2013

# Plots And Testing For Multivariate Linear Regression

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# PLOTS AND TESTING FOR MULTIVARIATE LINEAR REGRESSION

by

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Bachelor of Science, University of Sri Jayewardenepura, 2010

A Research Paper
Submitted in Partial Fulfillment of the Requirements for the
Master of Science Degree

Department of Mathematics
in the Graduate School
Southern Illinois University Carbondale
June 2013

**RESEARCH PAPER APPROVAL**

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June 20, 2013

## AN ABSTRACT OF THE RESEARCH PAPER OF

PELAWA WATAGODA LASANTHI CHATHURIKA RANASINGHE, for the Master of Science degree in MATHEMATICS, presented on June 20, 2013, at Southern Illinois University Carbondale.

TITLE: PLOTS AND TESTING FOR THE
MULTIVARIATE LINEAR MODEL

MAJOR PROFESSOR: Dr. D. Olive

To use the multivariate linear regression model, tests of hypotheses are needed. This paper gives $F$ approximations to the widely used Wilk, Pillai, and Hotelling Lawley test statistics, and gives plots to check goodness and lack of fit, to check for outliers and influential cases, and to check whether the error distribution is multivariate normal.

## ACKNOWLEDGMENTS

I acknowledge with gratitude to Dr. David J. Olive for his in invaluable assistance and insights leading to the writing of this paper. He has been supportive and patient throughout the production of this paper. I have found that he is not only an outstanding professor, but an excellent advisor too.

I would like to express my gratitude to the members of my graduate committee Dr. Bhaskar Bhattacharya and Dr. Mohammed, Salah-Eldin. I would also like to thank the authors of whose books and research papers were used in the writing of this paper.

A special thanks goes to my parents and my husband for their support and encouragement.

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# CHAPTER 1

# INTRODUCTION

For the multivariate linear regression model, this paper provides useful plots, and gives $F$ approximations to the widely used Wilk, Pillai, and Hotelling Lawley test statistics. First notation is given for the multivariate linear regression model. We follow Olive (2013) closely.

The *multivariate linear regression model* $\boldsymbol{y}_i = \boldsymbol{B}^T \boldsymbol{x}_i + \boldsymbol{\epsilon}_i$ for $i = 1, ..., n$ has $m \geq 2$ response variables $Y_1, ..., Y_m$ and $p$ predictor variables $x_1, x_2, ..., x_p$, with $x_{i1} = 1$. The $i$th case is $(\boldsymbol{x}_i^T, \boldsymbol{y}_i^T) = (x_{i1}, x_{i2}, ..., x_{ip}, Y_{i1}, ..., Y_{im})$. The model is written in matrix form as $\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{B} + \boldsymbol{E}$ where the matrices are defined below. The model has $E(\boldsymbol{\epsilon}_k) = \boldsymbol{0}$ and $\text{Cov}(\boldsymbol{\epsilon}_k) = \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} = (\sigma_{ij})$ for $k = 1, ..., n$. Also $E(\boldsymbol{e}_i) = \boldsymbol{0}$ while $\text{Cov}(\boldsymbol{e}_i, \boldsymbol{e}_j) = \sigma_{ij} \boldsymbol{I}_n$ for $i, j = 1, ..., m$ where $\boldsymbol{I}_n$ is the $n \times n$ identity matrix and $\boldsymbol{e}_i$ is defined below. Then the $p \times m$ coefficient matrix $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{\beta}_1 & \boldsymbol{\beta}_2 & \dots & \boldsymbol{\beta}_m \end{bmatrix}$ and the $m \times m$ covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ are to be estimated, and $E(\boldsymbol{Z}) = \boldsymbol{X}\boldsymbol{B}$ while $E(Y_{ij}) = \boldsymbol{x}_i^T \boldsymbol{\beta}_j$.

The $n \times m$ matrix of response variables

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Y}_1 & \boldsymbol{Y}_2 & \dots & \boldsymbol{Y}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_1^T \\ \vdots \\ \boldsymbol{y}_n^T \end{bmatrix}.$$

The $n \times p$ design matrix of predictor variables

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \dots & \boldsymbol{v}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_1^T \\ \vdots \\ \boldsymbol{x}_n^T \end{bmatrix}$$

where $\boldsymbol{v}_1 = \boldsymbol{1}$.

The $n \times m$ matrix of errors

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 & \dots & \boldsymbol{e}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\epsilon}_1^T \\ \vdots \\ \boldsymbol{\epsilon}_n^T \end{bmatrix}.$$

Least squares is the classical method for fitting the multivariate linear model. The *least squares estimators* are $\hat{\boldsymbol{B}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Z} = \begin{bmatrix} \hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\beta}}_2 & \dots & \hat{\boldsymbol{\beta}}_m \end{bmatrix}$. The *predicted values* or *fitted values* $\hat{\boldsymbol{Z}} = \boldsymbol{X}\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{\boldsymbol{Y}}_1 & \hat{\boldsymbol{Y}}_2 & \dots & \hat{\boldsymbol{Y}}_m \end{bmatrix}$. The *residuals*

$$\hat{\boldsymbol{E}} = \boldsymbol{Z} - \hat{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{\boldsymbol{\epsilon}}_1^T \\ \hat{\boldsymbol{\epsilon}}_2^T \\ \vdots \\ \hat{\boldsymbol{\epsilon}}_n^T \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{r}}_1 & \hat{\boldsymbol{r}}_2 & \dots & \hat{\boldsymbol{r}}_m \end{bmatrix}.$$

These quantities can be found from the $m$ multiple linear regressions of $Y_j$ on the predictors: $\widehat{\boldsymbol{\beta}}_j = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}_j$, $\widehat{\boldsymbol{Y}}_j = \boldsymbol{X}\widehat{\boldsymbol{\beta}}_j$ and $\hat{\boldsymbol{r}}_j = \boldsymbol{Y}_j - \hat{\boldsymbol{Y}}_j$ for $j = 1, ..., m$. Hence $\hat{\epsilon}_{i,j} = Y_{i,j} - \hat{Y}_{i,j}$ where $\hat{\boldsymbol{Y}}_j = (\hat{Y}_{1,j}, ..., \hat{Y}_{n,j})^T$. Finally,

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d} = \frac{(\boldsymbol{Z} - \hat{\boldsymbol{Z}})^T(\boldsymbol{Z} - \hat{\boldsymbol{Z}})}{n-d} = \frac{(\boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}})^T(\boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}})}{n-d} = \frac{\hat{\boldsymbol{E}}^T\hat{\boldsymbol{E}}}{n-d} = \frac{1}{n-d}\sum_{i=1}^n \hat{\boldsymbol{\epsilon}}_i \hat{\boldsymbol{\epsilon}}_i^T.$$

The choices $d = 0$ and $d = p$ are common. If $d = 1$, then $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d=1} = \boldsymbol{S}_r$, the sample covariance matrix of the residual vectors $\hat{\boldsymbol{\epsilon}}_i$ since the sample mean of the $\hat{\boldsymbol{\epsilon}}_i$ is $\boldsymbol{0}$. Let $\hat{\boldsymbol{\Sigma}}_\epsilon = \hat{\boldsymbol{\Sigma}}_{\epsilon,p}$ be the unbiased estimator of $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$.

The $\boldsymbol{\epsilon}_i$ are assumed to be iid. Some important joint distributions for $\boldsymbol{\epsilon}$ are completely specified by an $m \times 1$ population *location* vector $\boldsymbol{\mu}$ and an $m \times m$ symmetric positive definite population *dispersion* matrix $\boldsymbol{\Sigma}$. An important model is the elliptically contoured $EC_m(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g)$ distribution with probability density function

$$f(\boldsymbol{z}) = k_m |\boldsymbol{\Sigma}|^{-1/2} g[(\boldsymbol{z} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{z} - \boldsymbol{\mu})]$$

where $k_m > 0$ is some constant and $g$ is some known function. The multivariate normal (MVN) $N_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution is a special case.

Some additional notation will be useful. Assume that $\boldsymbol{x}_1$, ..., $\boldsymbol{x}_n$ are iid from a multivariate distribution. The classical estimator $(\overline{\boldsymbol{x}}, \boldsymbol{S})$ of multivariate location and dispersion is the sample mean and sample covariance matrix where

$$\overline{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i \quad \text{and} \quad \boldsymbol{S} = \frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}} (\boldsymbol{x}_{\mathrm{i}} - \overline{\boldsymbol{x}})(\boldsymbol{x}_{\mathrm{i}} - \overline{\boldsymbol{x}})^{\mathrm{T}}. \tag{1.1}$$

Let the $p \times 1$ column vector $T$ be a multivariate location estimator, and let the $p \times p$ symmetric positive definite matrix $\boldsymbol{C}$ be a dispersion estimator. Then the $i$th *squared sample Mahalanobis distance* is the scalar

$$D_i^2 = D_i^2(T, \boldsymbol{C}) = (\boldsymbol{x}_i - T)^T \boldsymbol{C}^{-1} (\boldsymbol{x}_i - T) \tag{1.2}$$

for each observation $\boldsymbol{x}_i$. Notice that the Euclidean distance of $\boldsymbol{x}_i$ from the estimate of center $T$ is $D_i(T, \boldsymbol{I}_p)$. The classical Mahalanobis distance uses $(T, \boldsymbol{C}) = (\overline{\boldsymbol{x}}, \boldsymbol{S})$. Following Johnson (1987, pp. 107-108), the population squared Mahalanobis distance

$$U \equiv D^2(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = (\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu}), \tag{1.3}$$

and for elliptically contoured distributions, $U$ has probability density function (pdf)

$$h(u) = \frac{\pi^{p/2}}{\Gamma(p/2)} k_p u^{p/2-1} g(u). \tag{1.4}$$

The next Section suggests plots for multivariate linear models, Chapter 2 shows that the Hotelling Lawley test statistic performs well asymptotically for a large class of zero mean error distributions with fourth moments, and Chapter 3 gives some examples and simulations.

## 1.1 PLOTS FOR THE MULTIVARIATE LINEAR MODEL

A *response plot* for the $j$th response variable is a plot of the fitted values $\widehat{Y}_{ij}$ versus the response $Y_{ij}$ where $i = 1, ..., n$. The identity line with slope one and zero intercept is added to the plot as a visual aid. A *residual plot* corresponding to the $j$th response variable is a plot of $\hat{Y}_{ij}$ versus $r_{ij}$.

Make the $m$ response and residual plots for the multivariate linear model. In a response plot, the vertical deviations from the identity line are the residuals $r_{ij} = Y_{ij} - \hat{Y}_{ij}$. Suppose the model is good, the error distribution is not highly skewed, and $n \geq 10p$. Then the plotted points should cluster about the identity line in each of the $m$ response plots. If outliers are present or if the plot is not linear, then the current model or data need to be changed or corrected. Each of the $m$ residual plots should be ellipsoidal with no trend and should be centered about the $r = 0$ line. There should not be any pattern in the residual plot: as a narrow vertical strip is moved from left to right, the behavior of the residuals within the strip should show little change. Outliers and patterns such as curvature or a fan shaped plot are bad. In other words, the response and residual plots should be used as for multiple linear regression where $m = 1$. See Olive and Hawkins (2005), Cook and Weisberg (1999, p. 432), and Example 1 in chapter 3.

The Rousseeuw and Van Driessen (1999) DD plot is a plot of classical Mahalanobis distances versus robust Mahalanobis distances. Simulations and results from Olive (2002) suggest the plotted points in the DD plot will cluster about the identity line if the $\boldsymbol{\epsilon}_i$ are iid from a multivariate normal $N_m(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}})$ distribution and about some line through the origin with slope greater than one for a large class of elliptically contoured distributions. Hence make a DD plot of the residuals $\hat{\boldsymbol{\epsilon}}_i$ to check the error distribution. Make a DD plot of the continuous predictor variables to check for $\boldsymbol{x}$-outliers. See Example 1.

# CHAPTER 2

# TESTING HYPOTHESES

Consider testing a linear hypothesis $H_0 : \boldsymbol{LB} = \boldsymbol{0}$ versus $H_1 : \boldsymbol{LB} \neq \boldsymbol{0}$ where $\boldsymbol{L}$ is a full rank $r \times p$ matrix. Let $\boldsymbol{H} = \hat{\boldsymbol{B}}^T \boldsymbol{L}^T [\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T]^{-1}\boldsymbol{L}\hat{\boldsymbol{B}}$. Let the error or residual sum of squares and cross products matrix be

$$\boldsymbol{W}_e = \hat{\boldsymbol{E}}^T \hat{\boldsymbol{E}} = (\boldsymbol{Z} - \hat{\boldsymbol{Z}})^T(\boldsymbol{Z} - \hat{\boldsymbol{Z}}) = \boldsymbol{Z}^T\boldsymbol{Z} - \boldsymbol{Z}^T\boldsymbol{X}\hat{\boldsymbol{B}} = \boldsymbol{Z}^T[\boldsymbol{I}_n - \boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T]\boldsymbol{Z}.$$

Then $\boldsymbol{W}_e/(n-p) = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$. Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$ be the ordered eigenvalues of $\boldsymbol{W}_e^{-1}\boldsymbol{H}$. Then there are four commonly used test statistics.

The Roy's maximum root statistic is $\lambda_{max}(\boldsymbol{L}) = \lambda_1$.

The Wilk's $\Lambda$ statistic is $\Lambda(\boldsymbol{L}) = |(\boldsymbol{H} + \boldsymbol{W}_e)^{-1}\boldsymbol{W}_e| = |\boldsymbol{W}_e^{-1}\boldsymbol{H} + \boldsymbol{I}|^{-1} = \prod_{i=1}^m (1+\lambda_i)^{-1}$.

The Pillai's trace statistic is $V(\boldsymbol{L}) = tr[(\boldsymbol{H} + \boldsymbol{W}_e)^{-1}\boldsymbol{H}] = \sum_{i=1}^m \frac{\lambda_i}{1+\lambda_i}$.

The Hotelling-Lawley trace statistic is $U(\boldsymbol{L}) = tr[\boldsymbol{W}_e^{-1}\boldsymbol{H}] = \sum_{i=1}^m \lambda_i$.

*Theorem 1.* The Hotelling-Lawley trace statistic

$$U(\boldsymbol{L}) = \frac{1}{n-p}[vec(\boldsymbol{L}\hat{\boldsymbol{B}})]^T[\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1} \otimes (\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}][vec(\boldsymbol{L}\hat{\boldsymbol{B}})]. \tag{2.1}$$

The proof of Theorem 1 is in the appendix. Some notation is useful to show (2.1) and to show that $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$ under mild conditions if $H_0$ is true. Following Henderson and Searle (1979), let matrix $\boldsymbol{A} = [\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \ldots \quad \boldsymbol{a}_p]$. Then the vec operator stacks the columns of $\boldsymbol{A}$ on top of one another so

$$vec(\boldsymbol{A}) = \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_p \end{pmatrix}.$$

Let $\boldsymbol{A} = (a_{ij})$ be an $m \times n$ matrix and $\boldsymbol{B}$ a $p \times q$ matrix. Then the Kronecker product of $\boldsymbol{A}$ and $\boldsymbol{B}$ is the $mp \times nq$ matrix

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} & \cdots & a_{1n}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} & \cdots & a_{2n}\boldsymbol{B} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}\boldsymbol{B} & a_{m2}\boldsymbol{B} & \cdots & a_{mn}\boldsymbol{B} \end{bmatrix}.$$

An important fact is that if $\boldsymbol{A}$ and $\boldsymbol{B}$ are nonsingular square matrices, then $[\boldsymbol{A} \otimes \boldsymbol{B}]^{-1} = \boldsymbol{A}^{-1} \otimes \boldsymbol{B}^{-1}$. The following assumption is important.

Assumption D1: Let $h_i$ be the $i$th diagonal element of $\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T$. Assume $\max_{1 \leq i \leq n} h_i \to 0$ as $n \to \infty$, assume that the zero mean iid errors have finite fourth moments, and assume that $\frac{1}{n}\boldsymbol{X}^T\boldsymbol{X} \xrightarrow{P} \boldsymbol{W}^{-1}$.

Then for the least squares estimator, Su and Cook (2012) show that if assumption D1 holds, then $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$ is $\sqrt{n}$ consistent and $\sqrt{n}\ \ vec(\hat{\boldsymbol{B}} - \boldsymbol{B}) \xrightarrow{D} N_{pm}(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \otimes \boldsymbol{W})$.

*Theorem 2.* If assumption D1 holds and if $H_0$ is true, then $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$.

The proof of Theorem 2 is in the appendix. Kakizawa (2009) shows, under stronger assumptions than Theorem 2, that for a large class of iid error distributions, the following test statistics have the same $\chi^2_{rm}$ limiting distribution when $H_0$ is true, and the same noncentral $\chi^2_{rm}(\omega^2)$ limiting distribution with noncentrality parameter $\omega^2$ when $H_0$ is false under a local alternative. Hence the three tests are robust to the assumption of normality. The limiting null distribution is well known when the zero mean errors are iid from a multivariate normal distribution. See Khattree and Naik (1999, p. 68): $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$, $(n-p)V(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$, and $-[n-p-0.5(m-r+3)]\log(\Lambda(\boldsymbol{L})) \xrightarrow{D} \chi^2_{rm}$. Results from Kshirsagar (1972, p. 301) suggest that the chi-square approximation is very good if $n \geq 3(m^2 + p^2)$ for multivariate normal errors.

Theorems 1 and 2 are useful for relating multivariate tests with the partial $F$ test for multiple linear regression that tests whether a reduced model that omits some of the predictors can be used instead of the full model that uses all $p$ predictors. The partial $F$ test statistic is

$$F_R = \left[\frac{SSE(R) - SSE(F)}{df_R - df_F}\right] / MSE(F)$$

where the residual sums of squares $SSE(F)$ and $SSE(R)$ and degrees of freedom $df_F$ and $df_r$ are for the full and reduced model while the mean square error $MSE(F)$ is for the full model. Let the null hypothesis for the partial $F$ test be $H_0 : \boldsymbol{L\beta} = \boldsymbol{0}$ where $\boldsymbol{L}$ sets the coefficients of the predictors in the full model but not in the reduced model to 0. Seber and Lee (2003, p. 100) shows that

$$F_R = \frac{[\boldsymbol{L}\hat{\boldsymbol{\beta}}]^T (\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}[\boldsymbol{L}\hat{\boldsymbol{\beta}}]}{r\hat{\sigma}^2}$$

is distributed as $F_{r,n-p}$ if $H_0$ is true and the errors are iid $N(0, \sigma^2)$. Note that for multiple linear regression with $m = 1$, $F_R = (n - p)U(\boldsymbol{L})/r$ since $\hat{\boldsymbol{\Sigma}}_\epsilon^{-1} = 1/\hat{\sigma}^2$. Hence the scaled Hotelling Lawley test statistic is the partial $F$ test statistic extended to $m > 1$ predictor variables by Theorem 1.

