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An Investigation of the Beam-Column and the Finite-Element Formulations for Analyzing Geometrically Nonlinear Thermal Response of Plane Frames

Baikuntha Silwal
Southern Illinois University Carbondale, baikunthanp@hotmail.com

AN INVESTIGATION OF THE BEAM-COLUMN AND THE FINITE-ELEMENT FORMULATIONS FOR ANALYZING GEOMETRICALLY NONLINEAR THERMAL RESPONSE OF PLANE FRAMES

By

Baikuntha Silwal

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Approved by:

Dr. Aslam Kassimali, Chair

Dr. J. Kent Hsiao

Dr. Jale Tezcan

Graduate School

Southern Illinois University Carbondale

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MAJOR PROFESSOR: Dr. Aslam Kassimali, Ph.D.

The objective of this study is to investigate the accuracy and computational efficiency of two commonly used formulations for performing the geometrically nonlinear thermal analysis of plane framed structures. The formulations considered are the followings: the Beam-Column formulation and the updated Lagrangian version of the finite element formulation that has been adopted in the commercially well-known software SAP2000. These two formulations are used to generate extensive numerical data for three plane frame configurations, which are then compared to evaluate the performance of the two formulations.

The Beam-Column method is based on an Eulerian formulation that incorporates the effects of large joint displacements. In addition, local member force-deformation relationships are based on the Beam-Column approach that includes the axial strain, flexural bowing, and thermal strain. The other formulation, the SAP2000, is based on the updated Lagrangian finite element formulation. The results for nonlinear thermal responses were generated for three plane structures by these formulations. Then, the data were compared for accuracy of deflection

responses and for computational efficiency of the Newton-Raphson iteration cycles required for the thermal analysis.

The results of this study indicate that the Beam-Column method is quite efficient and powerful for the thermal analysis of plane frames since the method is based on the exact solution of the differential equations. In comparison to the SAP2000 software, the Beam-Column method requires fewer iteration cycles and fewer elements per natural member, even when the structures are subjected to significant curvature effects and to restrained support conditions. The accuracy of the SAP2000 generally depends on the number of steps and/or the number of elements per natural member (especially four or more elements per member may be needed when a structure member encounters a significant curvature effect). Succinctly, the Beam-Column formulation requires considerably fewer elements per member, fewer iteration cycles, and less time for thermal analysis than the SAP2000 when the structures are subjected to significant bending effects.

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TABLE OF CONTENTS

LIST OF TABLE

LIST OF FIGURES

NOTATION

A	Cross-Sectional Area of a Member
$\mathbf{B}$	Transformation Matrix
$\mathbf{B}^T$	Transpose of Transformation Matrix
b_1 and b_2	Bowing Functions
c_b	Length Correction Factor Due to Bowing
$c_{b'}$	Derivate of C_b
c_1 *and* c_2	Stability Functions
E	Modulus of Elasticity
$\mathbf{f}$	Resultant Internal Forces
$\mathbf{F}$	Member End Forces in Global Coordinates
$\Delta\mathbf{F}$	Incremental Member End Forces in Global Coordinates
$\Delta\mathbf{F_T}$	The Element End Forces due to Temperature Increment in Global Coordinates
G_i	Quantity Defined in Eq. 2.11
$\mathbf{g}^{(j)}$	Geometric Matrix
H	Quantity Defined in Eq. 2.11
I	Moment of Inertia
$J(q)$	Quantity Defined in Eq. 2.28
$J(q)'$	Derivate of J(q)
$\mathbf{K}$	The Element Tangent Stiffness Matrix in Global Coordinates
$\mathbf{k}$	The Element Tangent Stiffness Matrix in Local Coordinates
L	Undeformed Member Length
$\overline{L}$	Deformed Member Chord Length

$\mathbf{P}$	External Joint Load
$\Delta\mathbf{P}$	Incremental External Joint Load
$\mathbf{P_T}$	Structural Fixed-Joint Forces Due to Temperature Increment
$\mathbf{Q}$	Element Forces in Local Coordinates
$\Delta\mathbf{Q}$	Unbalanced Joint Forces
$\Delta\mathbf{Q_T}$	The Element End Forces due to Temperature Increment in Local Coordinates
q	Axial Force Parameter
Δq	Incremental Axial Force Parameter
R	Radius of a Circular Arc
$\mathbf{S}$	Structural Tangent Stiffness Matrix in Global Coordinates
T	Temperature Increase
T_b	Temperature Increase at the Bottom
T_t	Temperature Increase at the Top
$T_{bif.}$	The Temperature at a Bifurcation Point
u_1 and u_2	Relative Element Deformations
$\Delta\mathbf{u}$	Incremental Relative Member Deformations
$\Delta\mathbf{v}$	Incremental Member Deformations in Global Coordinates
$\mathbf{x}$	Generalized Coordinates Composed of Translations and Rotation
$\Delta\mathbf{x}$	Incremental Joint Displacements
y	deflection of the element's centroidal axis with respect to its chord
α	Coefficient of Thermal Expansion
θ	The Orientation of the Member in Undeformed Configuration
$\bar{\theta}$	The Orientation of the Chord in Deformed Configuration

λ	The Slenderness Ratio
δ_h	The Horizontal Deflection of the Tip of the Cantilever Beam
δ_r	The Rotation of the Tip of the Cantilever Beam
δ_v	The Vertical Deflection of the Tip of the Cantilever Beam

CHAPTER 1

INTRODUCTION

1.1 General

Fundamentally, structural analyses involve the assessment of the response characteristics of a structure under the applied load effects. Although the conventional linear analysis of structures seems acceptable for most common structures, it cannot precisely predict the structural response in the large deformation, instability, and failure range in which the applied external loads exceed the service limit. This limitation of the first-order analysis exists inherently based on the two fundamental assumptions adopted in the linear analysis: material linearity, represented by linear stress-strain laws; and geometric linearity, characterized by a linear strain-displacement relationship. Furthermore, the equilibrium equations are also expressed in the un-deformed state of a structure. Despite these limitations, the linear analysis could be a good approximation of the portions of the nonlinear response near the reference state. Hence, there is an increase in the application of the nonlinear analysis for the structural design in order to study the true response of the structure. In this study, the focus of the investigation is on the geometrically nonlinear analysis.

Two approaches are used to formulate the geometrically nonlinear analysis: the Beam-Column formulation and the finite element formulation. In the Beam-Column theory, the member force-deflection relations are based on the exact solution of the underlying differential equations (Kassimali, 2010), which are more complicated and require iterations to obtain the dimensionless axial force parameter q. In the finite element formulation, an approximate solution to the differential equation (shape function) is assumed, and force-deformation relations are simple, and are the functions of polynomial equations. The Beam-Column formulation generally requires fewer iterations and load/temperature steps with the natural structural members at the system level. However, the finite element formulation demands more elements, iterations and/or temperature steps to get adequate results. While the literature does contain studies comparing the performance of the Beam-Column and the finite element formulations for framed structures subjected to external loads (Kassimali, 1976), no such studies seem to have been reported for the frames subjected to temperature changes.

1.2 A Brief Review of Literature

An excellent review of the recent literature on the subject of nonlinear analysis subject to thermal effect can be found in the journal published by the American Society of Civil Engineers (ASCE) in 2010 (Kassimali, 2010). This nonlinear analysis procedure is an extension of a previous formulation for static loads (Kassimali, 1976) since it includes the thermal loading. The local element force-deformation relation with the inclusion of the stability and bowing functions originally expressed was extended to incorporate the effect of changes in chord lengths in the presence of thermal strain and bowing effects due to a temperature gradient. Using this method, the numerical studies were carried out on structures composed of prismatic, elastic, and straight elements that were subjected to temperature changes uniformly along their longitudinal axis and the linear temperature gradient across the cross-sectional depth. The external loads were applied only at the joints of the frames and these loads were considered non-follower load, that is, independent of the structural configuration. The results for the large displacements and stability of these plane frames were compared to the experimental (Rubert and Schaumann, 1986) and the analytical results (Chan and Chan, 2001) available in the literature, and they were found in general agreement with the previous results. However, it was noted that the procedure is not valid for an analysis involving yielding and material nonlinearity.

1.3 Objective and Scope

The objective of this study is to investigate the accuracy and computational efficiency of two commonly used formulations for performing the geometrically nonlinear thermal analysis of framed structures. The formulations considered are the followings: the Beam-Column formulation and the updated Lagrangian version of the finite element formulation that has been adopted in the widely used commercial software SAP2000. These two formulations are used to generate extensive numerical data for three plane frame configurations, which are then compared to evaluate the performance of the two formulations.

The Beam-Column formulation used in this study was recently published in ASCE journal (Kassimali, 2010). For finite element formulation, it was decided to use the well-known commercial software SAP2000, which is based on Updated Lagrangian formulation (CSI SAP2000).

The specific objectives are to examine: (1) the accuracy of the results yielded by the two formulations when a minimum number (one or two) of elements per natural members and/or temperature steps are used in the analysis, and (2) how the accuracy of the results improves as the number of elements and/or steps are increased. The number of Newton-Raphson iteration cycles required for convergence at each temperature step is also monitored, as well as the computer run time required for each analysis. The numerical results thus obtained for three benchmark structures are presented in this report.

CHAPTER 2

THE BEAM-COLUMN FORMULATION

The Beam-Column formulation used in this study for large displacement analysis of elastic plane frames subjected to temperature changes is presented in this chapter. This method of analysis was initially published by Kassimali (2010).

2.1 Element Force-Deformation Relations

Consider an arbitrary element of a plane frame structure subjected to a temperature increase varying linearly over its depth, d, from T_t at the top to T_b at the bottom as shown in Fig. 2.1. The element is assumed straight, prismatic, and elastic and is subjected to uniform temperature changes along the length L of the element. The relationships between the element end-forces, element end moments Q_1 and Q_2 as well as element axial force Q_3, and the relative end rotations, u_1 and u_2, can be determined by solving the differential equation for the bending of the element subjected to prescribed temperature effects, which can be written as follows:

$$EI\left(\frac{d^2y}{dx^2}\right) + Q_3 y = -Q_1 + \left(\frac{Q_1+Q_2}{L}\right)x + EI\alpha\left(\frac{T_b-T_t}{d}\right) \tag{2.1}$$

in which

E = modulus of elasticity;

I = moment of inertia;

α = coefficient of thermal expansion; and

y = deflection of the element's centroidal axis with respect to its chord, in the local coordinate system, due to the combined effect of end moments, axial force, and temperature change.

The relations between end moments, axial force, and end rotations can be obtained by solving Eq. 2.1 and applying the boundary conditions, which are as follows:

$$Q_1 = \frac{EI}{L}(c_1u_1 + c_2u_2) + EI\alpha\left(\frac{T_b - T_t}{d}\right) \tag{2.2}$$

$$Q_2 = \frac{EI}{L}(c_2u_1 + c_1u_2) - EI\alpha\left(\frac{T_b - T_t}{d}\right) \tag{2.3}$$

with $c_{1,}$ c_2 = stability functions Livesley and Chandler (1956).

The element axial force Q_3 can be expressed as

$$Q_3 = EA\left[\frac{u_3}{L} - c_b + \alpha\left(\frac{T_b + T_t}{2}\right)\right] \tag{2.4}$$

in which $c_b = b_1(u_1 + u_2)^2 + b_2(u_1 - u_2)^2$ (2.5)

The term c_b denotes the axial strain due to flexural bowing action, with b_1, b_2= bowing functions (Saafan, 1963). The axial strain (c_b) expressed in Eq. (2.5) comprises of the bowing effect due to temperature gradient via Eqs. (2.2) and (2.3): it seems analogous to the form used by (Oran, 1973). From a pragmatic viewpoint, the stability and bowing functions are explicitly expressed in terms of a dimensionless axial force parameter (q).

$$q = \frac{Q_3}{Q_{Eular}} = \frac{Q_3L^2}{\pi^2EI} \tag{2.6}$$

It is worthwhile mentioning that in the absence of an axial force, Q_3 = 0, the stability functions $c_1$and c_2 becomes 4 and 2 respectively. Then, the end moments so obtained become the familiar expression used in the slope-deflection equation for linear analysis.

The global member-forces **F** in Fig. 2.2 can be related to an element's local forces **Q** (Oran, 1973).

$$\mathbf{F}=\mathbf{B}\mathbf{Q} \tag{2.7}$$

in which **B**= Transformation matrix

$$B = \frac{1}{\bar{L}}\begin{bmatrix} -n & -n & m\bar{L} \\ m & m & n\bar{L} \\ \bar{L} & 0 & 0 \\ n & n & -m\bar{L} \\ -m & -m & -n\bar{L} \\ 0 & \bar{L} & 0 \end{bmatrix} \tag{2.8}$$

in which

$$m = \cos\bar{\theta}\ ; m = \sin\bar{\theta} \tag{2.9}$$

In Eqs. (2.8) and (2.9), $\bar{L}$ and $\bar{\theta}$ represent the length and orientation of the chord of the element in its deformed configuration, respectively, as shown in Fig. 2.2. The method to obtain $\bar{L}$, *m*, *n*, and **u** from the element's global end-displacements, **v**, has been previously published by (Kassimali, 1983).

2.2 Element Incremental Stiffness Relations

By differentiating the element force-deformation relations [Eqs. (2.2) – (2.4)], term by term, with respect to **u**, T_b, and T_t, the tangent-stiffness relationships in local coordinates (Fig. 2.1) can be determined and expressed in concise form as follows:

$$\Delta\mathbf{Q} = \mathbf{k}\ \ \Delta\mathbf{u} + \Delta\mathbf{Q_T} \tag{2.10}$$

in which **k** = element tangent-stiffness matrix in local coordinates (Oran 1973); and

$\mathbf{\Delta Q_T}$ = the element fixed end-forces due to temperature increment ΔT_b and ΔT_t

For an element with no hinges, the $\mathbf{\Delta Q_T}$ can be expressed as follows:

$$\Delta Q_T = EI\alpha \begin{bmatrix} \frac{G_1}{2LH}\ (\Delta T_b + \Delta T_t) + \frac{1}{d}(\Delta T_b - \Delta T_t) \\ \frac{G_2}{2LH}\ (\Delta T_b + \Delta T_t) - \frac{1}{d}(\Delta T_b - \Delta T_t) \\ \frac{\pi^2}{2L^2\,H}\ (\Delta T_b + \Delta T_t) \end{bmatrix} \tag{2.11}$$

in which

$$G_1 = c_1' u_1 + c'_2\, u_2 \tag{2.12}$$

$$G_2 = c_2' u_1 + c'_1\, u_2 \tag{2.13}$$

$$H = \frac{\pi^2}{\lambda^2} + b'_1(u_1 + u_2)^2 + b'_2\,(u_1 - u_2)^2 \tag{2.14}$$

Where $\lambda = \frac{L}{\sqrt{\frac{I}{A}}}$ (2.15)

The prime superscript used in above Equations shows a differentiation with respect to *q* (See Appendix).

2.3 Member Tangent-Stiffness Matrices

It is pertinent to establish element tangent-stiffness relationship between global end-forces and element's global end-displacements developed due to temperature effect. The incremental relationship between member end-forces, end-displacement, and member end-forces due to temperature effect can be expressed as

$$\Delta \mathbf{F} = \mathbf{K}\ \Delta \mathbf{v} + \Delta \mathbf{F}_{\boldsymbol{T}} \tag{2.16}$$

in which the element tangent-stiffness, **K**, is given by (Oran, 1973)

$$\mathbf{K} = \mathbf{BkB}^{\boldsymbol{T}} + \sum_{\boldsymbol{j}=\mathbf{1}}^{\mathbf{3}} Q_j \boldsymbol{g}^{(j)} \tag{2.17}$$

with $\boldsymbol{g}^{(j)}$ represents the geometric matrices (Appendix); the superscript T denotes the transpose; and

$$\Delta\mathbf{F_T} = \mathbf{B}\ \Delta\mathbf{Q_T} \tag{2.18}$$

With $\Delta\mathbf{F_T}$ indicates global end-forces due to temperature increments ΔT_b and ΔT_t.

2.4 Structural Equilibrium Equations

For a plane frame subjected to external joint loads **P** and temperature changes **T**, the following nonlinear equations of equilibrium of the entire structure can be written as

$$\mathbf{f}\ (\mathbf{x}, \mathbf{T}) = \mathbf{P} \tag{2.19}$$

in which **f(x, T)** represents resultant internal forces; **x** denotes generalized coordinates consisting of translations and rotations of the joints; and **P** refers to the applied external joint forces.

It has already mentioned that the member force-deformation relations are nonlinear that would lead to a highly nonlinear relationship between **f** and **x**. In order to perform the computations, a differential form of Eq. 2.19 can be expressed as

$$[\Delta\mathbf{P}] = [\mathbf{S}][\Delta\mathbf{x}] \tag{2.20}$$

In the presence of thermal effects, it can be rewritten as

$$\Delta\mathbf{P} - \Delta\mathbf{P_T} = \mathbf{S}\ \Delta\mathbf{x} \tag{2.21}$$

in which $\Delta\mathbf{P}$ and $\Delta\mathbf{x}$ refer to incremental values of external joint loads and joint displacements, respectively; **S** represents the structural tangent-stiffness matrix; and the vector $\Delta\mathbf{P_T}$ denotes the structural fixed-joint forces due to the effect of temperature increment. These matrices **S** and $\mathbf{P_T}$ can be expediently assembled from their element tangent-stiffness matrix **K** and global end-forces $\Delta\mathbf{F_T}$ respectively, with the aid of the element code number technique (Kassimali, 1999).

