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An Empirical Comparison of Four Data Generating Procedures in Parametric and Nonparametric ANOVA

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A Dissertation
Submitted in Partial Fulfillment of Requirements for the
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Department of Educational Psychology and Special Education
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DISSERTATION APPROVAL

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AN ABSTRACT OF THE DISSERTATION OF

Anquan Zhang, for the Doctor of Philosophy Degree in Educational Psychology and Special Education

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MAJOR PROFESSORS: Dr. Todd C. Headrick, Dr. Yanyan Sheng

The purpose of this dissertation was to empirically investigate the Type I error and power rates of four data transformations that produce a variety of non-normal distributions. Specifically, the transformations investigated were (a) the *g*-and-*h*, (b) the generalized lambda distribution (GLD), (c) the power method, and (d) the Burr families of distributions in the context of between-subjects and within-subjects analysis of variance (ANOVA). The traditional parametric *F* tests and their nonparametric counterparts, the Kruskal-Wallis (*KW*) and Friedman (*FR*) tests, were selected to be used in this investigation.

The four data transformations produce non-normal distributions that have either valid or invalid probability density functions (PDFs). Specifically, the data generating procedures will produce distributions with valid PDFs if and only if the transformations are strictly increasing – otherwise the distributions are considered to be associated with invalid PDFs. As such, the primary objective of this study was to isolate and investigate the behaviors of the four data transformation procedures themselves while holding all other conditions constant (i.e., sample sizes, effect sizes, correlation levels, skew, kurtosis, random seed numbers, etc. all remain the same).

The overall results of the Monte Carlo study generally suggest that when the distributions have valid probability density functions (PDFs) that the Type I error and power rates for the parametric (or nonparametric) tests were similar across all four data

transformations. It is noted that there were some dissimilar results when the distributions were very skewed and near their associated boundary conditions for a valid PDF. These dissimilarities were most pronounced in the context of the *KW* and *FR* tests.

In contrast, when the four transformations produced distributions with invalid PDFs, the Type I error and power rates were more frequently dissimilar for both the parametric *F* and nonparametric (*KW*, *FR*) tests. The dissimilarities were most pronounced when the distributions were skewed and heavy-tailed. For example, in the context of a parametric between subjects design, four groups of data were generated with (a) sample sizes of 10, (b) standardized effect size of 0.50 between groups, (c) skew of 2.5 and kurtosis of 60, (d) power method transformations generating distributions with invalid PDFs, and (e) *g*-and-*h* and GLD transformations both generating distributions with valid PDFs. The power results associated with the power method transformation showed that the *F*-test (*KW* test) was rejecting at a rate of .32 (.86). On the other hand, the power results associated with both the *g*-and-*h* and GLD transformations showed that the *F*-test (*KW* test) was rejecting at a rate of approximately .19 (.26).

The primary recommendation of this study is that researchers conducting Monte Carlo studies in the context described herein should use data transformation procedures that produce valid PDFs. This recommendation is important to the extent that researchers using transformations that produce invalid PDFs increase the likelihood of limiting their study to the data generating procedure being used i.e. Type I error and power results may be substantially disparate between different procedures. Further, it also recommended that *g*-and-*h*, GLD, Burr, and fifth-order power method transformations be used if it is desired to generate distributions with extreme skew and/or heavy-tails whereas third-order polynomials should be avoided in this context.

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CHAPTER 1

INTRODUCTON

The Monte Carlo Method

The Monte Carlo method (MCM) developed by the scientists at Los Alamos Laboratory in the 1940s used random sampling techniques and statistics to empirically obtain numerical solutions to quantitative problems (Metropolis, 1987). MCM has been widely used in combinatorial problems, development of statistical tests, eigenvalue problems, high energy particles, operations research, search and optimization procedures, and signal detection. Further, Fan, Felsővályi, Sivo, and Keenan (2002) described two typical situations where MCM is needed in quantitative methodology: (a) assessing the consequences of a statistical technique when the associated derivation assumptions are violated, and (b) determining the sampling distribution of a statistic that has no theoretical distribution (e.g., the mode). Moreover, the use of MCM to investigate Type I error and power properties of parametric (or nonparametric) statistics under non-normal conditions has become a "common practice" (Headrick, 2002, p. 685).

The general steps of a Monte Carlo study as described in Fan et al. (2002) are: (a) ask questions suitable for a Monte Carlo study (e.g., what are appropriate Type I error rates for the one-way ANOVA *F* test under varying degrees of non-normality?); (b) design a Monte Carlo study to provide answers to the questions (e.g., the number of factors, sample sizes, skewness, kurtosis, correlation levels, etc.); (c) generate pseudo-random deviates for the study; (d) implement, replicate, and accumulate the quantitative technique (e.g., calculate and evaluate *F* statistics); (e) compare the empirical Type I error rate with the nominal alpha; and (f) draw conclusions based on the empirical results from the previous step (e.g., is the *F* test robust?).

One of the most important steps in a Monte Carlo study is the data generation procedure. As Fan et al. (2002) pointed out, “the importance of data generation in a Monte Carlo study can never be overemphasized” (p. 16). Four commonly used data transformations considered herein are (a) the *g*-and-*h* distribution, (b) the Generalized Lambda Distribution (GLD), (c) the power method, and (d) the Burr family distributions. One primary reason for considering these data generating procedures is that they all produce non-normal distributions with associated valid probability density functions (PDFs). A valid PDF produced with the data transformation procedure is strictly increasing function. The other main characteristics of these four procedures are subsequently reviewed.

The Tukey *g*-and-*h* transformation (Tukey, 1977) generates non-normal distributions from standard normal deviates with specified *g* and *h* parameters (Headrick, Kowalchuk, & Sheng, 2008; Kowalchuk, & Headrick, 2010). These pseudo-random deviates are transformed to non-normal distributions using the following quantile functions (Kowalchuk, & Headrick, 2010, p. 63):

$$q(Z) = q_{g,h}(Z) = g^{-1}(\exp(gZ)-1)\exp(hZ^2/2), \tag{1.1}$$

$$q(Z) = q_{g,0}(Z) = g^{-1}(\exp(gZ)-1), \tag{1.2}$$

$$q(Z) = q_{0,h}(Z) = Z\exp(hZ^2/2). \tag{1.3}$$

The parameter *g* controls the direction and magnitude of the skewness of a distribution (i.e., the sign of *g* represents the direction, and its absolute value indicates the severity of the skewness). The parameter *h* controls the tail weight or elongation of a distribution and is positively related to kurtosis. Equation (1.2) is used to generate lognormal distribution (i.e., *g* distribution), and Equation (1.3) generates symmetric *h* distributions. The *g*-and-*h* family of distributions is easy to execute for the purpose of generating non-normal data with extreme skewness and kurtosis, with specified correlations. These three advantages make the *g*-and-*h* family distributions a popular tool for use in a Monte Carlo study.

Ramber and Schmeiser (1972, 1974) proposed the class of the generalized lambda distributions (GLD) as a generalization of Tukey's (1960) lambda distribution. For the univariate case, a GLD is expressed by the inverse distribution function (Headrick & Mugdadi, 2006, p. 3344; Ramber & Schmeiser, 1974, p. 78):

$$x = \lambda_1 + (p^{\lambda_3} - (1-p)^{\lambda_4}) / \lambda_2. \tag{1.4}$$

If $p \sim U(0,1)$ then x is a GLD where λ_1 and λ_2 are its location and scale parameters, respectively and where λ_3 and λ_4 are the shape parameters that determine its skewness and kurtosis. It is worth noting that Headrick and Mugdadi (2006) extended the univariate GLD for the purpose of simulating multivariate data with specified correlations.

The Fleishman (1978) power method uses a third-order polynomial to generate standardized (i.e., $\mu = 0$ and $\sigma = 1$) non-normal data. The univariate transformation is summarized by taking the sum of a linear combination of a standard normal variable as shown in the following equation (Fleishman, 1978, p. 522; Headrick & Sawilowsky, 1999, p. 27):

$$X = a + bZ + (-a)Z^2 + dZ^3, \tag{1.5}$$

where Z ~*N iid* (0, 1). In order to obtain the coefficients a, b, and d, a researcher has to solve simultaneously Fleishman Equations (5), (11), (17) and (18) (Fleishman, 1978) for the desired values of skewness (γ_1) and kurtosis (γ_2), while setting the first and second moments arbitrarily to zero and one, respectively.

Vale and Maurelli (1983) extended the Fleishman (1978) procedure to generate multivariate non-normal data. Headrick and Sawilowsky (1999) improved the procedure so that researchers could easily generate multivariate data. Headrick (2002) further extended the Fleishman power method by using the fifth and sixth moments in addition to the first four moments, so that the power method could generate correlated non-normal distributions that span a larger space in the skew and kurtosis plane.

Burr (1942) introduced 12 cumulative distribution functions (CDFs) for the primary purpose of fitting data. The Burr Type III and Type XII distributions attracted special attention because they include distributions with varying degrees of skewness and kurtosis (Burr, 1973; Tadikamalla, 1980). For example, the Type III distributions include characteristics of the normal, lognormal, gamma, logistic, and exponential distributions as well as other characteristics associated with the Pearson family of distributions (Headrick & Pant, 2010; Rodriguez, 1977). The quantile functions associated with Burr Type III and Type XII distributions (Equations 5 and 6 in Headrick & Pant, 2010) is presented as:

$$q(u)_{III} = (u^{-1/k} - 1)^{1/c}, \tag{1.6}$$

$$q(u)_{XII} = ((1-u)^{-1/k} - 1)^{1/c}, \tag{1.7}$$

where u follows a uniform distribution, and $q(u)$ is a strictly increasing monotonic function in u (i.e., the first derivative $q'(u) > 0$). The real valued parameters c and k control the shape of a distribution, where k is positive for both Type III and Type XII distributions, while Type III (Type XII) has a negative (positive) c value (Headrick & Pant). Headrick and Pant (2010) considered the Type III and Type XII distributions together as a single family and derived general parametric forms of a probability density function (PDF) and a cumulative distribution function (CDF) for this family, and further extended these families of Burr distributions from univariate to multivariate data generation. Thus, methodologists are able to use the Burr family distributions to generate not only univariate data, but also correlated multivariate data in their Monte Carlo studies.

Tadikamalla (1980) compared five alternative algorithms (the Johnson System, the Tadikamalla-Johnson System, the generalized lambda distribution (GLD), the Schmeiser-Deutch System, and the Burr Distribution) to generate non-normal distributions with the Fleishman power method in terms of speed, simplicity, and generality. Tadikamalla (1980) defined 'speed' as the microseconds needed for the CPU to generate 10,000 variates with each of the algorithms implemented with FORTRAN statements. The term 'generality' was measured by the region covered by each of the algorithms in the (γ_1^2, γ_2) plane, and 'simplicity' was measured by the combination of effort required to implement the algorithm (e.g., length of the program and special functions or subprograms) and the core storage required to store the program. Tadikamalla (1980) concluded that the six algorithms were similar in terms of simplicity. The Johnson System and the Tadikamalla-Johnson System cover the entire region in the (γ_1^2, γ_2) plane, but the Johnson System requires special tables to implement data generation (Tadikamalla, 1980, p. 275). At that time (1980), each of the six methods except the power method, the GLD and the Burr Distribution had "severe limitations or obstacles in constructing correlated data sets" (Headrick & Sawilowsky, 1999, p. 25).

The Burr Distribution, the GLD, and the power method cover approximately the same region in the (γ_1^2, γ_2) plane (Tadikamalla, 1980) while the *g*-and-*h* distribution is able to generate non-normal data with extreme skew and kurtosis. Each of the four procedures is able to generate correlated non-normal data due to more recent studies (review presented in Chapter Two of studies by Headrick, 2002; 2010; Headrick & Mugdadi, 2006; Headrick & Pant, 2010; Headrick, Pant, & sheng, 2010; Kowalchuk & Headrick, 2010).

Currently, there is a paucity of research that has evaluated how the four data generating procedures compare (and contrast) in the context of generating distributions with valid probability density functions (PDFs) or invalid PDFs. Thus, a methodologist may have difficulty in making decisions about data generation for simulation studies because the results maybe disparate between the generating procedures. As such, a Monte Carlo study to investigate the behavior of the four data generating procedures could provide not only an empirical comparison in terms of Type I error rate and power performances of selected statistical tests across the data generating procedures, but also provide further insight to the impact data generating procedures have, if any, on the results of Monte Carlo studies. To the best of my knowledge, there are no such investigations in the literature to date.

Purpose of the Study

In view of the above, the primary objective of this study is to empirically compare the four data generating procedures described above in the context of one-way analysis of variance. Specifically, the current study will assess Type I error and power properties of the four data generating procedures using Monte Carlo techniques in the context of parametric and nonparametric *F*-tests for both between- and within-subjects designs. A variety of different conditions are included in both designs, e.g. valid (invalid) PDFs, samples sizes, effects sizes, severity of non-normality, degree of correlation, etc.

This study includes three main purposes. More specifically, the achieved results will provide (a) conditions where the four data generating procedures provide consistent or inconsistent results; (b) a basis for algorithm developers and methodologists to improve the data generating procedures; and (c) a basis for applied methodologists to make appropriate decisions in selecting data transformation procedures for their Monte Carlo studies.

Relevance to Education and Psychology

The ANOVA *F* test has been the most used statistical technique in education and psychology (Howell, 2002). In addition to between-subjects designs, learning and developmental studies often involve repeated measures, and thus often may require within-subjects designs. It is also widely known that educational and behavioral data are typically non-normal (Micceri, 1989). A common practice is to choose nonparametric alternatives, for example, the Kruskal-Wallis (*KW*)(Kruskal & Wallis,1952) test for the one-way between-subjects ANOVA, or the Friedman (*FR*)(Friedman,1937) test for the repeated measures ANOVA, when the normality assumption is violated and the validity and power of the *F* tests are compromised. Therefore, the *F* tests and their nonparametric alternatives (*KW* and *FR* tests) are selected to investigate the Type I error and power properties in the context of the four data transformations described above.

Research Questions

The primary interests in the present study are Type I error and power rates of selected statistical tests across the four data transformation procedures. Conditions to be simulated are (a) four data transformation procedures (the *g*-and-*h* distribution, the GLD, the power method, and the Burr family of distributions), (b) two types of data distributions (those with valid PDFs vs. those with invalid PDFs), (c) two types of statistical tests (parametric and nonparametric), (d) two experimental designs (one-way between-subjects design and within-subjects design), and (e) varying levels of correlation for the within-subjects design. Specifically, there are six research questions:

1. What is the Type I error rate in the between-subjects parametric ANOVA *F* test or nonparametric *KW* test in each simulation condition? How do they compare across the four data generating procedures with other conditions being held the same?

