91236
Frt-GH-column effect	0.34028	0.71380	0.94904
Frt-GH-row effect	0.04532	0.04656	0.04752
Frt-GH- interaction effect	0.33664	0.71484	0.94972

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction

and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.6%, and for GH are from 84.8% to 94.7%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.7-94.3%, and 85-94.7% for GH scenario, being almost the same for 50 observations.

Table 4.4.16 Simulated Power rates for size 0.75 effect Triangular distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.63232	0.96016	0.99972
F-row effect	0.05460	0.05220	0.04988
F-interaction effect	0.63812	0.96056	0.99960
F-GH- column effect	0.64744	0.96852	0.99996
F-GH- row effect	0.03756	0.03668	0.03744
F-GH- interaction effect	0.64444	0.96668	0.99984
Frt- column effect	0.58532	0.93988	0.99880
Frt- row effect	0.05192	0.04784	0.04744
Frt-interaction effect	0.59052	0.93844	0.99904
Frt-GH-column effect	0.65612	0.96860	0.99996
Frt-GH-row effect	0.04116	0.04240	0.04448
Frt-GH- interaction effect	0.65612	0.96684	0.99972

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 94.99%, and for GH are from 92.2% to 94.99%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.4-94.99%, and 93.9-94.99% for GH scenario, being the same for 50 observations.

Table 4.4.17 Simulated Power rates for size 1.0 effect Triangular distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.86648	0.99820	1.00000
F-row effect	0.05356	0.05236	0.05172
F-interaction effect	0.86584	0.99860	1.00000
F-GH- column effect	0.88100	0.99928	1.00000
F-GH- row effect	0.03808	0.03832	0.03640
F-GH- interaction effect	0.88296	0.99940	1.00000
Frt- column effect	0.82756	0.99644	0.99996
Frt- row effect	0.04548	0.04416	0.04552
Frt-interaction effect	0.82532	0.99736	1.00000

Frt-GH-column effect	0.88608	0.99908	1.00000
Frt-GH-row effect	0.03816	0.03812	0.03692
Frt-GH- interaction effect	0.88704	0.99920	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95%, and for GH are from 94.3% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.9-94.99%, and 94.3-95% for GH scenario, being the same for 50 observations.

Table 4.4.18 Simulated Power rates for size 0.25 effect Triangular distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.07348	0.09544	0.14184
F-row effect	0.06932	0.09556	0.14084
F-interaction effect	0.07252	0.09872	0.14328
F-GH- column effect	0.05456	0.07960	0.12836
F-GH- row effect	0.05644	0.08352	0.12044
F-GH- interaction effect	0.05448	0.08312	0.12520

Frt- column effect	0.07112	0.09184	0.13208
Frt- row effect	0.06796	0.09008	0.13076
Frt-interaction effect	0.07164	0.09468	0.13188
Frt-GH-column effect	0.06016	0.09300	0.14508
Frt-GH-row effect	0.06328	0.09800	0. 13652
Frt-GH- interaction effect	0.05984	0.09500	0.14080

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 27.5% to 65%, and for GH are from 0% to 60.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 26.5%-62.1%, and 16.6%-65.5% for GH scenario, being almost the same for 50 observations.

Table 4.4.19 Simulated Power rates for size 0.5 effect Triangular distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.11920	0.23276	0.41808
F-row effect	0.11972	0.23424	0.42016

F-interaction effect	0.12360	0.23752	0.42288
F-GH- column effect	0.10368	0.22064	0.41700
F-GH- row effect	0.10584	0.21780	0.41292
F-GH- interaction effect	0.10464	0.22460	0.41236
Frt- column effect	0.11184	0.21228	0.37912
Frt- row effect	0.11412	0.21348	0.37916
Frt-interaction effect	0.11628	0.21920	0.38232
Frt-GH-column effect	0.11156	0.23708	0.42792
Frt-GH-row effect	0.11468	0.23456	0.42756
Frt-GH- interaction effect	0.11244	0.23884	0.42436

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 58.3% to 88%, and for GH are from 51.4% to 87.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 56.1%-86.8%, and 55%-88.3% for GH scenario, being almost the same for 50 observations.

Table 4.4.20 Simulated Power rates for size 0.75 effect Triangular distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.21096	0.45940	0.74420
F-row effect	0.21440	0.45808	0.74740
F-interaction effect	0.21568	0.45508	0.75336
F-GH- column effect	0.20476	0.45324	0.76380
F-GH- row effect	0.19964	0.45360	0.76656
F-GH- interaction effect	0.19948	0.45116	0.76396
Frt- column effect	0.19416	0.41456	0.69576
Frt- row effect	0.19900	0.41628	0.69540
Frt-interaction effect	0.19932	0.41424	0.70404
Frt-GH-column effect	0.21132	0.46552	0.76416
Frt-GH-row effect	0.20640	0.45924	0.76756
Frt-GH- interaction effect	0.20544	0.46220	0.76352

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 76.3% to 93.3%, and for GH are from 74.9% to 92.2%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 74.2%-92.9%, and 75.6%-93.5% for GH scenario, being almost the same for 50 observations.

Table 4.4.21 Simulated Power rates for size 1.0 effect Triangular distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33736	0.69100	0.93976
F-row effect	0.33332	0.69276	0.94212
F-interaction effect	0.33892	0.69356	0.94276
F-GH- column effect	0.33080	0.71024	0.95036
F-GH- row effect	0.33524	0.71148	0.95080
F-GH- interaction effect	0.32996	0.70756	0.95344
Frt- column effect	0.31116	0.64084	0.91288
Frt- row effect	0.30240	0.64260	0.91312
Frt-interaction effect	0.31116	0.64252	0.91428
Frt-GH-column effect	0.33364	0.70720	0.94520
Frt-GH-row effect	0.33972	0.70860	0.94740
Frt-GH- interaction effect	0.33464	0.70772	0.94816

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 76.3% to 93.3%, and for GH are from 74.9% to 92.2%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 74.2%-92.9%, and 75.6%-93.5% for GH scenario, being almost the same for 50 observations.