By Theorem 2, for example, $rF_R \xrightarrow{D} \chi^2_r$ for a large class of nonnormal error distribution. If $Z_n \sim F_{k,d_n}$, then $Z_n \xrightarrow{D} \chi^2_k/k$ as $d_n \to \infty$. Hence using the $F_{r,n-p}$ approximation gives a large sample test with correct asymptotic level, and the partial $F$ test is robust to nonnormality.

Similarly, using an $F_{rm,n-pm}$ approximation for the following test statistics gives large sample tests with correct asymptotic level by Kakizawa (2009) and similar power for large $n$. The large sample test will have correct asymptotic level as long as the denominator degrees of freedom $d_n \to \infty$ as $n \to \infty$, and $d_n = n - pm$ reduces to the partial $F$ test if $m = 1$ and $U(\boldsymbol{L})$ is used. Then the three test statistics are

$$\frac{-[n - p - 0.5(m - r + 3)]}{rm} \log(\Lambda(\boldsymbol{L}), \quad \frac{n - p}{rm} V(\boldsymbol{L}), \text{ and } \frac{\mathrm{n - p}}{\mathrm{rm}} \mathrm{U}(\boldsymbol{L}).$$

Following Khattree and Naik (1999, p. 67) for the Roy's largest root test, if $h = \max(r, m)$, use

$$\frac{n - p - h + r}{h} \lambda_{max}(\boldsymbol{L}) \approx F(h, n - p - h + r).$$

The simulations in chapter 3 suggest that this approximation is good for $r = 1$ but poor for $r > 1$. Anderson (1984, p. 333) states that Roy's largest root test has the greatest power if $r = 1$ but is an inferior test for $r > 1$.

Multivariate analogs of tests for multiple linear regression can be derived with appropriate choice of $\boldsymbol{L}$. Assume a constant $x_1 = 1$ is in the model. The analog of the ANOVA $F$ test for multiple linear regression is the MANOVA $F$ test that uses $\boldsymbol{L} = [\boldsymbol{0} \;\; \boldsymbol{I}_{p-1}]$ to test whether the nontrivial predictors are needed in the model. This test should reject $H_0$ if the response and residual plots look good, $n$ is large enough and at least one response plot does not look like the corresponding residual plot. A response plot for $Y_j$ will look like a residual plot if the identity line appears almost horizontal, hence the range of $\hat{Y}_j$ is small.

The $F_j$ test of hypotheses uses $\boldsymbol{L}_j = [0, ..., 0, 1, 0, ..., 0]$, where the 1 is in the $j$th position, to test whether the $j$th predictor is needed in the model given that the other $p - 1$ predictors are in the model. This test is an analog of the $t$ tests for multiple linear regression.

The MANOVA partial F test is used to test whether a reduced model is good where the reduced model deletes $r$ of the variables from the full model. For this test, the $i$th row of $\boldsymbol{L}$ has a 1 in the position corresponding to the $i$th variable to be deleted. Omitting the $j$th variable corresponds to the $F_j$ test while omitting variables $x_2, ..., x_p$ corresponds to the MANOVA F test. Using $\boldsymbol{L} = [\boldsymbol{0} \;\; \boldsymbol{I}_k]$ tests whether the last $k$ predictors are needed in the multivariate linear regression model given that the remaining predictors are in the model.

# CHAPTER 3

# EXAMPLES AND SIMULATIONS

**Example 1.** Cook and Weisberg (1999, p. 351, 433, 447) give a data set on 82 mussels sampled off the coast of New Zealand. Let $Y_1 = \log(S)$ and $Y_2 = \log(M)$ where $S$ is the shell mass and $M$ is the muscle mass. The predictors are $X_2 = L$, $X_3 = \log(W)$ and $X_4 = H$: the shell length, log(width) and height. Figures 3.1 and 3.2 give the response and residual plots for $Y_1$ and $Y_2$. For $Y_1$, case 79 sticks out while for $Y_2$, cases 8, 25 and 48 are not fit well. Highlighted cases had Cook's distance $> \min(0.5, 2p/n)$. See Cook (1977). Figure 3.3 shows the DD plot of the residual vectors. Cases 8, 48 and 79 have especially large distances. The response, residual and DD plots are effective for finding influential cases, for checking linearity and whether the error distribution is multivariate normal or some other elliptically contoured distribution.

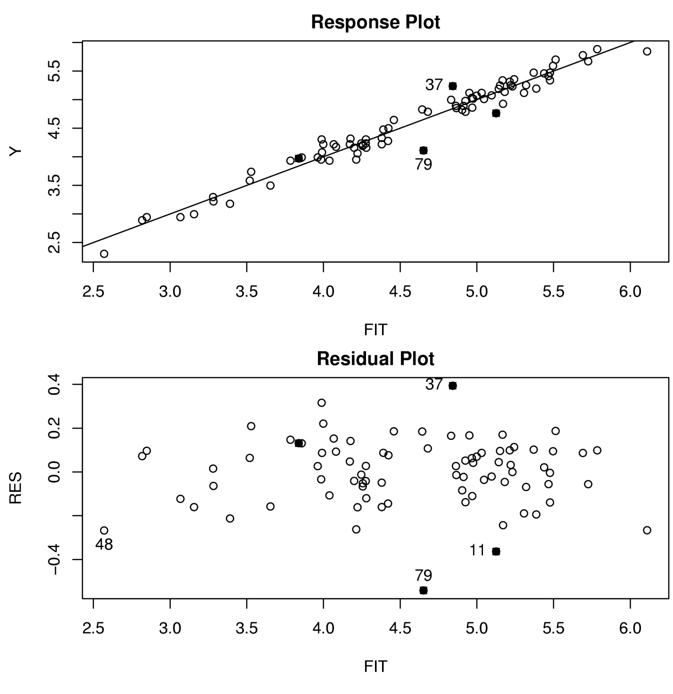


Figure 3.1. Plots for $Y_1 = \log(W)$.

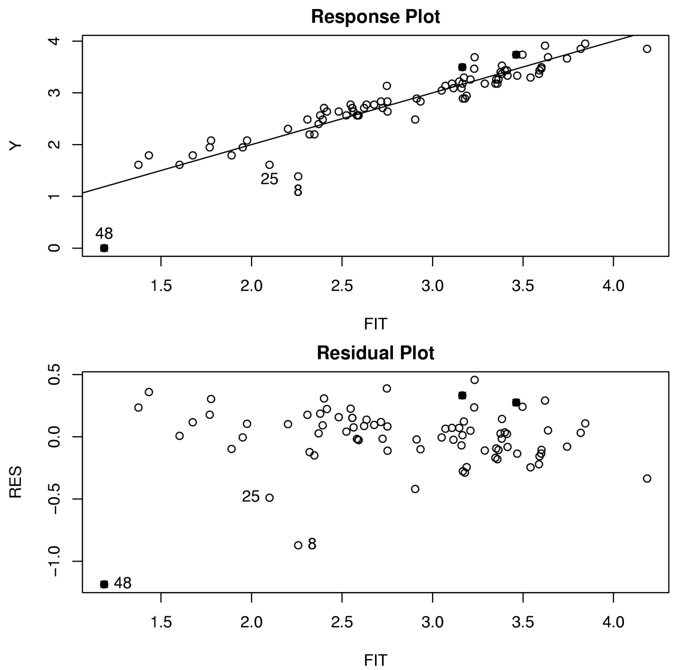


Figure 3.2. Plots for $Y_2 = \log(M)$.

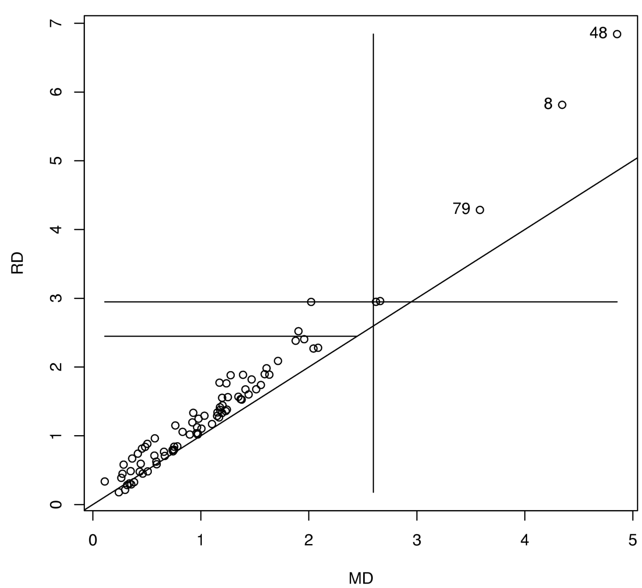


Figure 3.3. DD Plot of the Residual Vectors for the Mussel Data.

A simulation was used to study the Wilk's Lambda test, the Pillai's trace test, the Hotelling Lawley trace test, and the Roy's largest root test for the $F_j$ tests and the MANOVA $F$ test for multivariate linear regression. The first row of $\boldsymbol{B}$ was always $\mathbf{1}^T$ and the last row of $\boldsymbol{B}$ was always $\mathbf{0}^T$. When the null hypothesis for the MANOVA F test is true, all but the first row corresponding to the constant are equal to $\mathbf{0}^T$. When $p \geq 3$ and the null hypothesis for the MANOVA $F$ test is false, then the second to last row of $\boldsymbol{B}$ is (1, 0, ..., 0), the third to last row is (1, 1, 0, ..., 0) etcetera as long as the first row is not changed from $\mathbf{1}^T$. First $m$ iid errors $\boldsymbol{w}_i$ are generated such that the $m$ errors are iid with variance $\sigma^2$. Let the $m \times m$ matrix $\boldsymbol{A} = (a_{ij})$ with $a_{ii} = 1$ and $a_{ij} = \psi$ where $0 \leq \psi < 1$ for $i \neq j$. Then $\boldsymbol{\epsilon}_i = \boldsymbol{A}\boldsymbol{w}_i$ so that $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}} = \sigma^2 \boldsymbol{A}\boldsymbol{A}^T = (\sigma_{ij})$ where the diagonal entries $\sigma_{ii} = \sigma^2[1 + (m-1)\psi^2]$ and the off diagonal entries $\sigma_{ij} = \sigma^2[2\psi + (m-2)\psi^2]$ where $\psi = 0.10$. Hence the correlations $(2\psi + (m-2)\psi^2)/(1 + (m-1)\psi^2)$. As $\psi$ gets close to 1, the data clusters about the line in the direction of $(1, ..., 1)^T$. Used $\boldsymbol{w}_i \sim N_m(\mathbf{0}, \boldsymbol{I}), \boldsymbol{w}_i \sim (1-\tau)N_m(\mathbf{0}, \boldsymbol{I}) + \tau N_m(\mathbf{0}, 25\boldsymbol{I})$ with $0 < \tau < 1$ and $\tau = 0.25$ in the simulation, $\boldsymbol{w}_i \sim$ multivariate $t_d$ with $d = 7$ degrees of freedom, or $\boldsymbol{w}_i \sim$ lognormal - E(lognormal): where the $m$ components of $\boldsymbol{w}_i$ were iid with distribution $e^z - E(e^z)$ where $z \sim N(0, 1)$. Only the lognormal distribution is not elliptically contoured.

The simulation used 5000 runs, and $H_0$ was rejected if the $F$ statistic was greater than $F_{d_1,d_2}(0.95)$ where $P(F_{d_1,d_2} < F_{d_1,d_2}(0.95)) = 0.95$ with $d_1 = rm$ and $d_2 = n - mp$ for the test statistics

$$\frac{-[n-p-0.5(m-r+3)]}{rm} \ \log(\Lambda(\boldsymbol{L}), \quad \frac{n-p}{rm} \ V(\boldsymbol{L}), \text{ and } \frac{\text{n} - \text{p}}{\text{rm}} \ \text{U}(\boldsymbol{L})$$

while $d_1 = h = \max(r, m)$ and $d_2 = n - p - h + r$ for the test statistic

$$\frac{n-p-h+r}{h} \lambda_{max}(\boldsymbol{L}).$$

Denote these statistics by *Wilk's test*, *Pillai's test*, *Hotelling-Lawley* and *Roy's test*. Let the coverage be the proportion of times that $H_0$ is rejected. Want coverage near 0.05 when

$H_0$ is true and coverage close to 1 for good power when $H_0$ is false. With 5000 runs, coverage outside of (0.04,0.06) suggests that the true coverage is not 0.05. Coverages are tabled for the $b_1, b_2, b_{p-1}$, and $b_p$ test and for the MANOVA F test denoted by MANOVA $F$. Error distributions are denoted as follows; $etypeI$ for MVN $N_m(0, I)$, $etypeII$ for $(1 - eps)N_m(0, I) + epsN_m(0, 25I)$ eps = 0.1, 0.25, 0.4, and 0.6 are interesting $etypeIII$ for multivariate $t_d$ with $d = dd$ degrees of freedom dd = 1, 2, 3, 5, 7 are interesting $etypeIV$ for $lognormal - E(lognormal)$. The null hypothesis $H_0$ was always true for the $b_p$ test and always false for the $b_1$ test. When the MANOVA $F$ test was true, $H_0$ was true for the $b_j$ tests with $j \neq 1$. When the MANOVA $F$ test was false, $H_0$ was false for the $b_j$ tests with $j \neq p$, but the $b_{p-1}$ test should be hardest to reject for $j \neq p$ by construction of $\boldsymbol{B}$ and the error vectors.

Results from Berndt and Savin (1977) suggest that Pillai's test will reject $H_0$ less often than Wilk's test which will reject less often than the Hotelling Lawley test.

# CHAPTER 4

# COVERAGES: $H_0$ IS NOT TRUE FOR ERROR TYPE I

Table 4.1. H0 false - Error typeI For m=3, p=3, n=30

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9998 | 0.9674 | | 0.0266 | 0.8904 |
| Pillai's test | 0.9994 | 0.9414 | | 0.0128 | 0.6946 |
| Hotelling-Lawley | 1.0000 | 0.9850 | | 0.0586 | 0.9616 |
| Roy's test | 1.0000 | 0.9826 | | 0.0502 | 0.9914 |

Table 4.2. H0 false - Error typeI For m=3, p=3, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | | 0.0420 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | | 0.0368 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | | 0.0520 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | | 0.0488 | 1.0000 |

Table 4.3. H0 false - Error typeI For m=4, p=4, n=40

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.9996 | 0.9928 | 0.0214 | 1.0000 |
| Pillai's test | 1.0000 | 0.9988 | 0.9830 | 0.0118 | 0.9994 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 0.9970 | 0.0570 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 0.9960 | 0.0464 | 1.0000 |

Table 4.4. H0 false - Error typeI For m=4, p=4, n=90

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0384 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0334 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0528 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0472 | 1.0000 |

Table 4.5. H0 false - Error typeI For m=5, p=5, n=50

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 0.9986 | 0.0266 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 0.9954 | 0.0154 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 0.9998 | 0.0540 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 0.9998 | 0.0502 | 1.0000 |

Table 4.6. H0 false - Error typeI For m=6, p=6, n=60

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 0.9990 | 0.0180 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 0.9982 | 0.0072 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 0.9994 | 0.0476 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 0.9994 | 0.0464 | 1.0000 |

Table 4.7. H0 false - Error typeI For m=7, p=7, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0146 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 0.9996 | 0.0072 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0430 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0502 | 1.0000 |

Table 4.8. H0 false - Error typeI For m=8, p=8, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 0.9998 | 0.0088 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 0.9996 | 0.0038 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0280 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0498 | 1.0000 |

Table 4.9. H0 false - Error typeI For m=9, p=9, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0102 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0046 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0308 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0540 | 1.0000 |

Table 4.10. H0 false - Error typeI For m=10, p=10, n=110

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0006 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 0.9998 | 0.0000 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0060 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0558 | 1.0000 |

Table 4.11. H0 false - Error typeI For m=11, p=11, n=140

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0068 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0036 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0206 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0484 | 1.0000 |

Table 4.12. H0 false - Error typeI For m=12, p=12, n=160

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0030 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0014 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0110 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0442 | 1.0000 |

Table 4.13. H0 false - Error typeI For m=13, p=13, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0006 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0034 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0460 | 1.0000 |

Table 4.14. H0 false - Error typeI For m=14, p=14, n=210

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0006 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0002 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0050 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0514 | 1.0000 |

Table 4.15. H0 false - Error typeI For m=15, p=15, n=240

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 1.0000 | 1.0000 | 0.0026 | 1.0000 |
| Pillai's test | 1.0000 | 1.0000 | 1.0000 | 0.0020 | 1.0000 |
| Hotelling-Lawley | 1.0000 | 1.0000 | 1.0000 | 0.0054 | 1.0000 |
| Roy's test | 1.0000 | 1.0000 | 1.0000 | 0.0532 | 1.0000 |

# CHAPTER 5

# COVERAGES: $H_0$ IS TRUE FOR ERROR TYPES I, II, III AND IV

## 5.1 ERROR TYPE I

Table 5.1. Error type I For m=2, p=2, n=20

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9944 | | | 0.0278 | 0.0278 |
| Pillai's test | 0.9824 | | | 0.0132 | 0.0132 |
| Hotelling -Lawley test | 0.9986 | | | 0.0624 | 0.0624 |
| Roy's test | 0.9986 | | | 0.0552 | 0.0552 |

Table 5.2. Error typeI For m=2, p=2, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0464 | 0.0464 |
| Pillai's test | 1.0000 | | | 0.0418 | 0.0418 |
| Hotelling -Lawley test | 1.0000 | | | 0.0544 | 0.0544 |
| Roy's test | 1.0000 | | | 0.0516 | 0.0516 |

Table 5.3. Error typeI For m=2, p=2, n=120

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0474 | 0.0474 |
| Pillai's test | 1.0000 | | | 0.0450 | 0.0450 |
| Hotelling -Lawley test | 1.0000 | | | 0.0514 | 0.0514 |
| Roy's test | 1.0000 | | | 0.0506 | 0.0506 |