2.5 Computational Procedure

It has been mentioned in the literature that two types of computational techniques are commonly applied to solve the nonlinear problems: linearized incremental procedure and the Newton-Raphson iteration technique. In this study, the geometrically nonlinear analysis of plane frame structures is performed by using an incremental load approach. The above-mentioned conventional Newton-Raphson type of iteration technique is applied at each load level or temperature step until the joint equilibrium equations on the deformed configuration are satisfied within a prescribed tolerance limit.

Consider a specified set of external joint loads represented by $\{\mathbf{P}\}$ is applied on a plane frame, and our objective is to determine the deformed configuration i.e. the joint displacement $\mathbf{x}$ of a plane frame. The solution technique is shown in Fig. 2.3.

From above mentioned Eq. 2.19,

$$\{\mathbf{P}\} = \{\mathbf{f}\ (\mathbf{x}, \mathbf{T})\}$$

By analogy to a first order Taylor series expansion, we can write as

$$\{\mathbf{P}\} = \left\{\mathbf{f}\ (\mathbf{x}, \mathbf{T})_{\mathbf{j}}\right\} + \left[\mathbf{S}_{\mathbf{j}}\right]\left\{\Delta\mathbf{x}_{\mathbf{j}}\right\} \tag{2.22}$$

in which $\left\{\Delta\mathbf{x}_{\mathbf{j}}\right\}$ denotes a small increment to previous displacement and $\left\{\mathbf{f}(\mathbf{x}, \mathbf{T})_{\mathbf{j}}\right\}$ represent internal joint forces corresponding to $\mathbf{x}$. The above equation can be written as

$$\left[\mathbf{S}_{\mathbf{j}}\right]\left\{\Delta\mathbf{x}_{\mathbf{j}}\right\} = \left\{\Delta\mathbf{Q}_{\mathbf{j}}\right\} \tag{2.23}$$

in which $[\mathbf{S}]$= tangent-stiffness matrix for the structure; and

$$\left\{\Delta\mathbf{Q}_{\mathbf{j}}\right\} = \{\mathbf{P}\} - \left\{\mathbf{f}(\mathbf{x}, \mathbf{T})_{\mathbf{j}}\right\} \tag{2.24}$$

in which $\Delta\mathbf{Q_j}$ denotes the updated unbalanced joint force vector. These unbalanced joint forces are treated as a load increment and the corresponding correction vector $\Delta\mathbf{x}$ is determined by applying the linearized incremental relationship used in Eq. 2.23.

$$\{\Delta\mathbf{Q_j}\} = [\mathbf{S_j}]\{\Delta\mathbf{x}_j\} \tag{2.25}$$

$$\{\mathbf{x_{j+1}}\} = \{\mathbf{x_j}\} + \{\Delta\mathbf{x_j}\} \tag{2.26}$$

In this step, $[\boldsymbol{S_j}]$ represented the updated tangent-stiffness matrix corresponding to joint displacement vector $\Delta\mathbf{x_j}$, and a new configuration $\Delta\mathbf{x_{j+1}}$ is then evaluated by adding the correction vector $\Delta\mathbf{x_j}$ to the current configuration $\mathbf{x_j}$. This iterative process is repeated until the updated correction vector $\Delta\mathbf{x}$ becomes sufficiently small so that the equations of equilibrium are satisfied within the prescribed tolerance.

Various criteria can be used to check whether the iterative process has converged. In this study, we adopted a convergence criteria based on a comparison of the changes in joint displacement, $\Delta\mathbf{x_j}$, to their cumulative displacement $\mathbf{x_j}$. In applying this criteria, translations and rotations of the joints are treated as separate groups, and convergence criteria is assumed to have occurred when the following inequality is satisfied independently for each groups.

$$\frac{\sum_j^{NDOF}(\Delta\mathbf{x_j})^2}{\sum_j^{NDOF}(\mathbf{x}_j)^2} \leq e \tag{2.27}$$

in which the dimensionless quantity *e* denotes a specified tolerance adopted in the analysis. A value of *e* = 0.001 has been used in all the numerical solutions for the bench marked structures selected in this study.

In this nonlinear thermal analysis, we have performed two different types of iterations: the first iteration is employed to calculate the axial forces of elements from their known deformed configuration at the element level; and the other iteration is applied to determine the joint displacements of the deformed structure after determining the joint load vectors. The first iteration becomes essential to determine axial forces because the equation for element axial forces Q_3 [Eq. 2.4] consist of bowing functions, which are highly nonlinear function of axial force parameter q. At the element level, the iteration is required to solve Eq. 2.4, which can be expediently rewritten in terms of q, as

$$J(q) = \frac{\pi^2}{\lambda^2} q + c_b - \frac{u_3}{L} - \alpha \left(\frac{T_b + T_t}{2}\right) \tag{2.28}$$

Initially, an appropriate q_i can be selected, and is updated successively using the relation developed by Kassimali (1983)

$$q_{i+1} = q_i + \Delta q_i = q_i - \frac{J(q_i)}{J'(q_i)} \tag{2.29}$$

until the increment Δq is sufficiently small

$$|\Delta q| \le e \tag{2.30}$$

in which

$$J'(q) = \frac{\pi^2}{\lambda^2} + c'_b \tag{2.31}$$

and

$$c'_b = b'_1 (u_1 + u_2)^2 + b'_2 (u_1 - u_2)^2 + 2b_1 (u_1 + u_2)(u'_1 + u'_2) + 2b_2 (u_1 - u_2)(u'_1 - u'_2) \tag{2.32}$$

In which u'_i terms are zero at elements ends rigidly connected to the joints. After evaluating the axial force using the forgoing iteration, the element end moments can be obtained using Eqs. 2.2 and 2.3.

2.6 Computer Program

The computer program was initially developed by Kassimali (2010) for the geometrically nonlinear thermal analysis of plane framed-structures adopted in this study. This program was developed to implement the general method of geometrically nonlinear thermal analysis based on the Beam-Column formulation including stability and bowing functions. This program was developed to incorporated temperature effect for fire analysis (Kassimali, 2010), in which total temperature loading and the number of temperature increments, the convergence tolerance, and the maximum number of iteration cycles that can be performed at any given temperature level can be specified. The temperature-depended material properties are adopted from (AISC 2005). It is imperative to mention that this program contributes results explicitly and efficiently at prescribed temperature range in a single analysis. It means that the program can update the temperature-depended material properties automatically as long as temperature changes. In this study, the computer program was used for two different phenomenon: one nonlinear thermal analysis for temperature change without the inclusion of strength degradation, and the other for thermal analysis incorporating the temperature-depended material properties. The former phenomenon is considered to generate data for the cantilever beam and the axially restrained column, and the later one is employed for the one-story frame analysis.

CHAPTER 3

FINITE-ELEMENT FORMULATION (SAP2000)

The commercial software SAP2000 was selected to perform the finite element analysis because it is widely used in industries. The SAP2000 uses the updated Lagrangian finite element formulation to track the deformed position of the elements. As this software SAP2000 is very sensitive to specified convergence tolerance in large displacement analysis, it necessities smaller steps and convergence tolerance to get the adequate results (CSI SAP2000). This chapter describes some of the options for nonlinear static analysis available in the SAP2000 which were used to generate the nonlinear data in this study.

3.1 Nonlinear Solution Control Parameters

The SAP2000 has several nonlinear solution control parameters available to control the iteration and sub-stepping process. However, the following solution control parameters were selected to simulate the identical nonlinear parameters for the two formulations used in this study: Maximum Total Steps per Stage, Maximum Null (Zero) Steps per Stage, Maximum Constant-Stiffness Iterations per Step, Maximum Newton-Raphson Iterations per Step, and Iteration Convergence Tolerance (Relative).

Maximum Total Steps is the maximum number of steps allowed in the analysis that includes the saved steps as well as intermediate sub-steps. In this study, the maximum number of total steps was assigned to apply the each temperature step in single step.

Maximum Null (Zero) steps are the total null (zero) steps that occur in presence of catastrophic failure or numerical instability during the nonlinear solution procedure. In this study, Maximum Null (Zero) step was assigned to zero that means there is no convergence trouble due to unexpected causes in solution.

Two types of iteration are used to satisfy the equilibrium equations in the deformed state at each step of the analysis in the SAP2000: a Constant-Stiffness Iteration and a Newton-Raphson Iteration. For each step, the former one is tried first in the SAP2000, and then the Newton-Raphson iteration is applied if the convergence is not achieved with the former. In this study, the maximum constant-stiffness iteration per step parameter is set to zero to prevent that type of iterations from being performed in each step since the computational technique adopted in this study is the Newton-Raphson iteration. The literature shows that the Newton-Raphson iteration is more effective for geometric nonlinearity. It is imperative that the maximum number of Newton-Raphson iteration specified must be greater than the actual number of iteration required in each step.

Iteration Convergence Tolerance available in the SAP2000 is the relative tolerance limit which compares the magnitude of force error with the magnitude of the load acting on the structure. For large-displacement analysis, a small value of iteration convergence tolerance is desirable to achieve good results.

3.2 Result Saved Options

Generally, after performing nonlinear analysis, the SAP2000 only saves the output for the final state of the structures after the full load has been applied to it. In this study to investigate the nonlinear response of the structures, the intermediate results were also saved by choosing the “multiple states” options in the results saved option menu available in the program. The software SAP2000 automatically determines the spacing of the steps saved by dividing total force or total displacement target by the specified Minimum Number of Saved Steps (CSI SAP2000).

CHAPTER 4

NUMERICAL RESULTS AND DISCUSSION

To compare the accuracy and computational efficiency of the Beam-Column and the finite element formulations, a large number of numerical solutions were generated for three benchmark structures shown in Fig. 4.1 under various thermal conditions, using the two formulations.

The main objectives of this numerical study were to examine: (1) the accuracy of the results yielded by the formulations when a minimum number (one or two) of elements per natural members and/or temperature steps are employed in the analysis, and (2) how the accuracy of the results improves as the number of elements and/or steps are increased. The number of Newton-Raphson iteration cycles required for convergence at each temperature step was monitored, as well as the computer run time required for each analysis. The numerical results thus obtained for the three benchmark structures are summarized in this chapter.

Before the SAP2000 software was used to perform any thermal analysis, it was validated by performing the large deformation analysis of a cantilever beam subjected to a concentrated load at its tip. The dimensions and properties of the beam were taken from a previous study by Kocaturk, Akbas, and Simsek (2010), who also analyzed the same structure by the SAP2000. From Fig. 4.2, it can be seen that the results obtained in the present study are almost identical to those reported in the literature.

4.1 Cantilever Beam

The first benchmark structure studied was the cantilever beam subjected to pure bending caused by a lateral temperature gradient, as shown in Fig. 4.1(a). The dimensions and the cross-sectional and material properties used in this study are identical to those used by Kassimali (2010) and are listed in Table 4.1(a). For this structure, the temperature dependent material properties were not considered.

The exact theoretical solution for this problem has been derived by Kassimali (2010). In this study, this exact solution was used a benchmark against which the numerical results yielded by the Beam-Column and the finite-element formulations were compared. The numerical results generated for the cantilever beam are presented in Figs. 4.3 to 4.6 and Table 4.2 to 4.5.

The temperature-deformation curves obtained for the cantilever beam, when subjected to a total temperature increase of 1600°C applied in 40 equal increments (of 40°C each), are depicted in Figs. 4.3 and 4.4. Of these, Fig. 4.3 shows the horizontal deflection of the free end, and Fig. 4.4 depicts the vertical deflection of the free end of the cantilever beam. It can be seen from these figures that, even when the deformations are very large, the Beam-Column formulation with only one element yields highly accurate results which are in close agreement with the exact solution.

The SAP2000 program failed to converge, even in the small deformation range, when one element was used to model the structure. It can be seen from Figs. 4.3 and 4.4 that when the beam was modeled with two elements, the SAP2000 results were in agreement with the exact solutions in the small deformation range, but tended to deviate away from the exact solutions as the deformations increased. Finally, when the 4-element and 10-element models were analyzed on the SAP2000, the deflection results obtained were in close agreement with the exact solutions (see Figs. 4.3 and 4.4).

A commonly used measure of the computational efficiency of any nonlinear algorithm is its rate of convergence, i.e., the number of iteration cycles it requires to reach the solution. To compare the computational efficiency of the two formulations considered in this study, the number of Newton-Raphson iteration cycles required by each formulation for each model, at each temperature level was traced. These numbers, based on a convergence tolerance of 0.001, are listed in Table 4.2, from which it can be seen that the Beam-Column formulation, with just one element, required significantly less iteration cycles for convergence than the 4-element model analyzed on the SAP2000. The number of cycles for the 10-element SAP2000 model is closer to those for the 1-element Beam-Column model. However, it should be recognized that 1-element model involves solving three simultaneous equations in each iteration cycle; whereas the 10-element model requires the solution of thirty equations per cycles.

Next, to examine the effect of step size on the performance of the Beam-Column and the finite element formulations, the analyses performed previously were repeated with the number of steps reduced by half. In other words, the total temperature increase of 1600°C was applied in 20 increments of 80°C each. The numerical results thus obtained are given in Figs. 4.5 and 4.6 and Table 4.3. From these figures, it can be seen that with the 1-element model, the Beam-Column formulation predicts the horizontal deflection in close agreement with the exact solution, but the vertical deflection tends to deviate from the exact in the large deformation range. This indicates using large step sizes with 1-element model can adversely affect the results. As shown in Table 4.4, when beam was divided into two elements, the results become very close to the exact solutions.

In addition to the 40- and 20- step temperature sequences, described in the preceding paragraphs, a number of other loading sequences were tried. Some of results thus obtained are summarized in Table 4.4. From this table, it can be seen that the Beam-Column formulation required only one or two elements to predict accurate results even when the bending deformations of the cantilever beam ware substantial, whereas the finite-element formulation of the SAP2000 required a minimum of four elements to provide adequately accurate results.

Finally, Table 4.5(a) lists the time required by the SAP2000 to run the various models of the cantilever beam. In contrast, the computer time required to perform the Beam-Column analyzes was significantly less.

4.2 Axially Restrained Column

The second benchmark structure studied was the axially restrained column subjected to axial temperature changes along its longitudinal axial with a small temperature gradient ($e = 0.10$) in lateral direction, as shown in Fig. 4.1(b). The dimensions and the cross-sectional and material properties used in this study are identical to those used by Kassimali (2010) and are listed in Table 4.1(b). For this structure, the temperature dependent material properties were not considered.

In this study, the numerical results yielded by the Beam-Column and the finite-element formulations were compared. The numerical results generated for the column are presented in Figs. 4.7 to 4.11 and Table 4.6 to 4.8.

The temperature-deformation curves obtained for the column, when subjected to a total temperature increase of 800°C applied in 40 equal increments (of 20°C each), are depicted in Figs. 4.6 and 4.7. Of these, Fig. 4.6 shows the rotation of the hinged support, and Fig. 4.7 depicts the mid-span deflection of the column. It can be seen from these figures that, even when the deformations are very large, the Beam-Column formulation with only one element yields highly accurate results which are in close agreement with the Beam-Column formulation with 2-element model.

The SAP2000 program failed to provide adequate results, even in the small deformation range beyond the bifurcation temperature of 107.5°C, when two elements was used to model the structure. It can be seen from Figs. 4.6 and 4.7 that when the column was modeled with two elements, the SAP2000 results were in agreement with the Beam-Column solutions in the small deformation range below the bifurcation temperature, but tended to deviate away from the Beam-Column solutions as the deformations increased. Finally, when the 4-element and 10-element models were analyzed on the SAP2000, the deflection results obtained were in close agreement with the Beam-Column solutions (see Figs. 4.6 and 4.7).

A commonly used measure of the computational efficiency of any nonlinear algorithm is its rate of convergence, i.e., the number of iteration cycles it requires to reach the solution. To compare the computational efficiency of the two formulations considered in this study, the number of Newton-Raphson iteration cycles required by each formulation for each model, at each temperature level was traced. These numbers, based on a convergence tolerance of 0.001, are listed in Table 4.8, from which it can be seen that the Beam-Column formulation, with just one element, required significantly less iteration cycles for convergence than the 4-element model analyzed on the SAP2000. The number of cycles for the 10-element SAP2000 model is closer to those for the 1-element Beam-Column model. However, it should be recalled that 1-element model involves solving three simultaneous equations in each iteration cycle; whereas the 10-element model requires the solution of thirty equations per cycles.