4.5. Uniform distribution.

Table 4.5.1 Simulated Type I error rates for size 0 effect Uniform distribution, pattern “All effects null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05144	0.05196	0.05160
F-row effect	0.05260	0.05308	0.05228
F-interaction effect	0.05236	0.05280	0.05060
F-GH- column effect	0.01008	0.01012	0.00968
F-GH- row effect	0.00996	0.00912	0.01064
F-GH- interaction effect	0.00892	0.00904	0.00980
Frt- column effect	0.05256	0.05124	0.05076
Frt- row effect	0.05264	0.05296	0.05184
Frt-interaction effect	0.05216	0.05280	0.05064
Frt-GH-column effect	0.04552	0.04904	0.04944
Frt-GH-row effect	0.04644	0.04876	0.05144
Frt-GH- interaction effect	0.04468	0.04668	0.04780

The rates are close to 0.05, which is in line with the theory. Only the GH parametric scenario gives lower rates in all cases.

It should be also noted that F-GH nonparametric scenarios in most cases are slightly lower than 0.05. In contrast, for power method cases the values are higher than 0.05.

In general, size 0 effect rates confirm the theory. Now, we need to look at treatment pattern by the effect size for Uniform distribution.

Table 4.5.2 Simulated Power rates for size 0.25 effect Uniform distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.12356	0.23188	0.41328
F-row effect	0.05392	0.05180	0.05024
F-interaction effect	0.05716	0.05208	0.05168
F-GH- column effect	0.06060	0.17548	0.39556
F-GH- row effect	0.01016	0.00944	0.00916
F-GH- interaction effect	0.01052	0.00928	0.01000
Frt- column effect	0.12268	0.22640	0.40088
Frt- row effect	0.05412	0.05176	0.05040
Frt-interaction effect	0.05664	0.05140	0.05236
Frt-GH-column effect	0.15408	0.34808	0.60484
Frt-GH-row effect	0.04548	0.04916	0.04792
Frt-GH- interaction effect	0.04440	0.04472	0.04780

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 59.3% to 87.9%, and for GH are from 20% to 87.3%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 59%-87.5%, and 67.5%-91.7% for GH scenario, being close for 50 observations.

Table 4.5.3 Simulated Power rates for size 0.5 effect Uniform distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.32696	0.69648	0.93944
F-row effect	0.05312	0.05084	0.05264
F-interaction effect	0.05312	0.05248	0.05212
F-GH- column effect	0.29256	0.75268	0.97964
F-GH- row effect	0.01128	0.01016	0.01032
F-GH- interaction effect	0.01060	0.00956	0.00848
Frt- column effect	0.30816	0.64876	0.91288
Frt- row effect	0.05348	0.05160	0.05276
Frt-interaction effect	0.05348	0.05108	0.05244

Frt-GH-column effect	0.46996	0.86844	0.99116
Frt-GH-row effect	0.04320	0.04324	0.04588
Frt-GH- interaction effect	0.04392	0.04452	0.04344

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.7% to 94.7%, and for GH are from 82.9% to 94.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.8%-94.5%, and 89.3%-94.95% for GH scenario, being close for 50 observations.

Table 4.5.4 Simulated Power rates for size 0.75 effect Uniform distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.62556	0.96280	0.99972
F-row effect	0.05568	0.05036	0.05008
F-interaction effect	0.05316	0.05012	0.04992
F-GH- column effect	0.68092	0.98984	1.00000
F-GH- row effect	0.00952	0.00952	0.01016
F-GH- interaction effect	0.00992	0.00952	0.01008

Frt- column effect	0.57140	0.93144	0.99824
Frt- row effect	0.05532	0.05028	0.04972
Frt-interaction effect	0.05236	0.05132	0.04960
Frt-GH-column effect	0.81512	0.99552	1.00000
Frt-GH-row effect	0.03928	0.03928	0.04184
Frt-GH- interaction effect	0.03936	0.04048	0.04012

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 94.99%, and for GH are from 92.7% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.2%-94.99%, and 93.86%-95% for GH scenario, being close for 50 observations.

Table 4.5.5 Simulated Power rates for size 1.0 effect Uniform distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86688	0.99876	1.00000
F-row effect	0.05420	0.05296	0.05208
F-interaction effect	0.05632	0.05284	0.05000

F-GH- column effect	0.92744	0.99996	1.00000
F-GH- row effect	0.01040	0.01004	0.00876
F-GH- interaction effect	0.01120	0.00908	0.00976
Frt- column effect	0.80920	0.99404	0.99992
Frt- row effect	0.05404	0.05232	0.05212
Frt-interaction effect	0.05536	0.05436	0.04984
Frt-GH-column effect	0.96824	1.00000	1.00000
Frt-GH-row effect	0.03724	0.03680	0.03456
Frt-GH- interaction effect	0.03820	0.03408	0.03624

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.2% to 95%, and for GH are from 94.6% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.8%-94.99%, and 94.83%-95% for GH scenario, being close for 50 observations.

Table 4.5.6 Simulated Power rates for size 0.25 effect Uniform distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50

F-column effect	0.33320	0.69720	0.94276
F-row effect	0.33016	0.69292	0.94172
F-interaction effect	0.05716	0.05208	0.05168
F-GH- column effect	0.29392	0.74784	0.98084
F-GH- row effect	0.29492	0.74676	0.98028
F-GH- interaction effect	0.01052	0.00928	0.01000
Frt- column effect	0.30476	0.63756	0.90724
Frt- row effect	0.30284	0.63684	0.90684
Frt-interaction effect	0.05556	0.05112	0.05132
Frt-GH-column effect	0.44976	0.84956	0.99068
Frt-GH-row effect	0.44384	0.84740	0.98980
Frt-GH- interaction effect	0.03920	0.03796	0.04024

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.8% to 94.7%, and for GH are from 83% to 94.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.4%-94.48%, and 88.7%-94.94% for GH scenario, being close for 50 observations.

Table 4.5.7 Simulated Power rates for size 0.5 effect Uniform distribution, pattern “Two main effects present, and Interaction is null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.87080	0.99864	1.00000
F-row effect	0.87088	0.99872	1.00000
F-interaction effect	0.05312	0.05248	0.05212
F-GH- column effect	0.93084	1.00000	1.00000
F-GH- row effect	0.92976	1.00000	1.00000
F-GH- interaction effect	0.01060	0.00956	0.00848
Frt- column effect	0.81064	0.99552	1.00000
Frt- row effect	0.80968	0.99500	1.00000
Frt-interaction effect	0.04524	0.04540	0.04632
Frt-GH-column effect	0.95780	1.00000	1.00000
Frt-GH-row effect	0.95744	0.99996	1.00000
Frt-GH- interaction effect	0.02292	0.01880	0.01864

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.25% to 95%, and for GH are from 94.6% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.83%-95%, and 94.78%-95% for GH scenario, being the same for 50 observations.