2. What is the power performance in the between-subjects parametric or nonparametric test in each simulation condition? How do they compare among the four data generating procedures with other conditions being held the same?

3. What is the Type I error rate in the within-subjects parametric ANOVA *F* test or nonparametric *FR* test in each simulation condition? How do they compare among the different data generating procedures while other conditions remain the same?

4. What is the power performance for the within-subjects parametric or nonparametric test in each simulation condition? How do they compare among the different data generating procedures while other conditions remain the same?

5. Is the nonparametric test more robust and powerful than the parametric competitor in the between-subjects design when distribution assumptions are violated and other conditions remain the same?

6. Is the nonparametric test more robust and powerful than the parametric competitor in the within-subjects design when distribution assumptions are violated and other conditions remain the same?

Definitions

Experiment. An experiment is the process by which an observation or measurement is made (Miller & Miller, 2004, p. 24; Wackerly, Mendenhall, & Scheaffer, 2002, p. 25).

Sample space. The sample space associated with an experiment is the set consisting of all possible points (i.e., the set of all possible outcomes) (Miller & Miller, 2004, p. 24; Wackerly, Mendenhall, & Scheaffer, 2002, p. 27).

Random variable. A random variable is a real valued function for which the domain is a sample space (Wackerly, Mendenhall, & Scheaffer, 2002, p. 73).

Continuous random variable. A continuous random variable is the type of random variable that takes on any value in an interval (Wackerly, Mendenhall, & Scheaffer, 2002, p. 151).

Probability density function (PDF) of a continuous random variable. A function with values *f(x)*, defined over the set of all real numbers, is called a probability density function (PDF) of the continuous variable *X* if and only if

$$p(a \le X \le b) = \int_a^b f(x)dx \tag{1.8}$$

for all real constants a and b with $a \le b$ (Miller & Miller, 2004, p. 24).

Cumulative distribution function (CDF) of a continuous random variable. If *X* is a continuous random variable and the value of its probability density at *t* is *f(t)*, then the function given by

$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(t)dt\,, \tag{1.9}$$

for $-\infty < x < \infty$ is called the distribution function or cumulative distribution function (CDF) of *X* (Miller & Miller, 2004, p. 86).

Quantile function of a continuous random variable. The quantile function or percentile function of a random variable *X*, denoted as $Q_X(y)$, is defined as the inverse function of the CDF of *X*. The quantile function of *X* gives the value of *x* such that $F_X(x) = y$, for each *y* between 0 and 1 (Karian & Dudewicz, 2000, p. 7).

Probability density function (PDF) and Cumulative distribution function (CDF) for the four data generating procedures. Let *Q(V)* be the general form of the quantile function associated with the transformations (a) the *g*-and-*h*, (b) power method, (c) the GLD, and (d) the Burr family distributions. The continuous variable *V*~*N*(0,1) for the first two transformations, while *V*~*U*(0,1) for the other two transformations. As such, the PDF and CDF associated with *Q(V)* is expressed in parametric form ($\mathbb{R}^2$) as (Headrick, 2010, p.3)

$$f_{Q(V)}(Q(v)) = f_{Q(V)}(Q(x,y)) = f_{Q(V)}[(Q(v), \frac{f_V(v)}{Q'(v)}], \tag{1.10}$$

$$F_{Q(V)}(Q(v)) = F_{Q(V)}(Q(x,y)) = F_{Q(V)}((Q(v), F_V(v)), \tag{1.11}$$

where Q*(V)* is an increasing monotonic function (i.e., the first derivative $Q'(v) > 0$).

Central moment of a continuous distribution (moment of a distribution about its mean). Miller and Miller (2004, p. 139) defined the *r*-th moment about the mean of a continuous random variable *X*, denoted by μ_r, as the expected value of $(X-\mu)^r$. Symbolically,

$$E(X-\mu)^r = \int_{-\infty}^{\infty} (x-\mu)^r f(x)dx. \tag{1.12}$$

Cumulants. Kendall and Stuart defined the cumulants associated with the first six central moments as follows (as cited in Headrick, 2002, p. 689):

$$k_1 = \mu_1 = 0, \tag{1.13}$$

$$k_2 = \mu_2, \tag{1.14}$$

$$k_3 = \mu_3, \tag{1.15}$$

$$k_4 = \mu_4 - 3\mu_2^2, \tag{1.16}$$

$$k_5 = \mu_5 - 10\mu_3\mu_2, \tag{1.17}$$

$$k_6 = \mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3. \tag{1.18}$$

Normalized cumulants. Because some values of the central moments may be very large for some densities, it is convenient to normalize the cumulants with $\sqrt{k_2^r} = \sigma^r$. Dividing the left hand sides of (1.11)-(1.16) by $\sqrt{k_2^r}$ and the right hand sides by σ^r gives the normalized cumulants (Headrick, 2002, p. 689):

$$\gamma_1 = k_1 / \sqrt{k_2^2} = \mu_1 / \sigma = 0, \tag{1.19}$$

$$\gamma_2 = k_2 / k_2 = \mu_2 / \sigma^2 = 1, \tag{1.20}$$

$$\gamma_3 = k_3 / \sqrt{k_2^3} = \mu_3 / \sigma^3, \tag{1.21}$$

$$\gamma_4 = k_4 / k_2^2 = \mu_4 / \sigma^4 - 3, \tag{1.22}$$

$$\gamma_5 = k_5 / \sqrt{k_2^5} = \mu_5 / \sigma^5 - 10\gamma_3, \tag{1.23}$$

$$\gamma_6 = k_6 / k_2^3 = \mu_6 / \sigma^6 - 15\gamma_4 - 10\gamma_3^2 - 15. \tag{1.24}$$

Skewness of a distribution. Skewness refers to "the degree to which a frequency distribution is asymmetrical" (Glass & Hopkins, 1996, p. 24). The skewness is often measured by means of the quantity (Miller & Miller, 2004, p. 147):

$$\alpha_3 = \frac{\mu_3}{\sigma^3} = \frac{E(X-\mu)^3}{\sigma^3}. \tag{1.25}$$

Note that skewness is also the third normalized cumulant.

Kurtosis of a distribution. Kurtosis refers to the relative concentration of scores in the center, the upper and lower ends (tails), and the shoulders (between the center and tails) of a distribution (Howell, 2002, p. 23). In this study, the calculation of kurtosis is based on the ratio of the fourth central moment and the fourth power of standard deviation:

$$\alpha_4 = \frac{\mu_4}{\sigma_4} - 3 = \frac{E(X-\mu)^4}{\sigma^4} - 3. \tag{1.26}$$

Note that the kurtosis is also the fourth normalized cumulant.

Type I error and Type I error rate (α). Type I error is erroneously rejecting the null hypothesis (i.e. H_0) when the null hypothesis (i.e., H_0) is true. Type I error rate refers to the conditional probability of rejecting the null hypothesis (i.e., H_0) given that it is true (Howell, 2002, p. 105).

Type II error and Type II error rate (β). Type II error refers to failing to reject the null hypothesis (i.e. H_0) when it is in fact false. Type II error rate is the conditional probability of accepting the null hypothesis (i.e., H_0) given it is false (Howell, 2002, p. 105).

Power of a statistical test (1-β). The power of a statistical test or technique is the conditional probability of correctly rejecting the null hypothesis (i.e., H_0) when it is false. Mathematically, power is the complement of type II error rate (i.e., Power = 1-β).

Robustness of a statistical test. A test statistic is said to be robust if the sampling distribution of the test statistic is "not seriously affected" by violations of underlying assumptions (Miller & Miller, 2004, p. 520). In practice, three robust criteria are widely used: (a) Bradley's (1978) liberal criterion, that is, empirical estimate of Type I error rate $(\hat{\alpha})$ falls between 0.5 and 1.5 times the nominal Type I error rate (i.e., $0.5\alpha \le \hat{\alpha} \le 1.5\alpha$),

(b) Bradley's (1978) stringent criterion (i.e., $0.9\alpha \leq \hat{\alpha} \leq 1.1\alpha$), and (c) a binomial standard error approach (e.g., plus or minus two or three times *SE*, where *SE* = $[\alpha (1 - \alpha)/N]^{1/2}$, and N is the number of simulations).

Nonparametric tests. Nonparametric tests are alternatives that make fewer prior distribution assumptions about the population under investigation than the classical statistical-inference techniques. Sawilowsky (1990) noticed that nonparametric tests typically fall into three divisions based on the type of information they use: categorical, sign or rank. In the current study nonparametric tests are rank based.

Limitations of the Study

There are five limitations of the current investigation, all of which have potential threat to the generalization of conclusions of this study. First, the distributions were deliberately selected to suit the four data generating procedures with particular concerns on three groups of distributions: (a) distributions in which all the four data generation transformations have valid PDFs, (b) none of the data transformations have valid PDFs, and (c) some of the data transformations have valid PDFs while others do not have valid PDFs. The characteristics of data in situations of real research may be more complicated.

Second, in addition to the distributions, the conclusions of this investigation are limited to the parametric (nonparametric) statistical tests selected, although the statistical tests considered in this study are commonly used in educational and psychological literature. Third, the data generated for the parametric techniques (i.e., ANOVA *F* tests) consider that the distributions are (a) identical, (b) with equal variances, and (c) circular covariance matrix in the case of within-subjects design, across groups in each of the simulated conditions in this study. Other conditions, such as two correlated populations with different distributions, unequal variances, unbalanced sample sizes, or non-circular covariance matrices are not considered.

Fourth, the samples were drawn in such a way that tied observations are purposefully eliminated. This may affect the results in the nonparametric tests. In situations of real research, data may have tied observations. Finally, the critical values for the nonparametric tests used exact values in a controlled simulated condition, which affect Type I error and power properties in the current study. In a real research, tabled critical value or *p*-value is used assuming data follow a theoretical distribution.

CHAPTER 2

LITERATURE REVIEW

Systems, Models, Simulation, and the Monte Carlo Methods

The concepts in this section are mainly from Rubinstein (1981), who defined a system as a set of related entities (i.e., components or elements). The attributes of the system elements define its state. An adaptive system has the capacity to change its own state through reacting to the feedback from the changes in its environment(s).

The primary step in studying a system is to build a scientific model, in order to determine how changes in various aspects of the system under study may affect other aspects of the system or the whole system. The scientific model should be “a reasonable approximation to the real system and incorporate most of the aspects of the system”, but “must not be so complex that it is impossible to understand and manipulate” (Rubinstein, 1981, p. 4). Three typical models are iconic models (i.e., pictorial or visual representation of a system), analog models (using one set of properties to represent other set of properties of systems under consideration), and symbolic models (i.e., abstract models, or mathematical models, which are ones that apply mathematical and logical operations to formulate a solution to the problem). Mathematical models have many advantages over the others (for more details see Rubinstein, 1981, pp. 3-4). One important classification of mathematical models concerns deterministic versus stochastic models. In a deterministic model all mathematical and logical relationships between the elements are fixed. Thus, the solutions of the model are completely determined by these relationships. On the other hand, in a stochastic model "at least one variable is random" (p. 4).

After constructing a mathematical model, researchers need to derive a solution to the model for the problem(s) of interest. Quantitatively, there are analytic and numerical approaches to obtain solutions to the model. The analytic approach obtains solution(s) directly from the mathematical representation in the form of formula, while the numerical approach seeks for an approximate solution by substituting numerical values for the variables and parameters of the model. Two special numerical methods are simulation and Monte Carlo methods.

Naylor defined simulation as " a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behavior of business or economic system (or some component thereof) over extended periods of real time" (as cited in Rubinstein, 1981, p. 6). Simulation deals with both abstract and physical models. Operational gaming, war games, business games and military gaming are typical examples of simulation.

Rubinstein (1981) defined simulation in a wide sense as "a technique of performing sampling experiments on the model of the system" (p. 11) and in a narrow sense as "stochastic simulation", which is experimenting with the model over time and includes sampling stochastic variates from a probability distribution. Because sampling from a particular distribution involves the use of random numbers, "stochastic simulation is sometimes called Monte Carlo simulation" (p. 11). Rubinstein (1981) pointed out that the Monte Carlo method (MCM) can be used for "both solutions of stochastic problems and deterministic problems" (e.g. multi-dimensional integrals and differential equations). Another field of application of the MCM is sampling random variates from probability distributions (see also Dimov, 2008; Kalos & Whitlock, 2008; Rubinstein & Kroese, 2008).

According to Halton (1980), the early ideas of the MCM can be traced "as far back as Babylonian and Old Testament times" (p. 6). Student (i.e., Gossett) in 1908 used the MCM in estimating the correlation coefficient in his *t*-distribution. However, systematic use of the method and the name “Monte Carlo” were due to scientists who worked at Los Alamos during World War II, specifically, Von Neumann, Ulam, Metropolis, Kanhn, Fermi and their collaborators (Metropolis 1987; Metropolis & Ulam, 1949). Since then, the MCM has been found in a plethora of applications, and also extended to many other disciplines such as chemistry, biology, medical sciences, computer sciences, and business and economics. For instance, a Google Scholar search of Monte Carlo simulation or Monte Carlo methods identified more than one million results.

Although terminologies such as the Monte Carlo method, Monte Carlo methods, Monte Carlo simulation, simulation study, Monte Carlo techniques, and Monte Carlo algorithms have been used in more or less the same meaning in various literature. In this study, the researcher subjectively uses the Monte Carlo method, Monte Carlo techniques, or simulation study interchangeably to indicate obtaining a numerical approximation of a solution of problems of interest through random sampling and probability statistics, and uses the term “algorithms” for data generating or transformation procedures.

Requirements of Generating Data for the Simulation Study

Fleishman (1978) proposed five requirements for a data generating procedure. An algorithm and its generated distribution should (a) have known parameters, (b) enable the researcher to change distributions easily, (c) be realistic simulations of empirical distributions, (d) cover a wide range of distributions and (e) be computationally efficient (Fleishman, 1978). Another important criterion to consider is the algorithm’s ability to generate "multivariate non-normal data" (Headrick, 2002, p. 686). As mentioned in Chapter One, the four algorithms selected (i.e., the *g*-and-*h*, the GLD, the power method, and the Burr family distributions) satisfy the aforementioned criteria. The remaining portion of this chapter will briefly review the four data transformation procedures in terms of their history and evolution.