Next, to examine the effect of step size on the performance of the Beam-Column and the finite element formulations, the analyses performed previously were repeated with the number of steps reduced by half. In other words, the total temperature increase of 800°C was applied in 20 increments of 40°C each. The numerical results thus obtained are given in Figs. 4.9 and 4.10 and Table 4.7. From these figures, it can be seen that with the 1-element model, the Beam-Column formulation predicts the rotation at the hinged end and mid-span displacement in close agreement with the Beam-Column solution with 2-element model, but the SAP2000 results tends to deviate from the Beam-Column results in the large deformation range. This indicates using large step sizes with 1- and 2-element models in the SAP2000 can adversely affect the results. As shown in Table 4.8, when the column was divided into four or more elements in the SAP2000, the results become very close to the Beam-Column solutions.

In addition to the 40- and 20- step temperature sequences, described in the preceding paragraphs, a number of other loading sequences were tried. Some of results thus obtained are summarized in Table 4.8. From this table, it can be seen that the Beam-Column formulation required only one or two elements to predict accurate results even when the deformations of the column were substantial, whereas the finite-element formulation of the SAP2000 required a minimum of four elements to provide adequately accurate results.

Finally, the graph plotted between det **S** and temperature increase T illustrates in Fig. 4.11 that the stiffness of the column structure with imperfection (e = 0.10) becomes minimum at temperature lower than the bifurcation temperature for the perfect column (e = 0.0).

4.3 One-Story Frame

The third benchmark structure studied was the one-story frame subjected to the temperature increase uniformly along the longitudinal axis and with initial constant joint loads, as shown in Fig. 4.1(c). The dimensions and the cross-sectional and material properties used in this study are identical to those used by Kassimali (2010) and are listed in Table 4.1(c). For this structure, the temperature dependent material properties were incorporated.

The temperature-dependent material properties were assigned as specified in Appendix 4 of the AISC Specification for Structural Steel Buildings (AISC 2005). In this study, the influence of elevated temperature on the modulus of elasticity and the coefficient of thermal expansion was incorporated into the analysis by changing these two properties as the temperature was increased during the analysis.

In this study, this Beam-Column solution was used a benchmark against which the numerical results yielded by the Beam-Column and the finite-element formulations were compared. The numerical results generated for the one-story frame are presented in Figs. 4.12 to 4.17 and Table 4.9 to 4.11.

The temperature-deformation curves obtained for the one-story frame, when subjected to a total temperature increase of 600°C applied in 20 equal increments (of 30°C each), are depicted in Figs. 4.12 to 4.14. Of these, Figs. 4.12 and 4.13 shows the horizontal deflection ($\delta 1$, $\delta 2$) of the left and right joints at the top of the frame, and Fig. 4.14 depicts the horizontal deflection ($\delta 3$) at the midspan of the one-story frame. It can be seen from these figures that, the deformations of this frame are not very large, and consequently the Beam-Column formulation with only one element yields highly accurate results which are in close agreement with the SAP2000 solution.

The SAP2000 program provides the satisfactory results, even when one element was used to model the structure. It can be seen from Figs. 4.12 to 4.14 that when the one-story frame was modeled with different elements, the SAP2000 results were in agreement with the Beam-Column solutions in the nonlinear range. Finally, even when the 1-element and 2-element models were analyzed on the SAP2000, the deflection results obtained were in close agreement with the Beam-Column solutions (see Figs. 4.12 to 4.14).

A commonly used measure of the computational efficiency of any nonlinear algorithm is its rate of convergence, i.e., the number of iteration cycles it requires to reach the solution. To compare the computational efficiency of the two formulations considered in this study, the number of Newton-Raphson iteration cycles required by each formulation for each model, at each temperature level was traced. These numbers, based on a convergence tolerance of 0.001, are listed in Table 4.9, from which it can be seen that both the Beam-Column and the SAP2000 formulations required essentially the same iteration cycles for convergence.

Next, to examine the effect of step size on the performance of the Beam-Column and the finite element formulations, the analyses performed previously were repeated with the number of steps reduced by half. In other words, the total temperature increase of 600°C was applied in 10 increments of 60°C each. The numerical results thus obtained are given in Figs. 4.15 to 4.17 and Table 4.10. From these figures, it can be seen that with the 1-element model, the Beam-Column formulation predicts the horizontal deflection in close agreement with the SAP2000 solution, even in the large deformation range. This indicates the using large step sizes with 1-element model do not affect the results. As shown in Table 4.11, when member was divided into one or two elements, the results become very close to the SAP2000 solutions for different element models.

In addition to the 20- and 10- step temperature sequences, described in the preceding paragraphs, a number of other loading sequences were tried. Some of results thus obtained are summarized in Table 4.11. From this table, it can be seen that both the Beam-Column and the SAP2000 formulations required only one or two elements to predict accurate results.

Finally, both the Beam-Column and the SAP2000 formulations needed almost the same iterations in each temperature stage since the nonlinearity of the frame primarily depends on the strength degradation at elevated temperatures instead of on large deformations.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The objective of this study was to investigate the accuracy and computational efficiency of two commonly used formulations for performing the geometrically nonlinear thermal analysis of plane framed structures. The formulations considered are the followings: the Beam-Column formulation and the updated Lagrangian version of the finite element formulation that has been adopted in the commercially well-known software SAP2000. These two formulations were used to generate extensive numerical data for three plane frame configurations, which were then compared to evaluate the performance of the two formulations.

The Beam-Column method is based on an Eulerian formulation that incorporates the effects of large joint displacements. In addition, local member force-deformation relationships are based on the Beam-Column approach that includes the axial strain, flexural bowing, and thermal strain. The other formulation, the SAP2000, is based on the updated Lagrangian finite element formulation. The results for nonlinear thermal responses were generated for three plane structures by these formulations. Then, the data was compared for accuracy of deflection responses and for computational efficiency of the Newton-Raphson iterations required for the thermal analysis. The specific performance parameters considered were: (a) the number of elements per natural member required to obtain accurate results, (b) the number of steps in which the full thermal loading be applied, and (c) the number of Newton-Raphson iteration cycles required for convergence at each load step.

The results of this study indicate that the Beam-Column method is quite efficient and powerful for the geometrically nonlinear thermal analysis of plane frames since the method is based on the exact solution of the differential equations. In comparison to the SAP2000 software, the Beam-Column method requires fewer (one or two) elements per natural member, fewer temperature steps, and fewer iteration cycles, when the deformations of the structures are substantial. For most practical purpose, this formulation can be expected to yield accurate results in the large deformation range without dividing the natural members of the structure into smaller elements and applying the entire thermal load in just one step.

The finite-element formulation of the SAP2000, because of its approximate nature, generally requires the natural members of the structures to be divided into smaller elements to yield adequate results. For structures subjected to large deformations, four or more elements per member may be needed to obtain adequate results. Furthermore, this formulation generally requires that the total thermal load be applied to the structure in smaller steps in order for the Newton-Raphson iteration process to converge. Finally, the computer run-time required to analyze a structure by the SAP2000 is generally significantly larger than when the Beam-Column formulation is used.

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APPENDICES

APPENDIX

STABILITY AND BOWING FUNCTIONS AND THEIR DERIVATIES

1 Stability Functions (Oran 1973)

For compressive axial force, (q>0)

$$c_1 = \frac{\phi \sin\phi - \phi^2 \cos\phi}{2 - 2\cos\phi - \phi\sin\phi} \tag{A1-1}$$

$$c_2 = \frac{\phi^2 - \phi\sin\phi}{2 - 2\cos\phi - \phi\sin\phi} \tag{A1-2}$$

in which

$$\phi^2 = \pi^2 q \tag{A1-3}$$

For zero axial force, (q=0)

$$c_1 = 4 \tag{A1-4}$$

$$c_2 = 2 \tag{A1-5}$$

For tensile axial force, (q<0)

$$c_1 = \frac{\psi^2 \cosh\psi - \psi\sinh\psi}{2 - 2\cosh\psi + \psi\sinh\psi} \tag{A1-6}$$

$$c_2 = \frac{\psi\sinh\psi - \psi^2}{2 - 2\cosh\psi + \psi\sinh\psi} \tag{A1-7}$$

in which

$$\psi^2 = -\pi^2 q \tag{A1-8}$$

2 Bowing Functions (Oran 1973)

$$b_1 = \frac{(c_1 + c_2)(c_2 - 2)}{8\pi^2 q} \quad \text{(A1-9)}$$

$$b_2 = \frac{c_2}{8(c_1 + c_2)} \quad \text{(A1-10)}$$

3 Derivation of Stability and Bowing Functions with respect to *q* (Oran 1973: Kassimali 1976)

$$c_1' = -2\pi^2(b_1 + b_2) \quad \text{(A1-11)}$$

$$c_2' = -2\pi^2(b_1 - b_2) \quad \text{(A1-12)}$$

$$b_1' = -\frac{(b_1 - b_2)(c_1 + c_2) + 2c_2 b_1}{4q} \quad \text{(A1-13)}$$

$$b_2' = \frac{\pi^2(16b_1b_2 - b_1 + b_2)}{4(c_1 + c_2)} \quad \text{(A1-14)}$$

4 Series Expressions for Stability and Bowing Functions (Kassimali 1976)

$$c_1 \cong 4 - \frac{2}{15}\pi^2 q - \frac{11}{6{,}300}\pi^4 q^2 - \frac{1}{27{,}000}\pi^6 q^3 \quad \text{(A1-15)}$$

$$c_2 \cong 2 + \frac{1}{30}\pi^2 q + \frac{13}{12{,}600}\pi^4 q^2 + \frac{11}{378{,}000}\pi^6 q^3 \quad \text{(A1-16)}$$

$$b_1 \cong \frac{1}{40} + \frac{1}{2{,}800}\pi^2 q + \frac{1}{168{,}000}\pi^4 q^2 + \frac{37}{388{,}080{,}000}\pi^6 q^3 \quad \text{(A1-17)}$$

$$b_2 \cong \frac{1}{24} + \frac{1}{720}\pi^2 q + \frac{1}{20{,}160}\pi^4 q^2 + \frac{1}{604{,}800}\pi^6 q^3 \quad \text{(A1-18)}$$

$$c_1' \cong -\frac{2}{15}\pi^2 - \frac{11}{3{,}150}\pi^4 q - \frac{1}{9{,}000}\pi^6 q^2 \quad \text{(A1-19)}$$

$$c_2' \cong \frac{1}{30}\pi^2 + \frac{13}{6{,}300}\pi^4 q + \frac{11}{126{,}000}\pi^6 q^2 \tag{A1-20}$$

$$b_1' \cong \frac{1}{2{,}800}\pi^2 + \frac{1}{84{,}000}\pi^4 q + \frac{37}{129{,}360{,}000}\pi^6 q^2 \tag{A1-21}$$

$$b_2' \cong \frac{1}{720}\pi^2 + \frac{1}{10{,}080}\pi^4 q + \frac{1}{201{,}600}\pi^6 q^2 \tag{A1-22}$$

5 Element Tangent-stiffness Matrix in Local Coordinate (Oran 1973)

$$k = \frac{EI}{L}\begin{bmatrix} c_1 + \frac{G_1^2}{\pi^2 H} & c_2 + \frac{G_1 G_2}{\pi^2 H} & \frac{G_1}{LH} \\ c_2 + \frac{G_1 G_2}{\pi^2 H} & c_1 + \frac{G_2^2}{\pi^2 H} & \frac{G_2}{LH} \\ \frac{G_1}{LH} & \frac{G_2}{LH} & \frac{\pi^2}{L^2 H} \end{bmatrix} \tag{A1-23}$$

6 Element Geometric Matrices (Oran 1973)

$$g^{(1)} = g^{(2)} = \frac{1}{L^2}\begin{bmatrix} -2mn & m^2-n^2 & 0 & 2mn & -(m^2-n^2) & 0 \\ m^2-n^2 & 2mn & 0 & -(m^2-n^2) & -2mn & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2mn & -(m^2-n^2) & 0 & -2mn & m^2-n^2 & 0 \\ -(m^2-n^2) & -2mn & 0 & m^2-n^2 & 2mn & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(A1-24)

$$g^{(3)} = \frac{1}{L}\begin{bmatrix} -n^2 & mn & 0 & n^2 & -mn & 0 \\ mn & -m^2 & 0 & -mn & m^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ n^2 & -mn & 0 & -n^2 & mn & 0 \\ -mn & m^2 & 0 & mn & -m^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(A1-25)

TABLES

Table 4.1 Properties of Plane Frames Used for Numerical Study

(a) Cantilever Beam Properties

L	=	6100	mm
d	=	127	mm
A	=	6452	mm^2
I	=	4.16×10^7	mm^4
E	=	69	kN/mm^2
α	=	2.34×10^{-5}	/°C

(b) Axially Restrained Column Properties

L	=	11000	mm
d	=	203	mm
A	=	5860	mm^2
I	=	4.54×10^7	mm^4
E	=	210	kN/mm^2
α	=	1.20×10^{-5}	/°C

(c) One-Story Frame Properties

L	=	1220	mm
H	=	1170	mm
A	=	764	mm^2
I	=	801400	mm^4
E	=	210	kN/mm^2
α	=	1.20×10^{-5}	/°C

Table 4.2 Iteration Cycles for Cantilever Beam (40 Temperature Steps)

Temp. (T °C)	Temp. Step	Beam-Column (1 Element)	SAP2000 (10 Element)	SAP2000 (4 Element)	SAP2000 (2 Element)
40	1	3	2	3	3
80	2	4	2	3	3
120	3	4	2	3	3
160	4	3	3	3	3
200	5	3	3	3	3
240	6	3	3	3	3
280	7	3	3	3	5
320	8	3	3	3	3
360	9	3	3	3	5
400	10	3	3	3	5
440	11	2	3	3	5
480	12	2	3	3	7
520	13	2	3	3	7
560	14	2	3	3	7
600	15	2	3	3	7
640	16	2	3	5	7
680	17	2	3	3	7
720	18	2	3	3	9
760	19	2	3	5	11
800	20	2	3	3	11
840	21	2	3	5	11
880	22	2	3	3	11
920	23	2	3	5	12
960	24	2	3	3	14
1000	25	2	3	5	38
1040	26	2	3	5	257
1080	27	2	3	5	106
1120	28	2	3	5	63
1160	29	2	3	5	32
1200	30	2	3	5	100
1240	31	2	3	5	2031
1280	32	2	3	5	1555
1320	33	2	3	5	2510
1360	34	2	3	5	2855
1400	35	2	3	5	31749
1440	36	2	3	5	12853
1480	37	2	3	5	604
1520	38	2	3	5	55377
1560	39	2	3	5	32413
1600	40	2	3	7	65848

Table 4.3 Iterations Cycles for Cantilever Beam (20 Temperature Steps)

Temp. (T°C)	Temp. Steps (80 steps)	Beam-Column (1 Element)	SAP2000 (10 Element)	SAP2000 (4 Element)	SAP2000 (2 Element)
0	0				
80	1	4	3	3	3
160	2	5	3	3	3
240	3	4	3	3	5
320	4	4	3	3	7
400	5	4	3	5	9
480	6	3	3	3	9
560	7	3	3	5	10
640	8	3	3	3	14
720	9	3	3	5	21
800	10	2	3	3	>100,000
880	11	2	3	7	11
960	12	2	5	5	>100,000
1040	13	2	3	7	91
1120	14	2	3	7	34
1200	15	2	3	7	111
1280	16	2	5	7	221
1360	17	2	3	7	
1440	18	2	3	7	
1520	19	2	5	7	
1600	20	2	3	7	

Table 4.4 Numerical Results for Cantilever Beam Subjected to 1600°C Temperature Increase along Depth

S. N.	Formulation used	No. of Elements	Temp. steps	Hz. Disp. (mm)	V. Disp. (mm)	Iteration Cycles	Accuracy (%)		Remarks
							Hz. Disp.	V. Disp.	
1	Exact Solution			-6845.36	-3219.5		Exact		
2	Beam-Column	1	1	-6734.3	-2739.9	11	-1.62	-14.90	
			20	-6734.3	-2740.0	55	-1.62	-14.89	
		2	1	-6840.7	-3199.9	16	-0.07	-0.61	*
			20	-6840.7	-3200.0	64	-0.07	-0.61	
3	FEM in SAP2000	2	1						
		2	20	-6422.82	-4131.3	>100,000	-6.11	29.11	17 Temp. steps
			40			>100,000			Incomplete
			80	-6954.76	-3700.3	477	1.67	15.64	
		4	20	-6870.86	-3331.1	104	0.44	4.10	
			40	-6871.07	-3330.6	162	0.44	4.09	
			80	-6869.06	-3334.7	230	0.41	4.21	
		10	20	-6848.71	-3238.4	66	0.12	1.20	
			40	-6849.26	-3237.2	117	0.13	1.17	
			80	-6849.38	-3237.0	182	0.13	1.16	
*** Comparisons are with respect to the Beam-Column results for two elements**									

Table 4.5 Run Time Required for Plane Frame Analysis

Time (seconds) required for cantilever beam analysis in SAP2000

Temperature steps	10 elements	4 elements	2 elements	Remarks
20	7.48	13.33	98905.31	27.47 Hour for 2 elements
40	10.01	12.5	16799.34	4.66 Hour for 2 elements
80	15.1	16.49	32.05	

Table 4.6 Iteration Cycles for Column (40 Temperature Steps)

