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Comparing Optimal Government Enforcement Expenditures in a Pre- and Post-Legalization Marijuana Market

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COMPARING OPTIMAL GOVERNMENT ENFORCEMENT EXPENDITURES IN A PRE-
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by

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Approved by:

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TITLE: COMPARING OPTIMAL GOVERNMENT ENFORCEMENT EXPENDITURES IN A PRE- AND POST-LEGALIZATION MARIJUANA MARKET

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This paper analyzes optimal government expenditures for enforcement and interdiction of marijuana after the drug has been legalized in some capacity. It models a pre- and post-legalization market for marijuana in which two firms operate. In the pre-legalization model, both firms operate illegally and in the post-legalization market, only one firm operates illegally. Backward induction is used to solve for the government's optimal expenditures in period one and each firm's output decision in period two. Comparing both outcomes shows that expenditures are lower in the post-legalization regime and market share and profits are reduced for the firm operating illegally.

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CHAPTER 1

INTRODUCTION

Colorado and Washington became the first areas in the United States, and the world, to

legalize marijuana for recreational use following state ballot initiatives in 2012. Both states

legalized consumption of the drug for adults over the age of 21 and decriminalized the

possession of quantities below one ounce. Previously, a limited number of states had approved it

only for specific medical conditions. Since 2012, over two dozen other states and the District of

Columbia have legalized or decriminalized the drug in some capacity.[1] Many of those states

now, or soon will, allow the retail sale of marijuana to consumers over the age of 21.

This trend of legalization is a stark contrast to the prevailing social views of just a few

decades ago. Since President Richard Nixon's declaration of a "War on Drugs" in 1971, the U.S.

federal government has spent an estimated 1 trillion dollars[2] trying to reduce the production,

distribution, and consumption of drugs like marijuana, heroin, cocaine, etc. State governments

have also earmarked large sums of money, spending an estimated 3.6 billion dollars on

marijuana enforcement laws (arrest, conviction, imprisonment, etc.) in 2010 alone. They are

expected to spend a total of 20 billion by 2020.[3] This begs the question, do state legislatures that

have legalized consumption of marijuana in some capacity still need to spend large amounts of

money on enforcement and interdiction of the drug?

This paper attempts to answer the above question by modeling a market for marijuana

and comparing each firm's output and the government's optimal expenditures before and after

legalization. We use a game-theoretic approach to examine a government that has decided to

[1] http://www.governing.com/gov-data/state-marijuana-laws-map-medical-recreational.html

[2] http://www.foxnews.com/world/2010/05/13/ap-impact-years-trillion-war-drugs-failed-meet-goals.html

[3] http://www.cnbc.com/id/100791442

allow some production of marijuana within its boundaries, but continues its efforts to reduce

illegal production and trafficking. It may seem counterproductive that a government allows

some firms to legally produce a good while outlawing other firms from producing the same

good, but the government may have an interest in protecting citizens from undue harm by

regulating the process, ensuring safety and quality standards, etc. It may also be the case that the

government, as we assume later, wishes to protect the legal firms from unwanted competition

and reduce the market share and profits of the gang or cartel that is operating illegally.

The paper is organized as follows: Chapter 2 reviews related literature; Chapter 3

presents the models; Chapter 4 summarizes the findings; Chapter 5 summarizes and concludes

with extensions for future research.

CHAPTER 2

LITERATURE REVIEW

Most of the literature related to this paper examines government policies aimed at reducing cocaine or heroin consumption, both of which have been shown to be relatively more dangerous than marijuana. There are few papers that model government intervention in a market for a drug where some production is legal. However, this is not unexpected since the trend is relatively new. So, we look to literature related to drug production, government interdiction, and social welfare.

Daniel Mejia has extensively modeled the production and trafficking of drugs, and subsequent interdiction efforts by the U.S. and other governments. Much of his work focuses on *Plan Colombia* and the Colombian government's efforts to eliminate production of cocaine. Mejia and Restrepo (2016) model the supply chain of cocaine, from the farmers who cultivate the coca plants to consumption by the end user in the consumer country. They model the interdiction efforts of the Colombian government and an "interested outsider," which is the U.S. in the case of *Plan Colombia*. The Colombian government receives subsidies from the U.S. to aid in its efforts to control the supply of arable land to prevent the cultivation of coca plants and to aid with detecting and patrolling trafficking routes out of the country. They use backward induction to find the appropriate subsidies that the U.S. government should send. Mejia and Restrepo (2008), modeled this same topic, but more thoroughly. This paper takes a similar approach in that it attempts to determine the appropriate level or resources that a government should take to prevent illegal production, but allows for some production of the drug to be legalized.

Rydell, et al (1996) perform a cost-benefit analysis of four types of public policies to reduce cocaine consumption in the United States; source-country control, interdiction, domestic enforcement, and treatment of heavy users. They model for increased addiction rates after initial exposure to cocaine and find that treating heavy users for addiction can have the highest impact if the goal is to reduce total consumption, but caution that treatment does not work the same for every individual and has decreasing marginal returns. We allow for decreasing marginal returns to the government's investment in interdiction and enforcement in this paper as well.