Table 5.4. Error typeI For m=2, p=2, n=170

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0456 | 0.0456 |
| Pillai's test | 1.0000 | | | 0.0434 | 0.0434 |
| Hotelling -Lawley test | 1.0000 | | | 0.0492 | 0.0492 |
| Roy's test | 1.0000 | | | 0.0480 | 0.0480 |

Table 5.5. Error typeI For m=2, p=2, n=220

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0428 | 0.0428 |
| Pillai's test | 1.0000 | | | 0.0412 | 0.0412 |
| Hotelling -Lawley test | 1.0000 | | | 0.0466 | 0.0466 |
| Roy's test | 1.0000 | | | 0.0450 | 0.0450 |

Table 5.6. Error typeI For m=2, p=2, n=270

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0476 | 0.0476 |
| Pillai's test | 1.0000 | | | 0.0468 | 0.0468 |
| Hotelling -Lawley test | 1.0000 | | | 0.0496 | 0.0496 |
| Roy's test | 1.0000 | | | 0.0494 | 0.0494 |

Table 5.7. Error typeI For m=2, p=2, n=320

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0490 | 0.0490 |
| Pillai's test | 1.0000 | | | 0.0482 | 0.0482 |
| Hotelling -Lawley test | 1.0000 | | | 0.0512 | 0.0512 |
| Roy's test | 1.0000 | | | 0.0504 | 0.0504 |

Table 5.8. Error typeI For m=2, p=2, n=370

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | | | 0.0476 | 0.0476 |
| Pillai's test | 1.0000 | | | 0.0468 | 0.0468 |
| Hotelling -Lawley test | 1.0000 | | | 0.0488 | 0.0488 |
| Roy's test | 1.0000 | | | 0.0486 | 0.0486 |

Table 5.9. Error typeI For m=3, p=3, n=30

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0258 | | 0.0298 | 0.0192 |
| Pillai's test | 1.0000 | 0.0142 | | 0.0138 | 0.0050 |
| Hotelling -Lawley test | 1.0000 | 0.0560 | | 0.0620 | 0.0552 |
| Roy's test | 1.0000 | 0.0488 | | 0.0534 | 0.1704 |

Table 5.10. Error typeI For m=3, p=3, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0388 | | 0.0410 | 0.0392 |
| Pillai's test | 1.0000 | 0.0340 | | 0.0362 | 0.0296 |
| Hotelling -Lawley test | 1.0000 | 0.0478 | | 0.0532 | 0.0542 |
| Roy's test | 1.0000 | 0.0440 | | 0.0494 | 0.1482 |

Table 5.11. Error typeI For m=3, p=3, n=130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0468 | | 0.0474 | 0.0428 |
| Pillai's test | 1.0000 | 0.0438 | | 0.0448 | 0.0386 |
| Hotelling -Lawley test | 1.0000 | 0.0552 | | 0.0550 | 0.0542 |
| Roy's test | 1.0000 | 0.0530 | | 0.0530 | 0.1630 |

Table 5.12. Error typeI For m=3, p=3, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0516 | | 0.0398 | 0.0420 |
| Pillai's test | 1.0000 | 0.0490 | | 0.0382 | 0.0378 |
| Hotelling -Lawley test | 1.0000 | 0.0580 | | 0.0440 | 0.0494 |
| Roy's test | 1.0000 | 0.0558 | | 0.0426 | 0.1560 |

Table 5.13. Error typeI For m=3, p=3, n=230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0522 | | 0.0468 | 0.0464 |
| Pillai's test | 1.0000 | 0.0500 | | 0.0438 | 0.0430 |
| Hotelling -Lawley test | 1.0000 | 0.0558 | | 0.0502 | 0.0536 |
| Roy's test | 1.0000 | 0.0546 | | 0.0488 | 0.1626 |

Table 5.14. Error typeI For m=3, p=3, n=280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | | 0.0494 | 0.0464 |
| Pillai's test | 1.0000 | 0.0478 | | 0.0478 | 0.0434 |
| Hotelling -Lawley test | 1.0000 | 0.0514 | | 0.0512 | 0.0520 |
| Roy's test | 1.0000 | 0.0506 | | 0.0506 | 0.1600 |

Table 5.15. Error typeI For m=3, p=3, n=330

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0476 | | 0.0500 | 0.0496 |
| Pillai's test | 1.0000 | 0.0458 | | 0.0484 | 0.0474 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | | 0.0534 | 0.0526 |
| Roy's test | 1.0000 | 0.0506 | | 0.0520 | 0.1596 |

Table 5.16. Error typeI For m=3, p=3, n=380

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | | 0.0482 | 0.0450 |
| Pillai's test | 1.0000 | 0.0422 | | 0.0472 | 0.0422 |
| Hotelling -Lawley test | 1.0000 | 0.0456 | | 0.0512 | 0.0488 |
| Roy's test | 1.0000 | 0.0444 | | 0.0498 | 0.1564 |

Table 5.17. Error typeI For m=3, p=3, n=430

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0456 | | 0.0456 | 0.0452 |
| Pillai's test | 1.0000 | 0.0446 | | 0.0444 | 0.0424 |
| Hotelling -Lawley test | 1.0000 | 0.0486 | | 0.0468 | 0.0486 |
| Roy's test | 1.0000 | 0.0482 | | 0.0468 | 0.1538 |

Table 5.18. Error typeI For m=3, p=3, n=480

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | | 0.0468 | 0.0498 |
| Pillai's test | 1.0000 | 0.0480 | | 0.0460 | 0.0476 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | | 0.0496 | 0.0522 |
| Roy's test | 1.0000 | 0.0496 | | 0.0488 | 0.1604 |

Table 5.19. Error typeI For m=3, p=3, n=530

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0456 | | 0.0522 | 0.0498 |
| Pillai's test | 1.0000 | 0.0448 | | 0.0514 | 0.0488 |
| Hotelling -Lawley test | 1.0000 | 0.0476 | | 0.0540 | 0.0522 |
| Roy's test | 1.0000 | 0.0468 | | 0.0538 | 0.1580 |

Table 5.20. Error typeI For m=3, p=3, n=580

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0476 | | 0.0502 | 0.0484 |
| Pillai's test | 1.0000 | 0.0468 | | 0.0488 | 0.0476 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | | 0.0526 | 0.0506 |
| Roy's test | 1.0000 | 0.0486 | | 0.0512 | 0.1570 |

Table 5.21. Error typeI For m=4, p=4, n=40

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0256 | 0.0238 | 0.0240 | 0.0120 |
| Pillai's test | 1.0000 | 0.0134 | 0.0118 | 0.0128 | 0.0014 |
| Hotelling -Lawley test | 1.0000 | 0.0564 | 0.0562 | 0.0574 | 0.0496 |
| Roy's test | 1.0000 | 0.0494 | 0.0492 | 0.0482 | 0.3400 |

Table 5.22. Error typeI For m=4, p=4, n=90

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0422 | 0.0372 | 0.0424 | 0.0336 |
| Pillai's test | 1.0000 | 0.0370 | 0.0316 | 0.0358 | 0.0212 |
| Hotelling -Lawley test | 1.0000 | 0.0608 | 0.0530 | 0.0574 | 0.0538 |
| Roy's test | 1.0000 | 0.0534 | 0.0478 | 0.0520 | 0.3244 |

Table 5.23. Error typeI For m=4, p=4, n=140

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0432 | 0.0468 | 0.0442 | 0.0376 |
| Pillai's test | 1.0000 | 0.0392 | 0.0436 | 0.0384 | 0.0306 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0578 | 0.0530 | 0.0534 |
| Roy's test | 1.0000 | 0.0470 | 0.0540 | 0.0498 | 0.3266 |

Table 5.24. Error typeI For m=4, p=4, n=190

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0458 | 0.0486 | 0.0480 | 0.0458 |
| Pillai's test | 1.0000 | 0.0442 | 0.0444 | 0.0448 | 0.0382 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0554 | 0.0558 | 0.0556 |
| Roy's test | 1.0000 | 0.0500 | 0.0518 | 0.0524 | 0.3248 |

Table 5.25. Error typeI For m=4, p=4, n=240

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0464 | 0.0426 | 0.0462 | 0.0464 |
| Pillai's test | 1.0000 | 0.0436 | 0.0404 | 0.0442 | 0.0404 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0476 | 0.0518 | 0.0540 |
| Roy's test | 1.0000 | 0.0518 | 0.0452 | 0.0494 | 0.3236 |

Table 5.26. Error typeI For m=4, p=4, n=290

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0466 | 0.0462 | 0.0454 | 0.0452 |
| Pillai's test | 1.0000 | 0.0456 | 0.0444 | 0.0444 | 0.0410 |
| Hotelling -Lawley test | 1.0000 | 0.0512 | 0.0508 | 0.0496 | 0.0534 |
| Roy's test | 1.0000 | 0.0486 | 0.0490 | 0.0474 | 0.3262 |

Table 5.27. Error typeI For m=4, p=4, n=340

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | 0.0470 | 0.0478 | 0.0408 |
| Pillai's test | 1.0000 | 0.0466 | 0.0454 | 0.0470 | 0.0386 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0494 | 0.0528 | 0.0484 |
| Roy's test | 1.0000 | 0.0514 | 0.0492 | 0.0500 | 0.3218 |

Table 5.28. Error typeI For m=4, p=4, n=390

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0524 | 0.0496 | 0.0480 | 0.0498 |
| Pillai's test | 1.0000 | 0.0520 | 0.0480 | 0.0464 | 0.0458 |
| Hotelling -Lawley test | 1.0000 | 0.0550 | 0.0538 | 0.0524 | 0.0580 |
| Roy's test | 1.0000 | 0.0534 | 0.0522 | 0.0502 | 0.3300 |

Table 5.29. Error typeI For m=4, p=4, n=440

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0474 | 0.0524 | 0.0560 | 0.0498 |
| Pillai's test | 1.0000 | 0.0462 | 0.0508 | 0.0550 | 0.0464 |
| Hotelling -Lawley test | 1.0000 | 0.0506 | 0.0558 | 0.0588 | 0.0550 |
| Roy's test | 1.0000 | 0.0490 | 0.0538 | 0.0574 | 0.3308 |

Table 5.30. Error typeI For m=4, p=4, n=490

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0518 | 0.0474 | 0.0498 | 0.0476 |
| Pillai's test | 1.0000 | 0.0514 | 0.0460 | 0.0496 | 0.0436 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | 0.0496 | 0.0526 | 0.0508 |
| Roy's test | 1.0000 | 0.0526 | 0.0484 | 0.0518 | 0.3192 |

Table 5.31. Error typeI For m=4, p=4, n=540

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0494 | 0.0406 | 0.0476 | 0.0470 |
| Pillai's test | 1.0000 | 0.0478 | 0.0398 | 0.0470 | 0.0436 |
| Hotelling -Lawley test | 1.0000 | 0.0506 | 0.0434 | 0.0492 | 0.0500 |
| Roy's test | 1.0000 | 0.0502 | 0.0422 | 0.0482 | 0.3080 |

Table 5.32. Error typeI For m=4, p=4, n=590

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0530 | 0.0468 | 0.0468 | 0.0512 |
| Pillai's test | 1.0000 | 0.0520 | 0.0460 | 0.0464 | 0.0482 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0482 | 0.0490 | 0.0534 |
| Roy's test | 1.0000 | 0.0536 | 0.0478 | 0.0480 | 0.3278 |

Table 5.33. Error typeI For m=5, p=5, n=50

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0238 | 0.0222 | 0.0266 | 0.0052 |
| Pillai's test | 1.0000 | 0.0128 | 0.0120 | 0.0140 | 0.0004 |
| Hotelling -Lawley test | 1.0000 | 0.0522 | 0.0532 | 0.0554 | 0.0344 |
| Roy's test | 1.0000 | 0.0474 | 0.0490 | 0.0486 | 0.5506 |

Table 5.34. Error typeI For m=5, p=5, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0390 | 0.0358 | 0.0430 | 0.0222 |
| Pillai's test | 1.0000 | 0.0322 | 0.0300 | 0.0358 | 0.0114 |
| Hotelling -Lawley test | 1.0000 | 0.0564 | 0.0544 | 0.0626 | 0.0476 |
| Roy's test | 1.0000 | 0.0494 | 0.0458 | 0.0552 | 0.5348 |

Table 5.35. Error typeI For m=5, p=5, n=150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0412 | 0.0426 | 0.0430 | 0.0300 |
| Pillai's test | 1.0000 | 0.0366 | 0.0392 | 0.0362 | 0.0202 |
| Hotelling -Lawley test | 1.0000 | 0.0520 | 0.0554 | 0.0530 | 0.0478 |
| Roy's test | 1.0000 | 0.0472 | 0.0508 | 0.0498 | 0.5324 |

Table 5.36. Error typeI For m=5, p=5, n=200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0450 | 0.0456 | 0.0444 | 0.0338 |
| Pillai's test | 1.0000 | 0.0428 | 0.0420 | 0.0416 | 0.0248 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | 0.0548 | 0.0536 | 0.0500 |
| Roy's test | 1.0000 | 0.0500 | 0.0504 | 0.0492 | 0.5252 |

Table 5.37. Error typeI For m=5, p=5, n=250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0484 | 0.0440 | 0.0390 |
| Pillai's test | 1.0000 | 0.0452 | 0.0454 | 0.0414 | 0.0314 |
| Hotelling -Lawley test | 1.0000 | 0.0518 | 0.0564 | 0.0494 | 0.0502 |
| Roy's test | 1.0000 | 0.0494 | 0.0526 | 0.0474 | 0.5210 |

Table 5.38. Error typeI For m=5, p=5, n=300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0448 | 0.0420 | 0.0454 | 0.0422 |
| Pillai's test | 1.0000 | 0.0436 | 0.0396 | 0.0430 | 0.0348 |
| Hotelling -Lawley test | 1.0000 | 0.0496 | 0.0486 | 0.0498 | 0.0520 |
| Roy's test | 1.0000 | 0.0472 | 0.0454 | 0.0482 | 0.5202 |

Table 5.39. Error typeI For m=5, p=5, n=350

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0438 | 0.0484 | 0.0466 | 0.0434 |
| Pillai's test | 1.0000 | 0.0414 | 0.0476 | 0.0446 | 0.0374 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | 0.0528 | 0.0502 | 0.0478 |
| Roy's test | 1.0000 | 0.0472 | 0.0512 | 0.0484 | 0.5142 |

Table 5.40. Error typeI For m=5, p=5, n=400

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0500 | 0.0488 | 0.0448 | 0.0448 |
| Pillai's test | 1.0000 | 0.0484 | 0.0470 | 0.0440 | 0.0390 |
| Hotelling -Lawley test | 1.0000 | 0.0556 | 0.0532 | 0.0512 | 0.0546 |
| Roy's test | 1.0000 | 0.0526 | 0.0510 | 0.0478 | 0.5310 |

Table 5.41. Error typeI For m=5, p=5, n=450

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0522 | 0.0464 | 0.0462 | 0.0490 |
| Pillai's test | 1.0000 | 0.0504 | 0.0458 | 0.0450 | 0.0436 |
| Hotelling -Lawley test | 1.0000 | 0.0548 | 0.0488 | 0.0506 | 0.0548 |
| Roy's test | 1.0000 | 0.0538 | 0.0474 | 0.0486 | 0.5120 |

Table 5.42. Error typeI For m=5, p=5, n=500

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0498 | 0.0472 | 0.0496 | 0.0472 |
| Pillai's test | 1.0000 | 0.0484 | 0.0458 | 0.0484 | 0.0424 |
| Hotelling -Lawley test | 1.0000 | 0.0554 | 0.0516 | 0.0538 | 0.0540 |
| Roy's test | 1.0000 | 0.0530 | 0.0502 | 0.0522 | 0.5348 |

Table 5.43. Error typeI For m=5, p=5, n=550

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | 0.0502 | 0.0502 | 0.0412 |
| Pillai's test | 1.0000 | 0.0458 | 0.0494 | 0.0492 | 0.0364 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | 0.0522 | 0.0532 | 0.0464 |
| Roy's test | 1.0000 | 0.0478 | 0.0510 | 0.0518 | 0.5170 |

Table 5.44. Error typeI For m=5, p=5, n=600

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0500 | 0.0478 | 0.0510 | 0.0530 |
| Pillai's test | 1.0000 | 0.0482 | 0.0470 | 0.0488 | 0.0488 |
| Hotelling -Lawley test | 1.0000 | 0.0518 | 0.0492 | 0.0536 | 0.0582 |
| Roy's test | 1.0000 | 0.0514 | 0.0484 | 0.0520 | 0.5238 |

## 5.2 ERROR TYPE II

Table 5.45. Error typeII For m=2, p=2, n=20

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.4342 | 0.0264 | | | 0.0264 |
| Pillai's test | 0.3360 | 0.0114 | | | 0.0114 |
| Hotelling -Lawley test | 0.5418 | 0.0558 | | | 0.0558 |
| Roy's test | 0.5262 | 0.0498 | | | 0.0498 |

Table 5.46. Error typeII For m=2, p=2, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9402 | 0.0414 | | | 0.0414 |
| Pillai's test | 0.9346 | 0.0366 | | | 0.0366 |
| Hotelling -Lawley test | 0.9488 | 0.0486 | | | 0.0486 |
| Roy's test | 0.9466 | 0.0468 | | | 0.0468 |

Table 5.47. Error typeII For m=2, p=2, n=120

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9950 | 0.0500 | | | 0.0500 |
| Pillai's test | 0.9944 | 0.0486 | | | 0.0486 |
| Hotelling -Lawley test | 0.9956 | 0.0548 | | | 0.0548 |
| Roy's test | 0.9956 | 0.0542 | | | 0.0542 |

Table 5.48. Error typeII For m=2, p=2, n=170

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9998 | 0.0488 | | | 0.0488 |
| Pillai's test | 0.9998 | 0.0472 | | | 0.0472 |
| Hotelling -Lawley test | 0.9998 | 0.0518 | | | 0.0518 |
| Roy's test | 0.9998 | 0.0504 | | | 0.0504 |

Table 5.49. Error typeII For m=2, p=2, n=220

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0464 | | | 0.0464 |
| Pillai's test | 1.0000 | 0.0452 | | | 0.0452 |
| Hotelling -Lawley test | 1.0000 | 0.0496 | | | 0.0496 |
| Roy's test | 1.0000 | 0.0494 | | | 0.0494 |