Temp. (T °C)	Temp. Steps	Beam-Column		SAP2000 model		
		1 element	2 element	10 element	4 element	2 element
0	0					
20	1	1	2	1	1	2
40	2	2	2	1	2	2
60	3	2	2	1	2	2
80	4	3	3	1	3	2
100	5	4	4	4	3	3
120	6	2	2	3	3	5
140	7	3	3	3	3	6
160	8	2	2	3	3	7
180	9	2	2	3	3	7
200	10	2	2	3	3	7
220	11	2	2	3	3	7
240	12	2	2	2	3	8
260	13	2	2	2	3	8
280	14	2	2	2	3	8
300	15	2	2	2	3	8
320	16	2	2	2	3	8
340	17	1	1	2	3	8
360	18	1	1	2	3	8
380	19	1	1	2	3	8
400	20	1	1	2	3	8
420	21	1	1	2	3	8
440	22	1	1	2	3	8
460	23	1	1	2	3	8
480	24	1	1	2	3	8
500	25	1	1	2	3	8
520	26	1	1	2	3	8
540	27	1	1	2	4	8
560	28	1	1	2	4	8
580	29	1	1	2	4	8
600	30	1	1	2	4	8
620	31	1	1	2	4	8
640	32	1	1	2	4	8
660	33	1	1	2	4	8
680	34	1	1	2	4	8
700	35	1	1	2	4	8
720	36	1	1	2	4	8
740	37	1	1	2	4	8
760	38	1	1	2	4	8
780	39	1	1	2	4	8
800	40	1	1	1	1	8

Table 4.7 Iteration Cycles for Column (20 Temperature Steps)

Newton-Raphson Iteration required for Axially Restrained Column (20 Temperature steps)						
Temp. (T °C)	Temp. Steps	Beam-Column		SAP2000 model		
		1 element	2 element	10 element	4 element	2 element
0	0					
40	1	2	2	1	2	2
80	2	3	3	1	2	2
120	3	6	13	11	19	10
160	4	3	3	5	4	3
200	5	2	3	3	4	5
240	6	2	2	3	4	6
280	7	2	2	3	4	8
320	8	2	2	3	4	8
360	9	2	2	3	4	8
400	10	2	2	3	4	9
440	11	2	2	3	4	9
480	12	2	2	3	4	9
520	13	2	2	3	4	9
560	14	1	2	3	4	9
600	15	2	1	3	4	9
640	16	1	1	3	4	9
680	17	1	1	3	4	9
720	18	1	1	3	4	
760	19	1	1	3	4	
800	20	1	1	3	4	

Table 4.8 Numerical Results for Axially Restrained Column Subjected to 800°C Temperature Increase (*e*=0.10)

S. N.	Formulation used	No. of Elements	Temp. steps	Rotation at joint 2	Hz. Disp. At joint 3	Iteration Cycles	Accuracy (%)		Remarks
							Hz. Disp.	V. Disp.	
1	Beam-Column	1	1	-0.0913		4			
			10	0.2403		25			
			20	0.2403		40			
			40	-0.2403	0.00	59			
		2	1	0.2402	577.27	8			*
			10	-0.2135	-586.54	25			
			20	0.2402	577.27	48			
			40	-0.2402	577.27	60			
2	FEM in SAP2000	2	1	-0.0119	-2.23	3			
			10	0.2536	-705.85	64	5.59	-22.27	
			20			142			
			40	-0.2876	712.57	282	-19.73	23.44	
		4	1	0.0040	0.53	4			
			10	0.2254	-616.60	40	-6.15	-206.81	
			20	-0.2522	607.84	91	-5.00	5.30	
			40	-0.2522	607.84	127	-5.00	5.30	
		10	1	0.0031	0.12	4			
			10	-0.1391	-282.23	34	42.08	51.11	
			20	-0.2418	581.08	66	-0.68	0.66	
			40	-0.2418	581.08	83	-0.68	0.66	

*** Comparisons are with respect to the Beam-Column results for two elements**

Table 4.9 Iteration Cycles for One-Story Frame (20 Temperature Steps)

Temp T (°C)	Temp. step	Beam-Column		SAP2000 model			
		2 element	1 element	10 element	4 element	2 element	1 element
0	0	3	3			2	2
30	1	2	1	1	2	2	2
60	2	2	2	2	2	2	2
90	3	2	2	2	2	2	2
120	4	2	2	2	2	2	2
150	5	2	2	2	2	2	2
180	6	2	2	2	2	2	2
210	7	4	2	2	2	2	2
240	8	4	3	2	2	2	2
270	9	4	3	2	2	2	2
300	10	3	3	2	3	3	2
330	11	3	3	2	3	3	3
360	12	3	3	3	3	3	3
390	13	3	3	3	3	3	3
420	14	3	3	3	3	3	3
450	15	3	3	3	3	3	3
480	16	3	3	3	3	3	3
510	17	3	3	3	3	3	3
540	18	4	4	3	3	3	4
570	19	6	6	3	4	4	5
600	20						

Table 4.10 Iteration Cycles for One-Story Frame (10 Temperature Steps)

Temp T (°C)	Temp. step	Beam-Column		SAP2000 model			
		2 element	1 element	10 element	4 element	2 element	1 element
0	0	3	3				
60	1	2	2	2	2	2	2
120	2	4	3	2	2	2	2
180	3	4	3	2	2	3	2
240	4	4	3	2	3	3	3
300	5	4	3	3	3	3	3
360	6	3	3	3	3	3	3
420	7	3	3	3	3	3	3
480	8	4	3	3	3	3	3
540	9	4	4	3	3	3	4
600	10						

Table 4.11 Numerical Results for One-Story Frame Subjected to 540°C Temperature Increase

S. No.	Formul-ation used	No. of Elements	Temp. Steps	Hz. Displacement (mm) Joint 1	Joint 2	Joint 3	Iteration Cycles	Accuracy (%) Hz. Disp.	V. Disp.	Remarks
1	Beam-Column	1	1	41.09	50.33		7			
			10	41.09	50.33		30			
			20	41.09	50.33		53			
		2	1	41.13	50.39	31.65	7	0.09	0.13	*
			10	41.14	50.37	31.64	35	0.11	0.09	
			20	41.14	50.37	31.64	58	0.11	0.09	
2	FEM in SAP2000	1	1	39.79	50.07		7	-3.17	-0.52	
			10	40.04	49.29		25	-2.57	-2.06	
			20	40.04	49.29		50	-2.57	-2.06	
		2	1	40.00	50.27	31.54	6	-2.67	-0.11	
			10	40.23	49.50	31.08	25	-2.09	-1.64	
			20	40.24	49.49	31.07	49	-2.09	-1.67	
		4	1	40.03	50.29	31.55	5	-2.60	-0.08	
			10	40.27	49.50	31.08	24	-2.02	-1.64	
			20	40.27	49.50	31.08	49	-2.02	-1.64	
		10	1	40.03	50.29	31.55	4	-2.58	-0.07	
			10	40.27	49.50	31.08	23	-2.00	-1.63	
			20	40.27	49.50	31.08	45	-2.00	-1.63	