Table 4.5.8 Simulated Power rates for size 0.75 effect Uniform distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	0.99696	1.00000	1.00000
F-row effect	0.99672	1.00000	1.00000
F-interaction effect	0.05316	0.05012	0.04992
F-GH- column effect	0.99972	1.00000	1.00000
F-GH- row effect	0.99984	1.00000	1.00000
F-GH- interaction effect	0.00992	0.00952	0.01008
Frt- column effect	0.99208	1.00000	1.00000
Frt- row effect	0.99168	1.00000	1.00000
Frt-interaction effect	0.02784	0.02696	0.02896
Frt-GH-column effect	0.99988	1.00000	1.00000
Frt-GH-row effect	0.99984	1.00000	1.00000
Frt-GH- interaction effect	0.00456	0.00224	0.00164

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row

effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.98% to 95%, and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 94.99%-95% for GH scenario, being the same for 50 observations.

Table 4.5.9 Simulated Power rates for size 1.0 effect Uniform distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	1.00000	1.00000	1.00000
F-row effect	1.00000	1.00000	1.00000
F-interaction effect	0.05632	0.05284	0.05000
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.01120	0.00908	0.00976
Frt- column effect	0.99996	1.00000	1.00000
Frt- row effect	0.99984	1.00000	1.00000
Frt-interaction effect	0.00652	0.00660	0.00572
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00004	0.00000	0.00000

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 95%, and for GH are 95%. Thus, for all observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.5.10 Simulated Power rates for size 0.25 effect Uniform distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05552	0.05436	0.04864
F-row effect	0.05392	0.05180	0.05024
F-interaction effect	0.33300	0.69620	0.93948
F-GH- column effect	0.01048	0.00956	0.00920
F-GH- row effect	0.01016	0.00944	0.00916
F-GH- interaction effect	0.29236	0.75328	0.97884
Frt- column effect	0.05556	0.05428	0.04824
Frt- row effect	0.05304	0.05236	0.05040

Frt-interaction effect	0.31224	0.65204	0.91180
Frt-GH-column effect	0.04284	0.04376	0.04516
Frt-GH-row effect	0.04312	0.04660	0.04680
Frt-GH- interaction effect	0.46760	0.86608	0.99144

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are slightly lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 85% to 94.7%, and for GH are from 82.9% to 94.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84%-94.5%, and 89.3%-94.95% for GH scenario, being the almost the same for 50 observations.

Table 4.5.11 Simulated Power rates for size 0.5 effect Uniform distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05044	0.05000	0.05012
F-row effect	0.05312	0.05084	0.05264
F-interaction effect	0.87296	0.99864	1.00000
F-GH- column effect	0.01080	0.00872	0.00900
F-GH- row effect	0.01128	0.01016	0.01032

F-GH- interaction effect	0.93096	0.99996	1.00000
Frt- column effect	0.05096	0.05000	0.05024
Frt- row effect	0.05236	0.05080	0.05340
Frt-interaction effect	0.81340	0.99496	0.99996
Frt-GH-column effect	0.03756	0.03356	0.03592
Frt-GH-row effect	0.03684	0.03452	0.03556
Frt-GH- interaction effect	0.96952	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are slightly lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.27% to 95%, and for GH are from 94.6% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.8%-94.99%, and 94.84%-95% for GH scenario, being almost the same for 50 observations.

Table 4.5.12 Simulated Power rates for size 0.75 effect Uniform distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.05476	0.05060	0.05396
F-row effect	0.05568	0.05036	0.05008

F-interaction effect	0.99716	1.00000	1.00000
F-GH- column effect	0.00960	0.00976	0.01040
F-GH- row effect	0.00952	0.00952	0.01016
F-GH- interaction effect	0.99980	1.00000	1.00000
Frt- column effect	0.05452	0.05032	0.05324
Frt- row effect	0.05576	0.05112	0.05016
Frt-interaction effect	0.98764	1.00000	1.00000
Frt-GH-column effect	0.02848	0.02516	0.02644
Frt-GH-row effect	0.02612	0.02588	0.02568
Frt-GH- interaction effect	0.99992	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.98% to 95%, and for GH are from 94.99% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.93%-95%, and 94.99%-95% for GH scenario, being the same for 50 observations.

Table 4.5.13 Simulated Power rates for size 1.0 effect Uniform distribution, pattern “Two main effects null and Interaction effect present”.

Statistic	Sample size

	10	25	50
F-column effect	0.05364	0.05120	0.05072
F-row effect	0.05420	0.05296	0.05208
F-interaction effect	0.99996	1.00000	1.00000
F-GH- column effect	0.00976	0.00976	0.01120
F-GH- row effect	0.01040	0.01004	0.00876
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05400	0.05220	0.05068
Frt- row effect	0.05280	0.05380	0.05208
Frt-interaction effect	0.99984	1.00000	1.00000
Frt-GH-column effect	0.02004	0.00184	0.01908
Frt-GH-row effect	0.01952	0.01836	0.01752
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.99% to 95%, and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.5.14 Simulated Power rates for size 0.25 effect Uniform distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.12356	0.23188	0.41328
F-row effect	0.05392	0.05180	0.05024
F-interaction effect	0.12328	0.23756	0.42152
F-GH- column effect	0.06060	0.17548	0.39556
F-GH- row effect	0.01016	0.00944	0.00916
F-GH- interaction effect	0.06000	0.17324	0.39332
Frt- column effect	0.12064	0.21944	0.39012
Frt- row effect	0.05320	0.05100	0.05016
Frt-interaction effect	0.12144	0.22552	0.39512
Frt-GH-column effect	0.14756	0.33480	0.58936
Frt-GH-row effect	0.04452	0.04780	0.04720
Frt-GH- interaction effect	0.14648	0.33192	0.58444

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 59.35% to 88.12%, and for GH are from 20% to 87.34%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 58.68%-87.34%, and 66%-91.5% for GH scenario, being close for 50 observations.