Table 5.50. Error typeII For m=3, p=3, n=30

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.6218 | 0.0268 | | 0.0244 | 0.0178 |
| Pillai's test | 0.5378 | 0.0148 | | 0.0120 | 0.0034 |
| Hotelling -Lawley test | 0.7288 | 0.0586 | | 0.0574 | 0.0506 |
| Roy's test | 0.7084 | 0.0520 | | 0.0474 | 0.1628 |

Table 5.51. Error typeII For m=3, p=3, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9770 | 0.0448 | | 0.0456 | 0.0380 |
| Pillai's test | 0.9746 | 0.0392 | | 0.0410 | 0.0282 |
| Hotelling -Lawley test | 0.9820 | 0.0586 | | 0.0558 | 0.0542 |
| Roy's test | 0.9796 | 0.0538 | | 0.0524 | 0.1650 |

Table 5.52. Error typeII For m=3, p=3, n=130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9994 | 0.0444 | | 0.0422 | 0.0424 |
| Pillai's test | 0.9994 | 0.0404 | | 0.0386 | 0.0358 |
| Hotelling -Lawley test | 0.9994 | 0.0492 | | 0.0498 | 0.0536 |
| Roy's test | 0.9994 | 0.0476 | | 0.0474 | 0.1586 |

Table 5.53. Error typeII For m=3, p=3, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | | 0.0464 | 0.0422 |
| Pillai's test | 1.0000 | 0.0436 | | 0.0446 | 0.0376 |
| Hotelling -Lawley test | 1.0000 | 0.0508 | | 0.0508 | 0.0484 |
| Roy's test | 1.0000 | 0.0488 | | 0.0488 | 0.1574 |

Table 5.54. Error typeII For m=3, p=3, n=230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0460 | | 0.0488 | 0.0450 |
| Pillai's test | 1.0000 | 0.0436 | | 0.0466 | 0.0430 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | | 0.0524 | 0.0516 |
| Roy's test | 1.0000 | 0.0492 | | 0.0510 | 0.1544 |

Table 5.55. Error typeII For m=3, p=3, n=280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0482 | | 0.0428 | 0.0482 |
| Pillai's test | 1.0000 | 0.0470 | | 0.0416 | 0.0444 |
| Hotelling -Lawley test | 1.0000 | 0.0516 | | 0.0462 | 0.0534 |
| Roy's test | 1.0000 | 0.0502 | | 0.0446 | 0.1522 |

Table 5.56. Error typeII For m=4, p=4, n=40

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.7606 | 0.0280 | 0.0258 | 0.0290 | 0.0108 |
| Pillai's test | 0.6924 | 0.0180 | 0.0136 | 0.0162 | 0.0014 |
| Hotelling -Lawley test | 0.8438 | 0.0602 | 0.0580 | 0.0608 | 0.0540 |
| Roy's test | 0.8302 | 0.0540 | 0.0490 | 0.0530 | 0.3506 |

Table 5.57. Error typeII For m=4, p=4, n=90

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9894 | 0.0416 | 0.0414 | 0.0414 | 0.0304 |
| Pillai's test | 0.9874 | 0.0344 | 0.0352 | 0.0366 | 0.0146 |
| Hotelling -Lawley test | 0.9924 | 0.0556 | 0.0538 | 0.0550 | 0.0542 |
| Roy's test | 0.9914 | 0.0510 | 0.0492 | 0.0502 | 0.3356 |

Table 5.58. Error typeII For m=4, p=4, n=140

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0438 | 0.0438 | 0.0476 | 0.0362 |
| Pillai's test | 1.0000 | 0.0386 | 0.0406 | 0.0440 | 0.0282 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | 0.0528 | 0.0584 | 0.0550 |
| Roy's test | 1.0000 | 0.0496 | 0.0490 | 0.0546 | 0.3350 |

Table 5.59. Error typeII For m=4, p=4, n=190

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | 0.0500 | 0.0392 | 0.0414 |
| Pillai's test | 1.0000 | 0.0424 | 0.0478 | 0.0374 | 0.0362 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0582 | 0.0476 | 0.0518 |
| Roy's test | 1.0000 | 0.0498 | 0.0542 | 0.0444 | 0.3296 |

Table 5.60. Error typeII For m=4, p=4, n=240

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0450 | 0.0492 | 0.0446 | 0.0422 |
| Pillai's test | 1.0000 | 0.0428 | 0.0474 | 0.0426 | 0.0380 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0554 | 0.0516 | 0.0508 |
| Roy's test | 1.0000 | 0.0506 | 0.0530 | 0.0478 | 0.3236 |

Table 5.61. Error typeII For m=4, p=4, n=290

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0452 | 0.0444 | 0.0436 |
| Pillai's test | 1.0000 | 0.0456 | 0.0436 | 0.0432 | 0.0384 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0494 | 0.0474 | 0.0492 |
| Roy's test | 1.0000 | 0.0500 | 0.0478 | 0.0460 | 0.3208 |

Table 5.62. Error typeII For m=4, p=4, n=340

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0488 | 0.0578 | 0.0518 | 0.0508 |
| Pillai's test | 1.0000 | 0.0472 | 0.0564 | 0.0502 | 0.0456 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0628 | 0.0546 | 0.0576 |
| Roy's test | 1.0000 | 0.0512 | 0.0618 | 0.0538 | 0.3326 |

Table 5.63. Error typeII For m=4, p=4, n=390

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0456 | 0.0516 | 0.0478 | 0.0468 |
| Pillai's test | 1.0000 | 0.0442 | 0.0508 | 0.0472 | 0.0430 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | 0.0544 | 0.0518 | 0.0520 |
| Roy's test | 1.0000 | 0.0478 | 0.0534 | 0.0504 | 0.3264 |

Table 5.64. Error typeII For m=4, p=4, n=440

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0500 | 0.0484 | 0.0490 | 0.0448 |
| Pillai's test | 1.0000 | 0.0494 | 0.0478 | 0.0480 | 0.0414 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0522 | 0.0524 | 0.0498 |
| Roy's test | 1.0000 | 0.0516 | 0.0506 | 0.0514 | 0.3250 |

Table 5.65. Error typeII For m=5, p=5, n=50

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.8400 | 0.0218 | 0.0258 | 0.0270 | 0.0056 |
| Pillai's test | 0.7828 | 0.0128 | 0.0154 | 0.0154 | 0.0000 |
| Hotelling -Lawley test | 0.9070 | 0.0540 | 0.0574 | 0.0622 | 0.0378 |
| Roy's test | 0.8970 | 0.0478 | 0.0544 | 0.0548 | 0.5718 |

Table 5.66. Error typeII For m=5, p=5, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9942 | 0.0352 | 0.0370 | 0.0364 | 0.0248 |
| Pillai's test | 0.9932 | 0.0304 | 0.0310 | 0.0300 | 0.0122 |
| Hotelling -Lawley test | 0.9968 | 0.0548 | 0.0512 | 0.0544 | 0.0514 |
| Roy's test | 0.9956 | 0.0480 | 0.0468 | 0.0472 | 0.5268 |

Table 5.67. Error typeII For m=5, p=5, n=150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0406 | 0.0408 | 0.0474 | 0.0344 |
| Pillai's test | 1.0000 | 0.0376 | 0.0368 | 0.0424 | 0.0232 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | 0.0534 | 0.0594 | 0.0568 |
| Roy's test | 1.0000 | 0.0480 | 0.0484 | 0.0542 | 0.5274 |

Table 5.68. Error typeII For m=5, p=5, n=200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0490 | 0.0460 | 0.0410 |
| Pillai's test | 1.0000 | 0.0424 | 0.0444 | 0.0430 | 0.0328 |
| Hotelling -Lawley test | 1.0000 | 0.0574 | 0.0572 | 0.0548 | 0.0548 |
| Roy's test | 1.0000 | 0.0532 | 0.0538 | 0.0514 | 0.5444 |

Table 5.69. Error typeII For m=5, p=5, n=250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0420 | 0.0478 | 0.0426 | 0.0372 |
| Pillai's test | 1.0000 | 0.0400 | 0.0442 | 0.0410 | 0.0324 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0560 | 0.0500 | 0.0478 |
| Roy's test | 1.0000 | 0.0470 | 0.0526 | 0.0466 | 0.5118 |

Table 5.70. Error typeII For m=5, p=5, n=300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0468 | 0.0484 | 0.0544 | 0.0386 |
| Pillai's test | 1.0000 | 0.0440 | 0.0458 | 0.0520 | 0.0386 |
| Hotelling -Lawley test | 1.0000 | 0.0532 | 0.0512 | 0.0598 | 0.0386 |
| Roy's test | 1.0000 | 0.0500 | 0.0500 | 0.0568 | 0.0386 |

Table 5.71. Error typeII For m=5, p=5, n=350

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0480 | 0.0484 | 0.0432 | 0.0400 |
| Pillai's test | 1.0000 | 0.0454 | 0.0472 | 0.0414 | 0.0336 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0526 | 0.0470 | 0.0458 |
| Roy's test | 1.0000 | 0.0498 | 0.0506 | 0.0454 | 0.5202 |

Table 5.72. Error typeII For m=5, p=5, n=400

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0490 | 0.0492 | 0.0450 | 0.0410 |
| Pillai's test | 1.0000 | 0.0474 | 0.0474 | 0.0436 | 0.0364 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0536 | 0.0490 | 0.0498 |
| Roy's test | 1.0000 | 0.0510 | 0.0520 | 0.0464 | 0.5262 |

Table 5.73. Error typeII For m=5, p=5, n=450

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | 0.0450 | 0.0456 | 0.0460 |
| Pillai's test | 1.0000 | 0.0448 | 0.0438 | 0.0440 | 0.0414 |
| Hotelling -Lawley test | 1.0000 | 0.0496 | 0.0488 | 0.0504 | 0.0518 |
| Roy's test | 1.0000 | 0.0478 | 0.0470 | 0.0482 | 0.5238 |

Table 5.74. Error typeII For m=5, p=5, n=500

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | 0.0450 | 0.0468 | 0.0442 |
| Pillai's test | 1.0000 | 0.0484 | 0.0442 | 0.0458 | 0.0398 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0488 | 0.0498 | 0.0510 |
| Roy's test | 1.0000 | 0.0510 | 0.0474 | 0.0486 | 0.5272 |

Table 5.75. Error typeII For m=5, p=5, n=550

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0482 | 0.0446 | 0.0446 | 0.0426 |
| Pillai's test | 1.0000 | 0.0470 | 0.0432 | 0.0438 | 0.0382 |
| Hotelling -Lawley test | 1.0000 | 0.0514 | 0.0466 | 0.0474 | 0.0492 |
| Roy's test | 1.0000 | 0.0500 | 0.0452 | 0.0458 | 0.5230 |

Table 5.76. Error typeII For m=5, p=5, n=600

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0496 | 0.0522 | 0.0508 |
| Pillai's test | 1.0000 | 0.0462 | 0.0486 | 0.0520 | 0.0488 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0530 | 0.0546 | 0.0552 |
| Roy's test | 1.0000 | 0.0486 | 0.0510 | 0.0532 | 0.5236 |

Table 5.77. Error typeII For m=5, p=5, n=650

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0520 | 0.0444 | 0.0480 | 0.0454 |
| Pillai's test | 1.0000 | 0.0514 | 0.0430 | 0.0466 | 0.0428 |
| Hotelling -Lawley test | 1.0000 | 0.0548 | 0.0468 | 0.0502 | 0.0498 |
| Roy's test | 1.0000 | 0.0534 | 0.0450 | 0.0486 | 0.5326 |

## 5.3 ERROR TYPE III

Table 5.78. Error typeIII For m=2, p=2, n=20

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9492 | 0.0260 | | | 0.0260 |
| Pillai's test | 0.9070 | 0.0124 | | | 0.0124 |
| Hotelling -Lawley test | 0.9752 | 0.0524 | | | 0.0524 |
| Roy's test | 0.9720 | 0.0466 | | | 0.0466 |

Table 5.79. Error typeIII For m=2, p=2, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0466 | | | 0.0466 |
| Pillai's test | 1.0000 | 0.0402 | | | 0.0402 |
| Hotelling -Lawley test | 1.0000 | 0.0550 | | | 0.0550 |
| Roy's test | 1.0000 | 0.0538 | | | 0.0538 |

Table 5.80. Error typeIII For m=2, p=2, n=120

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0448 | | | 0.0448 |
| Pillai's test | 1.0000 | 0.0428 | | | 0.0428 |
| Hotelling -Lawley test | 1.0000 | 0.0492 | | | 0.0492 |
| Roy's test | 1.0000 | 0.0482 | | | 0.0482 |

Table 5.81. Error typeIII For m=3, p=3, n=30

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0426 | | 0.0408 | 0.0378 |
| Pillai's test | 1.0000 | 0.0366 | | 0.0344 | 0.0274 |
| Hotelling -Lawley test | 1.0000 | 0.0504 | | 0.0536 | 0.0550 |
| Roy's test | 1.0000 | 0.0482 | | 0.0498 | 0.1610 |

Table 5.82. Error typeIII For m=3, p=3, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0426 | | 0.0408 | 0.0378 |
| Pillai's test | 1.0000 | 0.0366 | | 0.0344 | 0.0274 |
| Hotelling -Lawley test | 1.0000 | 0.0504 | | 0.0536 | 0.0550 |
| Roy's test | 1.0000 | 0.0482 | | 0.0498 | 0.1610 |

Table 5.83. Error typeIII For m=3, p=3, n=130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0428 | | 0.0426 | 0.0404 |
| Pillai's test | 1.0000 | 0.0394 | | 0.0404 | 0.0346 |
| Hotelling -Lawley test | 1.0000 | 0.0506 | | 0.0506 | 0.0508 |
| Roy's test | 1.0000 | 0.0472 | | 0.0466 | 0.1538 |

Table 5.84. Error typeIII For m=3, p=3, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0426 | | 0.0488 | 0.0460 |
| Pillai's test | 1.0000 | 0.0398 | | 0.0460 | 0.0398 |
| Hotelling -Lawley test | 1.0000 | 0.0468 | | 0.0524 | 0.0518 |
| Roy's test | 1.0000 | 0.0456 | | 0.0514 | 0.1548 |

Table 5.85. Error typeIII For m=3, p=3, n=230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | | 0.0478 | 0.0476 |
| Pillai's test | 1.0000 | 0.0460 | | 0.0458 | 0.0440 |
| Hotelling -Lawley test | 1.0000 | 0.0514 | | 0.0520 | 0.0524 |
| Roy's test | 1.0000 | 0.0502 | | 0.0500 | 0.1636 |

Table 5.86. Error typeIII For m=3, p=3, n=280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | | 0.0476 | 0.0452 |
| Pillai's test | 1.0000 | 0.0488 | | 0.0462 | 0.0416 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | | 0.0512 | 0.0498 |
| Roy's test | 1.0000 | 0.0518 | | 0.0490 | 0.1634 |

Table 5.87. Error typeIII For m=3, p=3, n=330

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0500 | | 0.0476 | 0.0520 |
| Pillai's test | 1.0000 | 0.0490 | | 0.0460 | 0.0492 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | | 0.0500 | 0.0560 |
| Roy's test | 1.0000 | 0.0526 | | 0.0490 | 0.1578 |

Table 5.88. Error typeIII For m=4, p=4, n=40

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9998 | 0.0252 | 0.0266 | 0.0318 | 0.0126 |
| Pillai's test | 0.9998 | 0.0132 | 0.0140 | 0.0144 | 0.0008 |
| Hotelling -Lawley test | 1.0000 | 0.0604 | 0.0620 | 0.0596 | 0.0532 |
| Roy's test | 1.0000 | 0.0524 | 0.0542 | 0.0534 | 0.3536 |

Table 5.89. Error typeIII For m=4, p=4, n=90

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0422 | 0.0410 | 0.0406 | 0.0314 |
| Pillai's test | 1.0000 | 0.0378 | 0.0350 | 0.0358 | 0.0214 |
| Hotelling -Lawley test | 1.0000 | 0.0586 | 0.0534 | 0.0532 | 0.0534 |
| Roy's test | 1.0000 | 0.0524 | 0.0482 | 0.0484 | 0.3318 |

Table 5.90. Error typeIII For m=4, p=4, n=140

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0394 | 0.0422 | 0.0462 | 0.0356 |
| Pillai's test | 1.0000 | 0.0368 | 0.0368 | 0.0434 | 0.0260 |
| Hotelling -Lawley test | 1.0000 | 0.0518 | 0.0522 | 0.0548 | 0.0514 |
| Roy's test | 1.0000 | 0.0460 | 0.0478 | 0.0522 | 0.3290 |

Table 5.91. Error typeIII For m=4, p=4, n=190

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0438 | 0.0466 | 0.0492 | 0.0428 |
| Pillai's test | 1.0000 | 0.0416 | 0.0436 | 0.0454 | 0.0346 |
| Hotelling -Lawley test | 1.0000 | 0.0500 | 0.0528 | 0.0550 | 0.0512 |
| Roy's test | 1.0000 | 0.0474 | 0.0500 | 0.0530 | 0.3320 |

Table 5.92. Error typeIII For m=4, p=4, n=240

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0416 | 0.0486 | 0.0440 |
| Pillai's test | 1.0000 | 0.0456 | 0.0402 | 0.0466 | 0.0370 |
| Hotelling -Lawley test | 1.0000 | 0.0514 | 0.0466 | 0.0554 | 0.0514 |
| Roy's test | 1.0000 | 0.0506 | 0.0446 | 0.0530 | 0.3258 |

Table 5.93. Error typeIII For m=4, p=4, n=290

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0526 | 0.0492 | 0.0472 |
| Pillai's test | 1.0000 | 0.0462 | 0.0494 | 0.0474 | 0.0432 |
| Hotelling -Lawley test | 1.0000 | 0.0522 | 0.0578 | 0.0518 | 0.0552 |
| Roy's test | 1.0000 | 0.0498 | 0.0560 | 0.0506 | 0.3282 |

Table 5.94. Error typeIII For m=4, p=4, n=340

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0442 | 0.0498 | 0.0468 | 0.0508 |
| Pillai's test | 1.0000 | 0.0426 | 0.0490 | 0.0456 | 0.0468 |
| Hotelling -Lawley test | 1.0000 | 0.0494 | 0.0558 | 0.0506 | 0.0566 |
| Roy's test | 1.0000 | 0.0472 | 0.0530 | 0.0486 | 0.3354 |