*** Comparisons are with respect to the Beam-Column results for two elements**

FIGURES

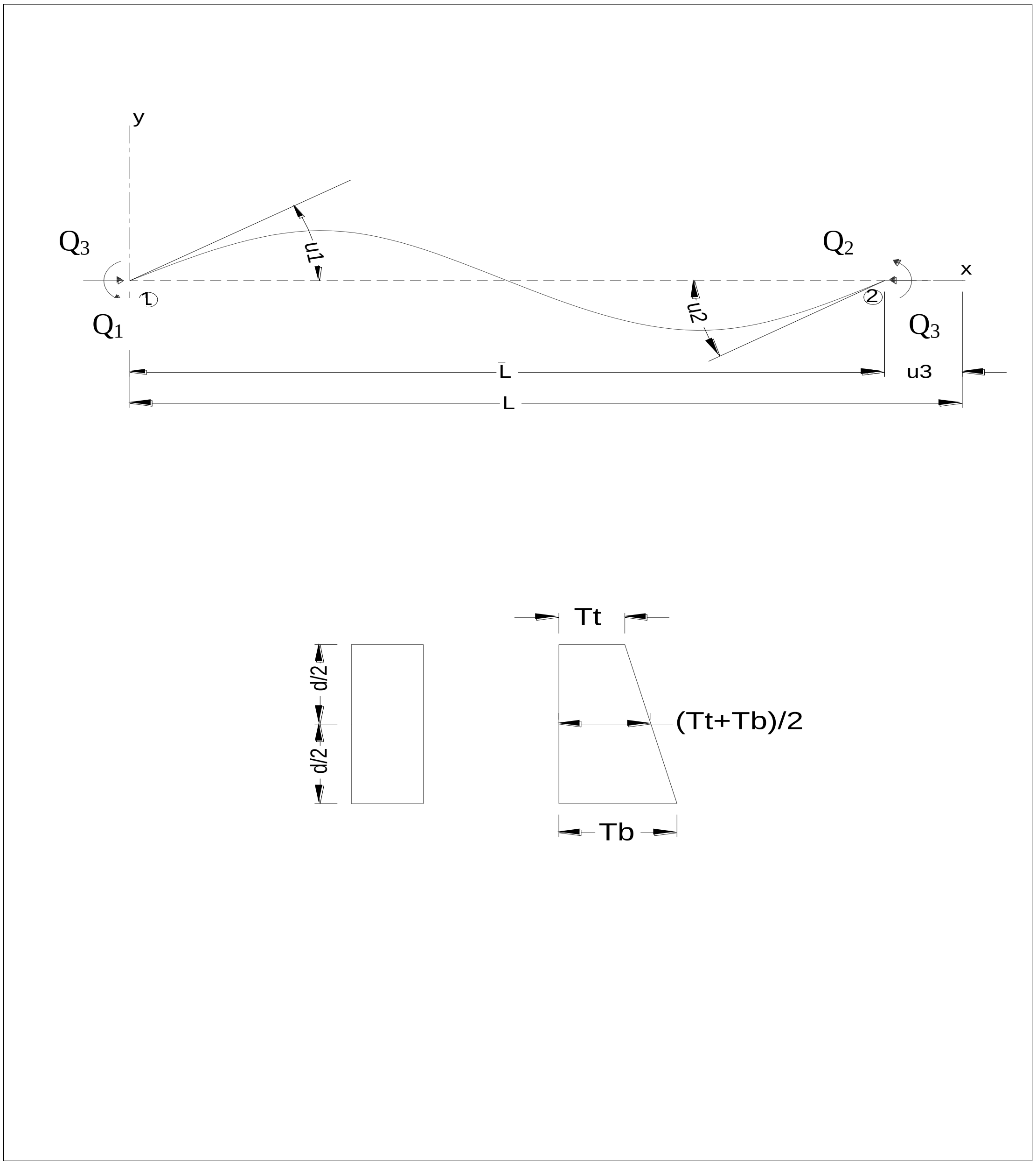

Fig. 2.1 Element in Local Coordinate System Subjected to Temperature Change

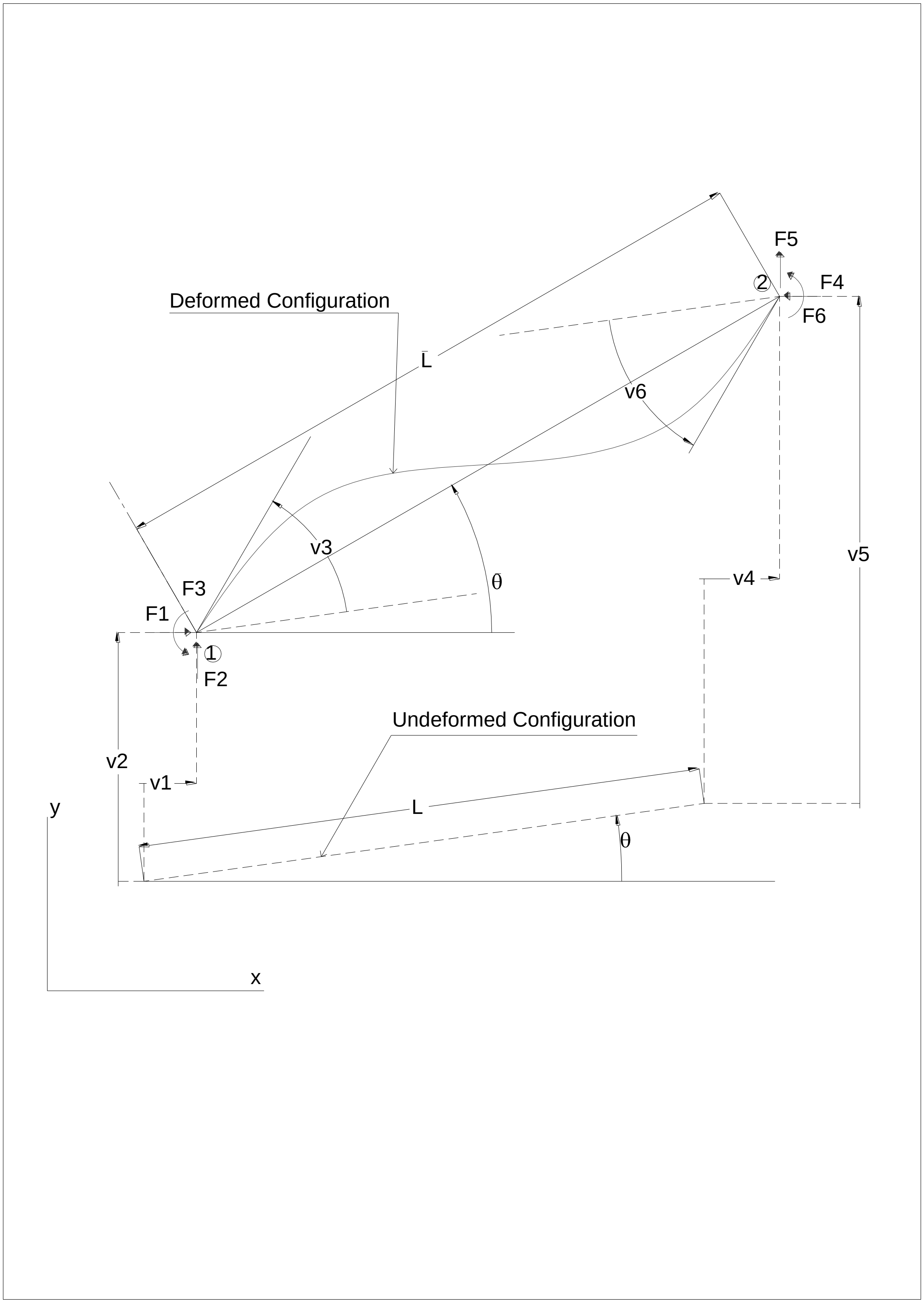

Fig. 2.2 Element Forces and Displacements in Global Coordinate System

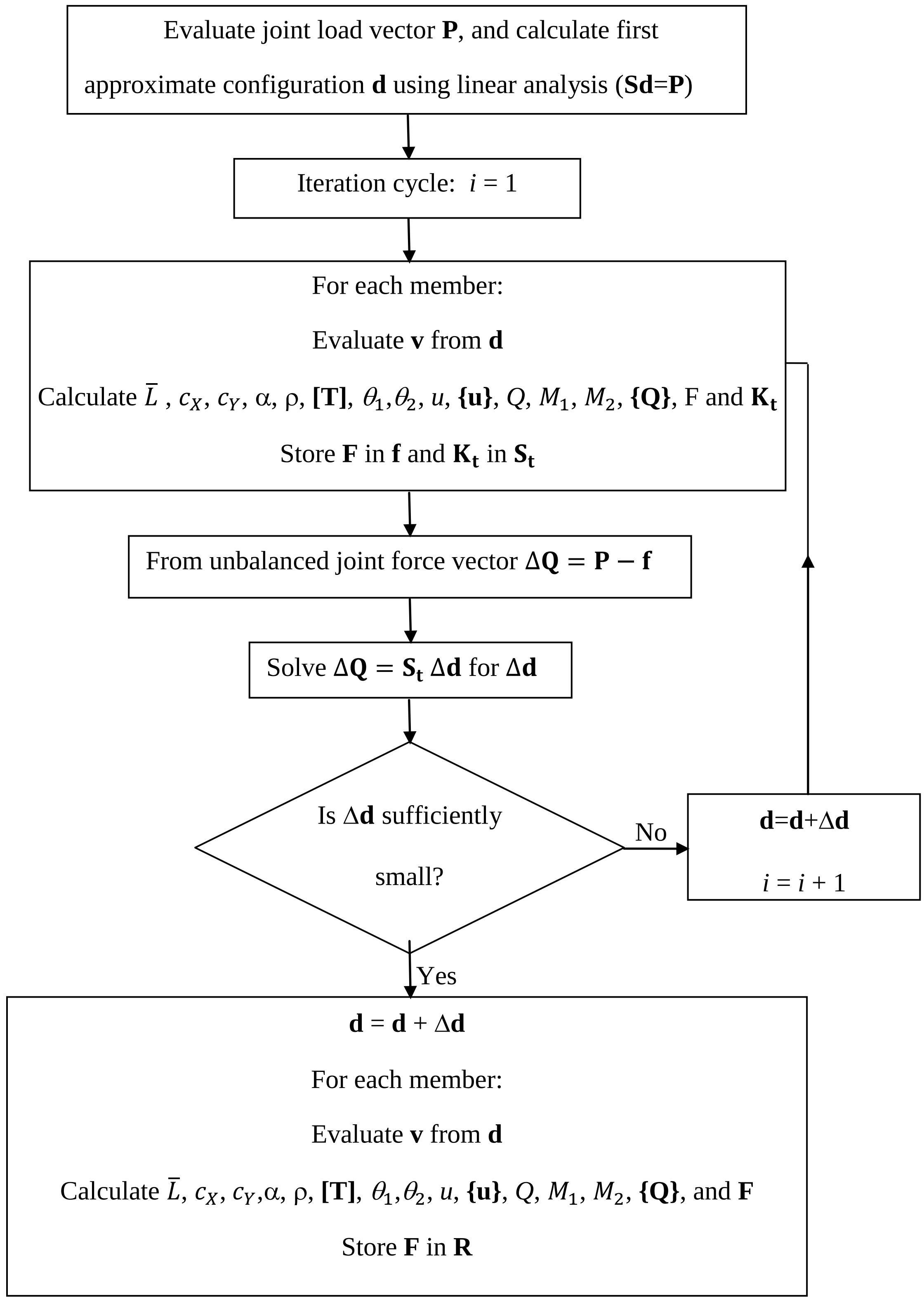

Fig. 2. 3 Block Diagram for Nonlinear Thermal Analysis for Plane Frames

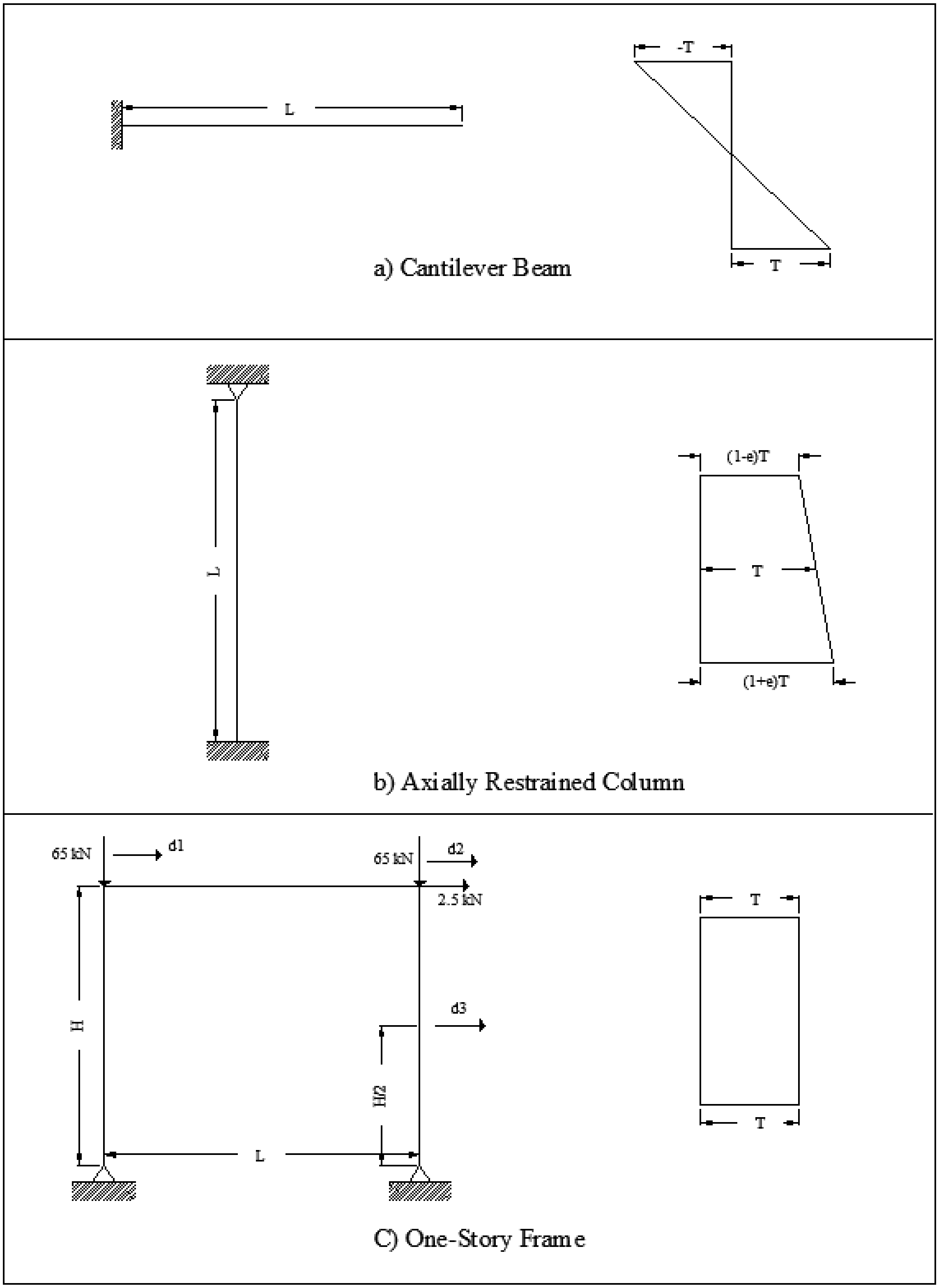

Fig. 4.1 Plane Frame Structures Selected for Numerical Study

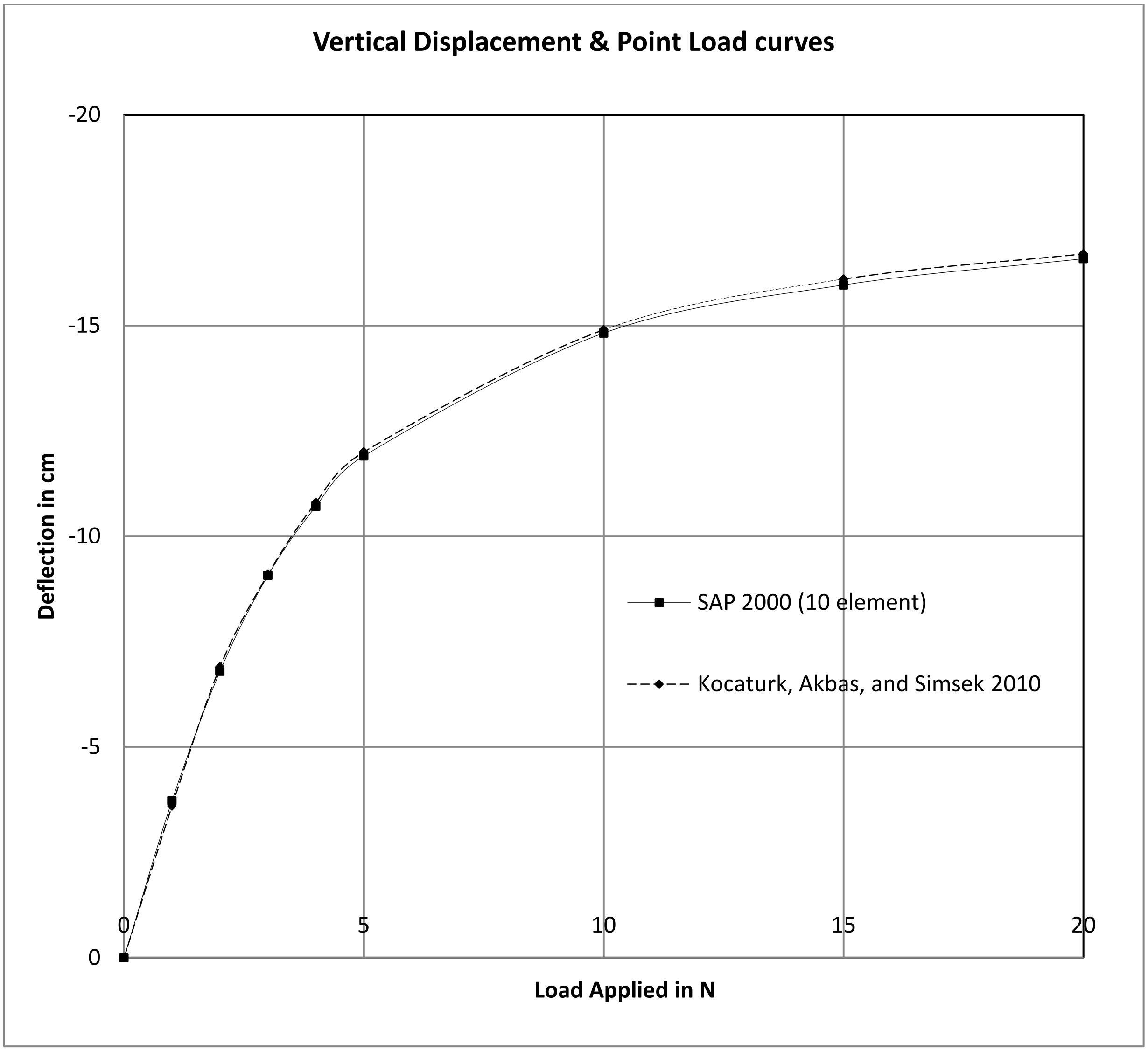

Fig. 4.2 Point Load versus Vertical-Deflection Curves for Cantilever Beam

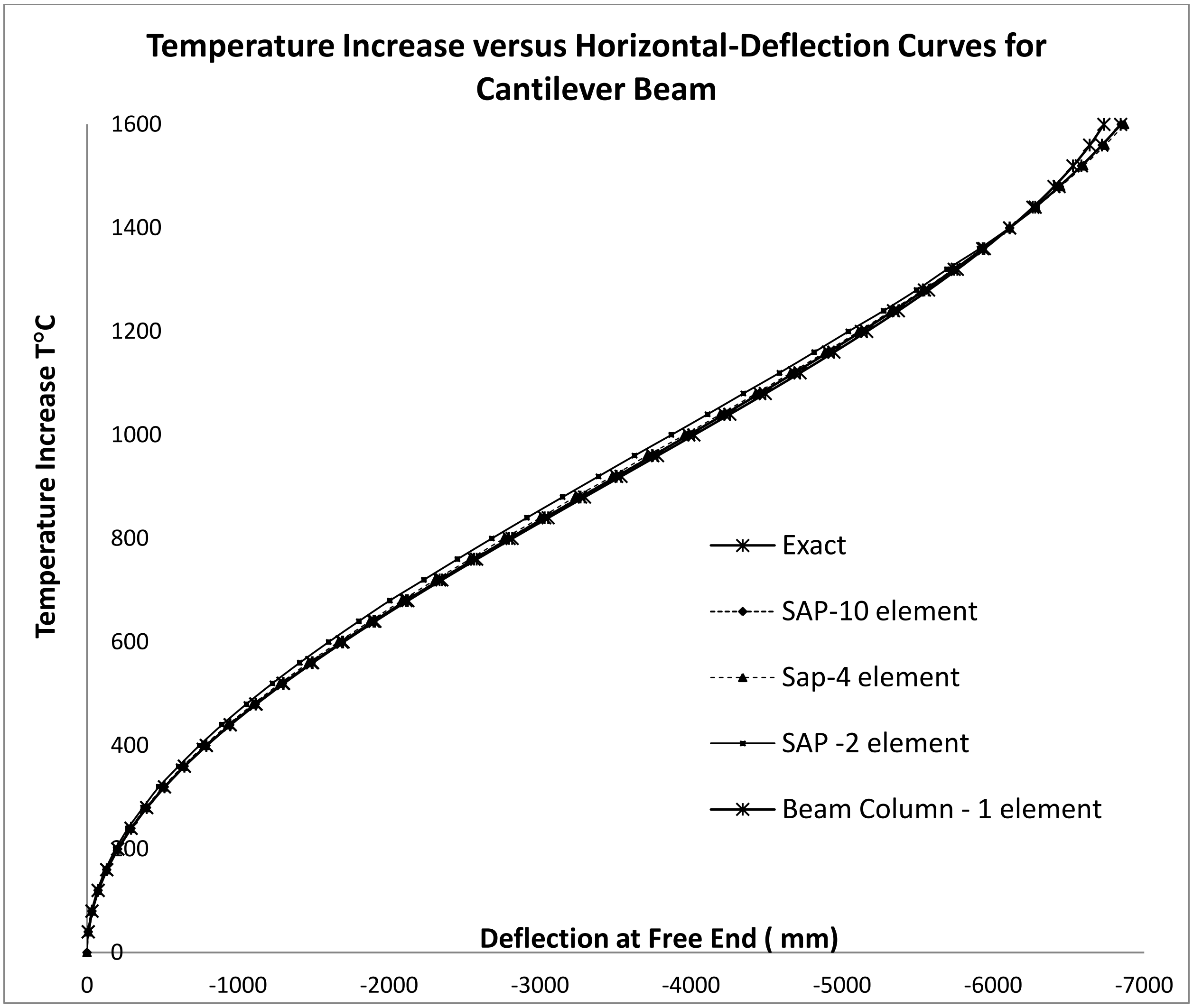

Fig. 4.3 Temperature Increase versus Horizontal-Deflection Curves for Cantilever Beam (40 Temperature Steps)

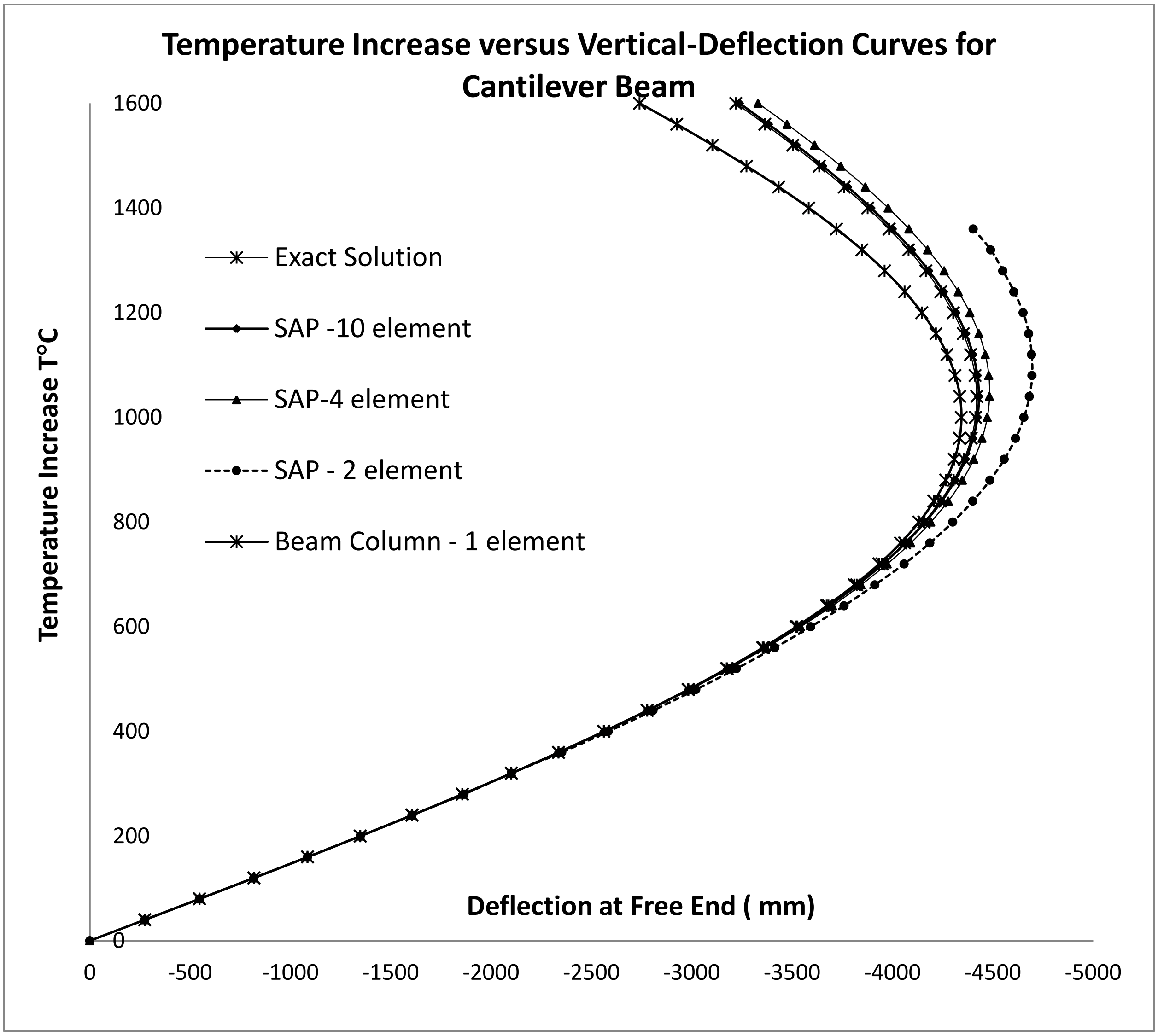

Fig. 4.4 Temperature versus Vertical-Deflection Curves for Cantilever Beam (40 Temperature Steps)