The *g*-and-*h* Distributions

The Tukey (1977) *g*-and-*h* family distributions (see also, Field and Genton, 2006; Headrick, Kowalchuk, & Sheng, 2008; Hoaglin, 1985; Kowalchuk & Headrick, 2010) are commonly used in Monte Carlo studies and distribution fitting. Kowalchuk and Headrick (2010) pointed out that two primary advantages associated with this family are its ease of execution (i.e., only two parameters and standard normal deviates are required) and generality (i.e., able to generate distributions with a variety of shapes including extreme skewness and kurtosis). Some examples of where this family of distributions have been used include indices of association and correlation (Wilcox, 2001); effect sizes (Algina, Keselman, & Penfield, 2005a, 2005b); the general linear model (Keselman, Lix, & Kowalchuk, 1998; Keselman & Wilcox, 1999); robust procedures (Guo & Luh, 2000; Keselman, Kowalchuk, & Lix, 1998; Keselman, Wilcox, Algina, Othman, & Fradette, 2008; Wilcox, 1994, 1995; Wilcox & Keselman, 2003); modeling wind speeds (Dupuis & Field, 2004); and option pricing of interest rate, and security and stock returns (Badrinath & Chatterjee, 1988, 1991; Dutta & Babbel, 2005; Mills, 1995).

Hoaglin (1985) made a very insightful investigation of the *g*-and-*h* family of distributions in the context of a univariate case. The author observed that data distributions in research practice are often skewed and elongated at the same time, which requires a researcher to handle both of the aspects simultaneously. The *g*-and-*h* distributions can be considered as "reshaping the unit Gaussian random variable *Z*" by multiplying it with two functions G(Z) and H(Z) (i.e., Y = G(Z)H(Z)Z, where G(Z) = $(e^{gZ}-1)/gZ$ is for skewness, and H(Z) = $e^{hZ^2/2}$ is for elongation), thus, $Y_{g,h}(Z)=g^{-1}(e^{gZ}-1)e^{hZ^2/2}$. The main advantage of combining skewness and elongation in this way is that it extends the definition of "neutral elongation" (p. 486). For a symmetrical distribution (i.e., *h* distribution, where g = 0), the neutrally elongated distribution is the unit normal. Reshaping by handling *g* and *h* in a separate, multiplicative way is very convenient so that it avoids the possible confounding of elongation with skewness by "first summarizing the skewness and allowing for it and then treating as elongation any tail heaviness not associated with skewness" (p.486). As Hoaglin (1985) pointed out, "in a sense this approach extends the customary requirements that measures of scale be free of location and that measures of shape be free of location and scale" (p. 486).

Headrick et al. (2008) derived the parametric forms of the probability density and distribution functions (i.e., PDFs and CDFs) associated with the *g*-and-*h* family distributions, so that the calculation of parameters *g* and *h*, and fitting *g*-and-*h* distributions to data have been largely simplified. The Headrick et al. (2008) procedure is subsequently reviewed.

Let Z be a random variable that has unit normal distribution with PDF and CDF respectively as (Heardick et al., 2008, p. 451, Equations 1 and 2):

$$f_Z(z)=(2\pi)^{-1/2}EXP(-z^2/2), \tag{2.1}$$

$$F_Z(z) = \Pr(Z \le z) = \int_{-\infty}^{z} (2\pi)^{-1/2} EXP(-\omega^2/2)d\omega, \quad -\infty < z < \infty. \tag{2.2}$$

Let $z = (x, y)$ be the auxiliary variable that maps the parametric curves of (2.1) and (2.2) as (Headrick et al., 2008, p. 451, Equations 3 and 4)

$$f : z = f_Z(z) = f_Z(x, y) = f_Z(z, f_Z(z)), \tag{2.3}$$

$$F : z = F_Z(z) = F_Z(x, y) = F_Z(z, F_Z(z)). \tag{2.4}$$

The analytical and empirical forms of the quantile function for *g*-and-*h* distributions are defined as (Headrick et al., 2008, p. 451, Equations 5 and 6)

$$q(z) = q_{g,h}(z) = g^{-1}(EXP(gz) - 1)EXP(hz^2/2), \tag{2.5}$$

$$q(Z) = q_{g,h}(Z) = g^{-1}(EXP(gZ) - 1)EXP(hZ^2/2), \tag{2.6}$$

where $q_{g,h}(z)$ is a strictly increasing monotonic function in z (i.e., derivative $q'_{g,h}(z)$ >0, $g \neq 0$, and h > 0). The parameter $\pm g$ controls the skewness of a distribution in terms of both direction and strength; *h* parameter controls the elongation of a distribution and is directly related to kurtosis. Two subclasses, the *g* distributions and the *h* distributions (Headrick et al., 2008, p. 451, Equations 7 and 8), can be derived by taking the respective limit of Equation (2.5)

$$q(z) = q_{g,0}(z) = \lim_{h \to 0} q_{g,h}(z) = g^{-1}(EXP(gz) - 1), \tag{2.7}$$

$$q(z) = q_{0,h}(z) = \lim_{g \to 0} q_{g,h}(z) = zEXP(hz^2/2), \tag{2.8}$$

where Equation (2.7) is a *g* distribution, which is asymmetric; and Equation (2.8) is an *h* distribution but symmetric. Note that from (2.8), it is easy to see that when $q_{0,0}(z) = z$, the unit normal distribution results, where skewness and kurtosis are zero. Headrick et al.

(2008) proved that if $q(z) = q(x, y)$ maps the parametric curves of $f_q(z)(q(z))$ and $F_q(z)(q(z))$ as (p. 452, Equations 12 and 13)

$$f_q(z)(q(z)) = f_q(z)(q(x, y)) = f_q(z)(q(z), \frac{f_Z(z)}{q'(z)}), \tag{2.9}$$

$$F_q(z)(q(z)) = F_q(z)(q(x, y)) = F_q(z)(q(z), F_Z(z)), \tag{2.10}$$

then $f_q(z)(q(z), f_Z(z)/q'(z))$ and $F_q(z)(q(z), F_Z(z))$ in (2.9) and (2.10) are the PDF and CDF associated with the quantile function $q(z)$. In the equations $q'(z)$ is the first derivative for the quantile functions (Headrick et al., 2008, pp. 451-452, Equations 9-11). Specifically:

$$q'(z) = q'_{g,h}(z) = EXP(gz + hz^2/2) + g^{-1}(EXP(hz^2/2)(EXP(gz) - 1))hz, \tag{2.11}$$

$$q'(z) = q'_{g,0}(z) = \lim_{h \to 0} q'_{g,h}(z) = EXP(gz), \tag{2.12}$$

$$q'(z) = q'_{0,h}(z) = \lim_{g \to 0} q'_{g,h}(z) = EXP(hz^2/2)(1 + hz^2). \tag{2.13}$$

The *k*-th moment of the quantile function can be determined by (Headrick et al., 2008, p. 454, Equation 16)

$$E[q(z)^k] = \int_{-\infty}^{+\infty} q(z)^k \quad f_Z(z)dz, \tag{2.14}$$

where $0 \leq h < 1/k$ for the *k*-th moment to exist. Given that the first four moments are solved based on their corresponding quantile functions, the mean, variance, skewness, and kurtosis of a distribution can be obtained analytically from these moments. Consequently, the *g*-and-*h* values can be solved by setting the desired numerical values of skewness and kurtosis of a distribution under consideration using an equation solver such as FindRoot in Mathematica (Wolfram, 2003).

Field and Genton (2006) extended the *g*-and-*h* family of quantile functions from univariate to multivariate distributions in the context of distribution fitting. Kowalchuk and Headrick (2010) derived the multivariate *g*-and-*h* distributions on the basis of the parametric forms of the PDF and CDF (i.e., Equations 4 and 5). More specifically, in the case of multivariate data generation, let $x_i(q_i(z_i))$ and $x_j(q_j(z_j))$ be standardized *g*-and-*h* distributions, the (post) correlation between these two distributions (Kowalchuk & Headrick, 2010, Equation 8) is

$$\rho_{xi,xj} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_i(q_i(z_i))x_j(q_j(z_j)) f_{ij} dz_i dz_j \,, \qquad (2.15)$$

where f_{ij} is defined as the standard bivariate normal density (Kowalchuk & Headrick, 2010, Equation 9)

$$f_{ij} = (2\pi\sqrt{1-\rho^2{}_{z_i z_i}})^{-1} EXP(-(2(1-\rho^2{}_{z_i z_i}))^{-1}(z_i{}^2 - 2\rho_{z_i z_i} z_i z_j + z_j{}^2)), \qquad (2.16)$$

where $\rho_{z_i z_i}$ is defined as the intermediate correlation. Kowalchuk and Headrick (2010) developed algorithms and source code for estimating the intermediate correlation. After the intermediate correlations are determined they are assembled into a matrix and decomposed using a Cholesky decomposition. Standard normal variables Z_i are subsequently correlated at the intermediate levels. The Z_i are then substituted back in the corresponding quantile functions in the forms of (2.5), (2.7), and (2.8) (i.e., Equations 1 - 3, in Kowalchuk & Headrick, 2010) to generate the *g*-and-*h* distributions with their desired shapes and post correlations.

The Generalized Lambda Distribution (GLD)

Tukey proposed a one-parameter lambda distribution in 1960, expressed as (Ramberg & Schmeiser, 1972)

$$R(p) = [p^{\lambda} - (1-p)^{\lambda}]/\lambda \qquad (0 \le \lambda \le 1), \tag{2.17}$$

which is defined for all nonzero lambda values. This Lambda distribution has been used to approximate many symmetric distributions. Ramberg and Schmeiser (1974) subsequently generalized (2.17) to a four-parameter distribution. The generalized lambda distribution (GLD) has the inverse function $x = \lambda_1 + (p^{\lambda_3} - (1-p)^{\lambda_4})/\lambda_2$, which is also given in Equation (1.4). Note that the GLD includes the original lambda distribution when $\lambda_3 = \lambda_4$.

One of the advantages that makes the univariate GLD attractive is that it enables a researcher to generate not only data with symmetric distributions, but also asymmetric distribution as well (Ramberg & Schmeiser, 1974). In addition to the ability to generate normal and non-normal data, the GLD has a known PDF and inverse distribution function (Headrick & Mugdadi, 2006; Karian & Dudewicz, 2000). Further, because data generation uses only one function, the algorithms can be implemented efficiently. The application of the GLD can be found in topics or statistical techniques such as corrosion processes and fatigue of materials (Bigerelle, Najjar, Fournier, Rupin, & Iost, 2005; Najjar, Bigerelle, Lefebvre, & Iost, 2003), discriminant analysis (Broffitt, Randles, & Hogg, 1976; Randles, Broffitt, Ramberg, & Hogg, 1978), independent component analysis (Eriksson, Karvanen, & Koivunen, 2000; Karvanen, Eriksson, & Koivunen, 2002), modeling non-normal process capability indices in manufacturing process (Pal, 2004), option pricing (Corrado, 2001), solar radiation (Öztürk & Dale, 1982), and structural equation modeling (Mattson, 1997; Reinartz, Echambadi, & Chin, 2002).

Karian, Dudewicz, and McDonald (1996) extended the GLD to the EGLD, which is a combination of the GLD and the generalized beta distribution (GBD). In terms of generality, the EGLD covers all possible combinations of skewness (α_3) and kurtosis (α_4) defined for a continuous PDF to exist as $\alpha_4 > \alpha_3^2 + 1$(Headrick & Mugdadi, 2006, p. 3345), which is a larger area defined by (α_3^2, α_4) space than the GLD covers. Note that the notations of skewness and kurtosis used here are consistent with the original authors'; but, in Chapter One of this dissertation γ_1 and γ_2 were used to denote skewness and kurtosis, respectively. In the context of multivariate data generation, the GLD and EGLD have computational difficulties in that they need (a) to use several steps to overcome the problem of generating biased correlation coefficients, (b) to solve an appropriate mixing matrix and its associated moments for the GBD, or (c) to use commercial software packages (e.g., IMSL) and numerical solutions of complicated integrals to ensure their accuracy (p. 3346).

Headrick and Mugdadi (2006) provided Mathematica source code for solving the λ_i in their Table 1 (p. 3344) on the basis of Equations (3)–(6) in Ramberg and Schmeiser (1974), and proposed methodology and algorithms that simplify the extension of the univariate GLD to multivariate data generation. The Headrick and Mugdadi's (2006) procedure involves the following steps: (a) obtain the λ_i parameters for each of the variates to be generated based on the desired skewness and kurtosis, (b) solve the intermediate correlation matrix, (c) factor the intermediate correlation matrix, (d) generate standard normal deviates based on entries of the decomposed intermediate correlation matrix, (e) transform the standard normal deviates into uniform deviates, and (f) transform the uniform deviates into GLDs. The Headrick and Mugdadi's (2006) procedures (pp. 3346-3351) are presented as follows:

Define $Z_{1,\ldots,}Z_T$ continuous variables of which Z_i and Z_j are any pair and have univariate and bivariate PDFs as (Headrick & Mugdadi, 2006, p. 3346, Equations 2-4)

$$f_{z_i}(z_i) = (2\pi)^{-1/2} EXP(-z_i^2 / 2), \quad (2.18)$$

$$f_{z_j}(z_j) = (2\pi)^{-1/2} EXP(-z_j^2 / 2), \quad (2.19)$$

$$f_{ij} = f_{z_i z_j}(z_i, z_j, \rho_{z_i z_j}) = (2\pi\sqrt{1-\rho^2{}_{z_i z_j}})^{-1} EXP(-(2(1-\rho^2{}_{z_i z_j}))^{-1}(z_i{}^2 - 2\rho_{z_i z_j} z_i z_j + z_j{}^2) . \quad (2.20)$$

The CDFs associated with (2.18) and (2.19) are denoted as (Headrick & Mugdadi, 2006, p. 3346, Equations 5 and 6)

$$\Phi(z_i) = \int_{-\infty}^{z_i} (2\pi)^{-1/2} EXP(-\mu_i^2 / 2) d\mu_i , \quad (2.21)$$

$$\Phi(z_j) = \int_{-\infty}^{z_j} (2\pi)^{-1/2} EXP(-\mu_j^2 / 2) d\mu_j , \quad (2.22)$$

where $\Phi(z_i) \sim U_i[0,1]$, and $\Phi(z_j) \sim U_j[0,1]$.

Let $x_i(z_i, \lambda_{ik})$ and $x_j(z_j, \lambda_{jk})$ where k = 1, … , 4 be standardized GLDs that take the form of (2.16) in the bivariate case (Headrick & Mugdadi, 2006, p. 3347, Equations 7 and 8) as below

$$x_i(z_i, \lambda_{ik}) = \lambda_{i1} + ((\Phi(z_i))^{\lambda_{i3}} - (1-\Phi(z_i))^{\lambda_{i4}}) / \lambda_{i2}, \quad (2.23)$$

$$x_j(z_j, \lambda_{jk}) = \lambda_{j1} + ((\Phi(z_j))^{\lambda_{j3}} - (1-\Phi(z_j))^{\lambda_{j4}}) / \lambda_{j2}. \quad (2.24)$$

As such, the correlation between $x_i(z_i, \lambda_{ik})$ and $x_j(z_j, \lambda_{jk})$ based on Headrick and Mugdadi (2006, p. 3347, Equation 9) is

$$\rho_{x_i x_j} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x_i(z_i, \lambda_{ik}) * x_j(z_j, \lambda_{jk})) f_{ij} dz_i dz_j \; . \quad (2.25)$$

The $\rho_{z_i z_j}$ in (2.20) and (2.23) is referred to as the intermediate correlation. Headrick and Mugdadi (2006) programmed the solution of the intermediate correlation in (2.23) into Mathematica code. Alternatively, the intermediate correlation can be estimated based on Riemann sums about which Headrick and Mugdadi (2006) provided the algorithm in Fortran 77 source code.

