ON THE FOUNDATION AND TECHNIC OF ARITHMETIC.*

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THE PSYCHOLOGY OF READING A NUMBER.

O UR marvelous positional notation for number is built of three elements, digit, base, column. The base it is which interprets the column. With base ten, 100 means a ten of tens. With base two, 100 means two twos. With base twelve, 100 means a dozen dozen.

The Romans had a base, or rather two bases, but neither digits nor columns. Their V is a trace of the more primitive base five, seen also in the Greek $\pi\epsilon\mu\pi\dot{a}\zeta\omega$, to finger fit by fives, to count. This, combining with the more final base ten, X, explains their having a separate symbol, L, for fifty.

Their ten of tens has its unitary symbol, C, and their ten of hundreds is M, a thousand.

Each basal number is a new unit, an atom, a monad, a neomon, squeezing into an individual the components, making thus one ball to be further played with.

Our present basal number-word, hundred, is properly a collective noun, a hundred, literally a count or tale of a hundred; for its red is the root in German Rede, talk, and its hund is the Old English word, cognate with Latin centum, Greek ἐκατόν, to be found in Bosworth's Anglo-Saxon Dictionary, but seldom used after A. D. 1200.

The *Century Dictionary*, to which I may be forgiven for being attached, says *hund* is from the root of ten, and this idea leads it far, into the postulating of an assumed type *kanta* which it gives as a reduced form of an equally hypothetical *dakanta* for an assumed original *dakan-dakan-ta*, "ten-ten-th," from assumed *dakan*, on the analogy of the Gothic *taihun-taihund*, *taihun-tāhund*, a hundred, of

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which it regards *hund* as an abbreviation or reduced form. The same original elements, it says, without the suffix d = th, appear in Old High German *zehanzo* = Anglo-Saxon *tcón-tig*, *ten-ty* = *ten-ten*.

The element hund occurring in the Anglo-Saxon hund-seofontig, seventy, etc., hund-endlefontig, eleventy, hund-twelftig, twelfty, it gives as representing "ten" or "tenth," and these words as developed by cumulation (hund and tig being ultimately from the same root, that of "ten") from the theoretically assumed hund-seofon, "tenth seven," etc. Murray is not well persuaded of all this, and says there is no satisfactory explanation of the use of hund in these Anglo-Saxon words.

For myself, even if the root of *hund* be that of ten, I can well conceive that *hund* should mean hundred without any first hypothetically postulated and hypothetically worn-away reduplication. Have we ourselves not "million," a simple augmentative of *mille*, a thousand?

Nor is the reduplication theory consonant with the fact that in Old Norse the word *hundrath*, "hundred," "tentale," originally meant 120; it was a tentale not of tens but of dozens, the rival base twelve, against which the bestial base ten, an Old-Man-of-the-Sea saddled upon us by our prehuman simian ancestors, has been continuously fighting down to this very day. And even in modern English remnants of this older usage remain. The *Glasgow Herald* of Sept. 13, 1886, says: "A mease [of herring]....is five hundreds of 120 each."

Chambers Cyclopædia says: "Deal boards are six score to the hundred."

This hundred was legal for balks, deals, eggs, spars, stone, etc.

Peacock, in the *Encyclopædia Metropolitana*, I, 381, says: "The technical meaning attached by merchants to the word 'hundred,' associated with certain objects, was six score—a usage which is commemorated in the popular distich:

"Five score of men, money, and pins, Six score of all other things."

All this abundantly proves that hundred is very far from being a simple numeral adjective, like, e. g., seventy; so that while we properly say seventy-five, to say a hundred-five is a hideous blunder.

Hundred is strictly not an adjective at all, but a collective noun; it is always preceded by a definitive, usually an article or a numeral, and if followed by a numeral, this must invariably be preceded by the word "and."

A following noun is, historically, a genitive partitive, in Old English a genitive plural, later a pural preceded by "of." Thus 1663, Gerbier, *Counsel*, "About one hundred of Leagues." Hale (1668): "These many hundred of years." Cowper (1782) *Loss* of *Royal George*: "Eight hundred of the brave." To-day: "A hundred of my friends," "A hundred of bricks," "Some hundreds of men were present." [Murray].