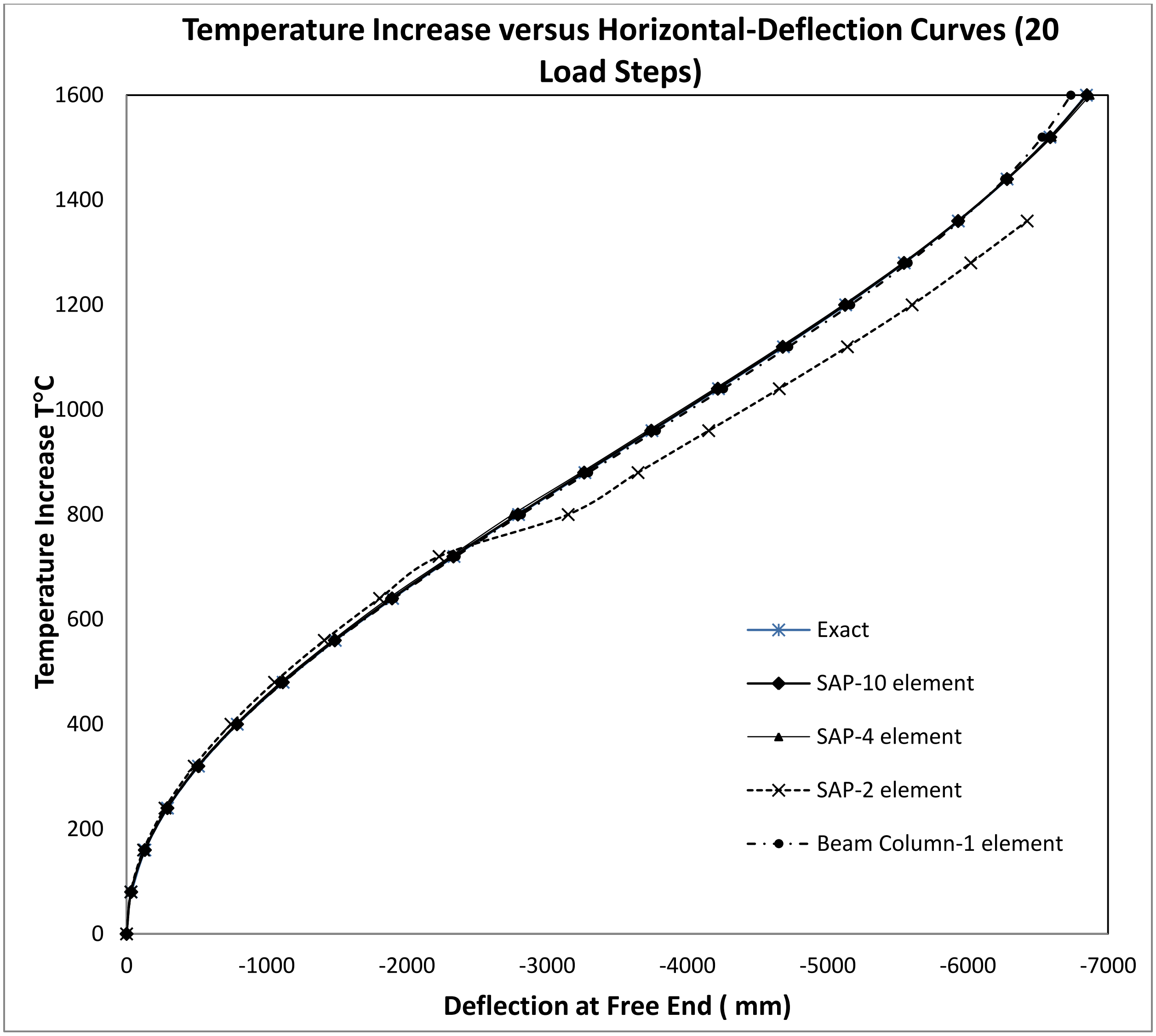

Fig. 4.5 Temperature versus Horizontal-Deflection Curves for Cantilever Beam (20 Temperature steps)

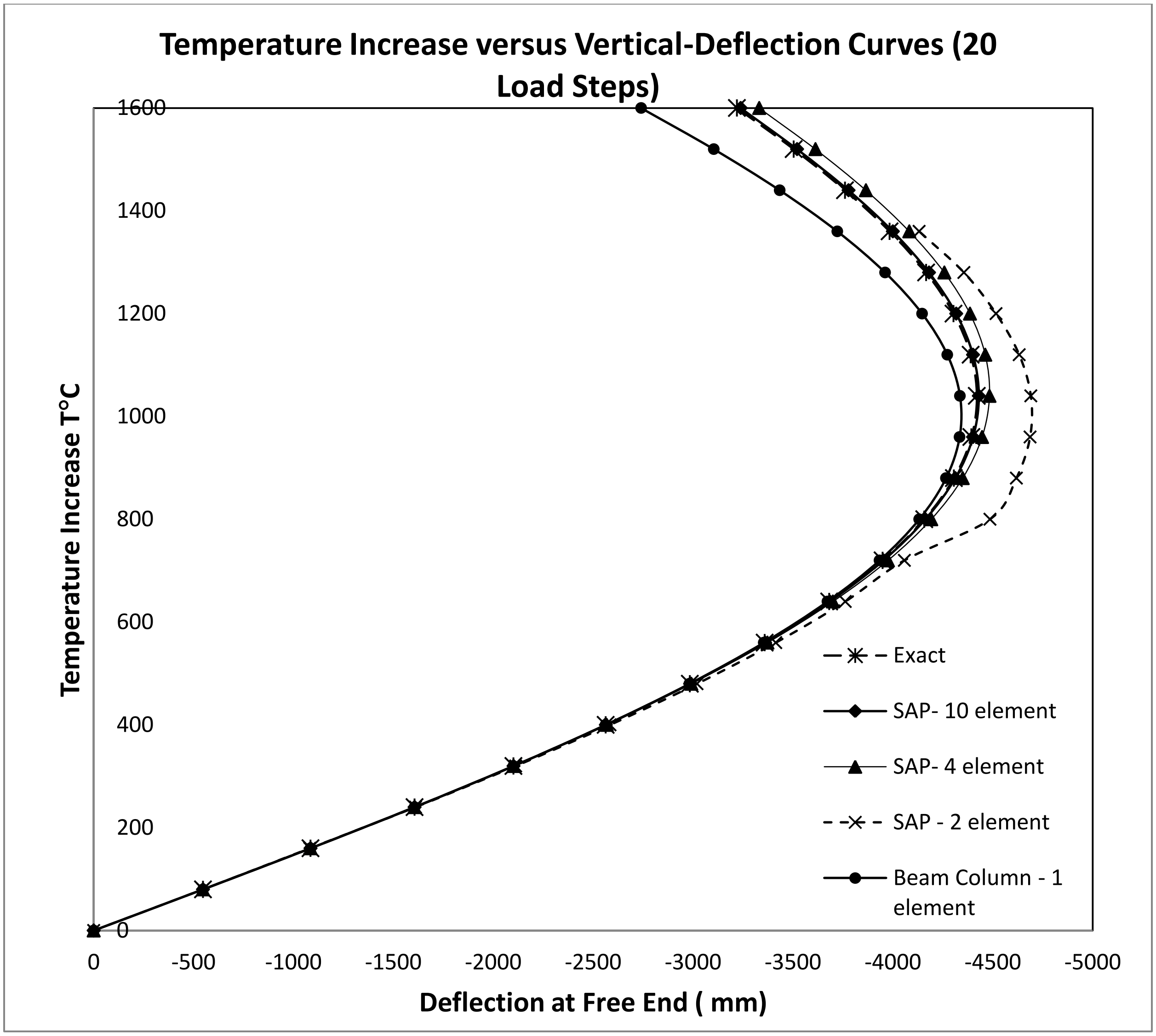

Fig. 4.6 Temperature versus Vertical-Deflection Curves for Cantilever Beam (20 Temperature Steps)

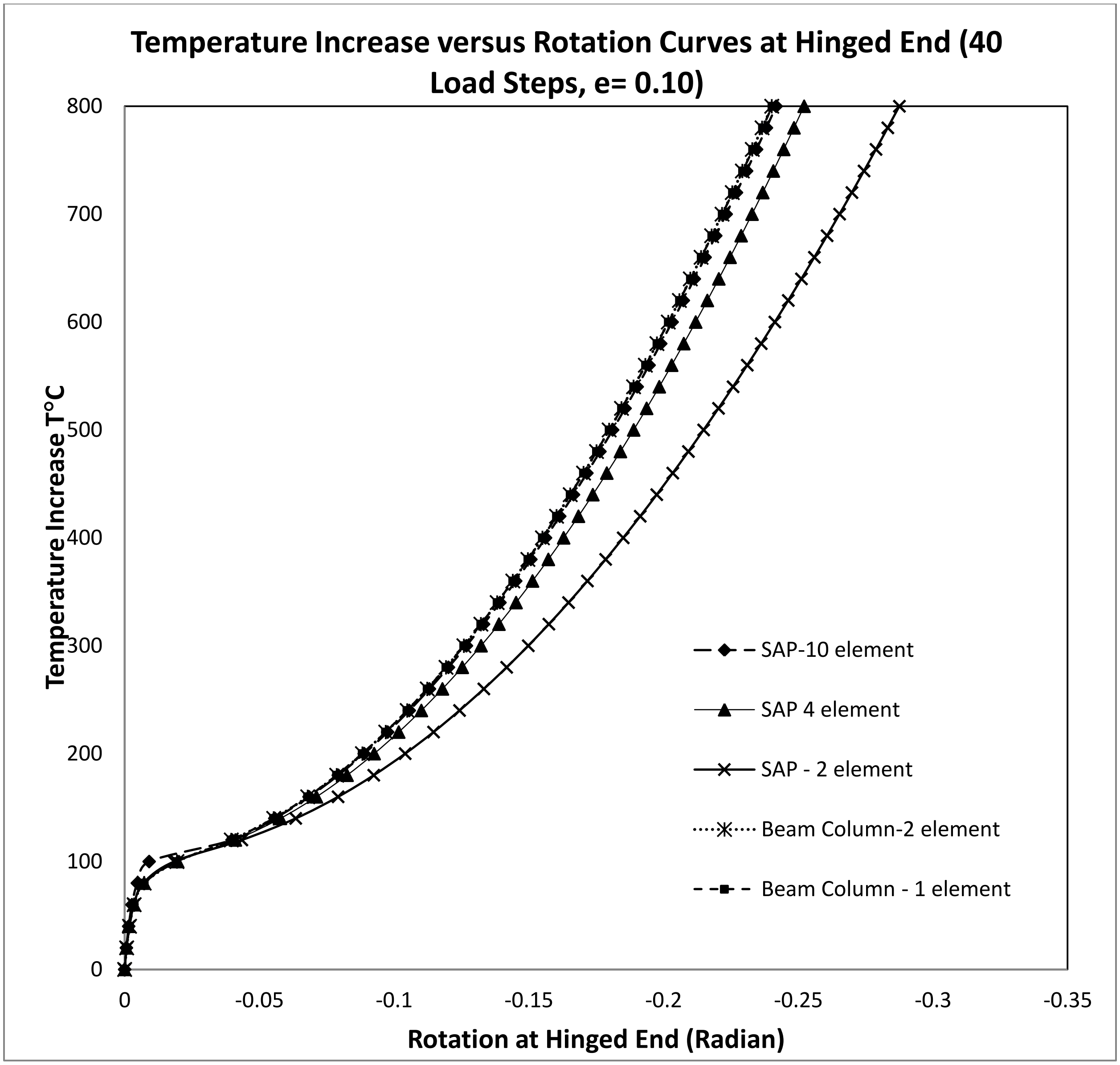

Fig. 4.7 Temperature Increase versus Rotation Curves at Hinged End for Axially Restrained Column (40 Temperature Steps, and *e*=0.10)

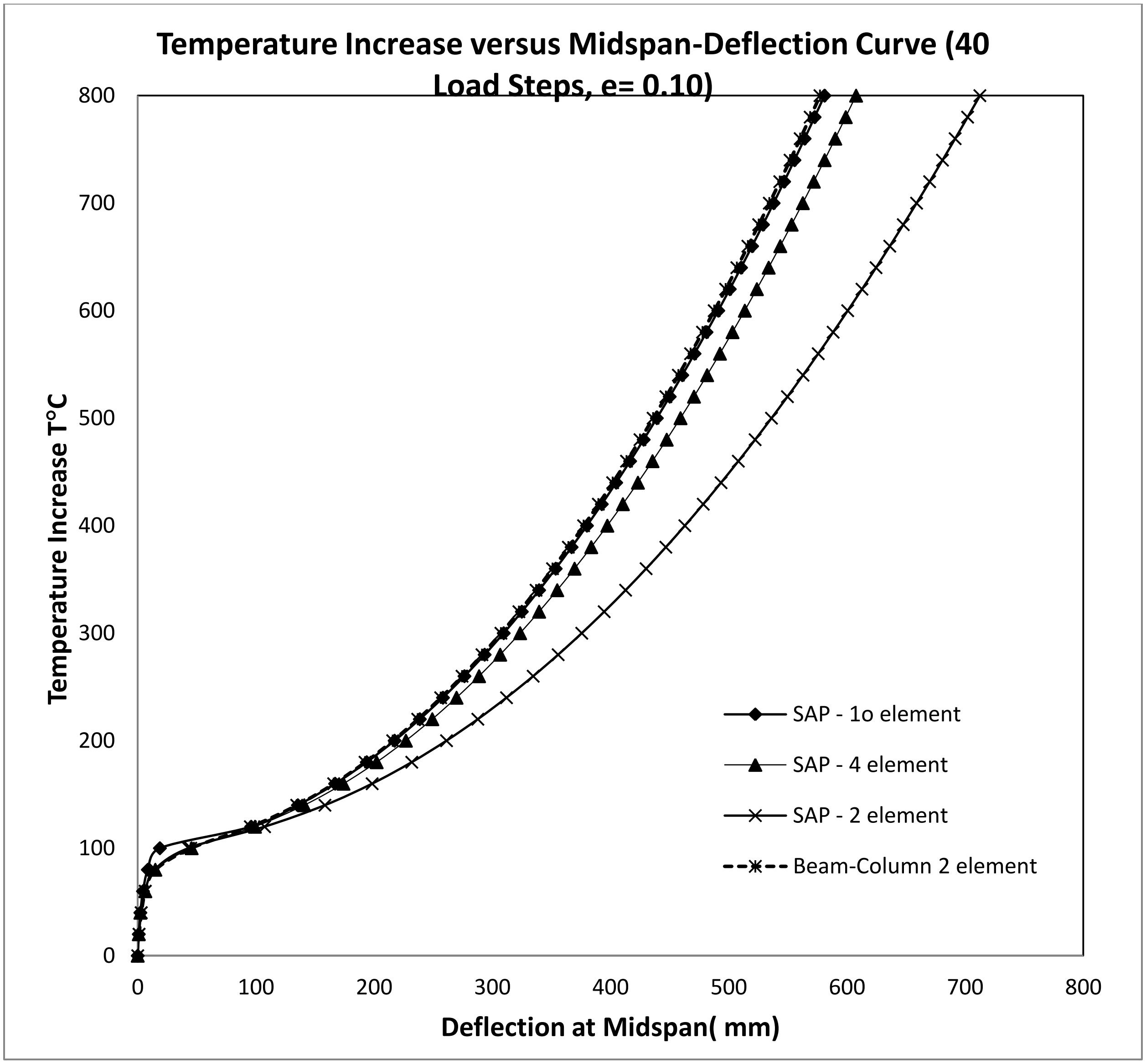

Fig. 4.8 Temperature Increase versus Midspan-Deflection Curves for Axially Restrained Column (40 Temperature Steps, and *e*=0.10)