Villa et al (2016) provide an extensive macroeconomic model on long-run growth from illicit activity in the Colombian economy and the government's subsequent welfare maximization problem in attempting to crack down on the illicit behavior and maximize public utility. This paper adapts a simplified model of the government's expenditure function, again utilizing the idea of decreasing marginal returns by selecting a certain functional form for interdiction efforts.

CHAPTER 3

THEORETICAL MODEL

To examine the effects of legalization on legitimate firms and interdiction efforts on illegitimate firms, we consider two models adapted from the sequential game of tariffs and foreign competition from McMillan (1986). In McMillan's model, demand for a good within a country is satisfied by a single domestic firm and by a single foreign firm. It is assumed the goods produced by both firms are homogenous. The home government maximizes social welfare and protects the profits of the domestic firm. It does so by playing the role of a Stackelberg leader and setting an optimal tariff for the imported good after anticipating the output from both firms. The two firms then serve as Stackelberg followers that must determine how much output they will produce in response to each other and the government's tariff.

The first model presented in this paper extends the above idea to the current market for marijuana in areas of the United States where it has been partially legalized (for medicinal and/or recreational use). For simplicity, we also aggregate production into two representative firms in the market. However, instead of differentiating by national boundaries, we assume one firm is legal and has been licensed by the government to produce marijuana, such as a grower or dispensary, while the other has not and is operating illegally, such as a cartel or gang. Both firms compete in the same market and can only exert control over the good's price by changing the quantity supplied. Additionally, the government does not charge a tariff on illegal production, but instead allocates resources (law enforcement officers, attorneys, judges, etc.) to combat illegal production and consumption, reduce profits of the illegal firm, and protect the profits of the legal firm. These interdiction and enforcement efforts increase the cost of production for the illegal firm to some degree and inversely affect the amount it can produce.

The timing of this game is as follows; (i) the government budgets for the level of resources to devote to interdiction and enforcement, (ii) the legal and illegal firms simultaneously choose their levels of output, and (iii) payoffs are realized in the form of profit for each firm and total social welfare to the government[4]. We refer to this game as the *Post-Legalization Model*.

The second model presented extends McMillan's game to the previous market for marijuana where all forms of production and consumption of the drug were illegal. Much of the setup is the same, except neither of the two firms are licensed to produce marijuana. The government still allocates resources to disrupt the firms' activities which increases the cost of production. We refer to this game as the *Pre-Legalization Model* and its timing is as follows; (i) the government budgets for the level of resources to devote to interdiction and enforcement, (ii) the firms simultaneously choose their levels of output, and (iii) payoffs are realized in the form of profit for each firm and total social welfare to the government.

We assume each player has complete and perfect information about every other player. We solve both models for the optimal level of government resources and each firm's optimal level of output by backward-induction, beginning with the quantity decisions faced by each firm in period two.

3.1 Post-Legalization Model

The Firms' Output Decision

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[4] This model does not consider any consequences that may accompany legalization (or decriminalization), such as increased healthcare costs associated with extended use or abuse, automobile accidents, etc., nor does it account for any taxes the legal firm may pay to the government. It is further assumed that if the government has legalized the drug, it expects its citizens to receive some net benefit from doing so.

Each of the firms must choose an optimal level of output in response to the other firm's level of output, and the level of resources budgeted by the government. However, the illegal firm faces an added marginal cost.

We begin by defining the linear inverse demand for marijuana faced by each firm, given as $P(Q) = a - Q$. Here price is a function of a, an exogenous scale parameter capturing the maximum price consumers are willing to pay,[5] and $Q = q_L + q_C$, is the total market supply of marijuana which is equal to the sum of marijuana produced by each firm.

It is important to note that we make a simplifying assumption that the marijuana produced by both firms is homogeneous. However, this may not be entirely true. Ignoring edibles, designer strains, and other boutique products currently produced by legal growers, the main difference between legal and illegal marijuana "flower" is usually potency, or the level of THC content. Since legalization, some growers have acquired the tools and skilled labor necessary to selectively breed cannabis plants with a higher THC content that may not be readily available to some gangs or cartels. However, the average THC content of illegal marijuana flower has still been increasing over the past few decades as well.[6] A model with differentiated products may allow for a more representative case, but we assume substitutability would be very high in the current market, so inference from the model presented in this paper is unlikely to be impacted.

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[5] It is assumed that the legal firm can produce enough marijuana to meet market demand, but due to reasons unrelated to price, such as loyalty to a particular drug dealer, distrust of the government, etc., a portion of consumers choose to purchase marijuana from the illegal firm. We also restrict $Q \leq a$ ensuring a positive price.
[6] A Colorado Department of Revenue report prepared in 2015 found the average THC content for marijuana produced legally in the state was 17.1 percent (2015). The Potency Monitoring Program at the University of Mississippi School of Pharmacy found average THC content from a 2014 sample of illegally grown marijuana to be approximately 12 percent (2016).