Table 4.5.15 Simulated Power rates for size 0.5 effect Uniform distribution, pattern ""One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.32696	0.69648	0.93944
F-row effect	0.05312	0.05084	0.05264
F-interaction effect	0.33352	0.69328	0.94000
F-GH- column effect	0.29256	0.75268	0.97964
F-GH- row effect	0.01128	0.01016	0.01032
F-GH- interaction effect	0.29180	0.75192	0.98184
Frt- column effect	0.29972	0.63676	0.90488
Frt- row effect	0.05172	0.05004	0.05188
Frt-interaction effect	0.30400	0.63508	0.90484
Frt-GH-column effect	0.44816	0.85432	0.98900
Frt-GH-row effect	0.03984	0.04036	0.04016
Frt-GH- interaction effect	0.44108	0.85036	0.99100

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction

and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 84.7% to 94.68%, and for GH are from 82.88% to 94.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 83.33%-94.48%, and 88.66%-94.95% for GH scenario, being close for 50 observations.

Table 4.5.16 Simulated Power rates for size 0.75 effect Uniform distribution, pattern “One main effect present, one main effect null, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.62556	0.96280	0.99972
F-row effect	0.05568	0.05036	0.05008
F-interaction effect	0.63124	0.96128	0.99968
F-GH- column effect	0.68092	0.98984	1.00000
F-GH- row effect	0.00952	0.00952	0.01016
F-GH- interaction effect	0.68008	0.98848	1.00000
Frt- column effect	0.56308	0.92728	0.99784
Frt- row effect	0.05132	0.04740	0.04808
Frt-interaction effect	0.56812	0.92576	0.99800
Frt-GH-column effect	0.78240	0.99404	1.00000
Frt-GH-row effect	0.03116	0.03084	0.03100
Frt-GH- interaction effect	0.78312	0.99400	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 94.99%, and for GH are from 92.65% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.2%-94.99%, and 93.6%-95% for GH scenario, being close for 50 observations.

Table 4.5.17 Simulated Power rates for size 1.0 effect Uniform distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.86688	0.99876	1.00000
F-row effect	0.05420	0.05296	0.05208
F-interaction effect	0.86616	0.99900	1.00000
F-GH- column effect	0.92744	0.99996	1.00000
F-GH- row effect	0.01040	0.01004	0.00876
F-GH- interaction effect	0.92956	0.99996	1.00000
Frt- column effect	0.80680	0.99452	0.99996
Frt- row effect	0.04716	0.04652	0.04704
Frt-interaction effect	0.80420	0.99560	1.00000

Frt-GH-column effect	0.95648	0.99996	1.00000
Frt-GH-row effect	0.02196	0.02000	0.01864
Frt-GH- interaction effect	0.95612	1.00000	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.23% to 95%, and for GH are from 94.6% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.78%-95%, and 94.77%-95% for GH scenario, being the same for 50 observations.

Table 4.5.18 Simulated Power rates for size 0.25 effect Uniform distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.07248	0.09500	0.14196
F-row effect	0.07028	0.09532	0.13948
F-interaction effect	0.07312	0.09804	0.14164
F-GH- column effect	0.02024	0.03768	0.07704
F-GH- row effect	0.02100	0.04020	0.07524
F-GH- interaction effect	0.02204	0.03916	0.07492

Frt- column effect	0.07164	0.09416	0.13856
Frt- row effect	0.06956	0.09388	0.13588
Frt-interaction effect	0.07332	0.09684	0.13792
Frt-GH-column effect	0.06788	0.11900	0.19624
Frt-GH-row effect	0.07216	0.12264	0.19048
Frt-GH- interaction effect	0.06872	0.12024	0.19512

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are not necessarily higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 28.57% to 64.79%, and for GH are from negative to 35.06%. In RT setting, the relative rejection rates for the power method are 26.47%-63.77%, and 26.3%-74.49% for GH scenario, being close for 10 observations.

Table 4.5.19 Simulated Power rates for size 0.5 effect Uniform distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.11708	0.23128	0.41492
F-row effect	0.11704	0.23264	0.41564
F-interaction effect	0.12180	0.23576	0.41888

F-GH- column effect	0.06048	0.17192	0.39632
F-GH- row effect	0.06104	0.16772	0.39432
F-GH- interaction effect	0.05956	0.17624	0.39228
Frt- column effect	0.11164	0.21248	0.37924
Frt- row effect	0.11372	0.21252	0.37996
Frt-interaction effect	0.11548	0.21860	0.38360
Frt-GH-column effect	0.13980	0.32144	0.57384
Frt-GH-row effect	0.14384	0.31808	0.57484
Frt-GH- interaction effect	0.14120	0.31864	0.56684

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 57.26% to 88.04%, and for GH are from 20% to 87.37%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 54.95%-86.95%, and 64.29%-91.29% for GH scenario, being close for 50 observations.

Table 4.5.20 Simulated Power rates for size 0.75 effect Uniform distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size

	10	25	50
F-column effect	0.20708	0.45512	0.74628
F-row effect	0.21152	0.45468	0.74700
F-interaction effect	0.21120	0.45420	0.75372
F-GH- column effect	0.15188	0.44616	0.81704
F-GH- row effect	0.14776	0.44336	0.81864
F-GH- interaction effect	0.14924	0.44028	0.81364
Frt- column effect	0.18884	0.40276	0.68176
Frt- row effect	0.19396	0.40228	0.68064
Frt-interaction effect	0.19400	0.40124	0.68932
Frt-GH-column effect	0.26880	0.59580	0.88664
Frt-GH-row effect	0.26308	0.59644	0.88892
Frt-GH- interaction effect	0.26596	0.59288	0.88612

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 75.85% to 93.3%, and for GH are from 66% to 93.85%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 73.54%-92.74%, and 81%-94.38% for GH scenario, being close for 50 observations.