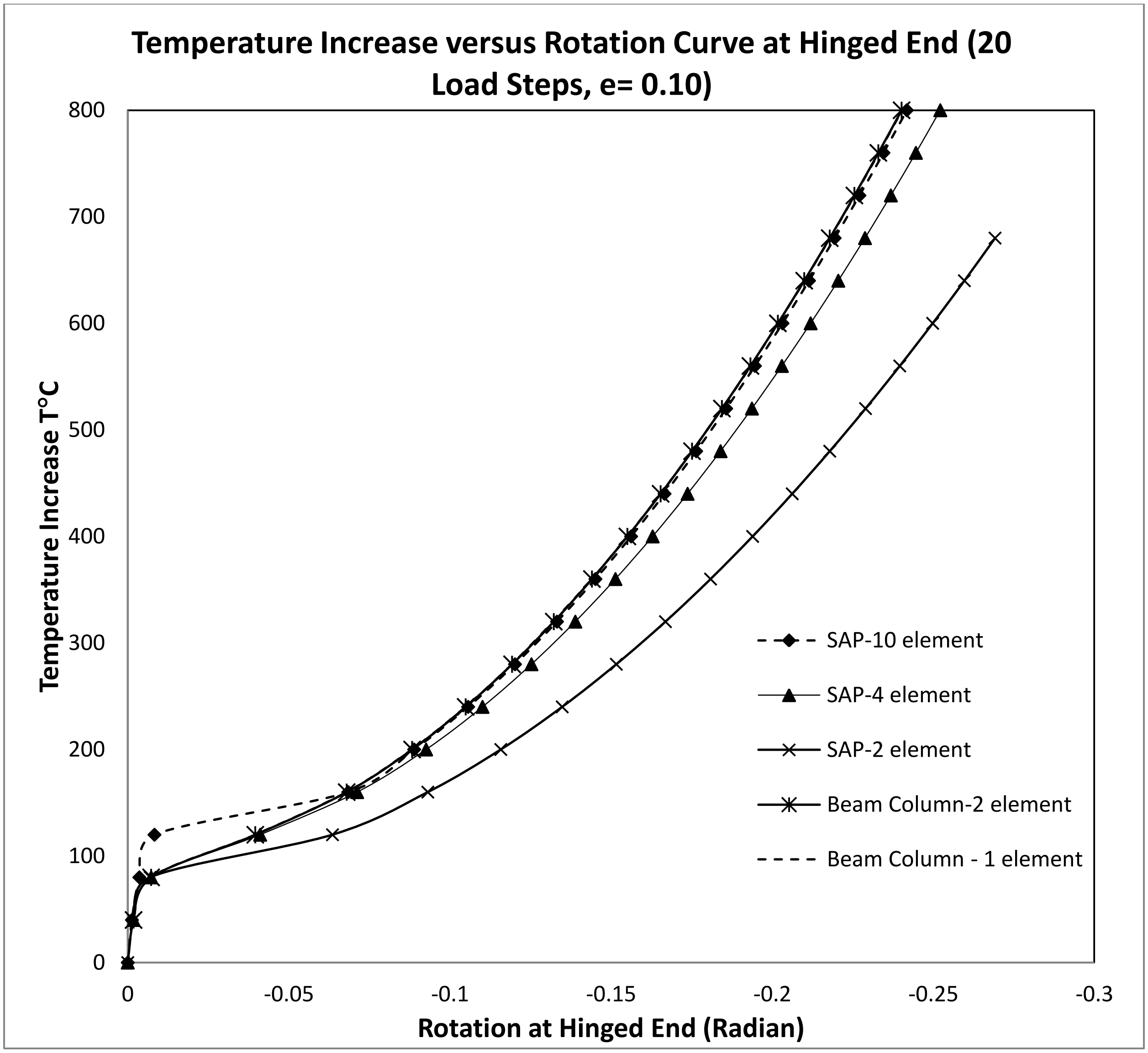

Fig. 4.9 Temperature Increase versus Rotation Curves at Hinged End for Axially Restrained Column (20 Temperature steps, and *e*=0.10)

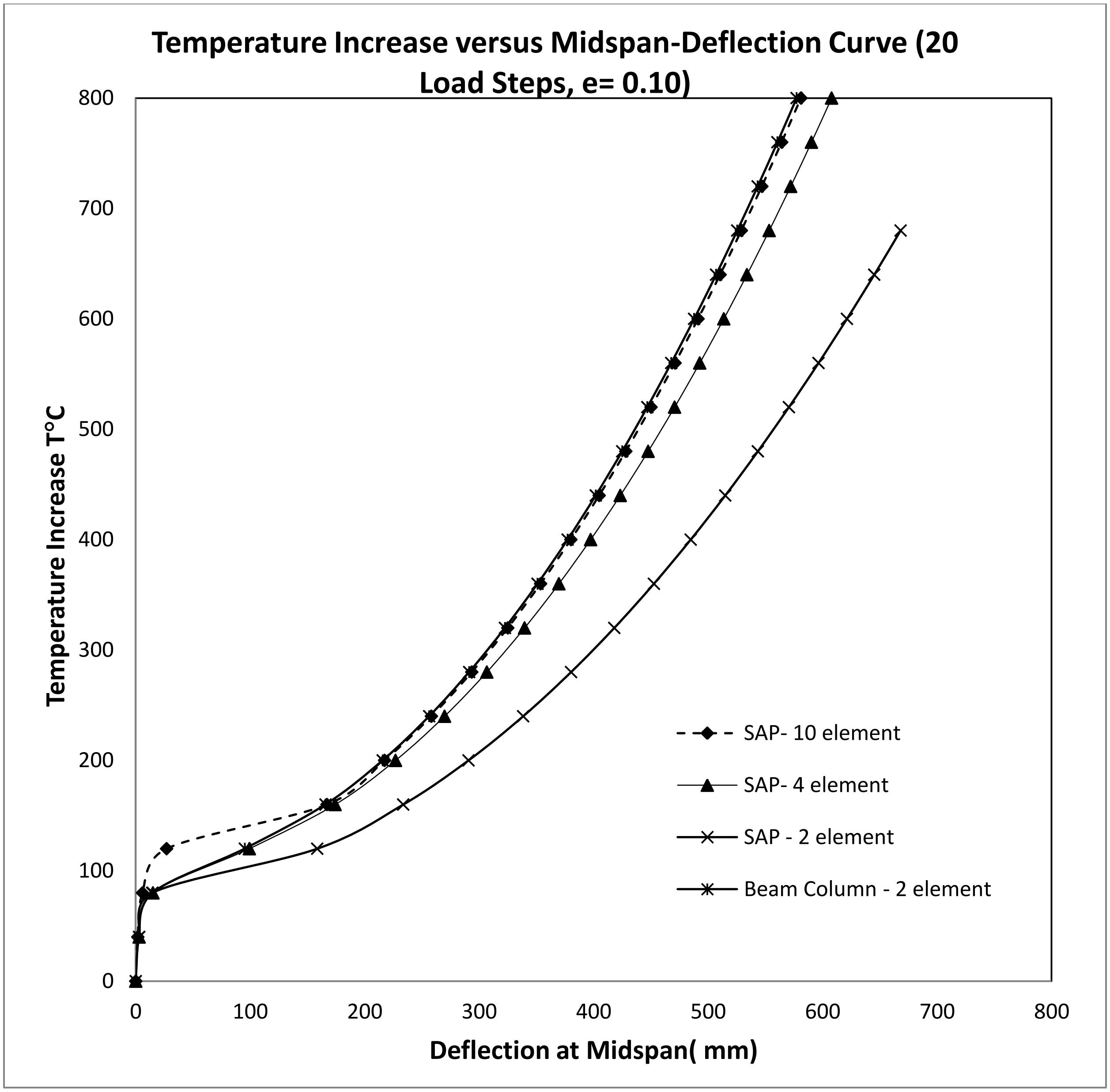

Fig. 4.10 Temperature Increase versus Midspan-Deflection Curves for Axially Restrained Column (20 Temperature steps, and *e*=0.10)

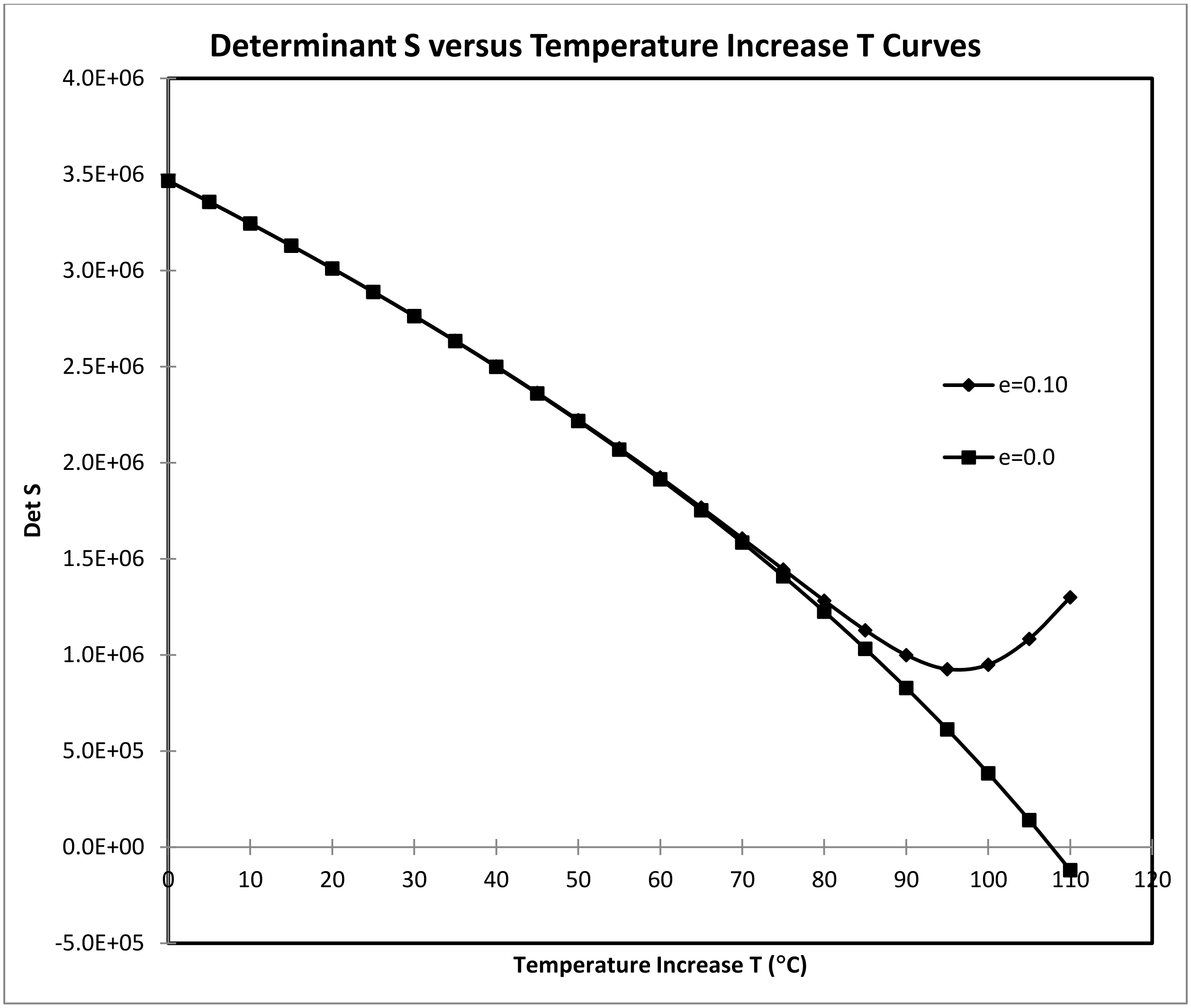

Fig. 4.11 Determinant S versus Temperature Increase Curves for Axially Restrained Column

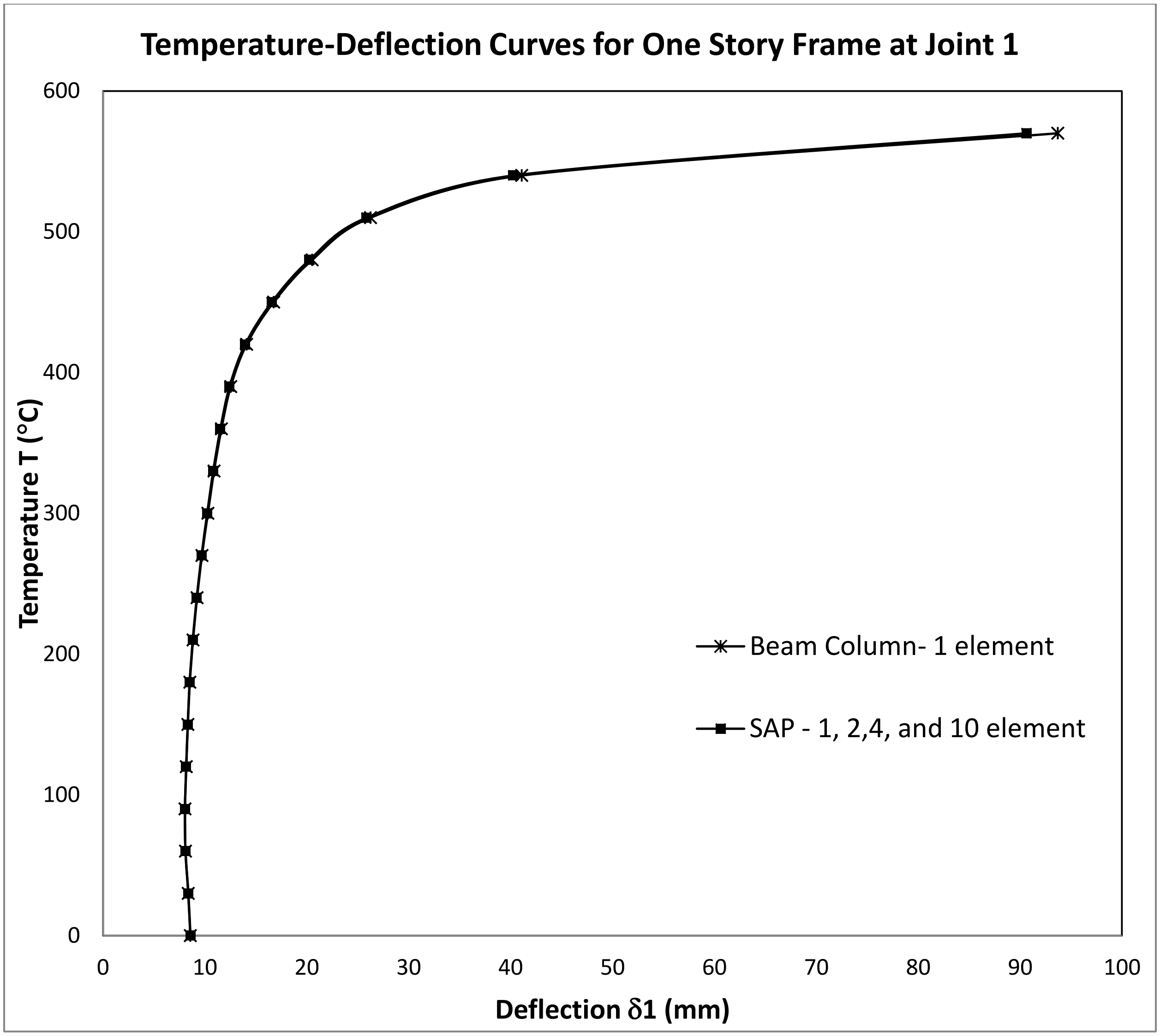

Fig. 4.12 Temperature-Deflection Curves for One-Story Frame in 20 Temperature steps (deflection δ_1)

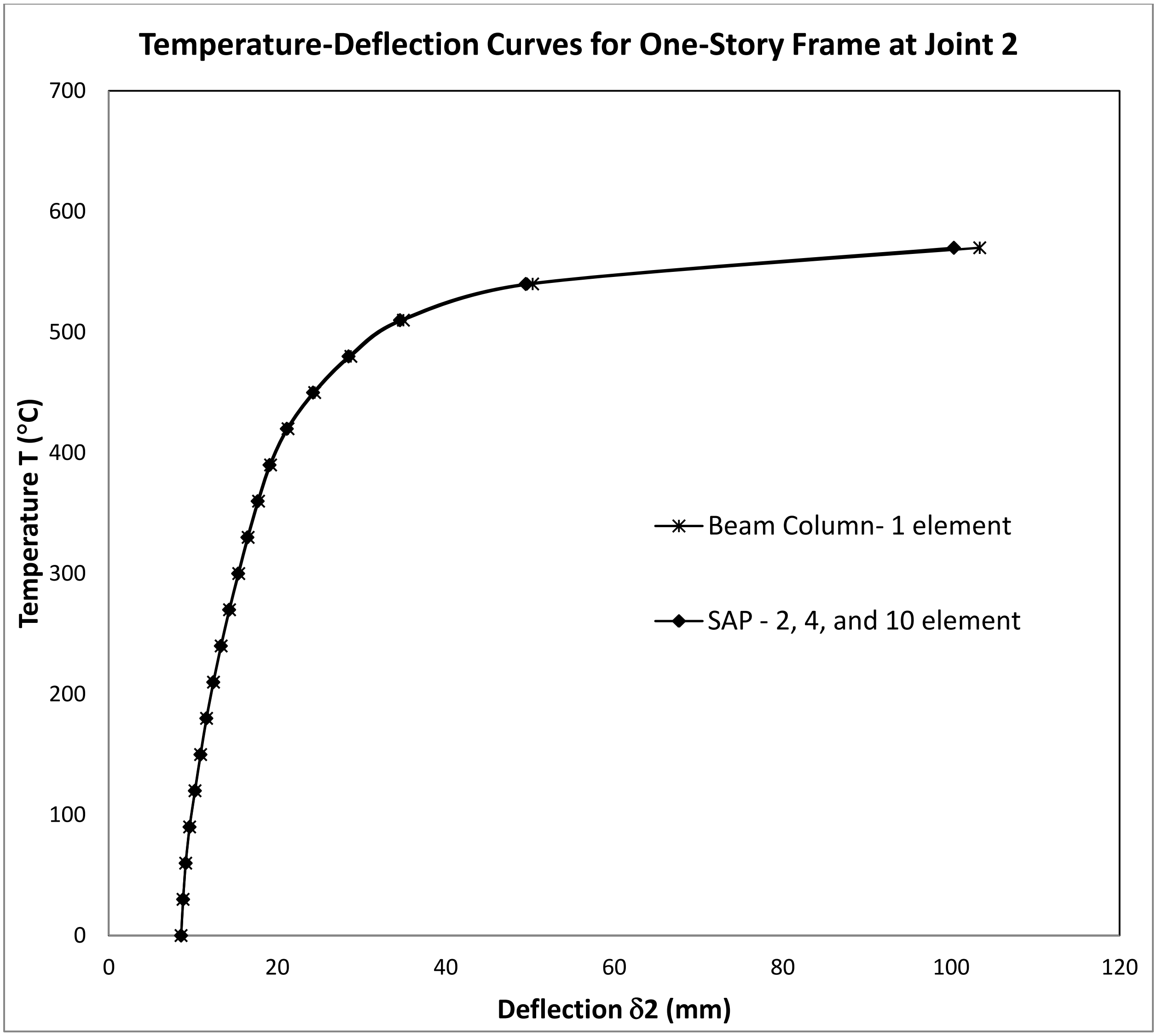

Fig. 4.13 Temperature Deflection Curves for One-Story Frame in 20 Temperature steps (deflection δ_2)