Each firm's objective is to maximize their profits by choosing an optimal level of output in response to the government's interdiction efforts and the other firm's level of output. The legal firm faces a standard profit maximization function of revenue minus costs, given by

$$\max_{q_L} \pi_L = pq_L - cq_L \tag{1}$$

The subscript L denotes the profit of and quantity produced by the legal firm and p and c are the respective price and constant marginal cost per unit of output.

The profit maximization problem faced by the illegal firm is similar to that of the legal firm, but with additional marginal costs. It is given by

$$\max_{q_C} \pi_C = pq_C - q_C\left(c + \theta\sqrt{T} + \gamma\sqrt{R}\right) \tag{2}$$

where the subscript C denotes the illegal or "criminal" firm's profit and output and p and c are again the price and constant marginal cost per unit[7]. For simplicity, we further assume c is the same for both firms. The variables T and R are the added marginal costs of production faced by the illegal firm.

T represents the level of resources the government devotes to interdicting illegal production and shipments of marijuana. Because its growing and distribution operation is likely to be decentralized to avoid detection, the illegal firm incurs a risk every time it moves the product from one location to another (e.g. from the grow site to the processing site to the storage site to the dealers, etc.). The government, through law enforcement officers, can exploit this supply chain and potentially seize the good during its transportation. Thus, T may be viewed from the firm's perspective as an added "transportation cost." This captures the idea that the

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[7] It is likely that a gang or cartel's revenue from selling drugs would be supplemented by other illicit activities, such as human trafficking, racketeering, extortion, etc. However, for simplicity, we allow them to only produce one "good."

more the illegal firm produces, the more difficult it can be to move the good around, and the greater the likelihood of being caught.

R represents the level of government resources dedicated to catching consumers who purchase marijuana illegally. Because the government prefers that consumers purchase marijuana legally (to avoid increasing profits for the illegal firm), this can be represented as an added "tax" or per unit cost to the illegal firm. The idea being that the risk of getting caught by law enforcement pushes some potential customers to obtain the drug through legal means.

Choosing these specific functional forms for T and R may result in a loss of generality, but simplifies our calculations and captures the intuitive idea that each additional dollar spent by the government on those resources has relatively less of an impact than the previous dollar spent. That is, as the government hires more officers, it has a greater chance of confiscating illegal marijuana, but also faces decreasing marginal returns. So, as the government increases the levels of T and/or R, costs are increasing for the firm, but in a decreasing manner.

The parameters $\theta, \gamma \in (0,1)$ capture the efficiency of the government in its interdiction and enforcement efforts respectively. We assume $\theta > \gamma$ by the nature of the market. Since consumers are far more likely to outnumber agents working for the illegal firm, it is not unreasonable to assume that law enforcement would be more efficient in targeting the supply-side of the market. However, this does not mean that law enforcement concentrates its resources only on the illegal firm. Allocating some resources to enforcement sends a signal to risk-averse consumers that they should purchase the drug legally to avoid arrest and/or prosecution. Moreover, one only needs to read the daily news to see that law enforcement can capture those who possess illegal drugs with some degree of effectiveness, so we restrict $\gamma > 0$.[8]

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[8] If both θ and γ were to equal zero, the firms would have identical profit maximization functions and we would have a duopoly market.

Returning to the maximization problems, P(Q) can be substituted for p in both (1) and

(2), and simplified, so the firms' objective functions are given by

$$\max_{q_L} \pi_L = q_L[a - q_L - q_C - c] \tag{3}$$

$$\max_{q_C} \pi_C = q_C[a - q_L - q_C - c - \theta\sqrt{T} - \gamma\sqrt{R}] \tag{4}$$

To find the quantity that maximizes each firm's profit, we take the derivative of π_i with respect

to q_i, set it equal to zero and solve for q_i. We then obtain the reaction function for each type of

firm given by

$$R_L(q_C^*) = q_L^* = \frac{a - q_C^* - c}{2} \tag{5}$$

$$R_C(q_L^*) = q_C^* = \frac{a - q_L^* - c - \theta\sqrt{T} - \gamma\sqrt{R}}{2} \tag{6}$$

This gives us each firm's best response in terms of the other firm's output decision and the

government's allocation of T and R. If we solve this system of equations simultaneously, we

find that the optimal quantity, denoted by q_i^*, for each type of firm is

$$q_L^* = \frac{(a - c) + (\theta\sqrt{T} + \gamma\sqrt{R})}{3} \tag{7}$$

$$q_C^* = \frac{(a - c) - 2(\theta\sqrt{T} + \gamma\sqrt{R})}{3} \tag{8}$$

Even though T and R do not appear in the legal firm's profit maximization problem, we can see

that the variables still positively affect its optimal quantity because they affect the illegal firm's

optimal quantity. For the illegal firm, increases in T and R will always negatively affect its

optimal quantity by a factor of two. And, as shown in Appendix A, the total profit for each firm

is equal to its optimal quantity squared. So, increases in T and/or R have the desired effect of

reducing the illegal firm's profit and increasing the legal firm's profit.