Table 4.5.21 Simulated Power rates for size 1.0 effect Uniform distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size		
	10	25	50
F-column effect	0.33272	0.68836	0.94184
F-row effect	0.32984	0.69048	0.94232
F-interaction effect	0.33484	0.69288	0.94384
F-GH- column effect	0.29616	0.75504	0.97928
F-GH- row effect	0.30124	0.75556	0.98084
F-GH- interaction effect	0.29332	0.75228	0.98160
Frt- column effect	0.29584	0.61432	0.89640
Frt- row effect	0.28948	0.61724	0.89708
Frt-interaction effect	0.29676	0.62000	0.89876
Frt-GH-column effect	0.41700	0.82908	0.98672
Frt-GH-row effect	0.42376	0.83268	0.98684
Frt-GH- interaction effect	0.41712	0.83152	0.98736

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 84.85% to 94.7%, and for GH are from 82.94% to 94.9%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 82.76%-94.43%, and 88%-94.93% for GH scenario, being close for 50 observations.

4.6. Beta (0.667, 0.667) distribution.

Table 4.6.1 Simulated Type I error rates for size 0 effect Beta (0.667, 0.667) distribution, pattern "All effects null".

Statistic	Sample size		
	10	25	50
F-column effect	0.05364	0.05104	0.05172
F-row effect	0.05328	0.05476	0.05152
F-interaction effect	0.05336	0.05356	0.05052
F-GH- column effect	0.00164	0.00216	0.00164
F-GH- row effect	0.00188	0.00168	0.00196
F-GH- interaction effect	0.00124	0.00168	0.00144
Frt- column effect	0.05364	0.05132	0.05116
Frt- row effect	0.05280	0.05440	0.05176
Frt-interaction effect	0.05412	0.05380	0.05064
Frt-GH-column effect	0.04504	0.04852	0.04916
Frt-GH-row effect	0.04676	0.04924	0.05148
Frt-GH- interaction effect	0.04576	0.04696	0.04904

The rates are close to 0.05, which is in line with the theory. Only the GH parametric scenario gives lower rates in all cases.

It should be also noted that F-GH nonparametric scenarios in most cases are slightly lower than 0.05. In contrast, for power method cases the values are higher than 0.05.

In general, size 0 effect rates confirm the theory. Now, we need to look at treatment pattern by the effect size for Beta (0.667, 0.667) distribution.

Table 4.6.2 Simulated Power rates for size 0.25 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.12152	0.23324	0.41504
F-row effect	0.05324	0.05092	0.04788
F-interaction effect	0.05696	0.05068	0.05236
F-GH- column effect	0.02988	0.12420	0.37504
F-GH- row effect	0.00164	0.00128	0.00148
F-GH- interaction effect	0.00216	0.00128	0.00180
Frt- column effect	0.13872	0.27472	0.48432
Frt- row effect	0.05252	0.05016	0.04796
Frt-interaction effect	0.05656	0.05012	0.05124
Frt-GH-column effect	0.24200	0.55408	0.84196
Frt-GH-row effect	0.04312	0.04704	0.04748
Frt-GH- interaction effect	0.04364	0.04264	0.04556

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 58.68% to 87.95%, and for GH are from negative to 86.67%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 64%-89.67%, and 79.34%-94.06% for GH scenario, being close for 50 observations.

Table 4.6.3 Simulated Power rates for size 0.5 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.32652	0.69460	0.93956
F-row effect	0.05304	0.04992	0.05268
F-interaction effect	0.05536	0.05212	0.05152
F-GH- column effect	0.25572	0.79800	0.99400
F-GH- row effect	0.00192	0.00128	0.00132
F-GH- interaction effect	0.00196	0.00124	0.00120
Frt- column effect	0.33732	0.69848	0.93692
Frt- row effect	0.05260	0.04860	0.05292
Frt-interaction effect	0.05408	0.05316	0.05192

Frt-GH-column effect	0.64704	0.96636	0.99968
Frt-GH-row effect	0.03976	0.03840	0.04096
Frt-GH- interaction effect	0.03996	0.03980	0.04068

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.66% to 94.68%, and for GH are from 80.47% to 94.97%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.2%-94.66%, and 92.27%-94.99% for GH scenario, being close for 50 observations.

Table 4.6.4 Simulated Power rates for size 0.75 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect null".

Statistic	Sample size		
	10	25	50
F-column effect	0.62324	0.96336	0.99964
F-row effect	0.05624	0.05084	0.04996
F-interaction effect	0.05304	0.05092	0.05048
F-GH- column effect	0.71952	0.99776	1.00000
F-GH- row effect	0.00204	0.00164	0.00164
F-GH- interaction effect	0.00204	0.00116	0.00140

Frt- column effect	0.58920	0.93956	0.99860
Frt- row effect	0.05692	0.05024	0.04916
Frt-interaction effect	0.05316	0.05112	0.05096
Frt-GH-column effect	0.92244	0.99956	1.00000
Frt-GH-row effect	0.03452	0.03164	0.03500
Frt-GH- interaction effect	0.03272	0.03284	0.03308

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 94.99%, and for GH are from 93.04% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.5%-94.99%, and 94.58%-95% for GH scenario, being close for 50 observations.

Table 4.6.5 Simulated Power rates for size 1.0 effect Beta (0.667, 0.667) distribution, pattern “One main effect present, one main effect null, and Interaction effect null”.

Statistic	Sample size		
	10	25	50
F-column effect	0.86720	0.99900	1.00000
F-row effect	0.05440	0.05160	0.05212
F-interaction effect	0.05692	0.05412	0.05076

F-GH- column effect	0.96332	1.00000	1.00000
F-GH- row effect	0.00176	0.00192	0.00164
F-GH- interaction effect	0.00212	0.00116	0.00168
Frt- column effect	0.80704	0.99376	0.99996
Frt- row effect	0.05496	0.05088	0.05300
Frt-interaction effect	0.05404	0.05312	0.05108
Frt-GH-column effect	0.99368	1.00000	1.00000
Frt-GH-row effect	0.02840	0.02716	0.02500
Frt-GH- interaction effect	0.02868	0.02584	0.02500

A column effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column effects are higher than alpha, but for interaction and row are lower than alpha. Besides, for nonparametric scenarios values for column effects are higher than alpha, but for interaction and row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.23% to 95%, and for GH are from 94.8% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.8%-94.99%, and 94.97%-95% for GH scenario, being close for 50 observations.