After the intermediate correlation matrix is estimated and decomposed, standard normal deviates are created from the decomposed intermediated correlation matrix. Headrick and Mugdadi (2006) recommended transforming the standard normal deviates by the following approximation to the integrals of the form in (2.21) from Bagaby (Headrick and Mugdadi, 2006, p. 3350, Equation 11)

$$\Phi(\pm z_i) = \frac{1}{2} \pm \frac{1}{2}\{1 - \frac{1}{30}[7e^{-(\pm z_i)^2/2} + 16e^{-(\pm z_i)^2(2-\sqrt{2})} + (7 + \frac{1}{4}\pi(\pm z_i)^2)e^{-(\pm z_i)^2}]\}^{1/2} + \varepsilon(\pm z_i), \tag{2.26}$$

where the absolute error is $\varepsilon(\pm z_i) < 3.04 \times 10^{-5}$. The uniform deviates generated from Equation (2.26) are then transformed with Equation (2.24) and the resulting GLDs will have desired shapes and specified correlations.

The Power Methods

Fleishman (1978) proposed the univariate power method as $Y = a + bZ + cZ^2 + dZ^3$, where Z ~$N(0,1)$, and $c = -a$. The Fleishman power method uses a moment matching technique to generate non-normal distributions. Specifically, Fleishman (1978) derived the first four moments of Y, and simplified by substituting the central moments of the unit normal distribution (note that the odd moments of Z are zero, and $E(Z^2) = 1$, $E(Z^4) = 3$, $E(Z^6) = 15$, $E(Z^8) = 105$, $E(Z^{10}) = 945$, $E(Z^{12}) = 10395$, and $E(Z^{14}) = 135135$). From the moments of Y, the mean, variance, skewness, and kurtosis of Y were subsequently derived. Next, the mean and variance were set to zero and one, and the skewness and kurtosis can be set to their desired values. The system of four equations is then simultaneously solved for the coefficients, *a, b, c*, and *d*. After the coefficients are obtained, they are substituted back into Equation (1.5) for transformation. As a result, the generated non-normal distribution will have a mean of zero, a variance of one, and the specified skewness and kurtosis.

In order to construct correlated variables, say X and Y, in Monte Carlo studies, researchers originally adopted a two-step procedure (Headrick & Sawilowsky, 1999; pp. 25-16): (a) use the Fleishman (1978) transformation or similar procedure to generate non-normal distributions which have means of zero and unit variances, and (b) use the algorithm (Headrick & Sawilowsky, 1999, p. 25, Equations 1 or 2)

$$Y_i = rX_i + \sqrt{1-r^2}E_i, \tag{2.27}$$

or the model

$$Y_i = \beta X_i + E_i, \tag{2.28}$$

to generate Y_i which are correlated with X_i at some specified level ($\rho = r$ in Equation 2.27, and $\rho = \beta / \sqrt{\beta^2 + 1}$ in Equation 2.28), where X_i and E_i are the independent pseudo-random normal variates. One problem with this two-step procedure is that the value of skewness and kurtosis for the Y_i are dependent on ρ (Headrick & Sawilowsky, 1999, p. 26).

Vale and Maurelli (1983) extended the Fleishman (1978) power method to the multivariate case with the idea of using an intermediate correlation, from which data could be generated without having the aforementioned problem. However, the Vale and

Maurelli (1983) method requires a preliminary decomposition step and fails to generate desired inter-correlations when the conditional distributions are extremely skewed, and /or heavily tailed (Headrick & Sawilowsky, 1999, p. 26). Headrick and Sawilowsky (1999) proposed an alternative approach to generate multivariate nonnormal data on the basis of the following lemma (p. 27).

Lemma 1. Let Z_1 and V be independent standard normal random variables with zero means and unit variances. Let $E_1, E_2, \ldots, E_j, E_k, \ldots, E_N$ be also a set of N independent standard normal random variables. Further, let $Z_2 = r_0 Z_1 + \sqrt{1-r_0^2}V$. If $X_j = r_j Z_1 + \sqrt{1-r_j^2}E_j$ and $X_k = r_k Z_2 + \sqrt{1-r_k^2}E_k$,then the correlation between X_j and X_k, $\rho_{x_j x_k}$ is equal to $r_0 r_j r_k$, where $r_0, r_1, r_2, \ldots, r_j, r_k \ldots, r_N \in [-1,1]$ and $j \neq k$ (p.27).

More generally, two nonnormal variables are generated by the Fleishman power method as (Headrick & Sawilowsky, 1999, p. 27, Equations 4 and 5)

$$X_j^* = a_j + b_j Z_j + c_j Z_j^2 + d_j Z_j^3, \tag{2.29}$$

$$X_k^* = a_k + b_k Z_k + c_k Z_k^2 + d_k Z_k^3. \tag{2.30}$$

Headrick and Sawilowsky (1999) derived the general relationship between the post correlation and the intermediate correlation (Headrick & Sawilowsky, 1999, p. 28, Equation 7b) as

$$\rho_{X_j^* X_k^*} = E[X_j^* X_k^*] = \rho_{X_j X_k}(b_j b_k + 3b_k d_j + 3b_j d_k + 9d_j d_k + 2a_j a_k \rho_{X_j X_k} + 6d_j d_k \rho^2{}_{X_j X_k}), \tag{2.31}$$

where X_j^*, and X_k^* are the resulting *j*-th and *k*-th nonnormal variables having the desired post correlation, $\rho_{X_j^* X_k^*}$, and $\rho_{X_j X_k}$ is the intermediate correlation. Headrick and

Sawilowsky (1999) developed algorithms and provided examples of generating two and four correlated non-normal variables. The Headrick and Sawilowsky's (1999) procedure avoids the preliminary decomposition step as required in Vale and Maurelli's (1983) and can generate correlated non-normal variables even when the sample sizes are as small as N = 10.

One limitation of the power methods is that they do not cover the entire ($\gamma_1^2, \gamma_2^{'}$) plane defined by the functional relationship $\gamma_2^{'} \geq \gamma_1^2 - 2$ given in Headrick and Sawilowsky (2000, p. 419). Thus, for any given value of skewness (γ_1), there is an associated lower bound of kurtosis (denoted as γ_2^*) above which the Fleishman constants are obtainable. A long standing problem was that approximations of the boundary given by Fleishman (1978, Equation 21) and Tadikamalla (1980, Fig. 1) are either incorrect, contradictory, or inadequate (Headrick & Sawilowsky, 2000, p. 420). Headrick and Sawilowsky (2000) derived the necessary and sufficient conditions for determining the boundaries of the power methods using the Lagrange multiplier techniques. The authors also provided FORTRAN 77 algorithms that determine γ_2^* for a given value of γ_1 and solve for power constants as well as tabled values of γ_2^* for given values of γ_1.

Headrick (2002) extended the Fleishman power method in both univariate and multivariate contexts by controlling two additional moments, which Headrick (2002) called "fast fifth-order polynomial transforms" (p. 685). In this study it will be called fifth-order power method. In contrast to the Fleishman power method, the Headrick (2002) fifth-order power method improved accuracy in data generation and derived a larger family of distributions that span a larger region in the γ_1^2, γ_2 plane defined by

$\gamma_2 \geq \gamma_1^2 - 2$. Headrick (2002) derived procedures, algorithms, and provided Mathematica codes that solve systems of equations to determine the polynomial constants, intermediate correlations, and lower boundaries of kurtosis.

In the univariate context, Headrick's (2002) fifth-order power method considers the following transformation (p. 690, Equation 16):

$$Y = c_0 + c_1 Z + c_2 Z^2 + c_3 Z^3 + c_4 Z^4 + c_5 Z^5, \qquad (2.32)$$

where $Z \sim$ iid N (0, 1). In order to obtain the values of the constants $c_0, \ldots, c_5$, the first six moments of Y are derived, from which the corresponding normalized cumulants are derived, and six equations are obtained as a result. The mean and variance are set to zero and one, respectively, the skewness, kurtosis, and fifth and sixth normalized cumulants are set to desired values, and the system of the six equations is simultaneously solved for the six polynomial constants (Headrick, 2002, p.692). The author also calculated $c_0, \ldots, c_5$ for some theoretical densities.

In the multivariate case, suppose two non-normal variables (Headrick, 2002, p. 693, Equations 23 and 24)

$$Y_i = c_{0i} + c_{1i} Z_{1i} + c_{2i} Z_{2i} + c_{3i} Z_{3i} + c_{4i} Z_{4i} + c_{5i} Z_{5i}, \qquad (2.33)$$

$$Y_j = c_{0j} + c_{1j} Z_{1j} + c_{2j} Z_{2j} + c_{3j} Z_{3j} + c_{4j} Z_{4j} + c_{5j} Z_{5j}, \qquad (2.34)$$

are needed, with zero means, unit variances, and the desired post correlation $\rho_{Y_i Y_j}$. $Z_{(i)}$ and $Z_{(j)}$ are generated and correlated according to the following theorem (p. 693):

Theorem 1. Let r_i be real-valued where $| r_i | \in [0; 1]$ $\forall i$=0; k and let $W_1, V, E_1, \ldots, E_k$ $\sim$ iid N(0; 1). Further, let $W_{t+1} = r_0 W_1 + \sqrt{1 - r_0^2}\, V$ where t = 0, if $r_0 < 1$, and t = 0 if $r_0 = 1$.

If $Z_i = r_i W_{i+1} + \sqrt{1 - r_i^2}\, E_i$, and $Z_j = r_j W_1 + \sqrt{1 - r_j^2}\, E_j$, then Z_i and $Z_j \sim N(0; 1)$,

$\rho_{z_i z_j} = r_0 r_i r_j$ when t =1; and $\rho_{z_i z_j} = r_i r_j$ when t =0. In particular, $\rho_{z_i z_j} = r^2$ when $r_i = r_j$ and t = 0 (Headrick, 2002, p. 693). The correlation between Y_i and Y_j (Headrick, 2002, p. 694, Equation 26) is

$$\begin{aligned}
\rho_{Y_iY_j} = E[Y_iY_j] &= 3c_{4i}c_{0j} + 3c_{4i}c_{2j} + 9c_{4i}c_{4j} + c_{0i}(c_{0j} + c_{2j} + 3c_{4j}) \\
&+ c_{1i}c_{1j}\ \rho_{Z_iZ_j} + 3c_{3i}c_{1j}\ \rho_{Z_iZ_j}\text{ j} + 15c_{5i}c_{1j}\ \rho_{Z_iZ_j} \\
&+ 3c_{1i}c_{3j}\ \rho_{Z_iZ_j} + 9c_{3i}c_{3j}\ \rho_{Z_iZ_j} + 45c_{5i}c_{3j}\ \rho_{Z_iZ_j} \\
&+ 15c_{1i}c_{5j}\ \rho_{Z_iZ_j} + 45c_{3i}c_{5j}\ \rho_{Z_iZ_j} + 225c_{5i}c_{5j}\ \rho_{Z_iZ_j} \\
&+ 12c_{4i}c_{2j}\ \rho^2{}_{Z_iZ_j} + 72c_{4i}c_{4j}\rho^2{}_{Z_iZ_j} + 6c_{3i}c_{3j}\ \rho^3{}_{Z_iZ_j} \\
&+ 60c_{5i}c_{3j}\rho^3{}_{Z_iZ_j} + 60c_{3i}c_{5j}\rho^3{}_{Z_iZ_j} + 600c_{5i}c_{5j}\rho^3{}_{Z_iZ_j} \\
&+ 24c_{4i}c_{4j}\rho^4{}_{Z_iZ_j} + 120c_{5i}c_{5j}\rho^5{}_{Z_iZ_j} \\
&+ c_{2i}(c_{0j} + c_{2j} + 3c_{4j} + 2c_{2j}\rho^2{}_{Z_iZ_j} + 12c_{4j}\rho^2{}_{Z_iZ_j})
\end{aligned} \tag{2.35}$$

where $\rho_{Z_iZ_j}$ is the intermediate correlation. The steps for generating correlated non-normal data are: (a) obtain the constants of the non-normal distribution based on the first six normalized cumulants, (b) determine the intermediate correlation with Equation (2.35) based on the desired post correlation and constants solved from step (a), and (c) follow the explanation and numerical example in Headrick (2002, pp. 694-696). If the number of correlated distributions is four (Headrick, Sheng & Hodis, 2007) and above (Headrick, 2002), the authors recommended to use Equation (2.35) to calculate an intermediate correlation matrix and impose a principle decomposition such as Cholesky decomposition. The decomposed intermediate correlation matrix takes the following form:

Table 2-1

Cholesky Decomposition on Intermediate Correlation Matrix

Entries			
a_{11}	a_{12}	a_{13}	a_{14}
0	a_{22}	a_{23}	a_{24}
0	*0*	a_{33}	a_{34}
0	*0*	*0*	a_{44}

And, subsequently use the formula (Headrick et al., 2007, p. 11)

$$Z_1 = a_{11}V_1, \tag{2.36}$$

$$Z_2 = a_{12}V_1 + a_{22}V_2,$$

$$Z_3 = a_{13}V_1 + a_{23}V_2 + a_{33}V_3,$$

$$Z_4 = a_{14}V_1 + a_{24}V_2 + a_{34}V_3 + a_{44}V_4,$$

where $V_1, \ldots, V_4$ are independent standard normal random deviates. The values of $Z_1, \ldots, Z_4$ are then used in equations (2.33) and (2.34) to produce non-normal variables $Y_1, \ldots, Y_4$ with specified shapes and desired correlation structure. Headrick (2002) also reported lower bounds of kurtosis (γ_2^*) for given values of γ_1, γ_3, and γ_4 and constants associated with γ_2^*, which is very convenient for reference.