Table 5.95. Error typeIII For m=4, p=4, n=390

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | 0.0562 | 0.0418 | 0.0478 |
| Pillai's test | 1.0000 | 0.0466 | 0.0544 | 0.0402 | 0.0442 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | 0.0590 | 0.0448 | 0.0516 |
| Roy's test | 1.0000 | 0.0516 | 0.0574 | 0.0436 | 0.3356 |

Table 5.96. Error typeIII For m=5, p=5, n=50

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0262 | 0.0230 | 0.0244 | 0.0030 |
| Pillai's test | 1.0000 | 0.0150 | 0.0112 | 0.0120 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0608 | 0.0554 | 0.0548 | 0.0346 |
| Roy's test | 1.0000 | 0.0528 | 0.0498 | 0.0490 | 0.5600 |

Table 5.97. Error typeIII For m=5, p=5, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0376 | 0.0426 | 0.0420 | 0.0244 |
| Pillai's test | 1.0000 | 0.0316 | 0.0350 | 0.0340 | 0.0120 |
| Hotelling -Lawley test | 1.0000 | 0.0550 | 0.0636 | 0.0616 | 0.0540 |
| Roy's test | 1.0000 | 0.0494 | 0.0550 | 0.0536 | 0.5400 |

Table 5.98. Error typeIII For m=5, p=5, n=150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | 0.0426 | 0.0410 | 0.0294 |
| Pillai's test | 1.0000 | 0.0386 | 0.0378 | 0.0358 | 0.0184 |
| Hotelling -Lawley test | 1.0000 | 0.0548 | 0.0562 | 0.0496 | 0.0486 |
| Roy's test | 1.0000 | 0.0498 | 0.0514 | 0.0460 | 0.5312 |

Table 5.99. Error typeIII For m=5, p=5, n=200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | 0.0442 | 0.0416 | 0.0378 |
| Pillai's test | 1.0000 | 0.0436 | 0.0410 | 0.0382 | 0.0308 |
| Hotelling -Lawley test | 1.0000 | 0.0586 | 0.0538 | 0.0502 | 0.0534 |
| Roy's test | 1.0000 | 0.0534 | 0.0476 | 0.0474 | 0.5264 |

Table 5.100. Error typeIII For m=5, p=5, n=250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | 0.0432 | 0.0490 | 0.0412 |
| Pillai's test | 1.0000 | 0.0474 | 0.0392 | 0.0466 | 0.0338 |
| Hotelling -Lawley test | 1.0000 | 0.0576 | 0.0498 | 0.0546 | 0.0510 |
| Roy's test | 1.0000 | 0.0558 | 0.0462 | 0.0516 | 0.5168 |

Table 5.101. Error typeIII For m=5, p=5, n=300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0492 | 0.0476 | 0.0434 | 0.0416 |
| Pillai's test | 1.0000 | 0.0480 | 0.0452 | 0.0414 | 0.0360 |
| Hotelling -Lawley test | 1.0000 | 0.0552 | 0.0536 | 0.0492 | 0.0494 |
| Roy's test | 1.0000 | 0.0514 | 0.0510 | 0.0460 | 0.5176 |

Table 5.102. Error typeIII For m=5, p=5, n=350

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0454 | 0.0456 | 0.0450 | 0.0454 |
| Pillai's test | 1.0000 | 0.0434 | 0.0442 | 0.0442 | 0.0388 |
| Hotelling -Lawley test | 1.0000 | 0.0498 | 0.0526 | 0.0500 | 0.0536 |
| Roy's test | 1.0000 | 0.0480 | 0.0492 | 0.0484 | 0.5106 |

Table 5.103. Error typeIII For m=5, p=5, n=400

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | 0.0426 | 0.0422 | 0.0404 |
| Pillai's test | 1.0000 | 0.0492 | 0.0404 | 0.0410 | 0.0350 |
| Hotelling -Lawley test | 1.0000 | 0.0554 | 0.0472 | 0.0466 | 0.0488 |
| Roy's test | 1.0000 | 0.0524 | 0.0444 | 0.0446 | 0.5212 |

Table 5.104. Error typeIII For m=5, p=5, n=450

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0452 | 0.0466 | 0.0458 | 0.0412 |
| Pillai's test | 1.0000 | 0.0426 | 0.0450 | 0.0452 | 0.0362 |
| Hotelling -Lawley test | 1.0000 | 0.0494 | 0.0518 | 0.0494 | 0.0472 |
| Roy's test | 1.0000 | 0.0474 | 0.0496 | 0.0474 | 0.5224 |

Table 5.105. Error typeIII For m=5, p=5, n=500

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | 0.0462 | 0.0448 | 0.0454 |
| Pillai's test | 1.0000 | 0.0474 | 0.0450 | 0.0432 | 0.0414 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | 0.0494 | 0.0486 | 0.0532 |
| Roy's test | 1.0000 | 0.0506 | 0.0478 | 0.0470 | 0.5152 |

Table 5.106. Error typeIII For m=6, p=6, n=60

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0194 | 0.0244 | 0.0198 | 0.0014 |
| Pillai's test | 1.0000 | 0.0126 | 0.0132 | 0.0122 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0498 | 0.0566 | 0.0512 | 0.0212 |
| Roy's test | 1.0000 | 0.0482 | 0.0556 | 0.0500 | 0.7532 |

Table 5.107. Error typeIII For m=6, p=6, n=110

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0384 | 0.0374 | 0.0348 | 0.0192 |
| Pillai's test | 1.0000 | 0.0314 | 0.0300 | 0.0276 | 0.0054 |
| Hotelling -Lawley test | 1.0000 | 0.0570 | 0.0574 | 0.0554 | 0.0462 |
| Roy's test | 1.0000 | 0.0502 | 0.0490 | 0.0462 | 0.7178 |

Table 5.108. Error typeIII For m=6, p=6, n=160

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | 0.0452 | 0.0414 | 0.0244 |
| Pillai's test | 1.0000 | 0.0378 | 0.0406 | 0.0368 | 0.0136 |
| Hotelling -Lawley test | 1.0000 | 0.0572 | 0.0590 | 0.0566 | 0.0476 |
| Roy's test | 1.0000 | 0.0514 | 0.0536 | 0.0510 | 0.7158 |

Table 5.109. Error typeIII For m=6, p=6, n=210

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0472 | 0.0514 | 0.0480 | 0.0376 |
| Pillai's test | 1.0000 | 0.0430 | 0.0468 | 0.0446 | 0.0250 |
| Hotelling -Lawley test | 1.0000 | 0.0572 | 0.0614 | 0.0566 | 0.0584 |
| Roy's test | 1.0000 | 0.0522 | 0.0564 | 0.0530 | 0.7142 |

Table 5.110. Error typeIII For m=6, p=6, n=260

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0464 | 0.0456 | 0.0426 | 0.0354 |
| Pillai's test | 1.0000 | 0.0442 | 0.0412 | 0.0404 | 0.0272 |
| Hotelling -Lawley test | 1.0000 | 0.0544 | 0.0518 | 0.0522 | 0.0502 |
| Roy's test | 1.0000 | 0.0506 | 0.0488 | 0.0486 | 0.7122 |

Table 5.111. Error typeIII For m=6, p=6, n=310

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0432 | 0.0422 | 0.0470 | 0.0390 |
| Pillai's test | 1.0000 | 0.0406 | 0.0398 | 0.0452 | 0.0318 |
| Hotelling -Lawley test | 1.0000 | 0.0486 | 0.0498 | 0.0554 | 0.0500 |
| Roy's test | 1.0000 | 0.0460 | 0.0464 | 0.0506 | 0.7106 |

Table 5.112. Error typeIII For m=6, p=6, n=360

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0452 | 0.0468 | 0.0432 | 0.0372 |
| Pillai's test | 1.0000 | 0.0428 | 0.0446 | 0.0414 | 0.0288 |
| Hotelling -Lawley test | 1.0000 | 0.0500 | 0.0528 | 0.0494 | 0.0510 |
| Roy's test | 1.0000 | 0.0474 | 0.0500 | 0.0456 | 0.7048 |

Table 5.113. Error typeIII For m=6, p=6, n=410

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | 0.0486 | 0.0488 | 0.0416 |
| Pillai's test | 1.0000 | 0.0436 | 0.0468 | 0.0466 | 0.0342 |
| Hotelling -Lawley test | 1.0000 | 0.0510 | 0.0526 | 0.0546 | 0.0520 |
| Roy's test | 1.0000 | 0.0490 | 0.0496 | 0.0516 | 0.6994 |

Table 5.114. Error typeIII For m=6, p=6, n=460

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0482 | 0.0414 | 0.0524 | 0.0404 |
| Pillai's test | 1.0000 | 0.0468 | 0.0382 | 0.0514 | 0.0350 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0464 | 0.0560 | 0.0474 |
| Roy's test | 1.0000 | 0.0508 | 0.0444 | 0.0536 | 0.7008 |

Table 5.115. Error typeIII For m=6, p=6, n=510

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0488 | 0.0484 | 0.0484 | 0.0426 |
| Pillai's test | 1.0000 | 0.0460 | 0.0476 | 0.0474 | 0.0366 |
| Hotelling -Lawley test | 1.0000 | 0.0520 | 0.0538 | 0.0528 | 0.0512 |
| Roy's test | 1.0000 | 0.0514 | 0.0508 | 0.0512 | 0.7012 |

Table 5.116. Error typeIII For m=6, p=6, n=560

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0444 | 0.0502 | 0.0528 | 0.0448 |
| Pillai's test | 1.0000 | 0.0438 | 0.0476 | 0.0522 | 0.0380 |
| Hotelling -Lawley test | 1.0000 | 0.0488 | 0.0542 | 0.0558 | 0.0496 |
| Roy's test | 1.0000 | 0.0462 | 0.0534 | 0.0534 | 0.6994 |

Table 5.117. Error typeIII For m=6, p=6, n=610

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0526 | 0.0498 | 0.0486 | 0.0452 |
| Pillai's test | 1.0000 | 0.0506 | 0.0492 | 0.0478 | 0.0384 |
| Hotelling -Lawley test | 1.0000 | 0.0556 | 0.0532 | 0.0520 | 0.0516 |
| Roy's test | 1.0000 | 0.0540 | 0.0508 | 0.0500 | 0.7052 |

Table 5.118. Error typeIII For m=6, p=6, n=660

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0466 | 0.0494 | 0.0506 | 0.0478 |
| Pillai's test | 1.0000 | 0.0462 | 0.0488 | 0.0490 | 0.0434 |
| Hotelling -Lawley test | 1.0000 | 0.0488 | 0.0528 | 0.0530 | 0.0526 |
| Roy's test | 1.0000 | 0.0478 | 0.0512 | 0.0514 | 0.7074 |

Table 5.119. Error typeIII For m=7, p=7, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0190 | 0.0182 | 0.0148 | 0.0002 |
| Pillai's test | 1.0000 | 0.0090 | 0.0110 | 0.0076 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0462 | 0.0450 | 0.0416 | 0.0114 |
| Roy's test | 1.0000 | 0.0548 | 0.0516 | 0.0474 | 0.8728 |

Table 5.120. Error typeIII For m=7, p=7, n=120

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0326 | 0.0384 | 0.0386 | 0.0096 |
| Pillai's test | 1.0000 | 0.0264 | 0.0312 | 0.0312 | 0.0022 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0562 | 0.0592 | 0.0406 |
| Roy's test | 1.0000 | 0.0452 | 0.0490 | 0.0510 | 0.8526 |

Table 5.121. Error typeIII For m=7, p=7, n=170

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0402 | 0.0420 | 0.0390 | 0.0230 |
| Pillai's test | 1.0000 | 0.0354 | 0.0368 | 0.0344 | 0.0118 |
| Hotelling -Lawley test | 1.0000 | 0.0570 | 0.0606 | 0.0510 | 0.0514 |
| Roy's test | 1.0000 | 0.0490 | 0.0510 | 0.0454 | 0.8598 |

Table 5.122. Error typeIII For m=7, p=7, n=220

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0444 | 0.0436 | 0.0430 | 0.0286 |
| Pillai's test | 1.0000 | 0.0408 | 0.0396 | 0.0390 | 0.0170 |
| Hotelling -Lawley test | 1.0000 | 0.0582 | 0.0558 | 0.0544 | 0.0524 |
| Roy's test | 1.0000 | 0.0504 | 0.0496 | 0.0476 | 0.8506 |

Table 5.123. Error typeIII For m=7, p=7, n=270

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0462 | 0.0448 | 0.0458 | 0.0362 |
| Pillai's test | 1.0000 | 0.0448 | 0.0406 | 0.0418 | 0.0242 |
| Hotelling -Lawley test | 1.0000 | 0.0560 | 0.0542 | 0.0562 | 0.0542 |
| Roy's test | 1.0000 | 0.0514 | 0.0488 | 0.0516 | 0.8516 |

Table 5.124. Error typeIII For m=7, p=7, n=320

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0416 | 0.0440 | 0.0432 | 0.0362 |
| Pillai's test | 1.0000 | 0.0388 | 0.0406 | 0.0410 | 0.0264 |
| Hotelling -Lawley test | 1.0000 | 0.0482 | 0.0522 | 0.0512 | 0.0516 |
| Roy's test | 1.0000 | 0.0438 | 0.0482 | 0.0472 | 0.8436 |

Table 5.125. Error typeIII For m=7, p=7, n=370

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0458 | 0.0476 | 0.0462 | 0.0394 |
| Pillai's test | 1.0000 | 0.0442 | 0.0460 | 0.0043 | 0.0282 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0526 | 0.0534 | 0.0534 |
| Roy's test | 1.0000 | 0.0486 | 0.0502 | 0.0496 | 0.8536 |

Table 5.126. Error typeIII For m=7, p=7, n=420

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | 0.0408 | 0.0458 | 0.0352 |
| Pillai's test | 1.0000 | 0.0422 | 0.0396 | 0.0434 | 0.0276 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0474 | 0.0538 | 0.0468 |
| Roy's test | 1.0000 | 0.0480 | 0.0448 | 0.0498 | 0.8484 |

Table 5.127. Error typeIII For m=7, p=7, n=470

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0492 | 0.0418 | 0.0460 | 0.0412 |
| Pillai's test | 1.0000 | 0.0474 | 0.0406 | 0.0448 | 0.0320 |
| Hotelling -Lawley test | 1.0000 | 0.0558 | 0.0450 | 0.0506 | 0.0514 |
| Roy's test | 1.0000 | 0.0518 | 0.0438 | 0.0482 | 0.8446 |

Table 5.128. Error typeIII For m=7, p=7, n=520

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | 0.0440 | 0.0472 | 0.0396 |
| Pillai's test | 1.0000 | 0.0484 | 0.0422 | 0.0464 | 0.0328 |
| Hotelling -Lawley test | 1.0000 | 0.0570 | 0.0486 | 0.0512 | 0.0498 |
| Roy's test | 1.0000 | 0.0542 | 0.0458 | 0.0494 | 0.8470 |

Table 5.129. Error typeIII For m=7, p=7, n=570

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0454 | 0.0476 | 0.0430 | 0.0436 |
| Pillai's test | 1.0000 | 0.0440 | 0.0452 | 0.0420 | 0.0376 |
| Hotelling -Lawley test | 1.0000 | 0.0506 | 0.0524 | 0.0476 | 0.0514 |
| Roy's test | 1.0000 | 0.0472 | 0.0490 | 0.0452 | 0.8506 |

Table 5.130. Error typeIII For m=7, p=7, n=620

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0456 | 0.0460 | 0.0384 |
| Pillai's test | 1.0000 | 0.0464 | 0.0444 | 0.0448 | 0.0324 |
| Hotelling -Lawley test | 1.0000 | 0.0544 | 0.0498 | 0.0512 | 0.0472 |
| Roy's test | 1.0000 | 0.0502 | 0.0474 | 0.0484 | 0.8408 |

Table 5.131. Error typeIII For m=7, p=7, n=670

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0472 | 0.0518 | 0.0484 | 0.0450 |
| Pillai's test | 1.0000 | 0.0460 | 0.0502 | 0.0468 | 0.0382 |
| Hotelling -Lawley test | 1.0000 | 0.0522 | 0.0544 | 0.0528 | 0.0522 |
| Roy's test | 1.0000 | 0.0494 | 0.0524 | 0.0500 | 0.8390 |

Table 5.132. Error typeIII For m=7, p=7, n=720

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0482 | 0.0482 | 0.0480 | 0.0392 |
| Pillai's test | 1.0000 | 0.0468 | 0.0474 | 0.0476 | 0.0348 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0516 | 0.0510 | 0.0466 |
| Roy's test | 1.0000 | 0.0498 | 0.0492 | 0.0492 | 0.8506 |

Table 5.133. Error typeIII For m=7, p=7, n=770

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0518 | 0.0468 | 0.0508 | 0.0432 |
| Pillai's test | 1.0000 | 0.0504 | 0.0454 | 0.0496 | 0.0382 |
| Hotelling -Lawley test | 1.0000 | 0.0560 | 0.0488 | 0.0548 | 0.0494 |
| Roy's test | 1.0000 | 0.0538 | 0.0478 | 0.0522 | 0.8466 |

Table 5.134. Error typeIII For m=7, p=7, n=820

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0518 | 0.0494 | 0.0478 |
| Pillai's test | 1.0000 | 0.0466 | 0.0510 | 0.0490 | 0.0428 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | 0.0544 | 0.0520 | 0.0552 |
| Roy's test | 1.0000 | 0.0478 | 0.0530 | 0.0506 | 0.8502 |

Table 5.135. Error typeIII For m=7, p=7, n=870

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0494 | 0.0478 | 0.0526 | 0.0404 |
| Pillai's test | 1.0000 | 0.0482 | 0.0462 | 0.0514 | 0.0356 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0498 | 0.0552 | 0.0472 |
| Roy's test | 1.0000 | 0.0512 | 0.0490 | 0.0534 | 0.8528 |

Table 5.136. Error typeIII For m=7, p=7, n=920

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0476 | 0.0502 | 0.0422 | 0.0458 |
| Pillai's test | 1.0000 | 0.0464 | 0.0490 | 0.0412 | 0.0418 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0544 | 0.0448 | 0.0526 |
| Roy's test | 1.0000 | 0.0490 | 0.0520 | 0.0432 | 0.8450 |