Table 4.6.6 Simulated Power rates for size 0.25 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50

F-column effect	0.33196	0.69568	0.94304
F-row effect	0.32832	0.69468	0.94140
F-interaction effect	0.05696	0.05068	0.05236
F-GH- column effect	0.25604	0.79292	0.99476
F-GH- row effect	0.25808	0.79300	0.99340
F-GH- interaction effect	0.00216	0.00128	0.00180
Frt- column effect	0.31928	0.66520	0.92172
Frt- row effect	0.31816	0.66140	0.92252
Frt-interaction effect	0.05608	0.05040	0.05116
Frt-GH-column effect	0.59028	0.95212	0.99940
Frt-GH-row effect	0.58560	0.94760	0.99932
Frt-GH- interaction effect	0.03360	0.03332	0.03248

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.94% to 94.7%, and for GH are from 80.47% to 94.97%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.27%-94.58%, and 91.46%-94.99% for GH scenario, being close for 50 observations.

Table 4.6.7 Simulated Power rates for size 0.5 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	0.87004	0.99872	1.00000
F-row effect	0.87260	0.99860	1.00000
F-interaction effect	0.05536	0.05212	0.05152
F-GH- column effect	0.96448	1.00000	1.00000
F-GH- row effect	0.96388	1.00000	1.00000
F-GH- interaction effect	0.00196	0.00124	0.00120
Frt- column effect	0.80332	0.99416	1.00000
Frt- row effect	0.80288	0.99468	1.00000
Frt-interaction effect	0.04644	0.04728	0.04672
Frt-GH-column effect	0.98912	1.00000	1.00000
Frt-GH-row effect	0.98804	1.00000	1.00000
Frt-GH- interaction effect	0.01028	0.00772	0.00668

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.25% to 95%, and for GH are from 94.8% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.77%-95%, and 94.94%-95% for GH scenario, being the same for 50 observations.

Table 4.6.8 Simulated Power rates for size 0.75 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	0.99700	1.00000	1.00000
F-row effect	0.99664	1.00000	1.00000
F-interaction effect	0.05304	0.05092	0.05048
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00204	0.00116	0.00140
Frt- column effect	0.99288	1.00000	1.00000
Frt- row effect	0.99204	1.00000	1.00000
Frt-interaction effect	0.03104	0.03152	0.03392
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00056	0.00004	0.00000

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row

effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.98% to 95%, and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.96%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.6.9 Simulated Power rates for size 1.0 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction is null".

Statistic	Sample size		
	10	25	50
F-column effect	1.00000	1.00000	1.00000
F-row effect	1.00000	1.00000	1.00000
F-interaction effect	0.05692	0.05412	0.05076
F-GH- column effect	1.00000	1.00000	1.00000
F-GH- row effect	1.00000	1.00000	1.00000
F-GH- interaction effect	0.00212	0.00116	0.00168
Frt- column effect	0.99992	1.00000	1.00000
Frt- row effect	0.99992	1.00000	1.00000
Frt-interaction effect	0.00824	0.00800	0.00848
Frt-GH-column effect	1.00000	1.00000	1.00000
Frt-GH-row effect	1.00000	1.00000	1.00000
Frt-GH- interaction effect	0.00000	0.00000	0.00000

Row and column effect are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for column and row effects are higher than alpha, but for interaction are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are higher than alpha, but for interaction are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 95%, and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.6.10 Simulated Power rates for size 0.25 effect Beta (0.667, 0.667) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05552	0.05428	0.04868
F-row effect	0.05324	0.05092	0.04788
F-interaction effect	0.32928	0.69764	0.94020
F-GH- column effect	0.00128	0.00128	0.00160
F-GH- row effect	0.00164	0.00128	0.00148
F-GH- interaction effect	0.25716	0.79632	0.99380
Frt- column effect	0.05624	0.05444	0.04856
Frt- row effect	0.05296	0.05044	0.04824
Frt-interaction effect	0.34056	0.69772	0.93744

Frt-GH-column effect	0.03924	0.03948	0.04044
Frt-GH-row effect	0.03908	0.04056	0.04204
Frt-GH- interaction effect	0.65028	0.96668	0.99960

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are slightly lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.8% to 94.68 and for GH are 80.54% to 94.97%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 85.29%-94.66%, and 92.3%-94.99 for GH scenario, being almost the same for 50 observations.

Table 4.6.11 Simulated Power rates for size 0.5 effect Beta (0.667, 0.667) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05068	0.04996	0.05044
F-row effect	0.05304	0.04992	0.05268
F-interaction effect	0.87016	0.99900	1.00000
F-GH- column effect	0.00192	0.00152	0.00144
F-GH- row effect	0.00192	0.00128	0.00132
F-GH- interaction effect	0.96512	1.00000	1.00000

Frt- column effect	0.05188	0.04996	0.05004
Frt- row effect	0.05176	0.05072	0.05240
Frt-interaction effect	0.81192	0.99444	1.00000
Frt-GH-column effect	0.02756	0.02532	0.02552
Frt-GH-row effect	0.02692	0.02512	0.02608
Frt-GH- interaction effect	0.99316	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 25 and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.25% to 95% and for GH are 94.82% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.84%-95%, and 94.96%-95% for GH scenario, being the same for 50 observations.

Table 4.6.12 Simulated Power rates for size 0.75 effect Beta (0.667, 0.667) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.05476	0.05072	0.05196
F-row effect	0.05624	0.05084	0.04996
F-interaction effect	0.99712	1.00000	1.00000

F-GH- column effect	0.00196	0.00212	0.00172
F-GH- row effect	0.00204	0.00164	0.00164
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05472	0.05076	0.05100
Frt- row effect	0.05592	0.05160	0.04972
Frt-interaction effect	0.98324	1.00000	1.00000
Frt-GH-column effect	0.01296	0.01072	0.01036
Frt-GH-row effect	0.01256	0.01052	0.01164
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are slightly lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92.98% to 95% and for GH are 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.9%-95%, and 95% for GH scenario, being the same for 50 observations.

Table 4.6.13 Simulated Power rates for size 1.0 effect Beta (0.667, 0.667) distribution, pattern "Two main effects null and Interaction effect present".