The primary advantages of the power method are its computationally efficient algorithms and ability to generate both univariate and correlated non-normal data (Headrick & Kowalchuk, 2007). It is not a surprise that the power method has been widely used in studies including topics or statistical techniques as analysis of variances (Algina, Blair, & Coombs, 1995; Algina, Olejnik, & Ocanto, 1989; Keselman, Wilcox, Algina, Othman, & Fradette, 2008); analysis of covariance (Harwell & Serlin, 1988; Olejnik & Algina, 1984; Seaman, Algina, & Olejnik, 1985); data mining (Rajagopalan &

Krovi, 2002); educational and psychological measurement (Bandalos & Enders, 1996; Enders & Bandalos, 1999; Kirisci, Hsu, & Yu, 2001; Nandakumar & Yu, 1996; Stone, 2003); jacknife, bootstrap and effect size (Algina, Keselman, & Penfield, 2006; Fan & Wang, 1996; Hess, Olejnik, & Huberty, 2001; Kelly, 2005); linear discriminant analysis, quadratic discriminant analysis, logistic regression, classification and regression trees (Finch & Schneider, 2006; Vaughn & Wang, 2008); multivariate nonparametric tests (Kabib & Harwell, 1989); repeated measures (Beasley, 2002); structural equation modeling and confirmatory factor analysis (Flora & Curran, 2004; Hwang, Malhotra, Kim, Marc Tomiuk, & Hong, 2010; Jedidi, Jagpal, & DeSarbo, 1997; Nevitt & Hancock, 2000); *t*-test (Hayes & Li, 2007); economics, finance and marketing literature (Affleck-Graves & McDonald, 1989; Cheung & Lai, 1993; Sharma, Durvasula, & Dillon, 1989); architecture (Zhao & Lu, 2007); and biological, genetics and medical literatures (Mehta, Tanik, & Allison, 2004; Powell, Anderson, Chen, & Alvord, 2002).

Headrick and Kowalchuk (2007) further derived the PDF and CDF for the power method transformation, a basic problem that prevailed for a long time. As such, given valid power method PDFs, a methodologist can (a) calculate percentage points and locate measures of central tendency, (b) compare and contrast the PDFs with other theoretical densities, and (c) estimate parameters and fit power method PDFs to data from real research (p. 230). Define Z a random variable, and $Z \sim N\,(0, 1)$ with PDF, CDF and moments based on Headrick and Kowalchuk (2007, p. 230, Equations 1-3) as

$$f_Z(z) = (2\pi)^{-1/2}EXP(-z^2/2)\,, \tag{2.37}$$

$$F_Z(z) = \Pr(Z \le z) = \int_{-\infty}^{z}(2\pi)^{-1/2}EXP(-\omega^2/2)d\omega\,, \tag{2.38}$$

$$\mu_t = \mu_t(Z) = \int_{-\infty}^{+\infty} (2\pi)^{-1/2} z^t dF_Z(z), \quad -\infty < z < +\infty. \tag{2.39}$$

Let $z = (x, y)$ be the auxiliary variable that maps the parametric equations (2.37) and (2.38) based on Headrick and Kowalchuk (2007, p. 231, Equations 4 and 5) as

$$f : z \to \mathbb{R}^2 = f_Z(z) = f_Z(x, y) = f_Z(z, f_Z(z)), \tag{2.40}$$

$$F : z \to \mathbb{R}^2 = F_Z(z) = F_Z(x, y) = F_Z(z, F_Z(z)), \tag{2.41}$$

and also let the analytical and stochastic forms of transformation *p* be represented (Headrick & Kowalchuk, 2007, p. 231, Equations 6a and 6b) as

$$p(z) = \sum_{i=1}^{r} c_i z^{i-1}, \tag{2.42}$$

$$p(Z) = \sum_{i=1}^{r} c_i Z^{i-1}, \tag{2.43}$$

where *r* is an even natural number. If the compositions of $f \circ p$ and $F \circ p$ based on (2.37) -(2.38) and (2.42)-(2.43) map the parametric curves of $f_{p(Z)}(p(z))$ and $F_{p(Z)}(p(z))$, where $p(z) = (x, y)$ as (Headrick & Kowalchuk, 2007, p. 232, Equations 11 and 12)

$$f \circ p \to \mathbb{R}^2 = f_{p(Z)}(p(z)) = f_{p(Z)}(p(x, y)) = f_{p(Z)}(p(z), \frac{f_Z(z)}{p'(z)}), \tag{2.44}$$

$$F \circ p \to \mathbb{R}^2 = F_{p(Z)}(p(z)) = F_{p(Z)}(p(x, y)) = F_{p(Z)}(p(z), F_Z(z)). \tag{2.45}$$

Headrick and Kowalchuk (2007) proved that $f_{p(Z)}(p(z), f_Z(z)/p'(z))$ and $F_{p(Z)}(p(z), F_Z(z))$ are the PDF and CDF associated with the stochastic form of the power method transformation *p(Z)* in equation (2.43).

Headrick and Kowalchuk (2007) also investigated five properties of the power method (pp. 235-238). More specifically, let $Y = c_1 + c_2 z + c_3 z^2 + c_4 z^3$ follow a distribution in the Fleishman (1978) class. Headrick and Kowalchuk (2007) showed that

(a) if Y has fixed values of c_2 and c_4, then simultaneous sign reversals of values of c_1 and c_3 will reverse sign of skewness (γ_3), but have no effect on kurtosis (γ_4), (b) if Y has fixed values of c_1 and c_3, then simultaneous sign reversals of non-zero values of c_2 and c_4 will have no effect on the first four standardized cumulants but will reverse the sign of the correlation coefficient, (c) if Y follows a non-normal distribution, then Y belongs to the Fleishman (1978) class if and only if $c_4 > (\sqrt{5+7c_2^2})/(5\sqrt{3}) - 2c2/5)$ and $0 < c_2 < 1$, and (d) if Y follows a non-normal distribution with a ratio of $\gamma_3^2/\gamma_4 > 9/14$, then Y does not belong to the Fleishman class. It is noted that (a) and (b) also extend to the fifth-order transformation (p. 135). Furthermore, Y does not belong to the Fleishman class if Y follows any chi-square distribution, because the chi-square distribution has a constant ratio of $\gamma_3^2/\gamma_4 = 2/3$(p. 236).

Let $Y = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 z^4 + c_6 z^5$ and where Y is a symmetric non-normal distribution, where $c_1 = c_3 = c_5 = 0$, $0 < \rho_{p(Z),Z} = c_2 + 3c_4 + 15c_6 < 1$ in the Headrick (2002) class of distributions. As such, if the complex numbers of z associated with Equation (22) in Headrick (2002) have non-zero imaginary parts, $Y' > 0$, and thus Y belongs to the Headrick (2002) fifth-order power transformation. Further, if we let Y be an asymmetric distribution in the Headrick (2002) class of distributions with $0 < \rho_{p(Z),Z} = c_2 + 3c_4 + 15c_6 < 1$ and if the complex numbers of z associated with Equation (23) in Headrick (2002) have non-zero imaginary parts, then $Y' > 0$, and thus Y belongs to the Headrick (2002) fifth-order power transformation. Headrick and Kowalchuk (2007) provided examples to demonstrate the steps to compare known theoretical PDFs with their fifth-order power method analogs. Headrick and Kowalchuk

(2007) also provided the steps to use fifth-order power method transformations to conduct parameter estimation and distribution fitting with real research data.

Headrick et al. (2007) developed a software package with Mathematica (Wolfram Research, 2003) to perform numerical computations and graphics with examples associated with the fifth-order power method. The software provides methodologists with three flexible choices to model theoretical PDFs, empirical data, or users' own selected distributions. The primary functions of the software packge include (a) computing standardized cumulants and solving polynomial coefficients, (b) adjusting the sixth standardized cumulants to ensure the polynomial transformations have valid PDFs, and (c) plotting the power method PDFs and CDFs. The software can also compute cumulative probabilities, modes, trimmed means, intermediate correlations, graphs with fitted power method PDFs against either empirical or theoretical distributions.

It is worth noting that the power method transformation has also been extended to generate multivariate distributions with specific measurement scales. For example, continuous non-normal distribution(s) correlated with ordinal distribution(s) (Headrick & Beasley, 2003); ranked data (Headrick, 2004); systems of linear statistical equations (Headrick & Beasley, 2004); and distributions with specified intraclass correlations (Headrick & Zumbo, 2004). It is very important to note that the Fleishman (1978) power method has been extended by using logistic or uniform random variables (Hodis & Headrick, 2007; 2008) so that the power method can generate univariate and multivariate distributions that cover a much wider region in the skewness-kurtosis plane.

The Burr Distributions

As mentioned in Chapter One, Burr (1942) introduced 12 cumulative frequency functions (Equations 9-20, p. 217) in the context of distribution fitting. The Type III and Type XII distributions (Burr 1942, p. 217, Equations 11 and 20) are defined as

$$F(x) = (x^{-k} + 1)^{-r} \quad (0, \infty), \tag{2.46}$$

$$F(x) = 1 - (1 + x^{c})^{-k} \quad (0, \infty). \tag{2.47}$$

Burr (1973) and Tadikamalla (1980) paid "extra attention" to the two distributions because they include a variety of distributions with various combinations of skewness and kurtosis (see Headrick & Pant, 2010, p. 3). For instance, Type XII distributions include characteristics of the normal, lognormal, gamma, logistic and exponential distributions and other characteristics associated with the Pearson family of distributions (Headrick & Pant, 2010, p. 3; Tadikamalla, 1980, p. 338). Applications of the Type III and Type XII distributions have been found in statistical modeling in disciplines and topics such as forestry (Gove, Ducey, Leak, & Zhang, 2008; Lindsay, Wood, & Woollons, 1996); fracture roughness (Nadarajah, & Kotz, 2006); life testing (Wingo, 1983; 1993); option market price distributions (Sherrick, Garcia, & Tirupattur, 1996); meteorology (Mielke, 1973); modeling crop prices (Tejeda, & Goodwin, 2008); and reliability (Mokhlis, 2005). Possible reasons why the Burr Type III and Type XII distributions are not as popular as other competing methods such as the three transformations described above are the computational difficulties associated with generating Burr distributions with desired correlations (Headrick & Pant, 2010). Fortunately, Headrick and Pant (2010) developed procedures, algorithms, and Mathematica codes to generate the Burr Type III and Type XII distributions in both univariate and multivariate contexts.

In order to derive PDFs, CDFs and the first four moments, Headrick and Pant (2010) defined the analytical forms of the quantile functions associated with Burr Type III and Type XII distributions (p. 6, Equations 5 and 6; see also Equations 1.5 and 1.6) as

$q(u)_{III} = (u^{-1/k} - 1)^{1/c}$,

$q(u)_{XII} = ((1-u)^{-1/k} - 1)^{1/c}$,

where the functions $q(u)$ are strictly monotonically increasing with $q'(u) > 0$, *u* follows uniform distribution (0, 1) with PDF $f_U(u) = 1$, and CDF

$F_U(u) = \Pr(U \le u) = \int_0^u 1du = u.$ The parameters *c* and *k* control the shape of a distribution. The value of *k* is positive in both of the distributions. Type III (Type XII) distribution has a negative (positive) *c* value. The authors proved that the PDF and CDF (p. 6, Equations 9 and 10) associated with the quantile functions with Burr Type III and Type XII distributions are, respectively

$$f_{q(U)}(q(u)) = f_{q(U)}(q(x,y)) = f_{q(U)}(q(u), \frac{1}{q'(u)}), \tag{2.48}$$

$$F_{q(U)}(q(u)) = F_{q(U)}(q(x,y)) = F_{q(U)}(q(u), u). \tag{2.49}$$

The moments (p. 8, Equation 11) of the Type III and Type XII distributions are

$$E[q(u)^r] = \int_0^1 q(u)^r du = \Gamma[(c+r)/c]\Gamma[k - r/c]/\Gamma[k]. \tag{2.50}$$

For Type III distributions, the *r*-th moment exists when c + r < 0; the *r*-th moment exists if *ck* > *r* for Type XII distributions. In order to solve for the *c* and *k* parameters, the first four moments determined from (2.50) are substituted into the equations for skewness and kurtosis (p. 9, Equations 16 and 17) for the Burr family distributions, which are expressed as

$$\alpha_1 = \{E[q(u)^3] - 3E[q(u)^2]E[q(u)] + 2(E[q(u))^3]\}/\{E[q(u)^2] - (E[q(u))^2]^{3/2}, \quad (2.51)$$

$$\alpha_2 = \{E[q(u)^4] - 4E[q(u)^3]E[q(u)] - 3(E[q(u)^2)^2] + 12E[q(u)^2](E[q(u)])^2 -$$

$$6(E[q(u)])^4\}/\{E[q(u)^2] - (E[q(u))^2\}^2. \quad (2.52)$$

Equations (2.51) and (2.52) are set to the desired values of skewness and kurtosis and the system is simultaneously solved to obtain the parameter values. Using the values of the *c* and *k* parameters, the mean and variance of the Burr family distributions can be determined from their Equations (14) and (15) (p. 8) expressed as

$$\mu = \{\Gamma[(c+1)/c]\Gamma k - 1/c\}/\Gamma[k], \quad (2.53)$$

$$\sigma^2 = \Gamma[k]^{-2}\{\Gamma[2+c)/c]\Gamma[k]\Gamma[k-2/c] - \Gamma[1+1/c]^2\Gamma[k-1/c]^2\}. \quad (2.54)$$

The authors also provided the values of shape parameters *c* and *k* for various combinations of skewness and kurtosis in their Appendix A, and approximate lower boundary and upper boundary values of kurtosis for given values of skewness in their Appendix B for convenient reference.

Headrick and Pant (2010) also extended the Burr family distributions to multivariate data generation. Suppose *T* correlated multivariate variables with *T* quantile functions $q(u)$ in the form of (1.6) and (1.7) need to be generated, their procedure (pp.13-17) is: Let $Z_1, \ldots, Z_T$ be standard normal variables with CDFs and bivariate density function (Equations 19-21) associated with any two of the variables Z_j and Z_k as

$$\Phi(z_j) = \Pr(Z_j \le z_j) = \int_{-\infty}^{z_j} (2\pi)^{-1/2} Exp(-\mu_j^2/2)d\mu_j\ , \quad (2.55)$$

$$\Phi(z_k) = \Pr(Z_k \le z_k) = \int_{-\infty}^{z_k} (2\pi)^{-1/2} Exp(-\mu_k^2/2)d\mu_k\ , \quad (2.56)$$

$$f_{jk} = f_{z_j z_k}(z_j, z_k, \rho_{z_j z_k}) = (2\pi\sqrt{1-\rho^2{}_{z_j z_k}})^{-1} EXP(-(2(1-\rho^2{}_{z_j z_k}))^{-1}(z_j{}^2 - 2\rho_{z_j z_k} z_j z_k + z_k{}^2),$$

(2.57)

because $(\Phi(z_j) \sim U(0,1)$ the quantile function of the form (1.6) or (1.7) can be expressed as $q_j(\Phi(z_j))$. The bivariate correlation between two standardized Burr distributions denoted as $x_j(q_j(\Phi(z_j)))$ and $x_k(q_k(\Phi(z_k)))$ can be determined by (Equation 22, p.16)

$$\rho_{x_j(q_j(\Phi(z_j))), x_k(q_k(\Phi(z_k)))} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_j(q_j(\Phi(z_j))) x_k(q_k(\Phi(z_k))) f_{jk} dz_j dz_k \,, \qquad (2.58)$$

for $j \neq k$ and the $\rho_{z_j z_k}$ in f_{jk} and hence in (2.58) is known as the intermediate correlation. After the pairwise intermediate correlations are solved, they are assembled into a matrix that is subsequently factored (e.g., using Cholesky decomposition) in order to produce standard normal deviates that are correlated at the intermediate levels. To generate the uniform deviates required for the quantile functions in (1.6) and (1.7), Headrick and Pant (2010) suggested the use of the Marsaglia (2004) expansion for the unit normal CDF to obtain uniform deviates as

$$\Phi(z_j) = 0.5 + \varphi(z_j)\{zj + z_j^3/3 + z_j^5/(3\times5) + z_j^7/(3\times5\times7) + z_j^9/(3\times5\times7\times9) + \ldots\} \pm \varepsilon \,,$$

(2.59)

where $\varphi(z_j)$ is the unit normal PDF and ε is less than 8×10^{-16}. Headrick and Pant (2010) also coded their algorithms into Mathematica and also provided a numerical example.

