IN THE MAZES OF MATHEMATICS.

A SERIES OF PERPLEXING QUESTIONS.

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III. GEOMETRIC PUZZLES.

"A RECTANGULAR hole 13 inches long and 5 inches wide was discovered in the bottom of a ship. The ship's carpenter had only one piece of board with which to make repairs, and that was but 8 inches square (64 square inches) while the hole contained 65 square inches. But he knew how to cut the board so as to make it fill the hole!" Or, in more prosaic form:

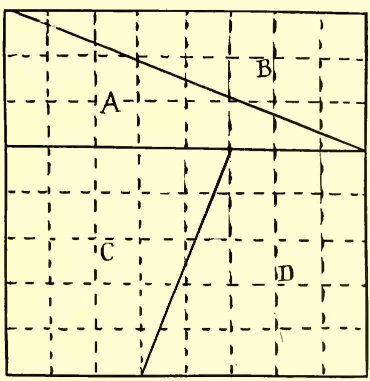

Fig. 1.

Fig. 1 is a square, 8 units on a side, area 64; cut it through the heavy lines and rearrange the pieces as indicated by the letters in Fig. 2, and you have a rectangle 5 by 13, area 65. Explain.

Fig. 3 explains. EH is a straight line, and HG is a straight line; but they are not parts of the same straight line. Proof:

Let X be the point at which EH produced meets GJ; then from the similarity of triangles EHK and EXJ

$$XJ : HK = EJ : EK$$
$$XJ : 3 = 13 : 8$$
$$XJ = 4.875$$
$$\text{But } GJ = 5$$

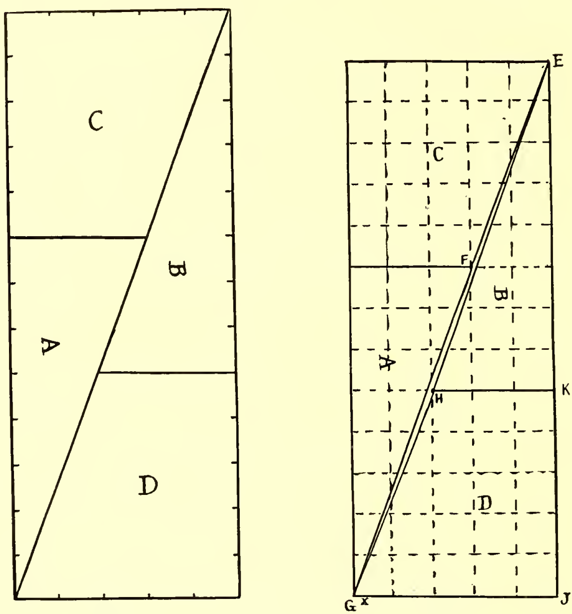

Fig. 2. Fig. 3.

Similarly, EFG is a broken line.

The area of the rectangle is, indeed, 65, but the area of the rhomboid $EFGH$ is 1.

This paradox is referred to as early as 1877, in the *Messenger of Mathematics*; cited by W. W. R. Ball (*Mathematical Recreations and Essays,* Macmillan, 1905, p. 49) who uses this to illustrate that proofs by dissection and superposition are to be regarded with suspicion until supplemented by mathematical reasoning.

Another puzzle is made by constructing a cardboard rectangle 13 by 11, cutting it through one of the diagonals (Fig. 4) and

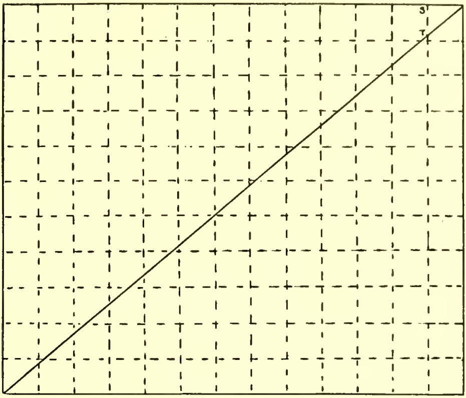

Fig. 4.

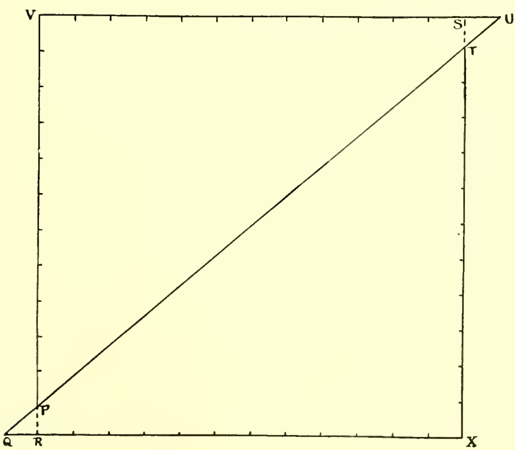

Fig. 5.

sliding one triangle against the other along their common hypotenuse to the position shown in Fig. 5. Query: How can Fig. 5

be made up of square $VRXS$, 12 on a side, area 144, + triangle PQR, area 0.5, + triangle STU, area 0.5, = total area 145; when the area of Fig. 4 is only 143?

Inspection of the figures, especially if aided by the cross lines, will show that $VRXS$ is not a square. VS is 12 long; but $SX < 12$. $TX = 11$ (the shorter side in Fig. 4, but $ST < 1$ (see ST in Fig. 4).

$$ST : VP = SU : VU$$

$$ST : 11 = 1 : 13$$

$$ST = {}^{11}/_{13}$$

$$\text{Square } VRXS = 12 \times 11\,{}^{11}/_{13} = 142\,{}^{2}/_{13}$$

$$\text{Triangle } PQR = \text{triangle } STU = {}^{1}/_{2} \cdot {}^{11}/_{13} \cdot 1 = {}^{11}/_{26}$$

$$\text{Fig. } 5 = \text{square} + 2 \text{ triangles} = 142\,{}^{2}/_{13} + {}^{11}/_{13} = 143.$$