Total market supply, Q, is given by the sum each firm's output, as shown below.

$$Q = \frac{2(a-c) - \left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3} \tag{9}$$

We can see that for every positive increase in T and/or R, market supply will decrease. Thus, considering our inverse demand function, we can expect market prices to increase with every increase in government spending. We need not worry about market supply being negative if T and R are sufficiently large. As we see in Chapter 4, this is not a concern given the government's optimal choices of T and R.

From here, we can use the above quantities in (7) and (8) to solve the government's problem of resource allocation.

The Government's Allocation of T and R

As mentioned earlier, the government seeks to maximize social welfare by allocating resources to increase consumer surplus, reduce the market share and profits of the illegitimate firm, and protect the profits of the legal firm. However, the government incurs a cost in doing so. Thus, anticipating each firm's output determined in the preceding section, the government faces the following societal welfare function

$$\max_{T,R} W = \frac{1}{2}Q^2 + \pi_L - \pi_C - T - R \tag{10}$$

We approach this maximization problem from the perspective of a social planner. That is, we do not constrain the combination of T and R, but are instead interested in finding the optimal amount of each resource. Examining the welfare function, we find the first term is the consumer surplus[9] in the market, π_L is profit for the legal firm, π_C is the profit for the illegal firm, and T and R together are the total resources budgeted by the government.

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[9] A proof of consumer surplus is offered in Appendix B.

It is important to note that in this model, the government does not discriminate between the surplus or utility gained by the user from *consuming* the legal or illegal marijuana. Hence, we allow consumer surplus to be determined from expenditures in the entire market. However, it does discriminate against marijuana illegally *purchased* because this increases profits for the cartel or gang, which may be used for any number of other criminal activities related to drug production and trafficking, such as murder, coercion, blackmail, etc. So, we allow for positive expenditures of R and view π_C as cost or detriment to society.

Substituting in the quantities q_L^* and q_C^* from (7) and (8) into the welfare function and using the property that $\pi_i = q_i^*$, we can see that for T^* and R^* to be a Nash equilibrium for the government, it must solve

$$\max_{T,R} W(T,R) = \frac{1}{2}\left(\frac{2(a-c)-\left(\theta\sqrt{T}+\gamma\sqrt{R}\right)}{3}\right)^2 + \left(\frac{(a-c)+\left(\theta\sqrt{T}+\gamma\sqrt{R}\right)}{3}\right)^2 -$$
$$\left(\frac{(a-c)-2\left(\theta\sqrt{T}+\gamma\sqrt{R}\right)}{3}\right)^2 - T - R \tag{11}$$

Solving first for the optimal quantity of T, we take the first order condition of the above function with respect to T. We can then find T as a function of R, given by

$$T^*(R) = \left(\frac{4\theta(a-c)-5\theta\gamma\sqrt{R}}{18+5\theta^2}\right)^2 \tag{12}$$

Solving now for the optimal quantity of R, we take the first order condition of *W* with respect to R and solve for R as a function of T. We obtain

$$R^*(T) = \left(\frac{4\gamma(a-c)-5\theta\gamma\sqrt{T}}{18+5\gamma^2}\right)^2 \tag{13}$$

We can see that the optimal quantities are both very similar, which is expected since T and R have the same functional form and vary by the parameters θ and γ. If we solve these functions simultaneously, we can get T^* and R^*, given below, in terms of the parameters only.

$$T^* = \left(\frac{4\theta(a-c)}{18 + 5(\theta^2 + \gamma^2)}\right)^2 \tag{14}$$

$$R^* = \left(\frac{4\gamma(a-c)}{18 + 5(\theta^2 + \gamma^2)}\right)^2 \tag{15}$$

From here, we can see that the optimal levels of T and R will always be positive because of the squared term. Additionally, T^* is increasing in θ and decreasing in γ, while the opposite is true for R^*. By substituting T^* and R^* into (7) and (8), we can get the subgame-perfect optimal quantities for each firm which are presented in Chapter 4.

3.2 Pre-Legalization Model

The Firms' Output Decision

To perform some comparative analysis of the marijuana market before legalization, we now consider a model in which all production of the drug is illegal. Much of the setup is the same as the previous model and we begin by looking at the two firms' maximization problems.

Inverse demand is given by $P(Q) = a - Q$, where a measures demand and Q is total market supply. Both firms again seek to maximize their profits by choosing optimal levels of output. However, in this case, each firm face the same maximization problem, given by

$$\max_{q_i} \pi_i = pq_i - q_i\left(c + \theta\sqrt{T} + \gamma\sqrt{R}\right) \tag{16}$$

This is because both firms now face the added marginal costs of the government's interdiction and enforcement efforts. Substituting P(Q) for p, we get

$$\max_{\substack{q_i \\ i \neq j}} \pi_i = q_i\left(a - q_i - q_j - c - \theta\sqrt{T} - \gamma\sqrt{R}\right) \tag{17}$$

Taking the first-order condition and solving for q_i, we find firm i's reaction function, given by

$$R_i(q_j) = \frac{(a-c) - q_j^* - \theta\sqrt{T} - \gamma\sqrt{R}}{2} \tag{18}$$

Since both firms are identical and are playing the same strategy, we can assume $q_i = q_j$, simplify the equation, and solve for q_i. Each firm's optimal quantity, q_i^* then becomes

$$q_i^* = \frac{(a-c) - \left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3} = q_j^* \tag{19}$$

We can see that quantities (and profits[10]) for *both* firms are now inversely related to T and R, as expected. However, each firm is producing more than the illegal firm in the first model because both are sharing the burden of the government's interdiction and enforcement efforts.