In summary, the transformations associated with (a) the *g*-and-*h*, (b) power method, (c) the GLD, and (d) the Burr family distributions are similar to the extent that they have a general form of a PDF and CDF. Specifically, let *Q(V)* be the general form of the quantile function associated with the four transformations. The continuous variable

V~N(0,1) for the first two transformations, while V~U(0,1) for the other two transformations. As such, the PDF and CDF associated with *Q(V)* take the parametric form ($\mathbb{R}^2$) of Equations (1.10) and (1.11) (Headrick, 2010, p.3), which are also listed below for convenience.

$$f_{Q(V)}(Q(v)) = f_{Q(V)}(Q(x,y)) = f_{Q(V)}[(Q(v), \frac{f_V(v)}{Q'(v)}],$$

$$F_{Q(V)}(Q(v)) = F_{Q(V)}(Q(x,y)) = F_{Q(V)}((Q(v), F_V(v)),$$

where Q*(V)* is an increasing monotonic function (i.e., the first derivative $Q'(v) > 0$). The quantile function *Q(V)* takes different forms in the four transformations. Specifically, in the *g*-and-*h* transformation, the quantile function takes the form of Equations (1.1)-(1.3); in the GLD transformation it takes the form of Equation (1.4); in the third-order power method it takes the form of Equation (1.5); in the fifth-order power method it takes the form of Equation (2.32); and in the Burr family transformation, it takes Equations (1.6)-(1.7). For generating univariate data, the corresponding coefficients or parameters are solved on the basis of desired skewness and kurtosis, and placed into the respective (quantile) equations. Let x_i and x_j be any two standardized variables of interest correlated at given (post) correlation ρ_{xixj} . Then, the general form of post correlation (by generalizing 2.25) for the four transformations can be expressed as

$$\rho_{x_i x_j} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_i(z_i, \theta_{ik}) * x_j(z_j, \theta_{jk}))\, f_{ij} dz_i dz_j \qquad (2.60)$$

where θ_{ik} and θ_{jk} are the coefficients or parameters in each of the four transformations, f_{ij} is the bivariate standard normal PDF in the form of (2.20). The general steps for the four data transformations are the same. In order to generate multivariate data one has to (a) solve the coefficients or parameters on the basis of desired skewness and kurtosis by moment matching as in the univariate case; (b) solve the intermediate correlations based on the specified correlations; (c) assemble the intermediate correlations into a matrix and factor this matrix by a Cholesky decomposition; (d) create standardized deviates of the form of (2.36); and (e) transform the deviates by using the corresponding quantile functions. More detailed steps about data generation are described in Chapter Three.

Analysis of Variance

The analysis of variance (ANOVA) is the most widely used statistical technique in psychology and education (Glass & Hopkins, 1996; Howell, 2002). Glass and Hopkins (1996) named ANOVA as the "workhorse" for comparative studies in educational and the behavioral sciences (p. 378). According to Keselman, Huberty et al. (1998), educational researchers "overwhelmingly favored" the ANOVA *F* test; and among the articles reviewed by these authors, more than 90% applied this technique (p. 355). In the context of the univariate model with fixed effects, ANOVA essentially includes two broad categories – the between-subjects and the repeated measures designs. The statistical model for the one-way between-subjects ANOVA (Glass & Hopkins, 1996, p. 388; Howell, 2002 p. 322) can be expressed as

$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}, \tag{2.61}$$

where Y_{ij} is the score for subject *i* in group *j*, μ is the grand mean, $\tau_j = \mu_j - \mu$ is the treatment effect, which represents the deviation between treatment *j* and the grand mean, and $\varepsilon_{ij} = Y_{ij} - \mu_j$ is the random error for score Y_{ij} when predicted from μ and τ_j. The expected mean squares for the error and treatment terms are respectively

$$E(MS_{error}) = \sigma_e^2, \tag{2.62}$$

$$E(MS_{treat}) = \sigma_e^2 + (n\sum \tau_j^2)/(k-1) = \sigma_e^2 + n\sigma_\tau^2 \text{ ,} \tag{2.63}$$

where σ_e^2 is the variance within each population (i.e., error variance), and σ_τ^2 is the variance of treatment populations (i.e., treatment variance). Under a true null of no treatment effect, the ratio of $E(MS_{treat})/E(MS_{error})$ is one and the expected F is about one when the null hypothesis (i.e., there is no treatment effect) is true.

In the one factor between-subjects ANOVA design, the subjects are randomly assigned to the different treatment conditions, and each subject is independently measured or observed on the dependent variable of interest. There are three important assumptions associated with the dependent variable Y_{ij} : (a) it follows a normal distribution in each group, (b) the observations are "independent" of each other, and (c) has "constant variance" across the different groups (Glass & Hopkins, pp. 402-403; Howell, pp. 341-342). It is well known that when one and/or more of the assumption(s) are violated, the AVOVA F test is not necessarily a valid technique. Specifically, the Type I and/or Type II errors (or the power) of the F test may be "distorted" (Keselman, Huberty et al., 1998, p. 353). It is also well known that when the normality assumption is violated, nonparametric procedures such as the Kruskal-Wallis test (Kruskal & Wallis, 1952) may be the optimal choice (Lix, Keselman, & Keselman, 1996; Potvin, & Roff, 1993).

The statistical model for the simple repeated measures ANOVA (Howell, 2002 p. 474; Keppel & Wickens, 2004, p. 374) can be expressed as

$$Y_{ij} = \mu + \pi_i + \tau_j + \pi\tau_{ij} + \varepsilon_{ij}, \tag{2.64}$$

where μ is the grand mean; π_i is the deviation of mean score of subject i from the grand mean, which is a measure of "the overall ability of the subject" (Keppel & Wickens, p.

374); $\pi\tau_{ij}$ represents subject by treatment interaction, which measures "the idiosyncratic response of the subject in a particular condition" (Keppel & Wickens, 2004, p. 374); ε_{ij} is the experimental error associated with the *i*-th subject under the *j*-th treatment condition. The expected mean squares for subjects, treatments, and error are denoted as

$$E(MS_{subject}) = \sigma_e^2 + k\sigma_\pi^2, \tag{2.65}$$

$$E(MS_{treatments}) = \sigma_e^2 + k\sigma_{\pi\tau}^2 + n\sigma_\tau^2, \tag{2.66}$$

$$E(MS_{error}) = \sigma_e^2 + k\sigma_{\pi\tau}^2, \tag{2.67}$$

where σ_e^2, σ_π^2, σ_τ^2, and $\sigma_{\pi\tau}^2$ are the error variance, subject variance, treatment variance, and variance due to subject by treatment interaction, respectively. Under a true null hypothesis, the ratio of the expected means squares $E(MS_{treatments}) / E(MS_{error})$ is one. It is easy to see that the expected value of *F* will be approximately one when there is no treatment effect.

In the simple repeated measures ANOVA design, a representative group of subjects is observed or measured "at each level of one repeated measures factor" (Keselman, Huberty et al., 1998, p. 365). It is widely known that repeated measures ANOVA has advantages over between-subjects ANOVA in that it can make economical use of the subjects; increased sensitivity (reduced error); and the ability to treat phenomenon under investigation that is time-related, such as developmental and attitude changes, learning and forgetting constructs, transfer of training, or the effects of repeatedly administering a drug or type of therapy (Keppel & Wickens, 2004; Keselman, Huberty et al.,1998).

In addition to normality, independence of observation, and homogeneity of variances between subjects (note that scores among the repeated measures are correlated), another important assumption for repeated measures ANOVA is the “sphericity” (Huynh & Feldt, 1970, 1976), which is equal “variances of differences for all the pairs of levels of repeated measures factor” (Glass & Hopkins, p. 575;). The conventional ANOVA *F* test is appropriate for testing repeated measures effects only if the assumptions are satisfied. It is a well known fact that when the normality assumption is violated, the nonparametric Friedman (1937) test is an option (Iman, Hora, & Conover, 1984; Skillings, & Mack, 1981).

Nonparametric Tests

Nonparametric tests are alternative procedures that do “not rely on parameter estimation” (Howell, 2002, p. 692) and “make no assumptions regarding normality” of the populations (Glass & Hopkins, 1996, p. 411). In this study, the two nonparametric procedures used are rank based and are subsequently described.

The first nonparametric procedure considered in this study is the alternative of between-subjects one-way ANOVA, Kruskal-Wallis test (Kruskal & Wallis, 1952). It tests the hypothesis that all samples were drawn from identical populations and is “particularly sensitive” to differences in central tendency (Howell, 2002, p. 719). In order to perform the Kruskal-Wallis test, researchers have to (a) rank all scores ignoring group membership, (b) compute the sum of ranks for each group (denoted by R_j), and (c) use equation (1.2) in Kruskal and Wallis (1952, p. 586; see also Howell, 2002, p. 719):

$$H = \frac{12}{N(N+1)}\sum_{j=1}^{k}\frac{R_j^2}{n_j} - 3(N+1)\,, \qquad (2.68)$$

where k is the number of groups, n_j is the number of scores in group J, R_j is sum of ranks group J, $N = \sum n_j$ is total sample size. H is then evaluated against $\chi^2_{(k-1)}$.

The other nonparametric procedure considered in this study is the Friedman test (Friedman, 1937), which is widely known as the alternative for simple repeated measures ANOVA. It is a test on the hypothesis that the score for each treatment condition was drawn from “identical populations” and is particularly sensitive to differences in the central tendency (Howell, 2002, p. 720). In order to apply the Friedman test, one has to rank the raw scores within each subject, calculate the sum of ranks for each condition (denoted as R_j), and use the equation (Freidman, 1937, p.679; Howell, 2002, p. 721)

$$\chi_F^2 = \frac{12}{Nk(k+1)}\sum_{j=1}^{k} R_j^2 - 3(N+1)\,, \qquad (2.69)$$

where R_j is the sum of the ranks for the j-th condition, N is the number of subjects, j is the number of conditions. The value χ_F^2 can be evaluated against $\chi_{(k-1)}^2$.

CHAPTER 3

METHODOLOGY

Design of the Study

A Monte Carlo study was conducted using the one-way between-subjects (with four groups) and the simple repeated measures (with four conditions) ANOVAs in order to compare the four data generation procedures in terms of Type I error and power rates. The ANOVA *F* tests and their corresponding nonparametric alternatives, Kruskal-Wallis (*KW*), and Friedman (*FR*) tests were considered. Simulated conditions included the type of data transformation, sample sizes, effect sizes, distributions, and correlation levels. Specifically, the data generation procedures had four levels (i.e., the *g*-and-*h*, the GLD, the power methods, and the Burr family distributions); sample sizes were 10, 20, 30, and 50 in terms of cell sizes of five levels; standardized effect sizes 0, .25, .5, .75, 1.0, 1.25, 1.5, where each level of standardized effect sizes was added to the fourth group (or the fourth condition in the case of repeated measures design). The interests of this simulation study were focused on small to medium sample sizes, combined with a wide range of effect sizes from small to large in terms of standardized effect sizes. Further, the range of sample sizes under consideration in this study were widely used in published literature of simulation studies (e.g., Feir-Walsh & Toothaker, 1974; Harwell & Serlin 1994; Kowalchuk, Keselman, Algina, & Wolfinger, 2004; Vallejo & Livacic-Rojas, 2005).

Distributions had 14 levels that were composed of three categories: (a) six distributions where all four data transformations had valid PDFs (denoted as D11, standard normal; five non-normal distributions: D12, with skewness (sk) = .0, kurtosis (kt) = 1.0; D13, sk = 1.0, kt =2.0; D14, sk = 2.0, kt = 8.0; D15, sk = 3.0, kt = 20.0; and D16, sk = 3.9, kt = 40.0); (b) four distributions where none of the three procedures had valid

PDFs (i.e., the fifth-order power method, *g*-and-*h*, and GLD; note that the Burr family distribution was unable to generate all distributions selected in this group and was excluded) (denoted as D21, with sk = .24, kt = -1.209981; D22, sk = .96, kt = .133374; D23, sk = 1.68, kt = 2.76236; and D24, sk = 2.40, kt = 6.606610); and (c) four distributions where the Burr family distributions, *g*-and-*h*, and GLD transformations had valid PDFs, while the power method had invalid PDFs (denoted as D31, sk = 2.50, kt = 60; D32, sk = 2.75, kt = 70; D33, sk = 3.00, kt = 80; and D34, sk = 3.25; kt = 90). The four factors generated 1456 simulation conditions (i.e., 4 data transformations, 4 cell sizes, 14 distributions, and 7 effect sizes, (4)(4)(14)(7) = 1568 – 112 missing conditions where the Burr distributions were unable to generate the second category of distributions) for each of the statistical techniques for the ANOVA *F* and *KW* tests. In addition, a correlation factor was considered with five levels (i.e., .25, .40, .55, .70, and .85 respectively) for the repeated measures design, which created a total of 7280 conditions for each of the repeated measures ANOVA *F* and *FR* tests. The statistical tests were replicated 50,000 times for each of the simulation conditions.

Type I error rates were compared on the basis of two criteria: (a) Bradley (1978) stringent criterion (i.e., if an estimated Type I error for any statistical test in a simulated condition was within the bound of .045 and .055, the transformation was considered as robust; if all the Type I error rates of the statistical test across the four data generating procedures were robust in the condition, the four transformations were considered similar or consistent; otherwise they are considered dissimilar or inconsistent given the simulated condition); and (b) a cut-off value of .004 (i.e., width of the 95% binomial confidence interval =$[2]1.96[(.05)(.95)/50{,}000]^{1/2}$). That is, in a case where Type I error rates of a statistical test in a simulated condition across all data generating procedures were all out of the interval in (a), if the range (i.e., maximum difference) of Type I error rates for the statistical test given the simulated condition across the four data transformations was within a limit of .004, the four transformations were considered similar or consistent; otherwise, they were considered dissimilar or inconsistent for the simulated condition. The power rates among data generating methods were compared on a criterion of .05. By this criterion, if the range (i.e. maximum difference in a condition) of power rates among the four data transformations given a simulated condition was equal to or less than .05 for a condition, the transformations were considered similar or consistent for the condition; otherwise they were considered dissimilar or inconsistent for the simulated condition.