Table 5.137. Error typeIII For m=8, p=8, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0080 | 0.0080 | 0.0074 | 0.0000 |
| Pillai's test | 1.0000 | 0.0032 | 0.0036 | 0.0028 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0300 | 0.0320 | 0.0246 | 0.0006 |
| Roy's test | 1.0000 | 0.0526 | 0.0510 | 0.0464 | 0.9574 |

Table 5.138. Error typeIII For m=8, p=8, n=130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0312 | 0.0338 | 0.0384 | 0.0072 |
| Pillai's test | 1.0000 | 0.0246 | 0.0248 | 0.0310 | 0.0012 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | 0.0562 | 0.0582 | 0.0312 |
| Roy's test | 1.0000 | 0.0476 | 0.0492 | 0.0524 | 0.9496 |

Table 5.139. Error typeIII For m=8, p=8, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0362 | 0.0442 | 0.0366 | 0.0186 |
| Pillai's test | 1.0000 | 0.0324 | 0.0384 | 0.0298 | 0.0060 |
| Hotelling -Lawley test | 1.0000 | 0.0532 | 0.0610 | 0.0552 | 0.0412 |
| Roy's test | 1.0000 | 0.0462 | 0.0542 | 0.0452 | 0.9386 |

Table 5.140. Error typeIII For m=8, p=8, n=230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0440 | 0.0424 | 0.0420 | 0.0232 |
| Pillai's test | 1.0000 | 0.0404 | 0.0378 | 0.0376 | 0.0110 |
| Hotelling -Lawley test | 1.0000 | 0.0594 | 0.0554 | 0.0566 | 0.0468 |
| Roy's test | 1.0000 | 0.0518 | 0.0496 | 0.0502 | 0.9348 |

Table 5.141. Error typeIII For m=8, p=8, n=280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0420 | 0.0472 | 0.0446 | 0.0236 |
| Pillai's test | 1.0000 | 0.0390 | 0.0426 | 0.0406 | 0.0148 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | 0.0626 | 0.0570 | 0.0450 |
| Roy's test | 1.0000 | 0.0470 | 0.0554 | 0.0502 | 0.9378 |

Table 5.142. Error typeIII For m=8, p=8, n=330

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0430 | 0.0424 | 0.0500 | 0.0292 |
| Pillai's test | 1.0000 | 0.0394 | 0.0406 | 0.0464 | 0.0174 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | 0.0518 | 0.0592 | 0.0418 |
| Roy's test | 1.0000 | 0.0466 | 0.0468 | 0.0536 | 0.9308 |

Table 5.143. Error typeIII For m=8, p=8, n=380

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0446 | 0.0432 | 0.0472 | 0.0298 |
| Pillai's test | 1.0000 | 0.0416 | 0.0412 | 0.0452 | 0.0204 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0520 | 0.0560 | 0.0446 |
| Roy's test | 1.0000 | 0.0484 | 0.0466 | 0.0514 | 0.9406 |

Table 5.144. Error typeIII For m=8, p=8, n=430

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0508 | 0.0446 | 0.0476 | 0.0390 |
| Pillai's test | 1.0000 | 0.0478 | 0.0432 | 0.0462 | 0.0286 |
| Hotelling -Lawley test | 1.0000 | 0.0552 | 0.0510 | 0.0550 | 0.0536 |
| Roy's test | 1.0000 | 0.0526 | 0.0478 | 0.0504 | 0.9318 |

Table 5.145. Error typeIII For m=8, p=8, n=480

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0464 | 0.0486 | 0.0384 |
| Pillai's test | 1.0000 | 0.0446 | 0.0450 | 0.0462 | 0.0296 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | 0.0506 | 0.0574 | 0.0526 |
| Roy's test | 1.0000 | 0.0492 | 0.0480 | 0.0540 | 0.9376 |

Table 5.146. Error typeIII For m=8, p=8, n=530

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0426 | 0.0406 | 0.0456 | 0.0346 |
| Pillai's test | 1.0000 | 0.0406 | 0.0394 | 0.0434 | 0.0272 |
| Hotelling -Lawley test | 1.0000 | 0.0492 | 0.0472 | 0.0544 | 0.0464 |
| Roy's test | 1.0000 | 0.0442 | 0.0442 | 0.0500 | 0.9342 |

Table 5.147. Error typeIII For m=8, p=8, n=580

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | 0.0516 | 0.0414 | 0.0386 |
| Pillai's test | 1.0000 | 0.0474 | 0.0500 | 0.0390 | 0.0310 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | 0.0560 | 0.0446 | 0.0492 |
| Roy's test | 1.0000 | 0.0504 | 0.0536 | 0.0428 | 0.9296 |

Table 5.148. Error typeIII For m=8, p=8, n=630

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0468 | 0.0466 | 0.0392 |
| Pillai's test | 1.0000 | 0.0456 | 0.0458 | 0.0456 | 0.0312 |
| Hotelling -Lawley test | 1.0000 | 0.0516 | 0.0510 | 0.0518 | 0.0496 |
| Roy's test | 1.0000 | 0.0484 | 0.0488 | 0.0486 | 0.9296 |

Table 5.149. Error typeIII For m=8, p=8, n=680

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0476 | 0.0382 | 0.0498 | 0.0394 |
| Pillai's test | 1.0000 | 0.0454 | 0.0366 | 0.0490 | 0.0328 |
| Hotelling -Lawley test | 1.0000 | 0.0508 | 0.0440 | 0.0544 | 0.0474 |
| Roy's test | 1.0000 | 0.0494 | 0.0412 | 0.0520 | 0.9414 |

Table 5.150. Error typeIII For m=8, p=8, n=730

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0436 | 0.0494 | 0.0484 | 0.0396 |
| Pillai's test | 1.0000 | 0.0420 | 0.0472 | 0.0474 | 0.0346 |
| Hotelling -Lawley test | 1.0000 | 0.0490 | 0.0540 | 0.0520 | 0.0498 |
| Roy's test | 1.0000 | 0.0466 | 0.0520 | 0.0498 | 0.9348 |

Table 5.151. Error typeIII For m=8, p=8, n=780

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0536 | 0.0426 | 0.0478 | 0.0390 |
| Pillai's test | 1.0000 | 0.0516 | 0.0416 | 0.0470 | 0.0316 |
| Hotelling -Lawley test | 1.0000 | 0.0580 | 0.0452 | 0.0516 | 0.0462 |
| Roy's test | 1.0000 | 0.0554 | 0.0448 | 0.0490 | 0.9298 |

Table 5.152. Error typeIII For m=8, p=8, n=830

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | 0.0514 | 0.0448 | 0.0404 |
| Pillai's test | 1.0000 | 0.0486 | 0.0502 | 0.0440 | 0.0364 |
| Hotelling -Lawley test | 1.0000 | 0.0544 | 0.0542 | 0.0492 | 0.0472 |
| Roy's test | 1.0000 | 0.0520 | 0.0526 | 0.0468 | 0.9292 |

Table 5.153. Error typeIII For m=8, p=8, n=880

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0464 | 0.0504 | 0.0486 | 0.0478 |
| Pillai's test | 1.0000 | 0.0454 | 0.0496 | 0.0478 | 0.0406 |
| Hotelling -Lawley test | 1.0000 | 0.0512 | 0.0528 | 0.0528 | 0.0532 |
| Roy's test | 1.0000 | 0.0486 | 0.0514 | 0.0512 | 0.9292 |

Table 5.154. Error typeIII For m=8, p=8, n=930

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0456 | 0.0470 | 0.0494 | 0.0428 |
| Pillai's test | 1.0000 | 0.0450 | 0.0460 | 0.0476 | 0.0374 |
| Hotelling -Lawley test | 1.0000 | 0.0492 | 0.0512 | 0.0534 | 0.0502 |
| Roy's test | 1.0000 | 0.0480 | 0.0484 | 0.0510 | 0.9296 |

Table 5.155. Error typeIII For m=8, p=8, n=980

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | 0.0462 | 0.0432 | 0.0450 |
| Pillai's test | 1.0000 | 0.0486 | 0.0452 | 0.0426 | 0.0390 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0502 | 0.0470 | 0.0516 |
| Roy's test | 1.0000 | 0.0520 | 0.0482 | 0.0450 | 0.9394 |

Table 5.156. Error typeIII For m=8, p=8, n=1030

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0526 | 0.0452 | 0.0512 | 0.0412 |
| Pillai's test | 1.0000 | 0.0514 | 0.0434 | 0.0502 | 0.0374 |
| Hotelling -Lawley test | 1.0000 | 0.0570 | 0.0484 | 0.0548 | 0.0480 |
| Roy's test | 1.0000 | 0.0538 | 0.0468 | 0.0528 | 0.9348 |

Table 5.157. Error typeIII For m=8, p=8, n=1080

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0502 | 0.0506 | 0.0462 |
| Pillai's test | 1.0000 | 0.0474 | 0.0500 | 0.0496 | 0.0410 |
| Hotelling -Lawley test | 1.0000 | 0.0498 | 0.0536 | 0.0530 | 0.0522 |
| Roy's test | 1.0000 | 0.0488 | 0.0516 | 0.0520 | 0.9340 |

Table 5.158. Error typeIII For m=8, p=8, n=1130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0508 | 0.0464 | 0.0488 | 0.0462 |
| Pillai's test | 1.0000 | 0.0500 | 0.0454 | 0.0482 | 0.0410 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0496 | 0.0520 | 0.0522 |
| Roy's test | 1.0000 | 0.0518 | 0.0474 | 0.0500 | 0.9340 |

Table 5.159. Error typeIII For m=8, p=8, n=1180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0496 | 0.0430 | 0.0416 | 0.0388 |
| Pillai's test | 1.0000 | 0.0490 | 0.0424 | 0.0410 | 0.0364 |
| Hotelling -Lawley test | 1.0000 | 0.0520 | 0.0450 | 0.0442 | 0.0440 |
| Roy's test | 1.0000 | 0.0504 | 0.0442 | 0.0428 | 0.9328 |

Table 5.160. Error typeIII For m=8, p=8, n=1230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0514 | 0.0578 | 0.0488 | 0.0496 |
| Pillai's test | 1.0000 | 0.0500 | 0.0572 | 0.0480 | 0.0442 |
| Hotelling -Lawley test | 1.0000 | 0.0538 | 0.0602 | 0.0512 | 0.0538 |
| Roy's test | 1.0000 | 0.0528 | 0.0588 | 0.0496 | 0.9320 |

Table 5.161. Error typeIII For m=8, p=8, n=1280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0442 | 0.0480 | 0.0514 | 0.0512 |
| Pillai's test | 1.0000 | 0.0430 | 0.0476 | 0.0512 | 0.0466 |
| Hotelling -Lawley test | 1.0000 | 0.0466 | 0.0520 | 0.0536 | 0.0564 |
| Roy's test | 1.0000 | 0.0448 | 0.0494 | 0.0524 | 0.9354 |

Table 5.162. Error typeIII For m=9, p=9, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0108 | 0.0086 | 0.0072 | 0.0000 |
| Pillai's test | 1.0000 | 0.0052 | 0.0040 | 0.0042 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0320 | 0.0282 | 0.0262 | 0.0002 |
| Roy's test | 1.0000 | 0.0510 | 0.0474 | 0.0440 | 0.9846 |

Table 5.163. Error typeIII For m=9, p=9, n=150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0346 | 0.0342 | 0.0340 | 0.0048 |
| Pillai's test | 1.0000 | 0.0290 | 0.0266 | 0.0264 | 0.0006 |
| Hotelling -Lawley test | 1.0000 | 0.0588 | 0.0568 | 0.0582 | 0.0288 |
| Roy's test | 1.0000 | 0.0528 | 0.0506 | 0.0506 | 0.9750 |

Table 5.164. Error typeIII For m=9, p=9, n=200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0358 | 0.0396 | 0.0436 | 0.0110 |
| Pillai's test | 1.0000 | 0.0318 | 0.0336 | 0.0366 | 0.0036 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0568 | 0.0604 | 0.0340 |
| Roy's test | 1.0000 | 0.0458 | 0.0510 | 0.0538 | 0.9790 |

Table 5.165. Error typeIII For m=9, p=9, n=250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0408 | 0.0472 | 0.0448 | 0.0194 |
| Pillai's test | 1.0000 | 0.0360 | 0.0424 | 0.0400 | 0.0076 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0598 | 0.0592 | 0.0428 |
| Roy's test | 1.0000 | 0.0464 | 0.0536 | 0.0516 | 0.9774 |

Table 5.166. Error typeIII For m=9, p=9, n=300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0428 | 0.0446 | 0.0498 | 0.0262 |
| Pillai's test | 1.0000 | 0.0400 | 0.0400 | 0.0460 | 0.0148 |
| Hotelling -Lawley test | 1.0000 | 0.0532 | 0.0570 | 0.0616 | 0.0438 |
| Roy's test | 1.0000 | 0.0474 | 0.0504 | 0.0554 | 0.9768 |

Table 5.167. Error typeIII For m=9, p=9, n=350

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0422 | 0.0498 | 0.0442 | 0.0314 |
| Pillai's test | 1.0000 | 0.0400 | 0.0472 | 0.0404 | 0.0198 |
| Hotelling -Lawley test | 1.0000 | 0.0532 | 0.0598 | 0.0560 | 0.0500 |
| Roy's test | 1.0000 | 0.0468 | 0.0542 | 0.0488 | 0.9768 |

Table 5.168. Error typeIII For m=9, p=9, n=400

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0452 | 0.0492 | 0.0460 | 0.0294 |
| Pillai's test | 1.0000 | 0.0428 | 0.0458 | 0.0426 | 0.0184 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0560 | 0.0550 | 0.0508 |
| Roy's test | 1.0000 | 0.0500 | 0.0520 | 0.0512 | 0.9782 |

Table 5.169. Error typeIII For m=9, p=9, n=450

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0454 | 0.0496 | 0.0464 | 0.0290 |
| Pillai's test | 1.0000 | 0.0418 | 0.0474 | 0.0440 | 0.0218 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0572 | 0.0538 | 0.0462 |
| Roy's test | 1.0000 | 0.0488 | 0.0536 | 0.0488 | 0.9760 |

Table 5.170. Error typeIII For m=9, p=9, n=500

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0414 | 0.0418 | 0.0466 | 0.0332 |
| Pillai's test | 1.0000 | 0.0386 | 0.0404 | 0.0436 | 0.0244 |
| Hotelling -Lawley test | 1.0000 | 0.0484 | 0.0478 | 0.0530 | 0.0472 |
| Roy's test | 1.0000 | 0.0438 | 0.0444 | 0.0488 | 0.9730 |

Table 5.171. Error typeIII For m=9, p=9, n=550

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | 0.0470 | 0.0520 | 0.0352 |
| Pillai's test | 1.0000 | 0.0490 | 0.0440 | 0.0504 | 0.0280 |
| Hotelling -Lawley test | 1.0000 | 0.0570 | 0.0530 | 0.0574 | 0.0486 |
| Roy's test | 1.0000 | 0.0534 | 0.0500 | 0.0538 | 0.9732 |

Table 5.172. Error typeIII For m=9, p=9, n=600

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0452 | 0.0504 | 0.0474 | 0.0426 |
| Pillai's test | 1.0000 | 0.0432 | 0.0486 | 0.0456 | 0.0330 |
| Hotelling -Lawley test | 1.0000 | 0.0492 | 0.0574 | 0.0534 | 0.0560 |
| Roy's test | 1.0000 | 0.0466 | 0.0520 | 0.0498 | 0.9748 |

Table 5.173. Error typeIII For m=9, p=9, n=650

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0466 | 0.0450 | 0.0480 | 0.0356 |
| Pillai's test | 1.0000 | 0.0452 | 0.0432 | 0.0468 | 0.0260 |
| Hotelling -Lawley test | 1.0000 | 0.0510 | 0.0522 | 0.0538 | 0.0482 |
| Roy's test | 1.0000 | 0.0480 | 0.0482 | 0.0510 | 0.9754 |

Table 5.174. Error typeIII For m=9, p=9, n=700

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0514 | 0.0502 | 0.0476 | 0.0406 |
| Pillai's test | 1.0000 | 0.0500 | 0.0486 | 0.0466 | 0.0328 |
| Hotelling -Lawley test | 1.0000 | 0.0574 | 0.0552 | 0.0514 | 0.0498 |
| Roy's test | 1.0000 | 0.0536 | 0.0534 | 0.0494 | 0.9772 |

Table 5.175. Error typeIII For m=9, p=9, n=750

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0468 | 0.0532 | 0.0488 | 0.0390 |
| Pillai's test | 1.0000 | 0.0456 | 0.0516 | 0.0476 | 0.0316 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0600 | 0.0532 | 0.0492 |
| Roy's test | 1.0000 | 0.0496 | 0.0562 | 0.0506 | 0.9780 |

Table 5.176. Error typeIII For m=9, p=9, n=800

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0464 | 0.0480 | 0.0518 | 0.0382 |
| Pillai's test | 1.0000 | 0.0450 | 0.0476 | 0.0508 | 0.0322 |
| Hotelling -Lawley test | 1.0000 | 0.0500 | 0.0538 | 0.0560 | 0.0482 |
| Roy's test | 1.0000 | 0.0484 | 0.0508 | 0.0540 | 0.9786 |

Table 5.177. Error typeIII For m=9, p=9, n=850

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | 0.0440 | 0.0470 | 0.0378 |
| Pillai's test | 1.0000 | 0.0494 | 0.0428 | 0.0458 | 0.0312 |
| Hotelling -Lawley test | 1.0000 | 0.0562 | 0.0496 | 0.0510 | 0.0468 |
| Roy's test | 1.0000 | 0.0528 | 0.0460 | 0.0492 | 0.9752 |

Table 5.178. Error typeIII For m=9, p=9, n=900

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0452 | 0.0458 | 0.0536 | 0.0460 |
| Pillai's test | 1.0000 | 0.0434 | 0.0446 | 0.0514 | 0.0378 |
| Hotelling -Lawley test | 1.0000 | 0.0502 | 0.0490 | 0.0588 | 0.0566 |
| Roy's test | 1.0000 | 0.0468 | 0.0470 | 0.0548 | 0.9752 |

Table 5.179. Error typeIII For m=9, p=9, n=950

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0412 | 0.0486 | 0.0452 | 0.0394 |
| Pillai's test | 1.0000 | 0.0398 | 0.0472 | 0.0436 | 0.0338 |
| Hotelling -Lawley test | 1.0000 | 0.0452 | 0.0520 | 0.0496 | 0.0482 |
| Roy's test | 1.0000 | 0.0430 | 0.0508 | 0.0468 | 0.9776 |