Statistic	Sample size		
	10	25	50

F-column effect	0.05412	0.05056	0.04848
F-row effect	0.05440	0.05160	0.05212
F-interaction effect	1.00000	1.00000	1.00000
F-GH- column effect	0.00180	0.00156	0.00152
F-GH- row effect	0.00176	0.00192	0.00164
F-GH- interaction effect	1.00000	1.00000	1.00000
Frt- column effect	0.05452	0.05096	0.04936
Frt- row effect	0.05292	0.05312	0.05240
Frt-interaction effect	0.99980	1.00000	1.00000
Frt-GH-column effect	0.00708	0.00604	0.00592
Frt-GH-row effect	0.00688	0.00616	0.00528
Frt-GH- interaction effect	1.00000	1.00000	1.00000

Interaction effect is present, and raises the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 10, 25, and 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction effects are higher than alpha, but for column and row effects are lower than alpha. Besides, for nonparametric scenarios values for column and row effects are lower than alpha, but for interaction are higher than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 95% and for GH are 95%. Thus, GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 94.99%-95%, and 95% for GH scenario, being the same for 25 and 50 observations.

Table 4.6.14 Simulated Power rates for size 0.25 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.12152	0.23324	0.41504
F-row effect	0.05324	0.05092	0.04788
F-interaction effect	0.12140	0.23680	0.41968
F-GH- column effect	0.02988	0.12420	0.37504
F-GH- row effect	0.00164	0.00128	0.00148
F-GH- interaction effect	0.02980	0.12500	0.37204
Frt- column effect	0.13136	0.25424	0.44928
Frt- row effect	0.05248	0.05016	0.04788
Frt-interaction effect	0.13016	0.26012	0.45516
Frt-GH-column effect	0.21584	0.50388	0.80068
Frt-GH-row effect	0.04152	0.04464	0.04508
Frt-GH- interaction effect	0.21588	0.49588	0.80256

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are not necessarily higher than alpha, but for row are necessarily lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are slightly lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 58.68% to 88.09% and for GH are from negative to 86.67%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 61.54%-89%, and 76.85%-93.77% for GH scenario, being close for 50 observations.

Table 4.6.15 Simulated Power rates for size 0.5 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.32652	0.69460	0.93956
F-row effect	0.05304	0.04992	0.05268
F-interaction effect	0.33028	0.69568	0.94112
F-GH- column effect	0.25572	0.79800	0.99400
F-GH- row effect	0.00192	0.00128	0.00132
F-GH- interaction effect	0.25224	0.79620	0.99492
Frt- column effect	0.31632	0.66288	0.91984
Frt- row effect	0.05068	0.04904	0.05272
Frt-interaction effect	0.31972	0.66448	0.92036
Frt-GH-column effect	0.58900	0.95048	0.99928
Frt-GH-row effect	0.03332	0.03160	0.00334
Frt-GH- interaction effect	0.58736	0.95112	0.99948

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction

and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 84.66% to 94.68% and for GH are from 80.2% to 94.97%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 84.2%-94.57%, and 91.48%-94.99% for GH scenario, being close for 50 observations.

Table 4.6.16 Simulated Power rates for size 0.75 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.62324	0.96336	0.99964
F-row effect	0.05624	0.05084	0.04996
F-interaction effect	0.63228	0.96308	0.99976
F-GH- column effect	0.71952	0.99776	1.00000
F-GH- row effect	0.00204	0.00164	0.00164
F-GH- interaction effect	0.71728	0.99816	1.00000
Frt- column effect	0.56612	0.92788	0.99768
Frt- row effect	0.05364	0.04784	0.04684
Frt-interaction effect	0.57136	0.92688	0.99808
Frt-GH-column effect	0.89528	0.99944	1.00000
Frt-GH-row effect	0.02140	0.02040	0.02056
Frt-GH- interaction effect	0.89884	0.99968	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 92% to 94.99% and for GH are from 93% to 95%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 91.2%-94.99%, and 94.4%-95% for GH scenario, being close for 50 observations.

Table 4.6.17 Simulated Power rates for size 1.0 effect Beta (0.667, 0.667) distribution, pattern "One main effect present, one main effect null, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.86720	0.99900	1.00000
F-row effect	0.05440	0.05160	0.05212
F-interaction effect	0.86824	0.99904	1.00000
F-GH- column effect	0.96332	1.00000	1.00000
F-GH- row effect	0.00176	0.00192	0.00164
F-GH- interaction effect	0.96288	1.00000	1.00000
Frt- column effect	0.79780	0.99372	1.00000
Frt- row effect	0.04712	0.04588	0.04704
Frt-interaction effect	0.79616	0.99432	1.00000

Frt-GH-column effect	0.98840	1.00000	1.00000
Frt-GH-row effect	0.01108	0.00908	0.00736
Frt-GH- interaction effect	0.98828	1.00000	1.00000

Interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for interaction and column effects are higher than alpha, but for row are lower than alpha. Besides, for nonparametric scenarios values for interaction and column effects are higher than alpha, but for row are lower than alpha.

Overall, in parametric setting, for the power method the relative rejection rates are 94.24% to 95% and for GH are from 94.8% to 95%. Thus, for 50 observations GH gives the same relative rejection. In RT setting, the relative rejection rates for the power method are 93.72%-95%, and 94.94%-95% for GH scenario, being the same for 50 observations.

Table 4.6.18 Simulated Power rates for size 0.25 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.07208	0.09596	0.14124
F-row effect	0.07036	0.09324	0.13980
F-interaction effect	0.07304	0.09616	0.14076
F-GH- column effect	0.00644	0.01460	0.04024
F-GH- row effect	0.00620	0.01572	0.04156
F-GH- interaction effect	0.00640	0.01464	0.03748

Frt- column effect	0.07696	0.10604	0.16056
Frt- row effect	0.07340	0.10512	0.15972
Frt-interaction effect	0.07688	0.10576	0.16144
Frt-GH-column effect	0.08900	0.17676	0.31368
Frt-GH-row effect	0.09196	0.18044	0.30772
Frt-GH- interaction effect	0.08748	0.17476	0.31076

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are lower than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 28.57% to 64.29% and for GH are negative. In RT setting, the relative rejection rates for the power method are 32%-68.75%, and 42.86%-83.77% for GH scenario.