Data Generation with the *g*-and-*h* Distribution Transformation

The generation of the univariate distributions used in the between-subjects one-way ANOVA *F* tests and *KW* tests followed the procedures proposed by Headrick et al. (2008). The derivation of the first four moments, the *g* and *h* parameters for the *g*-and-*h* distribution used for the non-normal distributions were coded with Mathematica and listed in Table 3-1. The *g* and *h* parameters were used to solve the mean, standard deviation, skewness, and kurtosis for the *g*-and-*h* distribution, using the Mathematica code displayed in Table 3-2, and were used to generate the desired *g*-and-*h* distribution(s) with Equation (2.5) in Chapter Two (i.e., Equation 5 in Headrick et al., 2008).

Similarly, the derivation of the first four moments, and the *h* parameter for the *h* distribution to be used for the two symmetric distributions (i.e., D11, standard normal; and D12, with sk = .0, kt = 1.0) in the study were coded with Mathematica and listed in Table 3-3. Separate Mathematica code was not created for the means and standard deviations for the *h* distributions, because there were only two *h* distributions. The means and standard deviations for the *h* distributions were obtained by copying the last part of Table 3-3 and pasting into a new notebook document, and substituting with the solved value of the *h* parameter.

The PDFs and CDFs of the generated distribution with *g*-and-*h* transformation were plotted using Equations (2.1), (2.2), (2.5), (2.9), and (2.10) (i.e., Equations 1, 2, 5, 12, and 13 in Headrick et al., 2008), and the Mathematica program was listed as Part I in Table 3-4. The PDFs and CDFs for the two *h* distributions generated were plotted using Equations (2.1), (2.2), (2.8), (2.9), and (2.10) (i.e., Equations 1, 2, 8, 12 and 13 in Headrick et al., 2008) and the Mathematica code was reported as Part II in Table 3-4. The plotted PDFs and CDFs were lengthy, thus presented in Appendix A.

Table 3-1

Solve for the g and h Parameters from Moments for the g-and-h Distributions

```
(*This program derives first four moments, skewness and kurtosis  for the
g-and-h distribution based on the quantile function, and solves for the values of g and h,
see Headrick, Kowalchuk,  & Sheng, 2008*)
(*Example shows solution for a distribution with skewness = 2.0, and kurtosis = 8.0*)
fz = (1/Sqrt[2*Pi])*Exp[-z^2/2];     (*Equation 1, standard normal*)
qz = (1/g)*( Exp[g*z] - 1)*   Exp[h*z^2/2];     (*equation 5, quantile function*)
(*the following are the first four moments*)
mu1 = Integrate[qz*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 1 ];
mu2 = Integrate[qz^2*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 0.5];
mu3 = Integrate[qz^3*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 1/3];
mu4 = Integrate[qz^4*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 1/4 ];
(*the following are for mean, variance, skewness, kurtosis and solving for  values of g and h*)
mu = mu1
var = mu2 - mu1^2
skew = (mu3 - 3*mu2*mu1 + 2*mu1^3)/var^(3/2)
kurt = (mu4 - 4*mu3*mu1 - 3*mu2^2 + 12*mu2*mu1^2 - 6*mu1^4)/var^2
(*set desired values of skewness  and kurtosis  for the g-and-h distribution in the equation below*)

FindRoot[{skew == 2.0, kurt == 8.0}, {g, .1}, {h, .01}]
```

Table 3-2

Solve for the Population Mean, Standard Deviation, Skewness, and Kurtosis of the g-and-h Distributions

---

```
(*This program solves for the population mean, standard deviation, skewness, and kurtosis of a g-and-h distribution given the g and h values *)

g = 0.7300649017932821;

h = 0.03337655402314846;

mu = (-1 + E^(g^2/(2 - 2 h)))/(g Sqrt[1 - h])

var = (1 - 2 E^(g^2/(2 - 4 h)) +  E^((2 g^2)/(1 - 2 h)))/(g^2 Sqrt[1 - 2 h]) - (-1 +
      E^(g^2/(2 - 2 h)))^2/(g^2 (1 - h));

sd = Sqrt[var]


skew = ((-1 + 3 E^(g^2/(2 - 6 h)) + E^((9 g^2)/(2 - 6 h)) -    3 E^((2 g^2)/(1 - 3 h)))/(g^3 Sqrt[1 - 3 h]) + (  2 (-1 + E^(g^2/(2 - 2 h)))^3)/(g^3 (1 - h)^(3/2)) - (  3 (1 - 2 E^(g^2/(2 - 4 h)) + E^((2 g^2)/(1 - 2 h))) (-1 + E^(g^2/(   2 - 2 h))))/(  g^3 Sqrt[1 - 2 h] Sqrt[1 - h]))/((   1 - 2 E^(g^2/(2 - 4 h)) + E^((2 g^2)/(1 - 2 h)))/(  g^2 Sqrt[1 - 2 h]) - (-1 + E^(g^2/(2 - 2 h)))^2/(g^2 (1 - h)))^(3/2)


kurt = ((1 - 4 E^(g^2/(2 - 8 h)) - 4 E^((9 g^2)/(2 - 8 h)) +  6 E^((2 g^2)/(1 - 4 h)) + E^((8 g^2)/(1 - 4 h)))/(  g^4 Sqrt[1 - 4 h]) - (  3 (1 - 2 E^(g^2/(2 - 4 h)) + E^((2 g^2)/(1 - 2 h)))^2)/(

  g^4 (1 - 2 h)) - (6 (-1 + E^(g^2/(2 - 2 h)))^4)/(g^4 (1 - h)^2) + (  12 (1 - 2 E^(g^2/(2 - 4 h)) + E^((2 g^2)/(1 - 2 h))) (-1 + E^(g^2/(   2 - 2 h)))^2)/(g^4 Sqrt[1 - 2 h] (1 - h)) - (

  4 (-1 + 3 E^(g^2/(2 - 6 h)) + E^((9 g^2)/(2 - 6 h)) -   3 E^((2 g^2)/(1 - 3 h))) (-1 + E^(g^2/(2 - 2 h))))/(  g^4 Sqrt[1 - 3 h] Sqrt[1 - h]))/((   1 - 2 E^(g^2/(2 - 4 h)) + E^((2 g^2)/(1 - 2 h)))/(

   g^2 Sqrt[1 - 2 h]) - (-1 + E^(g^2/(2 - 2 h)))^2/(g^2 (1 - h)))^2


0.442269918567528

1.63238840505551

3.9

40.
```

Table 3-3

Solve for the h Parameter from Moments for the h Distributions

```
(*This program derives first four moments, skewness and kurtosis  for the
h distribution based on the quantile function, and solves h parameter,  Headrick et al., 2008*)

fz = (1/Sqrt[2*Pi])*Exp[-z^2/2];(*Equation 1, standard normal PDF*)
qz = (1/g)*( Exp[g*z] - 1)*Exp[h*z^2/2];  (*Equation 5 g-and-h quantile function*)
qzh = Limit[qz, g -> 0]; (*Equation 8*)

(*the first four moments for the quantile function*)

mu1 = Integrate[qzh*fz, {z, -Infinity, Infinity},    Assumptions -> 0 <= h < 1 ];
mu2 = Integrate[qzh^2*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 0.5];
mu3 = Integrate[qzh^3*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 1/3];
mu4 = Integrate[qzh^4*fz, {z, -Infinity, Infinity},     Assumptions -> 0 <= h < 1/4 ];

(*calculate mean, variance, skewness and kurtosis*)

mu = mu1
var = mu2 - mu1^2;
sd = Sqrt[var]
skew = (mu3 - 3*mu2*mu1 + 2*mu1^3)/var^(3/2);
Simplify[%]
kurt = (mu4 - 4*mu3*mu1 - 3*mu2^2 + 12*mu2*mu1^2 - 6*mu1^4)/var^2;
Simplify[%]
FindRoot[{kurt == 1.0}, {h,    0.0}] (*set kurtosis and solve for h value*)
```

Table 3-4

Plot PDFs and CDFs for the g-and-h, and h Distributions

```
(*Part I. Plot the  g-and-h distribution PDF and CDF, Example distribution sk=2.50, kt = 60 *)

g = 0.23014757021789964;

h = 0.17722463345512662;

mu = 0.15669779406845993;

sig = 1.4824863375198063;

fz = (1/Sqrt[2*Pi])*Exp[-z^2/2];  (*unit normal PDF*)

qz = (1/g)*(Exp[g*z] - 1)*Exp[h*z^2/2];   (*quantile function of the g-and-h distribution  *)

sq = (qz - mu)/sig;   (*standardize the quantile function*)

t = D[qz, z]; (*derivative of the quantile function*)

Fy = Integrate[fz, {z, -Infinity, z}]; (*CDF of unit normal*)

SetOptions[ParametricPlot, AspectRatio -> 1/GoldenRatio];

ParametricPlot[{sq, (fz/t)}, {z, -3, 3}] (*plot the PDF*)

ParametricPlot[{sq, Fy}, {z, -3, 3}] (*plot the CDF*)


(*Part II. Plot the h distribution PDF and CDF, Example distribution: standard normal *)

h = 0.0; mu = 0.0; sig = 1.0;

fz = (1/Sqrt[2*Pi])*Exp[-z^2/2];

qz = z*Exp[h*z^2/2];(*quantile function  for h distribution*)

t = D[qz, z];

sq = (qz - mu)/sig; (*standardize the QF*)

Fy = Integrate[  fz, {z, -Infinity,    z}];(*CDF of unit normal*)

SetOptions[ParametricPlot,   AspectRatio -> 1/GoldenRatio];

SetOptions[ParametricPlot, AspectRatio -> 1/GoldenRatio];

ParametricPlot[{sq, (fz/t)}, {z, -3, 3}]

ParametricPlot[{sq, Fy}, {z, -3, 3}] (*plot the CDF*)
```

For multivariate distributions used for the simple repeated measures ANOVA *F* and the *FR* tests, the intermediate correlations of the two *h* distributions (i.e., D11, standard normal; and D12, with sk = .0, and kt = 1.0) were solved based on Equations (2.8), (2.15) and (2.16) for the *h* distribution (i.e., Equations 3, 8, 9, and Table 1, in Kowalchuk & Headrick, 2010). The Mathematica code was given in Table 3-5.

Table 3-5

Solve for Intermediate Correlation between Two h Distributions

```
 (*This program solves intermediate correlation for two h distributions.  Example shows solution for two
identical distributions with sk = 0.0, kt  =  1.0,  correlated at post correlation at 0.25 *)

h1 = 0.057624474958251;
mu1 = 0.0;
s1 = 1.096185832841500;

(*parameters for the second h distribution*)
h2 = 0.057624474958251;
mu2 = 0.0;
s2 = 1.096185832841500;

rho12 = 0.251319680830621; (*by manipulating this number such that the desired post correlation equals
the post correlation from the integral at the end of program*)

q1 = z1*Exp[(h1*z1^2)/ 2]; (*quantile function for the first h distribution*)
q2 = z2*Exp[(h2*z2^2)/ 2]; (*quantile function for the second h distribution*)

x1 = (q1 - mu1)/s1;
x2 = (q2 - mu2)/s2;

f12 = (2*Pi*Sqrt[(1 - rho12^2)])^-1*   Exp[-(2*(1 - rho12^2))^-1*(z1^2 - 2*rho12*z1*z2 + z2^2)];

int = NIntegrate[(x1*x2)*f12, {z1, -8, 8}, {z2, -8, 8},   Method ->
   "MultiDimensional"]  (*solve the desired post correlation given intermediate r*)

Out[491]= 0.25 (*desired post correlation*)
```

For other multivariate distributions, the *g*-and-*h* transformation procedure followed Equations (2.5), (2.15), and (2.16) (i.e., Equations 1, 8, 9, and Table 1 in Kowalchuk & Headrick, 2010). The calculation for the intermediate correlation (matrix) using the *g*-and-*h* transformation was coded into Mathematica and listed into Table 3-6.

Table 3-6

Solve for Intermediate Correlations between Two g-and-h Distributions

```
(*This program solves intermediate correlations for skewed distribution, given each pair of variables with g-and-h parameters calculated from a separate program. Example shows the inter-correlation is 0.3351539961888915, given post correlation 0.25*)

(*distribution1  with sk = 2.50, kt = 60.0*)
g1 = 0.23014757021789964;
h1 = 0.17722463345512662;

(*distribution2  with sk = 2.50, kt = 60.0*)
g2 = 0.23014757021789964;
h2 = 0.17722463345512662;

rho12 = 0.3351539961888915;  (*by manipulating this number such that the post correlation reached the desired value through test and trial, and put it into a Matrix to conduct Cholesky decomposition *)

mu1 = (g1*Sqrt[1-h1])^(-1) *(Exp[(1/2)*g1^2/(1-h1)]-1);
mu2 = (g2*Sqrt[1-h2])^(-1) *(Exp[(1/2)*g2^2/(1-h2)]-1);

std1 = Sqrt[(g1^2*Sqrt[1-2*h1])^(-1) * (Exp[2*g1^2/(1-2*h1)]- 2*Exp[(1/2)*g1^2/(1-2*h1)]+1) - (g1^2*(1-h1))^(-1)*(Exp[(1/2)*g1^2/(1-h1)]-1)^2];

std2 = Sqrt[(g2^2*Sqrt[1-2*h2])^(-1) *( Exp[2*g2^2/(1-2*h2)]- 2*Exp[(1/2)*g2^2/(1-2*h2)]+1) - (g2^2*(1-h2))^(-1)*(Exp[(1/2)*g2^2/(1-h2)]-1)^2];

q1 = (1/g1)*(Exp[g1*z1]-1)*Exp[h1*z1^2/2];
q2 = (1/g2)*(Exp[g2*z2]-1)*Exp[h2*z2^2/2];

zq1 = (q1-mu1)/std1;
zq2 = (q2-mu2)/std2;

f12 =(2*Pi*Sqrt[(1-rho12^2)])^-1*Exp[-(2(1-rho12^2))^-1*(z1^2-2*rho12*z1*z2+z2^2)];

rho= NIntegrate[(zq1*zq2)*f12,{z1,-8,8},{z2,-8,8},Method→MultiDimensional]

(*post correlation will be manually adjusted as required value, 0.25 in this case*)

0.25
```

The pairwise intermediate correlation coefficients for a desired distribution calculated from Tables 3-5 and 3-6 were assembled into a matrix. The correlation matrices were factored with Cholesky decomposition to create standard normal deviates following Equation (2.36) (i.e., Equation 10 in Kowalchuk & Headrick, 2010). These standard normal deviates were then placed back into Equations (2.5) and (2.8) for generating *g*-and-*h* distribution, and *h* distribution respectively. The generated distributions had the desired shapes and correlated at their desired (post) levels. The *g, h,* and descriptive parameters for the distributions were presented in Table 3-7. Because the Cholesky factored intermediate correlation matrices were lengthy, they were not reported. Instead, the intermediate correlations for each distribution (assuming the two correlated distributions were identical) were reported in Table 3-8. Note that the values of *g, h, M, SD* and the intermediated correlations were reported with eight decimal places in order to save space, and they were rounded to 14 decimal places in actual implementation.