Table 5.180. Error typeIII For m=9, p=9, n=1000

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0504 | 0.0490 | 0.0488 | 0.0382 |
| Pillai's test | 1.0000 | 0.0496 | 0.0482 | 0.0474 | 0.0314 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0522 | 0.0512 | 0.0468 |
| Roy's test | 1.0000 | 0.0516 | 0.0506 | 0.0502 | 0.9748 |

Table 5.181. Error typeIII For m=9, p=9, n=1050

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0538 | 0.0488 | 0.0470 | 0.0452 |
| Pillai's test | 1.0000 | 0.0530 | 0.0472 | 0.0466 | 0.0398 |
| Hotelling -Lawley test | 1.0000 | 0.0562 | 0.0514 | 0.0514 | 0.0518 |
| Roy's test | 1.0000 | 0.0550 | 0.0496 | 0.0500 | 0.9754 |

Table 5.182. Error typeIII For m=9, p=9, n=1100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0514 | 0.0484 | 0.0502 | 0.0482 |
| Pillai's test | 1.0000 | 0.0508 | 0.0476 | 0.0488 | 0.0408 |
| Hotelling -Lawley test | 1.0000 | 0.0532 | 0.0522 | 0.0538 | 0.0550 |
| Roy's test | 1.0000 | 0.0528 | 0.0506 | 0.0510 | 0.9794 |

Table 5.183. Error typeIII For m=9, p=9, n=1150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0480 | 0.0486 | 0.0520 | 0.0410 |
| Pillai's test | 1.0000 | 0.0468 | 0.0482 | 0.0514 | 0.0356 |
| Hotelling -Lawley test | 1.0000 | 0.0506 | 0.0510 | 0.0556 | 0.0482 |
| Roy's test | 1.0000 | 0.0494 | 0.0496 | 0.0534 | 0.9780 |

Table 5.184. Error typeIII For m=9, p=9, n=1200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0494 | 0.0496 | 0.0526 | 0.0460 |
| Pillai's test | 1.0000 | 0.0484 | 0.0488 | 0.0516 | 0.0412 |
| Hotelling -Lawley test | 1.0000 | 0.0518 | 0.0516 | 0.0578 | 0.0524 |
| Roy's test | 1.0000 | 0.0500 | 0.0504 | 0.0548 | 0.9714 |

Table 5.185. Error typeIII For m=9, p=9, n=1250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0544 | 0.0490 | 0.0534 | 0.0404 |
| Pillai's test | 1.0000 | 0.0538 | 0.0480 | 0.0528 | 0.0358 |
| Hotelling -Lawley test | 1.0000 | 0.0576 | 0.0508 | 0.0558 | 0.0484 |
| Roy's test | 1.0000 | 0.0558 | 0.0494 | 0.0544 | 0.9756 |

Table 5.186. Error typeIII For m=9, p=9, n=1300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0496 | 0.0538 | 0.0548 | 0.0452 |
| Pillai's test | 1.0000 | 0.0494 | 0.0530 | 0.0536 | 0.0404 |
| Hotelling -Lawley test | 1.0000 | 0.0522 | 0.0564 | 0.0568 | 0.0516 |
| Roy's test | 1.0000 | 0.0510 | 0.0544 | 0.0558 | 0.9742 |

Table 5.187. Error typeIII For m=10, p=10, n=110

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0002 | 0.0010 | 0.0012 | 0.0000 |
| Pillai's test | 1.0000 | 0.0000 | 0.0002 | 0.0002 | 0.0000 |
| Hotelling -Lawley test | 1.0000 | 0.0042 | 0.0068 | 0.0050 | 0.0000 |
| Roy's test | 1.0000 | 0.0464 | 0.0504 | 0.0460 | 0.9958 |

Table 5.188. Error typeIII For m=10, p=10, n=160

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0296 | 0.0284 | 0.0320 | 0.0020 |
| Pillai's test | 1.0000 | 0.0226 | 0.0210 | 0.0228 | 0.0004 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | 0.0530 | 0.0522 | 0.0124 |
| Roy's test | 1.0000 | 0.0504 | 0.0496 | 0.0492 | 0.9962 |

Table 5.189. Error typeIII For m=10, p=10, n=210

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0414 | 0.0420 | 0.0368 | 0.0084 |
| Pillai's test | 1.0000 | 0.0362 | 0.0358 | 0.0332 | 0.0018 |
| Hotelling -Lawley test | 1.0000 | 0.0614 | 0.0604 | 0.0536 | 0.0302 |
| Roy's test | 1.0000 | 0.0528 | 0.0544 | 0.0468 | 0.9928 |

Table 5.190. Error typeIII For m=10, p=10, n=260

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0406 | 0.0430 | 0.0400 | 0.0152 |
| Pillai's test | 1.0000 | 0.0360 | 0.0388 | 0.0344 | 0.0048 |
| Hotelling -Lawley test | 1.0000 | 0.0564 | 0.0594 | 0.0558 | 0.0378 |
| Roy's test | 1.0000 | 0.0478 | 0.0524 | 0.0498 | 0.9938 |

Table 5.191. Error typeIII For m=10, p=10, n=310

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0390 | 0.0366 | 0.0508 | 0.0228 |
| Pillai's test | 1.0000 | 0.0366 | 0.0328 | 0.0450 | 0.0090 |
| Hotelling -Lawley test | 1.0000 | 0.0534 | 0.0526 | 0.0658 | 0.0448 |
| Roy's test | 1.0000 | 0.0456 | 0.0438 | 0.0576 | 0.9930 |

Table 5.192. Error typeIII For m=10, p=10, n=360

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0432 | 0.0422 | 0.0464 | 0.0198 |
| Pillai's test | 1.0000 | 0.0384 | 0.0400 | 0.0436 | 0.0116 |
| Hotelling -Lawley test | 1.0000 | 0.0544 | 0.0520 | 0.0542 | 0.0446 |
| Roy's test | 1.0000 | 0.0492 | 0.0468 | 0.0506 | 0.9942 |

Table 5.193. Error typeIII For m=10, p=10, n=410

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0460 | 0.0438 | 0.0460 | 0.0264 |
| Pillai's test | 1.0000 | 0.0438 | 0.0402 | 0.0422 | 0.0138 |
| Hotelling -Lawley test | 1.0000 | 0.0552 | 0.0526 | 0.0552 | 0.0452 |
| Roy's test | 1.0000 | 0.0490 | 0.0480 | 0.0498 | 0.9948 |

Table 5.194. Error typeIII For m=10, p=10, n=460

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | 0.0428 | 0.0490 | 0.0290 |
| Pillai's test | 1.0000 | 0.0464 | 0.0398 | 0.0460 | 0.0188 |
| Hotelling -Lawley test | 1.0000 | 0.0576 | 0.0522 | 0.0592 | 0.0478 |
| Roy's test | 1.0000 | 0.0524 | 0.0470 | 0.0540 | 0.9936 |

Table 5.195. Error typeIII For m=10, p=10, n=510

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0488 | 0.0476 | 0.0450 | 0.0342 |
| Pillai's test | 1.0000 | 0.0462 | 0.0448 | 0.0430 | 0.0228 |
| Hotelling -Lawley test | 1.0000 | 0.0560 | 0.0556 | 0.0534 | 0.0500 |
| Roy's test | 1.0000 | 0.0528 | 0.0518 | 0.0470 | 0.9934 |

Table 5.196. Error typeIII For m=10, p=10, n=560

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0560 | 0.0434 | 0.0322 |
| Pillai's test | 1.0000 | 0.0448 | 0.0550 | 0.0416 | 0.0214 |
| Hotelling -Lawley test | 1.0000 | 0.0562 | 0.0646 | 0.0504 | 0.0506 |
| Roy's test | 1.0000 | 0.0516 | 0.0592 | 0.0470 | 0.9940 |

Table 5.197. Error typeIII For m=10, p=10, n=610

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0466 | 0.0480 | 0.0474 | 0.0356 |
| Pillai's test | 1.0000 | 0.0440 | 0.0458 | 0.0456 | 0.0262 |
| Hotelling -Lawley test | 1.0000 | 0.0558 | 0.0552 | 0.0556 | 0.0500 |
| Roy's test | 1.0000 | 0.0530 | 0.0508 | 0.0502 | 0.9912 |

Table 5.198. Error typeIII For m=10, p=10, n=660

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0502 | 0.0484 | 0.0468 | 0.0332 |
| Pillai's test | 1.0000 | 0.0488 | 0.0458 | 0.0458 | 0.0242 |
| Hotelling -Lawley test | 1.0000 | 0.0582 | 0.0546 | 0.0540 | 0.0454 |
| Roy's test | 1.0000 | 0.0534 | 0.0510 | 0.0494 | 0.9916 |

Table 5.199. Error typeIII For m=10, p=10, n=710

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0500 | 0.0500 | 0.0376 |
| Pillai's test | 1.0000 | 0.0462 | 0.0484 | 0.0492 | 0.0290 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0552 | 0.0556 | 0.0520 |
| Roy's test | 1.0000 | 0.0506 | 0.0522 | 0.0528 | 0.9930 |

Table 5.200. Error typeIII For m=10, p=10, n=760

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0428 | 0.0462 | 0.0450 | 0.0368 |
| Pillai's test | 1.0000 | 0.0412 | 0.0452 | 0.0436 | 0.0274 |
| Hotelling -Lawley test | 1.0000 | 0.0474 | 0.0516 | 0.0518 | 0.0480 |
| Roy's test | 1.0000 | 0.0444 | 0.0478 | 0.0470 | 0.9910 |

Table 5.201. Error typeIII For m=10, p=10, n=810

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0488 | 0.0446 | 0.0486 | 0.0420 |
| Pillai's test | 1.0000 | 0.0476 | 0.0440 | 0.0466 | 0.0332 |
| Hotelling -Lawley test | 1.0000 | 0.0542 | 0.0518 | 0.0532 | 0.0554 |
| Roy's test | 1.0000 | 0.0514 | 0.0486 | 0.0510 | 0.9910 |

Table 5.202. Error typeIII For m=10, p=10, n=860

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0516 | 0.0422 | 0.0540 | 0.0390 |
| Pillai's test | 1.0000 | 0.0506 | 0.0414 | 0.0518 | 0.0306 |
| Hotelling -Lawley test | 1.0000 | 0.0566 | 0.0476 | 0.0588 | 0.0506 |
| Roy's test | 1.0000 | 0.0538 | 0.0446 | 0.0552 | 0.9950 |

Table 5.203. Error typeIII For m=10, p=10, n=910

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0454 | 0.0484 | 0.0494 | 0.0382 |
| Pillai's test | 1.0000 | 0.0442 | 0.0474 | 0.0478 | 0.0304 |
| Hotelling -Lawley test | 1.0000 | 0.0514 | 0.0526 | 0.0524 | 0.0504 |
| Roy's test | 1.0000 | 0.0468 | 0.0494 | 0.0504 | 0.9946 |

Table 5.204. Error typeIII For m=10, p=10, n=960

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0486 | 0.0524 | 0.0468 | 0.0392 |
| Pillai's test | 1.0000 | 0.0474 | 0.0508 | 0.0456 | 0.0314 |
| Hotelling -Lawley test | 1.0000 | 0.0520 | 0.0548 | 0.0516 | 0.0478 |
| Roy's test | 1.0000 | 0.0498 | 0.0530 | 0.0488 | 0.9924 |

Table 5.205. Error typeIII For m=10, p=10, n=1010

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0532 | 0.0522 | 0.0498 | 0.0448 |
| Pillai's test | 1.0000 | 0.0524 | 0.0512 | 0.0494 | 0.0368 |
| Hotelling -Lawley test | 1.0000 | 0.0584 | 0.0560 | 0.0540 | 0.0532 |
| Roy's test | 1.0000 | 0.0554 | 0.0542 | 0.0510 | 0.9918 |

Table 5.206. Error typeIII For m=10, p=10, n=1060

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0422 | 0.0508 | 0.0454 | 0.0378 |
| Pillai's test | 1.0000 | 0.0414 | 0.0498 | 0.0442 | 0.0316 |
| Hotelling -Lawley test | 1.0000 | 0.0454 | 0.0560 | 0.0494 | 0.0458 |
| Roy's test | 1.0000 | 0.0440 | 0.0530 | 0.0472 | 0.9920 |

Table 5.207. Error typeIII For m=10, p=10, n=1110

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | 0.0486 | 0.0400 | 0.0436 |
| Pillai's test | 1.0000 | 0.0504 | 0.0482 | 0.0392 | 0.0372 |
| Hotelling -Lawley test | 1.0000 | 0.0548 | 0.0530 | 0.0434 | 0.0528 |
| Roy's test | 1.0000 | 0.0520 | 0.0502 | 0.0412 | 0.9906 |

Table 5.208. Error typeIII For m=10, p=10, n=1160

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0542 | 0.0528 | 0.0524 | 0.0436 |
| Pillai's test | 1.0000 | 0.0514 | 0.0526 | 0.0508 | 0.0382 |
| Hotelling -Lawley test | 1.0000 | 0.0582 | 0.0560 | 0.0550 | 0.0536 |
| Roy's test | 1.0000 | 0.0558 | 0.0534 | 0.0532 | 0.9940 |

Table 5.209. Error typeIII For m=10, p=10, n=1210

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0522 | 0.0526 | 0.0492 | 0.0428 |
| Pillai's test | 1.0000 | 0.0514 | 0.0518 | 0.0482 | 0.0376 |
| Hotelling -Lawley test | 1.0000 | 0.0568 | 0.0550 | 0.0532 | 0.0500 |
| Roy's test | 1.0000 | 0.0544 | 0.0538 | 0.0510 | 0.9908 |

Table 5.210. Error typeIII For m=10, p=10, n=1260

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0490 | 0.0468 | 0.0518 | 0.0402 |
| Pillai's test | 1.0000 | 0.0488 | 0.0464 | 0.0510 | 0.0352 |
| Hotelling -Lawley test | 1.0000 | 0.0512 | 0.0502 | 0.0550 | 0.0464 |
| Roy's test | 1.0000 | 0.0504 | 0.0476 | 0.0530 | 0.9904 |

Table 5.211. Error typeIII For m=10, p=10, n=1310

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0564 | 0.0510 | 0.0494 | 0.0430 |
| Pillai's test | 1.0000 | 0.0560 | 0.0502 | 0.0484 | 0.0394 |
| Hotelling -Lawley test | 1.0000 | 0.0596 | 0.0550 | 0.0524 | 0.0496 |
| Roy's test | 1.0000 | 0.0574 | 0.0528 | 0.0508 | 0.9924 |

Table 5.212. Error typeIII For m=10, p=10, n=1360

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0492 | 0.0432 | 0.0452 | 0.0416 |
| Pillai's test | 1.0000 | 0.0488 | 0.0418 | 0.0436 | 0.0362 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0462 | 0.0486 | 0.0462 |
| Roy's test | 1.0000 | 0.0498 | 0.0444 | 0.0464 | 0.9928 |

Table 5.213. Error typeIII For m=10, p=10, n=1410

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0552 | 0.0474 | 0.0536 | 0.0396 |
| Pillai's test | 1.0000 | 0.0536 | 0.0464 | 0.0524 | 0.0350 |
| Hotelling -Lawley test | 1.0000 | 0.0578 | 0.0504 | 0.0560 | 0.0468 |
| Roy's test | 1.0000 | 0.0558 | 0.0484 | 0.0546 | 0.9922 |

Table 5.214. Error typeIII For m=10, p=10, n=1460

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0478 | 0.0486 | 0.0472 | 0.0472 |
| Pillai's test | 1.0000 | 0.0466 | 0.0480 | 0.0460 | 0.0432 |
| Hotelling -Lawley test | 1.0000 | 0.0500 | 0.0498 | 0.0496 | 0.0536 |
| Roy's test | 1.0000 | 0.0480 | 0.0488 | 0.0490 | 0.9934 |

Table 5.215. Error typeIII For m=10, p=10, n=1510

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0504 | 0.0508 | 0.0492 | 0.0446 |
| Pillai's test | 1.0000 | 0.0486 | 0.0500 | 0.0482 | 0.0396 |
| Hotelling -Lawley test | 1.0000 | 0.0530 | 0.0552 | 0.0512 | 0.0532 |
| Roy's test | 1.0000 | 0.0514 | 0.0526 | 0.0502 | 0.9940 |

Table 5.216. Error typeIII For m=10, p=10, n=1560

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0500 | 0.0544 | 0.0422 | 0.0406 |
| Pillai's test | 1.0000 | 0.0488 | 0.0536 | 0.0416 | 0.0372 |
| Hotelling -Lawley test | 1.0000 | 0.0526 | 0.0566 | 0.0442 | 0.0488 |
| Roy's test | 1.0000 | 0.0514 | 0.0552 | 0.0432 | 0.9944 |

Table 5.217. Error typeIII For m=10, p=10, n=1610

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0496 | 0.0522 | 0.0502 | 0.0488 |
| Pillai's test | 1.0000 | 0.0476 | 0.0514 | 0.0500 | 0.0438 |
| Hotelling -Lawley test | 1.0000 | 0.0524 | 0.0550 | 0.0536 | 0.0540 |
| Roy's test | 1.0000 | 0.0510 | 0.0534 | 0.0516 | 0.9922 |

## 5.4 ERROR TYPE IV

Table 5.218. Error typeIV For m=2, p=2, n=20

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.5492 | 0.0266 | | | 0.0266 |
| Pillai's test | 0.3426 | 0.0136 | | | 0.0136 |
| Hotelling -Lawley test | 0.7590 | 0.0522 | | | 0.0522 |
| Roy's test | 0.7266 | 0.0468 | | | 0.0468 |

Table 5.219. Error typeIV For m=2, p=2, n=70

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0446 | | | 0.0446 |
| Pillai's test | 1.0000 | 0.0396 | | | 0.0396 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | | | 0.0528 |
| Roy's test | 1.0000 | 0.0512 | | | 0.0512 |

Table 5.220. Error typeIV For m=2, p=2, n=120

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0430 | | | 0.0430 |
| Pillai's test | 1.0000 | 0.0410 | | | 0.0410 |
| Hotelling -Lawley test | 1.0000 | 0.0476 | | | 0.0476 |
| Roy's test | 1.0000 | 0.0456 | | | 0.0456 |