Even if there be an ellipsis of "of" before the noun, the word hundred retains its substantival character so far as to be always preceded by "a" or some adjective. Compare "dozen," which has precisely parallel constructions, e. g., "a dozen of eggs." Hooke (1665) : "A hundred and twenty five thousand times bigger." Murray's *Dictionary* (1901) gives as model modern English: "Mod. The hundred and one odd chances." Again it says: "c. The cardinal form *hundred* is also used as an ordinal when followed by other numbers, the last of which alone takes the ordinal form: e. g., 'the hundred-and-first,' 'the hundred-and-twentieth,' 'the six-hundredand-fortieth part of a square mile.'" Goold Brown, *The Grammar of English Grammars*: "Four hundred and fiftieth."

All this furnishes complete explanation and warranty of the "and" which must always separate "hundred" from a following numeral. It marks a complete change of construction: "a hundred of leagues and three leagues"; "a hundred and three leagues." This fine English usage is unbroken throughout the centuries. Thus, Byrhtferth's *Handboc* (about 1050): "twa hundred & tyn"; Cursor Ms. 8886 (before 1300): "O quens had he [Solomon] hundrets seuen." *Myrr. our Ladye* (1450-1530) 309:

"Twyes syxe tymes ten, that ys to a hundereth and twenty." Oliver Wendell Holmes, "The Deacon's Masterpiece":

> "Seventeen hundred and fifty-five. Georgius Secundus was then alive,— Snuffy old drone from the German hive."

The London Times of Febr. 20, 1885: "The hundred and one forms of small craft used by the Chinese to gain an honest livelihood."

The new *Encyclopædia Britannica*, 11th Edition, 1911, Vol. 2, p. 523: "Thus we speak of one thousand eight hundred and seventysix, and represent it by MDCCCLXXVI or 1876." Again, p. 526: "A set of written symbols is sometimes read in more than one way. Thus 1820 might be read as one thousand eight hundred and twenty if it represented a number of men, but it would be read as eighteen hundred and twenty if it represented a year of the Christian era."

Though all the numerals up to a hundred belong in common to all the Indo-European languages, the word thousand is found only in the Teutonic and Slavonic languages, and maybe the Slavs borrowed the word in prehistoric times from the Teutons.

Very naturally thousand is construed precisely like hundred: "Land on him like a thousand of brick"; "The Thousand and One Nights."

And just so it is with that marvelous makeshift *million*, "big thousand," Old French augmentative of Latin *mille*, a thousand.

Says Piers Plowman (A), III, 255:

"Coneyte not his goodes For millions of moneye."

And the divine Shakespeare:

"Or may we cram Within this wooden O the very casques That did affright the air at Agincourt? O, pardon! since a crooked figure may Attest, in little place, a million! And let us, ciphers to this great accompt, On your imaginary forces work."

"Thus, we say six million three hundred and twenty thousand four hundred and thirty-six" [Whitney's *Essentials of English Grammar*, p. 94], which does not at all militate against our reading 10033 to the telephone girl as "one, double oh, double three." The word which specifies the local value of the digit is best omitted when this local value is unimportant or is otherwise determined. The date 1911 read "nineteen eleven." The approximation $\pi = 3.14159265$ read "*pi* is nearly equal to three, point, one, four, one, five, nine, two, six, five." Here, as in all decimals, the "point" fixes the local value of every subsequent digit.

The country schoolmaster's use of "and" solely to indicate the decimal point is not merely bad form and stupid; it is criminal. It introduces a completely unnecessary ambiguity, doubt, anxiety into the understanding even of oral whole numbers, since she (if it be a country schoolma'am who is reading them out) may end with a wretched fractional, such as hundredths, a retro-active dampener over all that has preceded it.

When that most spectacular of Frenchmen, who, like so many

great Frenchmen, was an Italian, witness Mazarin, Lagrange, etc., when the comparatively unlettered Corsican, Napoleon, sat upon his white horse at a German jubilee while an official opened at him an address of felicitation, the great Captain began to be puzzled at the silent strained attention of those listeners who were supposed to understand the German speech. He whispered to his aide, "Why do they not applaud?" "Sire," was the answer, "on attend le verbe." Just so when the country schoolmaster reads a number, one awaits the fractional!