Total market supply, Q, is then equal to

$$Q = \frac{2(a-c) - 2\left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3} \tag{20}$$

We can see that this is less than the market supply for the post-legalization model given by (9) because $(\theta\sqrt{T} + \gamma\sqrt{R})$ now has a coefficient of -2. As before, we can also expect prices to increase with T and R.

Using (19), we can now move to the government's welfare maximization problem.

The Government's Allocation of T and R

Like before, the government seeks to maximize social welfare by allocating resources to reduce the profits of the illegal firms. Though, in this model, it views the utility from the consumption of marijuana as a detriment to society. So, it seeks to maximize the welfare function given by

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[10] The author leaves it to the reader to prove that $\pi_i = q_i^2$ in this model as well.

$$\max_{T,R} W = -\frac{1}{2}Q^2 - \Pi - T - R \tag{21}$$

Here, Q is again the consumer surplus in the market, Π is industry profits (equal to the sum of profits for each firm), and T and R are expenditures on government resources for interdiction and enforcement. One immediate observation that we can make is that the government believes welfare can never be positive in a society where marijuana is produced and consumed. This was certainly the viewpoint of the federal government and many state governments in the mid-to-late twentieth century, so the notion should not seem too surprising.

Substituting (19) and (20) into the welfare function, we get

$$\max_{T,R} W(T,R) = \frac{1}{2}\left(\frac{2(a-c) - 2\left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3}\right)^2 - 2\left(\frac{(a-c) - \left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3}\right)^2$$
$$-T - R \tag{22}$$

This may be simplified further to

$$\max_{T,R} W(T,R) = -4\left(\frac{(a-c) - \left(\theta\sqrt{T} + \gamma\sqrt{R}\right)}{3}\right)^2 - T - R \tag{23}$$

Using the same steps as in the previous model, we find the optimal quantities of T and R, given as functions of the other:

$$T^*(R) = \left(\frac{4\theta(a-c) - 4\theta\gamma\sqrt{R}}{9 + 4\theta^2}\right)^2 \tag{24}$$

$$R^*(T) = \left(\frac{4\gamma(a-c) - 4\theta\gamma\sqrt{T}}{9 + 4\gamma^2}\right)^2 \tag{25}$$

Solving this system of equations simultaneously gives us

$$T^* = \left(\frac{4\theta(a-c)}{9 + 4(\theta^2 + \gamma^2)}\right)^2 \tag{26}$$

$$R^* = \left(\frac{4\gamma(a-c)}{9 + 4(\theta^2 + \gamma^2)}\right)^2 \tag{27}$$

We find that the government's choices of T^* and R^* are higher in this model than in the previous one (due to a smaller denominator). This is not unexpected since the government must now try to stop both firms from supplying the drug to the market and crack down on all consumers who purchase it. Substituting these quantities into (19) gives us the subgame-perfect outcome, presented in the next chapter.

CHAPTER 4

FINDINGS AND COMPARATIVE STATICS

4.1 Post-Legalization Model

Subgame-Perfect Outcome

Using the optimal amounts of T* and R* from (14) and (15), we substitute into (7) and (8)

and solve the post-legalization model completely to find the subgame-perfect outcome quantities

for each type of firm. They are given by

$$q_L^* = \frac{(a-c)[6 + 3(\theta^2 + \gamma^2)]}{18 + 5(\theta^2 + \gamma^2)} \tag{28}$$

$$q_C^* = \frac{(a-c)[6 - (\theta^2 + \gamma^2)]}{18 + 5(\theta^2 + \gamma^2)} \tag{29}$$

Because $\theta, \gamma \in (0,1)$, we can see that the quantity produced by each firm will be positive, with

$q_L^* \in (\frac{a-c}{3}, \frac{3(a-c)}{7})$, and $q_C^* \in (\frac{a-c}{7}, \frac{a-c}{3})$. If both θ and γ are approaching zero, which is

analogous to the government not allocating any resources for T and R, each firm produces the

standard Cournot duopoly quantity, $(a - c)/3$.[11] This is the lowest amount of output the legal

firm will produce, but the highest amount of output the illegal firm can achieve. On the other

hand, if both θ and γ are approaching one, or the government is totally efficient at using every

dollar spent on interdiction and enforcement, the illegal firm will still produce a positive,

nonzero quantity, but it is only 1/3 of what the legal firm will produce. Remember that even if θ

and γ are approximately one, the government still faces decreasing marginal returns in T and R,

hence, the gang or cartel can produce a positive quantity of marijuana. Nevertheless, we can see

that increases in the efficiency parameters still result in lower output (and profit) for the illegal

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[11] Output for each firm in Cournot competition facing a linear demand $P(Q) = a - Q$, and constant marginal costs $c,$ is equal to $(a - c)/(N+1)$ where N is the number of firms in the market.

firm. This allows the legal firm to boost production, obtain more market share, and ultimately, receive higher profits. It goes without saying that if the gap between demand and the constant marginal cost, $a - c$, is arbitrarily small, or approaching zero, both firms produce very little.