Table 5.221. Error typeIV For m=2, p=2, n=170

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | | | 0.0484 |
| Pillai's test | 1.0000 | 0.0464 | | | 0.0464 |
| Hotelling -Lawley test | 1.0000 | 0.0544 | | | 0.0544 |
| Roy's test | 1.0000 | 0.0528 | | | 0.0528 |

Table 5.222. Error typeIV For m=2, p=2, n=220

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0446 | | | 0.0446 |
| Pillai's test | 1.0000 | 0.0438 | | | 0.0438 |
| Hotelling -Lawley test | 1.0000 | 0.0480 | | | 0.0480 |
| Roy's test | 1.0000 | 0.0472 | | | 0.0472 |

Table 5.223. Error typeIV For m=2, p=2, n=270

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0476 | | | 0.0476 |
| Pillai's test | 1.0000 | 0.0464 | | | 0.0464 |
| Hotelling -Lawley test | 1.0000 | 0.0504 | | | 0.0504 |
| Roy's test | 1.0000 | 0.0504 | | | 0.0504 |

Table 5.224. Error typeIV For m=3, p=3, n=30

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.8446 | 0.0250 | | 0.0258 | 0.0202 |
| Pillai's test | 0.7280 | 0.0126 | | 0.0156 | 0.0056 |
| Hotelling -Lawley test | 0.9432 | 0.0594 | | 0.0592 | 0.0560 |
| Roy's test | 0.9290 | 0.0518 | | 0.0502 | 0.1670 |

Table 5.225. Error typeIV For m=3, p=3, n=80

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0390 | | 0.0434 | 0.0366 |
| Pillai's test | 1.0000 | 0.0344 | | 0.0358 | 0.0276 |
| Hotelling -Lawley test | 1.0000 | 0.0496 | | 0.0568 | 0.0532 |
| Roy's test | 1.0000 | 0.0468 | | 0.0534 | 0.1576 |

Table 5.226. Error typeIV For m=3, p=3, n=130

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | | 0.0432 | 0.0450 |
| Pillai's test | 1.0000 | 0.0432 | | 0.0416 | 0.0380 |
| Hotelling -Lawley test | 1.0000 | 0.0550 | | 0.0500 | 0.0550 |
| Roy's test | 1.0000 | 0.0520 | | 0.0474 | 0.1636 |

Table 5.227. Error typeIV For m=3, p=3, n=180

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0390 | | 0.0444 | 0.0458 |
| Pillai's test | 1.0000 | 0.0374 | | 0.0432 | 0.0398 |
| Hotelling -Lawley test | 1.0000 | 0.0448 | | 0.0508 | 0.0520 |
| Roy's test | 1.0000 | 0.0424 | | 0.0488 | 0.1488 |

Table 5.228. Error typeIV For m=3, p=3, n=230

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0524 | | 0.0514 | 0.0500 |
| Pillai's test | 1.0000 | 0.0498 | | 0.0498 | 0.0464 |
| Hotelling -Lawley test | 1.0000 | 0.0558 | | 0.0560 | 0.0560 |
| Roy's test | 1.0000 | 0.0552 | | 0.0542 | 0.1702 |

Table 5.229. Error typeIV For m=3, p=3, n=280

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0510 | | 0.0462 | 0.0478 |
| Pillai's test | 1.0000 | 0.0492 | | 0.0448 | 0.0452 |
| Hotelling -Lawley test | 1.0000 | 0.0536 | | 0.0494 | 0.0524 |
| Roy's test | 1.0000 | 0.0526 | | 0.0484 | 0.1648 |

Table 5.230. Error typeIV For m=3, p=3, n=330

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0506 | | 0.0446 | 0.0476 |
| Pillai's test | 1.0000 | 0.0490 | | 0.0434 | 0.0454 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | | 0.0482 | 0.0514 |
| Roy's test | 1.0000 | 0.0526 | | 0.0470 | 0.1584 |

Table 5.231. Error typeIV For m=4, p=4, n=40

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9628 | 0.0228 | 0.0252 | 0.0252 | 0.0090 |
| Pillai's test | 0.9196 | 0.0106 | 0.0118 | 0.0146 | 0.0016 |
| Hotelling -Lawley test | 0.9892 | 0.0536 | 0.0614 | 0.0600 | 0.0486 |
| Roy's test | 0.9864 | 0.0446 | 0.0526 | 0.0518 | 0.3524 |

Table 5.232. Error typeIV For m=4, p=4, n=90

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | 0.0398 | 0.0420 | 0.0316 |
| Pillai's test | 1.0000 | 0.0362 | 0.0338 | 0.0364 | 0.0188 |
| Hotelling -Lawley test | 1.0000 | 0.0600 | 0.0564 | 0.0546 | 0.0552 |
| Roy's test | 1.0000 | 0.0534 | 0.0502 | 0.0492 | 0.3342 |

Table 5.233. Error typeIV For m=4, p=4, n=140

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0428 | 0.0398 | 0.0466 | 0.0388 |
| Pillai's test | 1.0000 | 0.0382 | 0.0356 | 0.0424 | 0.0298 |
| Hotelling -Lawley test | 1.0000 | 0.0522 | 0.0496 | 0.0544 | 0.0560 |
| Roy's test | 1.0000 | 0.0500 | 0.0452 | 0.0516 | 0.3248 |

Table 5.234. Error typeIV For m=4, p=4, n=190

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0420 | 0.0488 | 0.0452 | 0.0396 |
| Pillai's test | 1.0000 | 0.0398 | 0.0454 | 0.0422 | 0.0334 |
| Hotelling -Lawley test | 1.0000 | 0.0472 | 0.0566 | 0.0530 | 0.0498 |
| Roy's test | 1.0000 | 0.0442 | 0.0534 | 0.0504 | 0.3242 |

Table 5.235. Error typeIV For m=4, p=4, n=240

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0520 | 0.0440 | 0.0448 |
| Pillai's test | 1.0000 | 0.0460 | 0.0490 | 0.0418 | 0.0366 |
| Hotelling -Lawley test | 1.0000 | 0.0518 | 0.0604 | 0.0482 | 0.0542 |
| Roy's test | 1.0000 | 0.0494 | 0.0564 | 0.0458 | 0.3274 |

Table 5.236. Error typeIV For m=4, p=4, n=290

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0484 | 0.0512 | 0.0482 | 0.0454 |
| Pillai's test | 1.0000 | 0.0458 | 0.0494 | 0.0476 | 0.0412 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | 0.0558 | 0.0534 | 0.0528 |
| Roy's test | 1.0000 | 0.0504 | 0.0544 | 0.0514 | 0.3320 |

Table 5.237. Error typeIV For m=4, p=4, n=340

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0480 | 0.0396 | 0.0442 | 0.0414 |
| Pillai's test | 1.0000 | 0.0462 | 0.0390 | 0.0422 | 0.0376 |
| Hotelling -Lawley test | 1.0000 | 0.0516 | 0.0426 | 0.0478 | 0.0490 |
| Roy's test | 1.0000 | 0.0504 | 0.0408 | 0.0466 | 0.3180 |

Table 5.238. Error typeIV For m=4, p=4, n=390

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0450 | 0.0490 | 0.0452 | 0.043 |
| Pillai's test | 1.0000 | 0.0434 | 0.0478 | 0.0440 | 0.0404 |
| Hotelling -Lawley test | 1.0000 | 0.0476 | 0.0524 | 0.0476 | 0.0490 |
| Roy's test | 1.0000 | 0.0468 | 0.0512 | 0.0458 | 0.3238 |

Table 5.239. Error typeIV For m=4, p=4, n=440

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0470 | 0.0472 | 0.0508 | 0.0488 |
| Pillai's test | 1.0000 | 0.0456 | 0.0462 | 0.0490 | 0.0464 |
| Hotelling -Lawley test | 1.0000 | 0.0496 | 0.0504 | 0.0546 | 0.0554 |
| Roy's test | 1.0000 | 0.0482 | 0.0490 | 0.0532 | 0.3168 |

Table 5.240. Error typeIV For m5, p=5, n=50

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 0.9906 | 0.0222 | 0.0256 | 0.0214 | 0.0050 |
| Pillai's test | 0.9750 | 0.0116 | 0.0150 | 0.0110 | 0.0000 |
| Hotelling -Lawley test | 0.9984 | 0.0568 | 0.0560 | 0.0520 | 0.0344 |
| Roy's test | 0.9976 | 0.0518 | 0.0514 | 0.0468 | 0.5542 |

Table 5.241. Error typeIV For m=5, p=5, n=100

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0354 | 0.0434 | 0.0416 | 0.0244 |
| Pillai's test | 1.0000 | 0.0286 | 0.0358 | 0.0354 | 0.0122 |
| Hotelling -Lawley test | 1.0000 | 0.0546 | 0.0608 | 0.0624 | 0.0508 |
| Roy's test | 1.0000 | 0.0452 | 0.0538 | 0.0528 | 0.5384 |

Table 5.242. Error typeIV For m=5, p=5, n=150

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0448 | 0.0378 | 0.0456 | 0.0318 |
| Pillai's test | 1.0000 | 0.0416 | 0.0338 | 0.0410 | 0.0208 |
| Hotelling -Lawley test | 1.0000 | 0.0548 | 0.0470 | 0.0576 | 0.0506 |
| Roy's test | 1.0000 | 0.0500 | 0.0432 | 0.0524 | 0.5302 |

Table 5.243. Error typeIV For m=5, p=5, n=200

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0448 | 0.0494 | 0.0454 | 0.0398 |
| Pillai's test | 1.0000 | 0.0408 | 0.0454 | 0.0426 | 0.0284 |
| Hotelling -Lawley test | 1.0000 | 0.0546 | 0.0596 | 0.0520 | 0.0530 |
| Roy's test | 1.0000 | 0.0512 | 0.0542 | 0.0490 | 0.5274 |

Table 5.244. Error typeIV For m=5, p=5, n=250

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0506 | 0.0472 | 0.0482 | 0.0382 |
| Pillai's test | 1.0000 | 0.0484 | 0.0454 | 0.0448 | 0.0328 |
| Hotelling -Lawley test | 1.0000 | 0.0562 | 0.0556 | 0.0536 | 0.0526 |
| Roy's test | 1.0000 | 0.0536 | 0.0516 | 0.0500 | 0.5238 |

Table 5.245. Error typeIV For m=5, p=5, n=300

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0434 | 0.0468 | 0.0470 | 0.0408 |
| Pillai's test | 1.0000 | 0.0416 | 0.0452 | 0.0444 | 0.0336 |
| Hotelling -Lawley test | 1.0000 | 0.0480 | 0.0518 | 0.0524 | 0.0490 |
| Roy's test | 1.0000 | 0.0462 | 0.0498 | 0.0506 | 0.5240 |

Table 5.246. Error typeIV For m=5, p=5, n=350

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0432 | 0.0506 | 0.0392 | 0.0386 |
| Pillai's test | 1.0000 | 0.0406 | 0.0484 | 0.0370 | 0.0322 |
| Hotelling -Lawley test | 1.0000 | 0.0492 | 0.0560 | 0.0432 | 0.0472 |
| Roy's test | 1.0000 | 0.0464 | 0.0528 | 0.0414 | 0.5158 |

Table 5.247. Error typeIV For m=5, p=5, n=400

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0480 | 0.0504 | 0.0486 | 0.0398 |
| Pillai's test | 1.0000 | 0.0446 | 0.0488 | 0.0462 | 0.0358 |
| Hotelling -Lawley test | 1.0000 | 0.0528 | 0.0540 | 0.0516 | 0.0466 |
| Roy's test | 1.0000 | 0.0504 | 0.0528 | 0.0496 | 0.5304 |

Table 5.248. Error typeIV For m=5, p=5, n=450

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0504 | 0.0464 | 0.0486 | 0.0484 |
| Pillai's test | 1.0000 | 0.0490 | 0.0446 | 0.0484 | 0.0424 |
| Hotelling -Lawley test | 1.0000 | 0.0540 | 0.0492 | 0.0516 | 0.0556 |
| Roy's test | 1.0000 | 0.0522 | 0.0480 | 0.0498 | 0.5276 |

Table 5.249. Error typeIV For m=5, p=5, n=500

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0460 | 0.0488 | 0.0476 | 0.0436 |
| Pillai's test | 1.0000 | 0.0444 | 0.0478 | 0.0466 | 0.0384 |
| Hotelling -Lawley test | 1.0000 | 0.0504 | 0.0506 | 0.0520 | 0.0474 |
| Roy's test | 1.0000 | 0.0476 | 0.0498 | 0.0504 | 0.5188 |

Table 5.250. Error typeIV For m=5, p=5, n=550

| | $b_1$ | $b_2$ | $b_{p-1}$ | $b_p$ | MANOVA F |
|---|---|---|---|---|---|
| Wilk's test | 1.0000 | 0.0522 | 0.0530 | 0.0506 | 0.0488 |
| Pillai's test | 1.0000 | 0.0518 | 0.0510 | 0.0502 | 0.0432 |
| Hotelling -Lawley test | 1.0000 | 0.0552 | 0.0558 | 0.0554 | 0.0550 |
| Roy's test | 1.0000 | 0.0536 | 0.0546 | 0.0534 | 0.5242 |

# CHAPTER 6

# CONCLUSIONS

Multivariate linear regression is a semiparametric method that is nearly as easy to use as multiple linear regression if $m$ is small. The $m$ response and residual plots should be made as well as the DD plot. Sample sizes are needed for determining when the tests and plots start to work well. Response and residual plots can look good for $n \geq 10p$, but for testing, may need $n \geq k(m+p)^2$ where $0.5 \leq k \leq 5$ even for well behave elliptically contoured error distributions. From the simulations needed $n \geq a+b(m+p)^2$ or $n \geq c(m+p)^2$ as shown below. These simulations used 5000 runs.

| | | | |
|---|---|---|---|
| *Hoteling Lawley's test-Ho true* | etype 1 | $2.5 + 0.773(m+p)^2$ | $0.783(m+p)^2$ |
| | etype 2 | $13.7 + 0.773(m+p)^2$ | $0.825(m+p)^2$ |
| | etype 3 | $2.02 + .743(m+p)^2$ | $0.80(m+p)^2$ |
| | etype 4 | $6.07 + 0.752(m+p)^2$ | $0.775(m+p)^2$ |

| | | | |
|---|---|---|---|
| *Wilk's test-Ho true* | etype 1 | $2.5 + 2.66(m+p)^2$ | $2.67(m+p)^2$ |
| | etype 2 | $-12 + 2.71(m+p)^2$ | $2.7(m+p)^2$ |
| | etype 3 | $27 + 2.70(m+p)^2$ | $2.80(m+p)^2$ |
| | etype 4 | $76.1 + 1.81(m+p)^2$ | $2.5(m+p)^2$ |

| | | | |
|---|---|---|---|
| *Pillai's test-Ho true* | etype 1 | $104 + 3.17(m+p)^2$ | $3.57(m+p)^2$ |
| | etype 2 | $95.4 + 3.58(m+p)^2$ | $4.0(m+p)^2$ |
| | etype 3 | $73.5 + 3.78(m+p)^2$ | $4.05(m+p)^2$ |
| | etype 4 | $80.4 + 3.87(m+p)^2$ | $4.2(m+p)^2$ |

The *R* software was used to make plots and software. See R Development Core Team (2011). `mregsim` was used to simulate the tests of hypotheses, and `mregddsim` simulated the DD plots for various distributions. The function `mltreg` makes the response and residual plots and computes the $F_j$, MANOVA $F$ and MANOVA partial $F$ test pvalues while the function `ddplot4` makes the DD plots.

# CHAPTER 7

# SUPPLEMENTARY MATERIAL

## Appendix, Technical Proofs:

*Proof of Theorem 1.*

Using the Searle (1982, p. 333) identity $tr(\boldsymbol{A}\boldsymbol{G}^T\boldsymbol{D}\boldsymbol{G}\boldsymbol{C}) = [vec(\boldsymbol{G})]^T[\boldsymbol{C}\boldsymbol{A} \otimes \boldsymbol{D}^T][vec(\boldsymbol{G})]$, it follows that $(n-p)U(\boldsymbol{L}) = tr[\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1}\hat{\boldsymbol{B}}^T\boldsymbol{L}^T[\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T]^{-1}\boldsymbol{L}\hat{\boldsymbol{B}}] = [vec(\boldsymbol{L}\hat{\boldsymbol{B}})]^T[\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1} \otimes (\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}][vec(\boldsymbol{L}\hat{\boldsymbol{B}})] = T$

where $\boldsymbol{A} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1}, \boldsymbol{G} = \boldsymbol{L}\hat{\boldsymbol{B}}, \boldsymbol{D} = [\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T]^{-1}$, and $\boldsymbol{C} = \boldsymbol{I}$. Hence (2.1) holds.

*Proof of Theorem 2.* By Su and Cook (2012), $\sqrt{n}\ \ vec(\hat{\boldsymbol{B}} - \boldsymbol{B}) \xrightarrow{D} N_{pm}(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \otimes \boldsymbol{W})$.

Then under $H_0$,

$$\sqrt{n}\ \ vec(\boldsymbol{L}\hat{\boldsymbol{B}}) = \sqrt{n} \begin{pmatrix} \boldsymbol{L}\hat{\boldsymbol{\beta}}_1 \\ \boldsymbol{L}\hat{\boldsymbol{\beta}}_2 \\ \vdots \\ \boldsymbol{L}\hat{\boldsymbol{\beta}}_m \end{pmatrix} \xrightarrow{D} N_{rm}(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \otimes \boldsymbol{L}\boldsymbol{W}\boldsymbol{L}^T),$$

and

$$n\ \ [vec(\boldsymbol{L}\hat{\boldsymbol{B}})]^T[\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}^{-1} \otimes (\boldsymbol{L}\boldsymbol{W}\boldsymbol{L}^T)^{-1}][vec(\boldsymbol{L}\hat{\boldsymbol{B}})] \xrightarrow{D} \chi^2_{rm}.$$

The above result also holds if $\boldsymbol{W}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ are replaced by $\hat{\boldsymbol{W}} = n(\boldsymbol{X}^T\boldsymbol{X})^{-1}$ and $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$. Hence under $H_0$ and using the proof of Theorem 1,

$$T = (n-p)U(\boldsymbol{L}) = [vec(\boldsymbol{L}\hat{\boldsymbol{B}})]^T[\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1} \otimes (\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}][vec(\boldsymbol{L}\hat{\boldsymbol{B}})] \xrightarrow{D} \chi^2_{rm}. \tag{7.1}$$

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Plots and Testing for Multivariate Linear Regression

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