Table 4.6.19 Simulated Power rates for size 0.5 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.11864	0.23288	0.41688
F-row effect	0.11548	0.23376	0.41368
F-interaction effect	0.12008	0.23560	0.41992

F-GH- column effect	0.02984	0.12412	0.37400
F-GH- row effect	0.03108	0.12124	0.37444
F-GH- interaction effect	0.03000	0.12728	0.36968
Frt- column effect	0.12084	0.23420	0.41768
Frt- row effect	0.11916	0.23612	0.41396
Frt-interaction effect	0.12320	0.23856	0.42032
Frt-GH-column effect	0.18932	0.45336	0.75400
Frt-GH-row effect	0.19492	0.44708	0.75140
Frt-GH- interaction effect	0.18976	0.45124	0.74676

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is not very high. It should be noted that, for GH scenarios the parametric values for row and column effects are not necessarily higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 56.52% to 87.9% and for GH are from negative to 86.63%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 58.33%-88.09%, and 73.54%-93.37% for GH scenario, being close for 50 observations.

Table 4.6.20 Simulated Power rates for size 0.75 effect Beta (0.667, 0.667) distribution, pattern “Two main effects present, and Interaction effect present”.

Statistic	Sample size

	10	25	50
F-column effect	0.20548	0.45252	0.74852
F-row effect	0.21120	0.45196	0.74596
F-interaction effect	0.21172	0.45136	0.75352
F-GH- column effect	0.10688	0.43608	0.86476
F-GH- row effect	0.10416	0.43120	0.86660
F-GH- interaction effect	0.10288	0.43056	0.86420
Frt- column effect	0.19432	0.41676	0.70052
Frt- row effect	0.20096	0.41604	0.69732
Frt-interaction effect	0.19984	0.41404	0.70864
Frt-GH-column effect	0.34780	0.73636	0.96136
Frt-GH-row effect	0.34080	0.73836	0.96248
Frt-GH- interaction effect	0.34132	0.73680	0.96304

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 75.6% to 93.36% and for GH are from 52% to 94.23%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 74.23%-92.94%, and 85.29%-94.8% for GH scenario, being close for 50 observations.

Table 4.6.21 Simulated Power rates for size 1.0 effect Beta (0.667, 0.667) distribution, pattern "Two main effects present, and Interaction effect present".

Statistic	Sample size		
	10	25	50
F-column effect	0.33112	0.68896	0.94316
F-row effect	0.33116	0.68928	0.94108
F-interaction effect	0.33316	0.69324	0.94320
F-GH- column effect	0.26016	0.79952	0.99320
F-GH- row effect	0.26380	0.80096	0.99400
F-GH- interaction effect	0.25760	0.80096	0.99412
Frt- column effect	0.29484	0.61192	0.89648
Frt- row effect	0.29180	0.61484	0.89456
Frt-interaction effect	0.29632	0.61992	0.89840
Frt-GH-column effect	0.51604	0.91964	0.99800
Frt-GH-row effect	0.51844	0.92096	0.99792
Frt-GH- interaction effect	0.51204	0.92268	0.99768

Row, interaction and column effects are present, and raise the Power for power method scenarios to the elevated levels for 10, 25, and 50 observations. Also, for 50 observations, the rate is very high. It should be noted that, for GH scenarios the parametric values for row and column effects are higher than alpha, same is true for the interaction effect. Besides, for nonparametric scenarios values for row and column effects are higher than alpha, and for interaction effect are also higher than alpha. There is also a need to describe relative rejection rates.

Overall, in parametric setting, for the power method the relative rejection rates are 84.89% to 94.7% and for GH are from 80.54% to 94.97%. Thus, for 50 observations GH gives almost the same relative rejection. In RT setting, the relative rejection rates for the power method are 82.88%-94.43%, and 90.23%-94.99% for GH scenario, being close for 50 observations.

CHAPTER 5

DISCUSSION

This chapter has two parts. First, in part 5.1, the findings from the simulation study are summarized. Next, in part 5.2, suggestions for future studies are discussed.

5.1. Findings

The main theoretical result and the findings of this dissertation are that MOP cumulants are analytically derived and discussed for HR, HQ, and HH distributions. Derivation of closed-form solutions eliminates the need for numerical methods for the researcher.

The simulation confirmed that the rank transform is appropriate in 2x2 between group designs. Thus, the simulation results confirm Akritas (1990), Headrick and Sawilowsky (2000), and Thompson (1991) theoretical results. Specifically, there was no inflation of Type 1 error when interaction is not present.

The results associated with the GH and power method are similar for strictly increasing monotonic distributions, but are dissimilar for nonmonotonic distributions. It should be noted that any Monte Carlo study is limited to the parameters, which includes the transformation types.

For Beta (4, 1.5) with no effect size, for the parametric GH scenario we are not rejecting at all, but for the rank transform GH rejection rate is around 0.05, which is unusual. With nonnull effect sizes the situation is different. Besides, for the “Two main effects present, interaction is null” scenario GH interaction power rate is 0.00 for both parametric and RT starting with the effect size of 0.5. The situation is the same for the scenario with column and interaction effect present, row effect null with size effect of 1.0. All features mentioned for Beta (4, 1.5) remain true for Beta (4, 2).

For Triangular distribution with no effect size parametric GH rate is lower than 0.05, but GH RT gives a rejection rate of approximately 0.05. For the scenario “Two main effects present, interaction is null” those rates for interaction are similar for a small effect size (0.25-0.5), but for a bigger effect size (0.75-1.0) parametric GH rate is higher than nonparametric.

For Uniform distribution with no effect size parametric GH rate is also lower than 0.05, but GH RT gives a rejection rate of approximately 0.05. Again, for the scenario “Two main effects present, interaction is null” for interaction for bigger effect size (0.75-1.0) the parametric GH rate is higher than the nonparametric.

For Beta (0.667, 0.667) distribution with no effect size parametric GH rate is also lower than 0.05, but GH RT gives a rejection rate of approximately 0.05. Once again, for the scenario “Two main effects present, interaction is null” for interaction for bigger effect size (0.75-1.0) the parametric GH rate is higher than the nonparametric (for effect size 1.0 GH RT rate for interaction is 0.00).

5.2 Suggestions for future studies.

Future research may be of interest in terms of Monte Carlo study with regard to other distributions, which are monotonic or nonmonotonic. Examples include Generalized Lambda Distribution, Burr distribution, etc.

There may be other possibilities of deriving other methods of translation (Johnson system, Burr system, etc.) in terms of MOP. MOP results demonstrate better relative bias and standard error than MOM results. Therefore, if somebody derives Johnson or Burr in terms of MOP, they will be useful.

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