From (28) and (29), we can derive total market supply, given by

$$Q^* = \frac{(a-c)[12 + 2(\theta^2 + \gamma^2)]}{18 + 5(\theta^2 + \gamma^2)} \tag{30}$$

We see that $Q^* \in (\frac{4(a-c)}{7}, \frac{2(a-c)}{3})$ because θ and γ are both between 0 and 1. As mentioned previously, total market supply is the largest when both θ and γ are approximately zero. In which case, supply will be equal to market output under Cournot duopoly competition, $2(a - c)/3$. As the efficiency parameters increase, or if the government allocates a nonzero amount of resources to T and R, the legal firm increases production, but total market supply is decreasing. Mathematically, this is easy to see in (9), but in (30) it may not be immediately obvious that the denominator is increasing at a faster rate than the numerator, resulting in the lower market supply. We may also notice that market supply is always larger than if there were only one firm operating in the market. In which case, Q would be equal to $(a - c)/2$. Relatedly, we can see that quantity supplied to the market will never equal that of perfect competition, or $(a - c)$, which is to be expected since neither firm has an incentive to increase their production and receive lower profits.

Effects on the variables of the model, given an increase in one parameter and holding the other parameters constant, are summarized in Table 1 below. Most of the results are expected and many of them have been discussed in the preceding sections, so we focus on some of the more interesting and potentially less obvious outcomes.

From the table, we can see that T is increasing in θ but decreasing in γ while the opposite is true for R. Looking back to (12) and (13) may shed some light on this result. In (12), T is

function of $-\sqrt{R}$ and in (13), R is a function of $-\sqrt{T}$. The negative coefficients capture the

trade-off the government faces when allocating funds between the two resources. That is, as

more funds are spent on policing the gang or cartel, less needs to be spent on policing consumers

who purchase from the gang or cartel. Additionally, if θ is increasing, or law enforcement is

becoming more efficient in its interdiction efforts, the coefficient in front of T in the illegal

firm's quantity decision, (8), is now higher, meaning T has a larger impact than before.

Therefore, the marginal returns to investing in interdiction are higher than for investing in

enforcement. From the government's perspective, each additional dollar it spends is costly, so it

makes sense to spend it where it can have the largest impact. Hence, if θ is increasing, the

government diverts funds from R and reallocates them to T, and if γ is increasing, it does the

opposite.

Table 1 – Sign Changes for Post-Legalization Model

	Variables					
Parameters	**T***	**R***	**q_L***	**q_C***	**Q**	**P**
θ	+	–	+	–	–	+
γ	–	+	+	–	–	+
a	+	+	+	+	+	+
c	–	–	–	–	–	+

Source: Author's calculations
Note: $\pi_i = q_i^2$, so we exclude it from the table.

We can also see that an increase in the scale parameter measuring demand, *a*, results in a

subsequent increase in every variable. This should seem intuitive. If demand is increasing and

consumers are willing to pay more, firms are going to expand their production efforts to increase

revenues and, because marginal costs have not increased, increase their profits. With more

marijuana in the market and no increases in the efficiency of law enforcement, the government must spend more to reduce the illegal firm's output and profits. This brings us to a limiting result of the models presented in this paper. Because the government's resources allocated to interdiction and enforcement are unconstrained in the welfare function, as $a \rightarrow \infty$, so too will T* and R*. In reality, this is not feasible. The government has a finite amount of resources and many other public programs to fund besides drug interdiction and enforcement. However, leaving T and R unconstrained should not change the inference mentioned above, only the optimal amounts of T* and R*.

Additionally, we can see that an increase in any parameter also increases prices in the market. Some of this can be attributed to government intervention. As θ or γ increases, the government spends more on T or R, leading to lower output for the illegal firm and lower overall market supply. The resulting scarcity leads to an increase in prices. As the marginal cost of production, c, increases, firms produce less to maximize their profits, and market supply decreases. In this case, the government no longer needs to spend as much on interdiction and enforcement. And finally, we see that an increase in demand, a, outweighs the resulting increase in market supply, resulting in higher prices. Interestingly, there would be no effect on prices if the government were totally efficient in its interdiction and enforcement efforts ($\theta = \gamma = 1$), as seen below.

$$\frac{\partial P(Q)}{\partial a} = 1 - \frac{[12 + 2(\theta^2 + \gamma^2)]}{18 + 5(\theta^2 + \gamma^2)} = \frac{6 - 3(\theta^2 + \gamma^2)}{18 + 5(\theta^2 + \gamma^2)} \tag{31}$$

As a note, the above results relating to price increases may not hold if more than two firms operate in the market, or if firms continue to enter the market. In this case, depending on the type and/or number of firms, we might see prices fall in some instances as they supply more marijuana.

