The Open Court

A MONTHLY MAGAZINE

Devoted to the Science of Religion, the Religion of Science, and the Extension of the Religious Parliament Idea

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THE CHARITY BALL.

By L. P. de Laubadere.

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THE NATURE OF MATHEMATICAL REASONING.[1]

BY WM. F. WHITE, PH. D.

WHY is mathematics "the exact science"? Because of its self-imposed limitations. Mathematics concerns itself, not with any problem of the nature of things in themselves, but with the simpler problems of the relations between things. Starting from certain definite assumptions, the mathematician seeks only to arrive by legitimate processes at conclusions that are surely right if the data are right; as in geometry. So the arithmetician is concerned only that the result of his computation shall be correct assuming the data to be correct; though if he is also a teacher, he is in that capacity concerned that the data of the problems set for his pupils shall correspond to actual commercial, industrial or scientific conditions of the present day.

Mathematics is usually occupied with the consideration of only one or a few of the phases of a situation. Of the many conditions involved, only a few of the most important and the most available are considered. All other variables are treated as constants. Take for illustration the "cistern problem," which as it occurs in the writings of Heron of Alexandria (c. 2d cent. B. C.) must be deemed very respectable on the score of age: given the time in which each pipe can fill a cistern separately, required the time in which they will fill it together. This assumes the flow to be constant. Other statements of the problem, in which one pipe fills while another empties, presuppose the outflow also to be constant whether the cistern is full or nearly empty; or at least the rate of outflow is

[1] Condensed from an address given by the author to the advanced section of teachers' institutes.

taken as an average rate and treated as a constant. Or the "days-work problem" (which is only the cistern problem disguised): given the time in which each man can do a piece of work separately, required the time in which they will do it together. This assumes that the men work at the same rate whether alone or together. Some persons who have employed labor know how violent an assumption this is, and are prepared to defend the position of the thoughtless schoolboy who says, "If A can do a piece of work in 5 days which B can do in 3 days, it will take them 8 days working together," as against the answer $1\frac{7}{8}$ days, which is deemed orthodox among arithmeticians. Or, to move up to the differential calculus for an illustration: "The differentials of variables which change non-uniformly are what *would be* their corresponding increments if at the corresponding values considered the change of each became and continued uniform with respect to the same variable."[2]

Mathematics resembles fine art in that each abstracts some one pertinent thing, or some few things, from the mass of things and concentrates attention on the element selected. The landscape painter gives us, not every blade of grass, but only those elements that serve to bring out the meaning of the scene. With mathematics also as with fine art, this may result in a more valuable product than any that could be obtained by taking into account every element. The portrait painted by the artist does not exactly reproduce the subject as he was at any one moment of his life, yet it may be a truer representation of the man than one or all of his photographs. So it is with one of Shakespeare's historical dramas and the annals which were its "source." "The truest things are things that never happened."

Mathematics is a science of the ideal. The magnitudes of geometry exist only as mental creations, a chalk mark being but a physical aid to the mind in holding the conception of a geometric line.

The concrete is of necessity complex; only the abstract can be simple. This is why mathematics is the simplest of all studies—simplest in proportion to the mastery attained. The same standard of mastery being applied, physics is much simpler than biology: it is more mathematical. As we rise in the scale mathematically, relations become simple, until in astronomy we find the nearest approach to conformity by physical nature to a *single* mathematical law, and we see a meaning in Plato's dictum, "God geometrizes continually."

Mathematics is thinking God's thought after him. When any-

[2] Taylor's *Calculus*, p. 8. Ginn, 1898.

thing is *understood,* it is found to be susceptible of mathematical statement. The vocabulary of mathematics "is the ultimate vocabulary of the material universe." The planets had for many centuries been recognized as "wanderers" among the heavenly bodies; much had come to be known about their movements; Tycho Brahe had made a series of careful observations of Mars; Kepler stated the law: Every planet moves in an elliptical orbit with the sun at one focus. When the motion was understood, it was expressed in the language of mathematics. Gravitation waited long for a Newton to state its law. When the statement came, it was in terms of "the ultimate vocabulary": Every particle of matter in the universe attracts every other particle with force varying directly as the masses, and inversely as the square of the distances. When any other science —say psychology—becomes as definite in its results, those results will be stated in as mathematical language. After many experiments to determine the measure of the increase of successive sensations of the same kind when the stimulus increases, and after tireless effort in the application of the "just perceptible increment" as a unit, Professor G. T. Fechner of Leipsic announced in 1860, in his *Psychophysik*, that the sensation varies as the logarithm of the stimulus. Fechner's law has not been established by subsequent investigations; but it was the expression of definiteness in thinking, whether that thinking was correct or not, and it illustrates mathematics as the language of precision.

Mathematics is ultimate in the generality of its reasoning. By the aid of symbols it transcends experience and the imaging power of the mind. It determines, for example, the number of diagonals in a polygon of 1000 sides to be 498500 by substitution in the easily deduced formula $n(n-3)/2$, although one never has occasion to draw a representation of a 1000-gon and could not make a distinct mental picture of its 498500 diagonals.

If there are other inhabited planets, doubtless "these all differ from one another in language, customs and laws." But one can not imagine a world in which 3×5 is not equal to $8 + 7$, or e not equal to $2.718281\ldots$, or π not equal to $3.1415926535\ldots$, though all the *symbols* for number might easily be very different.

In recent years a few "astronomers," with an enterprise that would reflect credit on an advertising bureau, have discussed in the newspapers plans for communicating with the inhabitants of Mars. What symbols could be used for such communication? Obviously those which must be common to rational beings everywhere. Accordingly it was proposed to lay out an equilateral triangle many kilometers on a side and illuminate it with powerful arc lights. If our Martian neighbors should reply with a triangle, we could then test them on other polygons. Apparently the courtesies exchanged would for some time have to be confined to the amenities of geometry.

Civilization is humanity's response to the first—not the last, or by any means greatest—command of its Maker, "Subdue the earth and have dominion over it." And the aim of applied mathematics is "the mastery of the world quantitively." "Science is only quantitative knowledge." Hence mathematics is an index of the advance of civilization.

The applications of mathematics have furnished the chief incentive to the investigation of pure mathematics and the best illustrations in the teaching of it; yet the mathematician must keep the abstract science in advance of the need for its application, and must even push his inquiry in directions that offer no prospect of any practical application, both from the point of view of truth for truth's sake and from a truly farsighted utilitarian viewpoint as well. Whewell said, "If the Greeks had not cultivated conic sections, Kepler could not have superseded Ptolemy." Behind the artisan is a chemist, "behind the chemist a physicist, behind the physicist a mathematician." It was Michael Faraday who said, "There is nothing so prolific in utilities as abstractions."