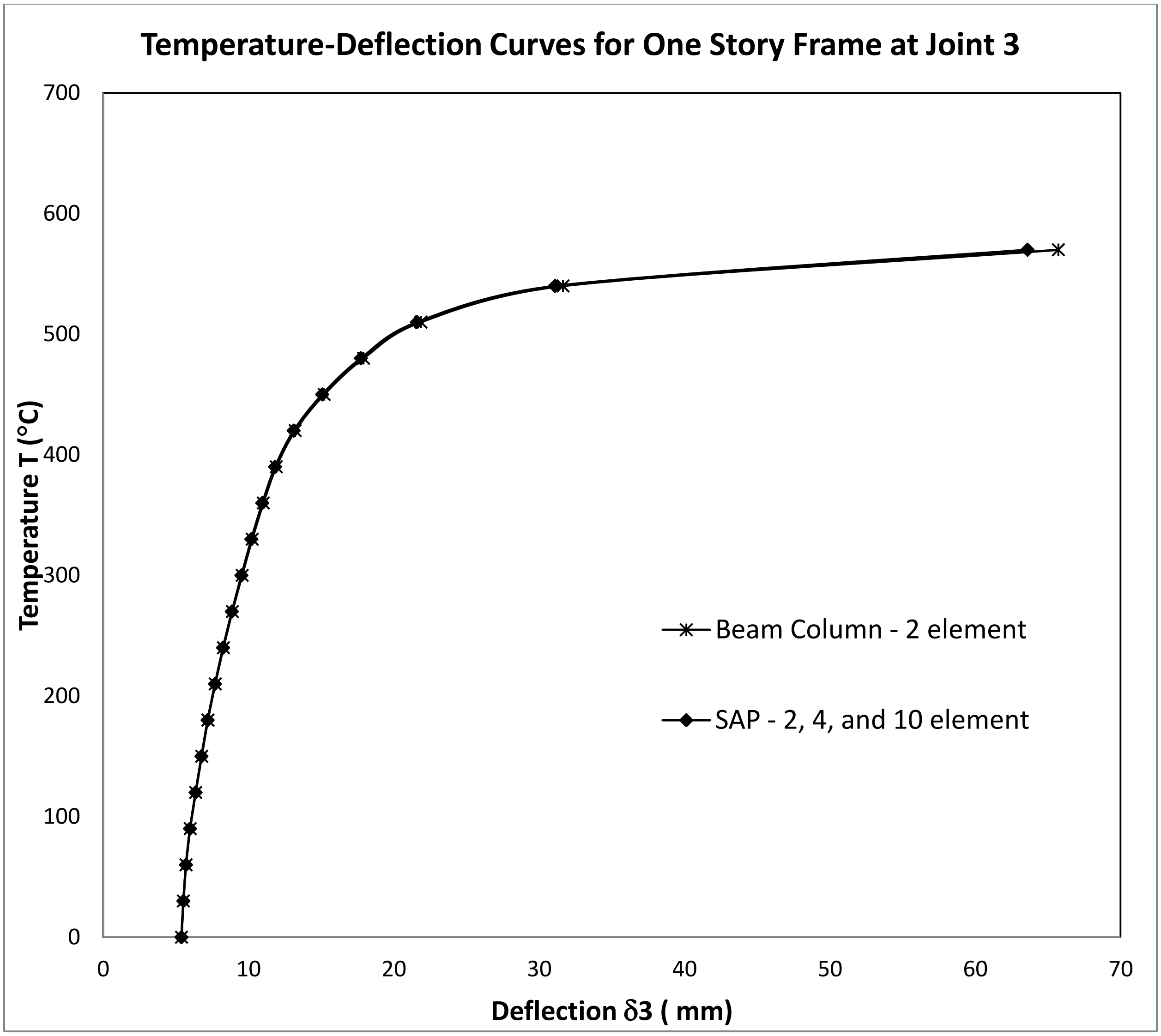

Fig. 4.14 Temperature Deflection Curves for One-Story Frame in 20 Temperature steps (deflection δ_3)

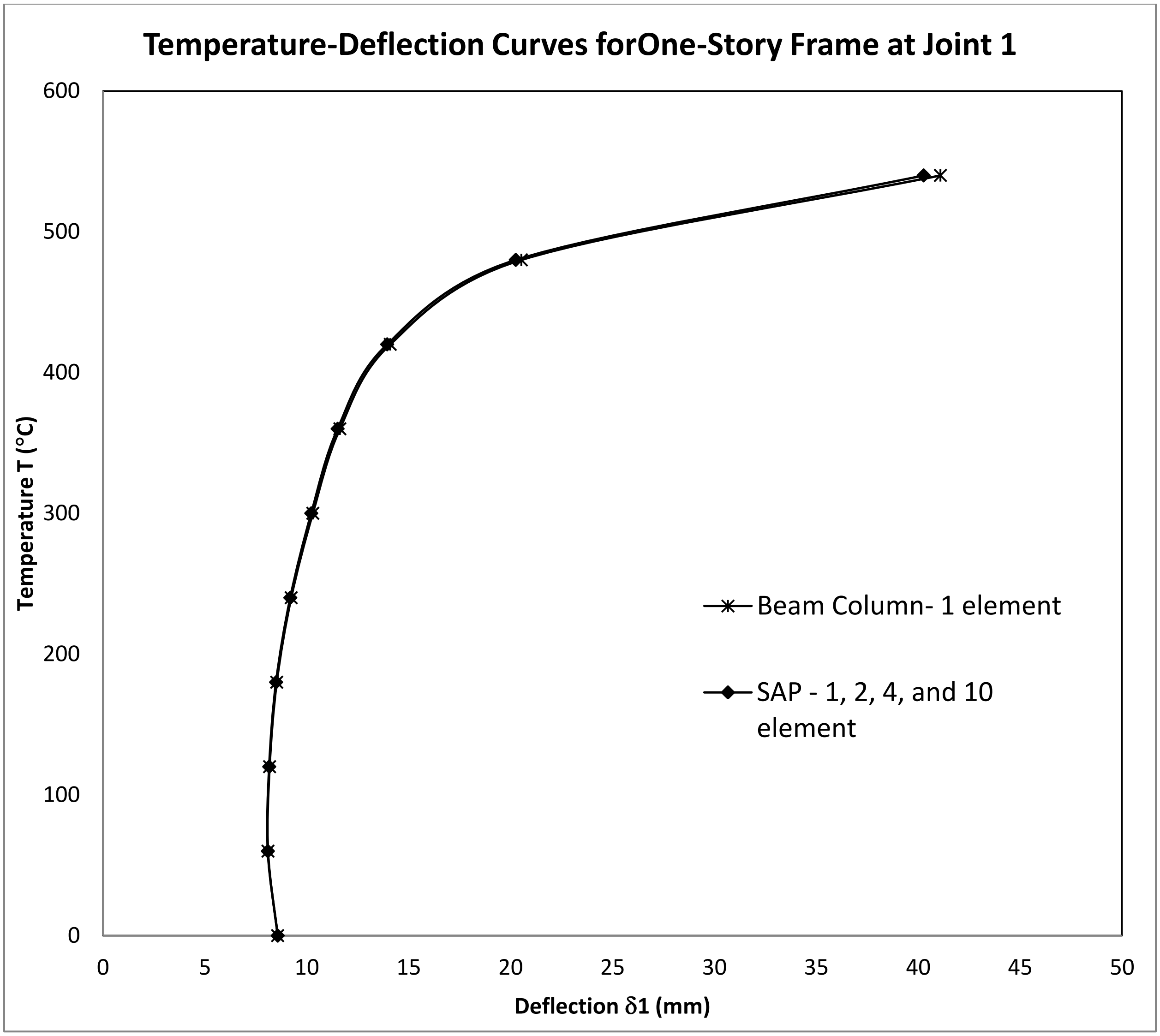

Fig. 4.15 Temperature Deflection Curves for One-Story Frame in 10 Temperature steps (deflection δ_1)

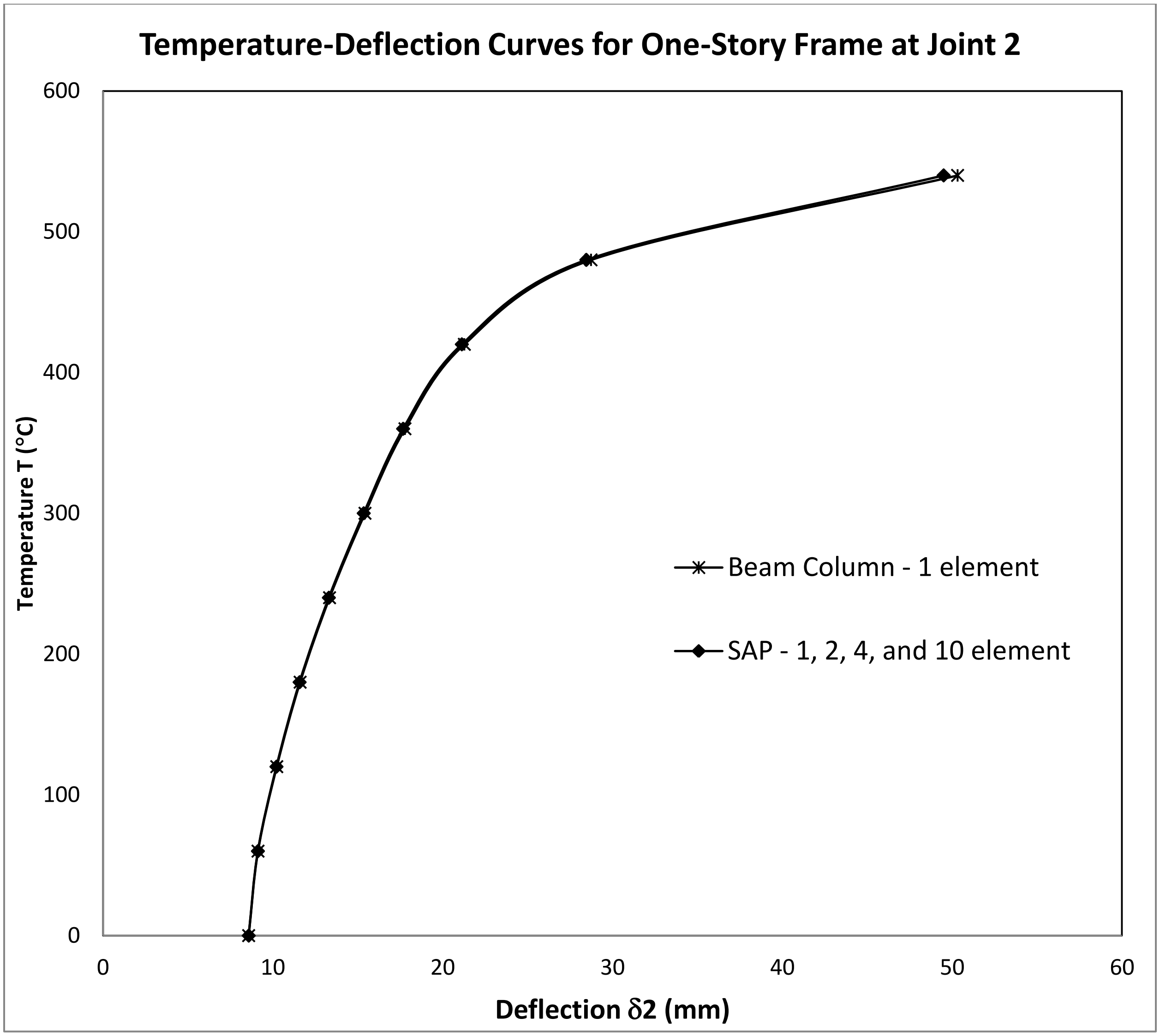

Fig. 4.16 Temperature Deflection Curves for One-Story Frame in 10 Temperature steps (deflection δ_2)

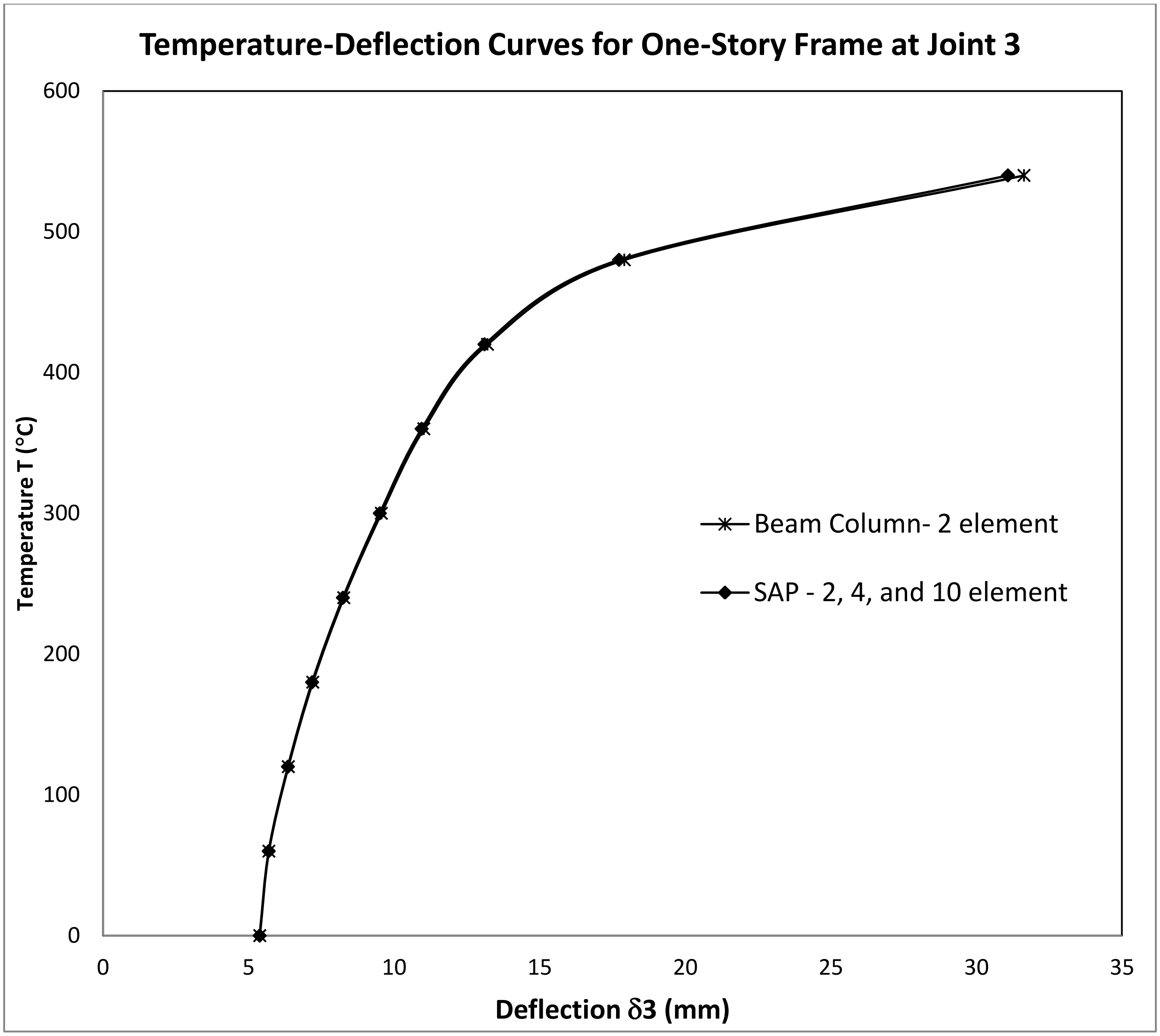

Fig. 4.17 Temperature Deflection Curves for One-Story Frame in 10 Temperature steps (deflection δ_3)

VITA

Graduate School

Southern Illinois University Carbondale

Baikuntha Silwal

baikunthanp@hotmail.com

Tribhuvan University Kathmandu, Nepal

Bachelor of Engineering, Civil Engineering, December 2000

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An Investigation of the Beam-Column and the Finite-Element Formulations for Analyzing Geometrically Nonlinear Thermal Response of Plane Frames

Major Professor: Dr. Aslam Kassimali, Ph.D.