4.2 Pre-Legalization Model

Subgame-perfect Outcome

We move now to examine the subgame-perfect outcome quantities for each firm in the

pre-legalization model after substituting (26) and (27) into (19) and solving. The optimal

quantities are given by

$$q_i^* = \frac{3(a - c)}{9 + 4(\theta^2 + \gamma^2)} = q_j^* \tag{32}$$

We see again that each firm will produce a positive, nonzero, quantity because the government's

effect on the firms is subject to decreasing marginal returns. We find that $q_i^* \in (\frac{3(a-c)}{17}, \frac{(a-c)}{3})$.

As was the case before, each firm produces the Cournot duopoly amount when θ and γ approach

zero. However, each firm produces more in this model, $\frac{3(a-c)}{17}$, than the illegal firm in previous

model, $\frac{a-c}{7}$, when θ and γ approach one. This is because each firm shares the burden of the

government's interdiction and enforcement efforts, while only the illegal firm is negatively

impacted after legalization.

The sum of output for each firm is given by

$$Q^* = \frac{6(a - c)}{9 + 4(\theta^2 + \gamma^2)} \tag{33}$$

and $Q^* \in (\frac{6(a-c)}{17}, \frac{2(a-c)}{3})$. Because θ and γ only appear in the denominator, we can see that in

this model, any increases in the two parameters result in a decrease in market supply. So, as

before, Q is the largest when θ and γ are approximately zero, and equals the market supply under

Cournot competition. And Q is the smallest when θ and γ are approximately one. In this case,

we find that market supply is less than $(a - c)/2$, or the monopoly quantity. In fact, so long as

$(\theta^2 + \gamma^2) \geq 3/4$, market supply will be less than the monopoly quantity. Additionally, neither firm

has any incentive to completely meet market demand and subsequently reduce their profits.

Table 2 below summarizes the comparative statics from the pre-legalization model. The

results and analysis are the same as in the post-legalization model, excluding q_L^*.

Table 2 – Sign Changes for Pre-Legalization Model

Parameters	Variables				
	T*	**R***	**q_i***	**Q**	**P**
θ	+	−	−	−	+
γ	−	+	−	−	+
a	+	+	+	+	+
c	−	−	−	−	+

Source: Author's calculations
Note: $\pi_i = q_i^2$, so we exclude it from the table.

Table 3 below lists the subgame-perfect outcomes for both models. Comparing some of

those outcomes we can see that the government will always spend more on T and R in the pre-

legalization model because the numerators for each variable are the same, but the denominators

will always be less than those in the post-legalization model. This makes sense given the

government's view of the drug as a detriment to society in the pre-legalization model, and

because it is allocating resources to reduce the output and profits of two firms instead of one.

Additionally, each of the illegal firms in the pre-legalization model will always produce more

than the illegal firm in the post-legalization model, but market supply is always lower in the pre-

legalization model. So, we can propose that if the government believes society can receive a net

benefit or utility from legalizing marijuana and allowing some production of the drug to be legal,

it may end up spending less than during prohibition, and reduce profits for the cartel or gangs.

Of course, with legalization, demand for the drug may increase as curious and previously law-abiding citizens seek to obtain it, which could instead result in the government needing to spend more on interdiction and enforcement as discussed at the end of Section 4.1.

Table 3 – Subgame-Perfect Outcomes for Both Models

	Pre-Legalization	Post-Legalization
q_i^*	$\dfrac{3(a-c)}{9+4(\theta^2+\gamma^2)}$	$q_L^* = \dfrac{(a-c)[6+3(\theta^2+\gamma^2)]}{18+5(\theta^2+\gamma^2)}$ $q_C^* = \dfrac{(a-c)[6-(\theta^2+\gamma^2)]}{18+5(\theta^2+\gamma^2)}$
Q^*	$\dfrac{6(a-c)}{9+4(\theta^2+\gamma^2)}.$	$\dfrac{(a-c)[12+2(\theta^2+\gamma^2)]}{18+5(\theta^2+\gamma^2)}$
T^*	$\left(\dfrac{4\theta(a-c)}{9+4(\theta^2+\gamma^2)}\right)^2$	$\left(\dfrac{4\theta(a-c)}{18+5(\theta^2+\gamma^2)}\right)^2$
R^*	$\left(\dfrac{4\gamma(a-c)}{9+4(\theta^2+\gamma^2)}\right)^2$	$\left(\dfrac{4\gamma(a-c)}{18+5(\theta^2+\gamma^2)}\right)^2$

Note: $\pi_i^* = q_i^2$ so it is excluded for brevity

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

As public support for marijuana legalization reaches new highs in the U.S.[12] and state legislatures are forced to reexamine the topic, it is likely that more states will not only decriminalize the drug, but also allow its retail sale. Revisiting the question posed at the beginning of the paper, we find that, ceteris paribus, a government who legalizes marijuana should indeed spend less on enforcement and interdiction. We also find that allowing legal production of the drug, while maintaining targeted enforcement and interdiction policies, can reduce a cartel or gang's profits. This is likely to limit their ability to engage in other criminal activities and increase social welfare. And for the government, finding ways to increase the efficiency of its interdiction and enforcement efforts can further suppress production by the illegal firm while preserving and increasing the legal firm's market share.

Because game theoretic literature, or economic literature at all, on the post-legalization markets for marijuana is scarce, this paper may be considered as a starting point for future research. There are numerous possibilities for extension and we examine a few below.

As mentioned in Chapter 3, a more general model might allow for substitutability based on THC content between the marijuana produced by the legal and illegal firm. While substitutability is presumed to be high in the current market, that may not be the case as the market matures. As access to the tools and labor necessary to breed cannabis plants with a higher THC content expands for the legal firm, it is all but inevitable that the substitutability gap will widen. This could have the effect of further reducing the illegal firm's output and profits if consumers preferred the drug with the higher potency, and reduce the optimal amount the

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[12] http://news.gallup.com/poll/221018/record-high-support-legalizing-marijuana.aspx

government should invest in interdiction and enforcement. However, this might not change any of the analysis offered in the models studied here. That is, changes in the other parameters are likely to have the same effects on variables in the model.

One might also consider the case of an N-firm market where a portion of the firms are licensed to operate and the others are not. This might provide a counterpoint to the comparative statics analysis in Chapter 4 that shows increases in any of the parameters will also increase prices. In the standard Cournot model, prices typically fall as competition increases and more firms supply to the market. It is likely that this would be the case in a multiple firm market for marijuana as well, although the outcome may depend on how many firms are producing legally or illegally.

Additionally, constraining the optimization of T and R to a set proportion of the government's overall budget would provide a realistic scenario of the government's limited resources. In the same vein, allowing the government to charge a per unit or sales tax on the marijuana produced by the legal firm might allow for a more representative case of markets in states where the drug has been legalized. A portion of the tax revenue could be combined with other appropriations to create the budgetary constraint the government faces. This would likely reduce the quantity supplied by the legal firm and allow the illegal firm to expand production which could result in the government needing to spend more on T and R.

And finally, it would also be reasonable to allow for differing cost structures between the two firms. Because they can rent or own large warehouses to cultivate the marijuana, legal producers can take advantage of economies of scale, while illegal producers must decentralize their growing operations to avoid government detection. It may also be unrealistic for the government to have perfect information about the cost structure of the illegal producers. Since

they have no taxes or regulatory paperwork to file, an element of uncertainty may be introduced regarding the criminal firm's marginal cost function.

While these are just a few possible extensions to enrich the models presented here, countless others exist. We may consider this paper just one small part of the comprehensive literature on the subject that is likely to appear as legalization expands to more states and even other countries.

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APPENDICES

APPENDIX A

Firm Profit Equal to Quantity Squared

Legal Firm:

$$\pi_L = q_L[a - q_L - q_C - c]$$

$$\pi_L = q_L\left[a - \frac{(a-c) + (\theta\sqrt{T} + \gamma\sqrt{R})}{3} - \frac{(a-c) - 2(\theta\sqrt{T} + \gamma\sqrt{R})}{3} - c\right]$$

$$\pi_L = q_L\left[\frac{3(a-c) - 2(a-c) - (\theta\sqrt{T} + \gamma\sqrt{R}) + 2(\theta\sqrt{T} + \gamma\sqrt{R})}{3}\right]$$

$$\pi_L = q_L\left[\frac{(a-c) + (\theta\sqrt{T} + \gamma\sqrt{R})}{3}\right]$$

$$\pi_L = q_L[q_L]$$

$$\pi_L = q_L^2$$

Illegal Firm:

$$\pi_C = q_C[a - q_L - q_C - c - \theta\sqrt{T} - \gamma\sqrt{R}]$$

$$\pi_C = q_C\left[a - \frac{(a-c) + (\theta\sqrt{T} + \gamma\sqrt{R})}{3} - \frac{(a-c) - 2(\theta\sqrt{T} + \gamma\sqrt{R})}{3} - c - (\theta\sqrt{T} + \gamma\sqrt{R})\right]$$

$$\pi_C = q_C\left[\frac{3(a-c) - 2(a-c) - (\theta\sqrt{T} + \gamma\sqrt{R}) + 2(\theta\sqrt{T} + \gamma\sqrt{R}) - 3(\theta\sqrt{T} + \gamma\sqrt{R})}{3}\right]$$

$$\pi_C = q_C\left[\frac{(a-c) - 2(\theta\sqrt{T} + \gamma\sqrt{R})}{3}\right]$$

$$\pi_C = q_C[q_C]$$

$$\pi_C = q_C^2$$

APPENDIX B

Proof of Consumer Surplus

Where P(Q) is market demand and QP(Q) is total expenditures for a certain good.

$$\int_0^Q P(Q)dQ - QP(Q)$$

$$\int_0^Q (a - Q)dQ - Q(a - Q)$$

$$[aQ - \frac{1}{2}Q^2]_0^Q - aQ - Q^2$$

$$aQ - \frac{1}{2}Q^2 - aQ + Q^2$$

$$= \frac{1}{2}Q^2$$

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