Table 3-7

Descriptive, g and h Parameters of Distributions Generated with the g-and-h Transformation

Distribution	Skew	Kurtosis	*g*	*h*	*M*	*SD*
D11	0.00	0.00	0.00000000	0.00000000	0.00000000	1.00000000
D12	0.00	1.00	0.00000000	0.05762447	0.00000000	1.09618583
D13	1.00	2.00	0.30060579	0.01025217	0.15618200	1.08885769
D14	2.00	8.00	0.54163584	0.00459620	0.29381272	1.25997529
D15	3.00	20.00	0.69067774	0.00983110	0.39631343	1.47049226
D16	3.90	40.00	0.73006490	0.03337655	0.44226992	1.63238841
D21	0.24	-1.209981	0.20428080	-0.30049139	0.06942645	0.71539349
D22	0.96	0.133374	0.63321888	-0.23860441	0.24931409	0.90589533
D23	1.68	2.762360	1.06907773	-0.30201654	0.45169182	1.17744989
D24	2.40	6.606610	1.61752258	-0.45366502	0.74833739	1.70844951
D31	2.50	60.00	0.23014757	0.17722463	0.15669779	1.48248634
D32	2.75	70.00	0.25117968	0.17609124	0.17118923	1.49671584
D33	3.00	80.00	0.27360053	0.17414895	0.18647169	1.51077764
D34	3.25	90.00	0.29726207	0.17156555	0.20246722	1.52534753

Note. D11-D16 are distributions where all four transformations have valid PDFs. D21-D24 are distributions where none distributions have valid PDFs. D31-D34 are distributions where the power method does not have valid PDFs, while the other three transformations have valid PDFs

Table 3-8

Intermediate Correlation of Distributions Generated with the g-and-h Transformation at Different Post Correlation Levels

Distribution	0.25	0.40	0.55	0.70	0.85
D11	0.25000000	0.40000001	0.54999998	0.69999996	0.85000004
D12	0.25131968	0.40188966	0.55215369	0.70199956	0.85131750
D13	0.25913127	0.41161405	0.55194931	0.71003351	0.85605306
D14	0.27954186	0.43669389	0.58675830	0.73032618	0.86791923
D15	0.30071117	0.46168693	0.61063146	0.74915918	0.87858655
D16	0.31499976	0.47834863	0.62635615	0.76142217	0.88545730
D21	0.27039041	0.42842998	0.58140538	0.72813892	0.86782354
D22	0.27388651	0.43048470	0.58134429	0.72650971	0.86603674
D23	0.29527832	0.45463334	0.60346477	0.74327486	0.87517030
D24	0.32314571	0.48499626	0.63070362	0.76370473	0.88627059
D31	0.28129147	0.44215259	0.59513087	0.73930314	0.87426059
D32	0.28361025	0.44500473	0.59791944	0.74152249	0.87551436
D33	0.28603623	0.44794654	0.59087036	0.74375250	0.87675936
D34	0.28861396	0.45103411	0.60370052	0.74604052	0.87802430

Note. For distributions D11-D16 all four transformations have valid PDFs.

D11: skewness (SK) = 0.0, kurtosis (KT) = 0.0; D12: sk = .0, kt = 1.0; D13: sk = 1.0, kt = 2.0; D14: sk = 2.0, kt = 8.0; D15: sk = 3.0, kt = 20.0; D16: sk = 3.9, kt = 40.0. For Distributions D21-D24 none of the transformations have valid PDFs. D21: sk = .24, kt = -1.209981; D22: sk = .96, kt = .133374; D23: sk = 1.68, kt = 2.76236; D24: sk = 2.40, kt = 6.60661. For Distributions D31-D34 all but the power method transformations have valid PDFs. D31: sk = 2.50, kt = 60; D32: sk =2.75, kt = 70; D33: sk = 3.00, kt = 80; D34: sk = 3.25, kt = 90.

Data Generation with the GLD Transformation

The univariate distributions generated by the GLD for the between-subjects one-way ANOVA *F* and *KW* tests followed the procedure proposed by Headrick and Mugdadi (2006). More specifically, the λ_i parameters for desired values of skewness (α_3) and kurtosis (α_4) were solved with Table 1 (p. 3344), and the parameters were replaced into Equation 1 in Headrick and Mugdadi (2006, p. 3344), and the generated distributions were standardized and have their desired shapes. It is noted that α_3 and α_4 were used in consistency with the original authors' notation to denote skewness and kurtosis, respectively. The Mathematica code based on Headrick and Mugdadi (2006) was given as Part I in Table 3-9. The Mathematica program plotting the PDFs and CDFs of generated distributions with the GLD procedure was listed as Part II in Table 3-9, and the plotted PDFs and CDFs were reported in Appendix A.

In order to generate the multivariate distributions for the simple repeated measures ANOVA *F* and *FR* tests, the λ_i parameters solved from Part I in Table 3-9 were placed into Mathematica code as proposed by Headrick and Mugdadi (2006, Table 2, p. 3348). The Mathematica program for solving the intermediate correlations as proposed by Headrick and Mugdadi (2006) was given in Table 3-10. The pairwise intermediate correlation coefficients for desired distributions calculated from Table 3-10 were assembled into a correlation matrix and subjected to a Cholesky decomposition. Standard normal deviates were generated in the form of Table 2-1 and Equation (2.36) (see, Headrick and Mugdadi, 2006, pp. 3348-3350). The obtained standard normal deviates were transformed into uniform deviates using the Marsaglian expansion given in Equation (2.59) (i.e., Equation 24 in Headrick and Pant, 2010) in order to increase precision. The generated uniform deviates were placed into Equation (2.23) to generate the distributions with their desired shapes and specified correlations. The calculated lambda parameters were presented in Table 3-11 and the intermediate correlations were given in Table 3-12. Again these values were rounded to the 14 decimal place in actual implementation.

Table 3-9

Solve the Parameters for the GLD Distributions and Plot the PDF and CDF

```
(*Part I. This program calculate the parameters for the generalized lambda distribution
given desired skewness and kurtosis, based on Table 1, in Headrick & Mugdadi, 2006, p.3344.
Example shows the solution for a distribution with sk = 3.25, and kt = 90.0*)

α3 = 3.25;
α4 = 90.0;
FindRoot[{λ1 +((1/(1+λ3))-(1/(1+ λ4)))/λ2 ==0,

  (-2*Beta[1+λ3,1+λ4]+ (1/(1+2*λ3)) + (1/(1+2*λ4))-((1/(1+λ3))-(1/(1+λ4)))^2)/λ2^2==1,

  (3*Beta[1+λ3,1+2*λ4]-3*Beta[1+2*λ3,1+λ4] + ( 1/(1+3*λ3))-(1/(1+3*λ4))
     +2*((1/(1+λ3))-(1/(1+λ4)))^3-3*((1/(1+λ3))-(1/(1+λ4)))*(-2*Beta[1+λ3,1+λ4]+
        (1/(1+2*λ3))+(1/(1+2*λ4))))/λ2^3 ==  α3,

  (-4*Beta[1+λ3,1+3*λ4] + 6*Beta[1+2*λ3,1+2*λ4]-4*Beta[1+3*λ3,1+λ4] +
      (1/(1+4*λ3))+(1/(1+4*λ4))- 3*((1/(1+λ3))-(1/(1+λ4)))^4 + 6*((1/(1+λ3))-
         (1/(1+λ4)))^2*(-2*Beta[1+λ3,1+λ4] +( 1/(1+2*λ3))+(1/(1+2*λ4)))- 4*((1/(1+λ3))-
        (1/(1+λ4)))*(3*Beta[1+λ3,1+2*λ4] -3*Beta[1+2*λ3,1+λ4]+ (1/(1+3*λ3))-
        (1/(1+3*λ4))))/λ2^4-3.0== α4},
 {λ1, -.75},{λ2,-.20},  {λ3,-0.014},{λ4,-0.14}, AccuracyGoal→10]
Solution:

{λ1→-0.3204460020046685`,λ2→-0.49591645287559305`,λ3→-0.1210407733734074`,λ4→-

0.2287661860409475`}

(Part II. This program plot the PDF and CDF of the generalized lambda distribution given the  Lambda
parameters solved from Part I. Example distribution sk = 3.25, kt = 90.0*)

λ1 = -0.3204460020046685;
λ2 = -0.49591645287559305;
λ3 = -0.1210407733734074;
λ4 = -0.2287661860409475;

x=λ1+(p^λ3-(1-p)^λ4)/λ2;

Dx=D[x,p];
SetOptions[ParametricPlot,AspectRatio→1/GoldenRatio];
ParametricPlot[{x,(1/Dx)},{p,0.001,1}]
ParametricPlot[{x,p},{p,0.001,1}]
```

Table 3-10

Solve the Intermediate Correlations with the GLD Distributions

---

```
(*This program solve the intermediate correlation given the lambda parameters of two GLD distributions
and desired post correlation based on Table 2, in Headrick & Mugdadi, 2006, p.3348. Example shows
solution of an intercorrelation 0.3205432340177 from two identical distributions correlated at a post
correlation 0.25*)

(*lambdas for the first distribution*)
λ11 = -0.7764156380750964;
λ12 = -0.2671576187306783;
λ13 = -0.005357043224460993;
λ14 = -0.17546938894209574;


(*lambdas for the second distribution*)
λ21 = -0.7764156380750964;
λ22 = -0.2671576187306783;
λ23 = -0.005357043224460993;
λ24 = -0.17546938894209574;

(*adjust this number so that post correlation from the integral below reaches desires value*)
ρz1z2 =0.3205432340177;

 fu1 = (1/Sqrt[2*Pi])*Exp[-u1^2/2];

fu2 = (1/Sqrt[2*Pi])*Exp[-u2^2/2];

Φ1= Integrate [fu1, {u1,-Infinity, z1}];
Φ2= Integrate [fu2, {u2,-Infinity, z2}] ;

x1=λ11  +(Φ1^λ13 -(1-Φ1)^ λ14)/λ12;
x2=λ21  +(Φ2^λ23 -(1-Φ2)^ λ24)/λ22;

f12 = Exp[(-1/(2*(1-(ρz1z2)^2)))*(z1^2-2*(ρz1z2)*(z1*z2)+ z2^2)]/(2*Pi*Sqrt[1-(ρz1z2)^2]);

int = NIntegrate[(x1*x2)*f12,{z1,-5,5},{z2,-5,5}, Method→ MultiDimensional]
```

Solution:

0.25 (*post correlation reaches the desired value*)

Table 3-11

Descriptive and Lambda Parameters of Distributions Generated with the GLD Transformation

Distribution	Skew	Kurtosis	λ_1	λ_2	λ_3	λ_4
D11	0.00	0.00	0.00000000	0.19745137	0.13491245	0.13491245
D12	0.00	1.00	0.00000000	0.02610270	0.01475973	0.01475973
D13	1.00	2.00	-0.53313951	0.03404451	0.00969812	0.02854779
D14	2.00	8.00	-0.66771114	-0.10096869	-0.01557577	-0.07684366
D15	3.00	20.00	-0.74361564	-0.19651555	-0.01375746	-0.13799152
D16	3.90	40.00	-0.77641564	-0.26715762	-0.00535704	-0.17546939
D21	0.24	-1.209981	-2.16009393	0.24493395	-0.04514353	0.92976624
D22	0.96	0.133374	-1.66170501	0.18878977	-0.02976604	0.39476563
D23	1.68	2.762360	7.81701879	0.12413373	5.86211850	-0.10400954
D24	2.40	6.606610	10.67741579	0.09547803	10.75912160	-0.09461212
D31	2.50	60.00	-0.23560581	-0.51581600	-0.14495316	-0.22544015
D32	2.75	70.00	-0.26158709	-0.51112551	-0.13794954	-0.22704044
D33	3.00	80.00	-0.28993142	-0.50420715	-0.12990588	-0.22808918
D34	3.25	90.00	-0.32044600	-0.49591645	-0.12104077	-0.22876619

Note. D11-D16 are distributions where all four transformations have valid PDFs. D21-D24 are distributions where none distributions have valid PDFs. D31-D34 are distributions where the power method does not have valid PDFs, while other transformations have valid PDFs.

Table 3-12

Intermediate Correlation of Distributions Generated with the GLD Transformation at Different Post Correlation Levels

Distribution	0.25	0.40	0.55	0.70	0.85
D11	0.25001262	0.40001993	0.55002631	0.70003029	0.70003029
D12	0.25148156	0.40212455	0.55242830	0.70226764	0.85151800
D13	0.25938869	0.41204313	0.56244436	0.71058311	0.85645349
D14	0.27915749	0.43644262	0.58670951	0.73045663	0.86814645
D15	0.30178013	0.46286504	0.61173155	0.75009943	0.87933593
D16	0.32054323	0.48375905	0.63078276	0.76463342	0.88749971
D21	0.26995988	0.42772927	0.57059546	0.72748375	0.86748594
D22	0.27688198	0.43405799	0.58476506	0.72919890	0.86755061
D23	0.43695714	0.61250642	0.74567170	0.84941249	0.93235954
D24	0.56290233	0.70729101	0.80996837	0.88801452	0.94980961
D31	0.28241335	0.44354159	0.59658558	0.74077870	0.87584086
D32	0.28468730	0.44624610	0.59915825	0.74279223	0.87699162
D33	0.28722612	0.44922337	0.60195186	0.74494863	0.87820361
D34	0.29009161	0.45254334	0.60503136	0.74729903	0.92138826

Note. For distributions D11-D16 all four transformations have valid PDFs. D11: skewness (SK) = 0.0, kurtosis (KT) = 0.0; D12: sk = .0, kt = 1.0; D13: sk = 1.0, kt = 2.0; D14: sk = 2.0, kt = 8.0; D15: sk = 3.0, kt = 20.0; D16: sk = 3.9, kt = 40.0. For Distributions D21-D24 none of the transformations have valid PDFs. D21: sk = .24, kt = -1.209981; D22: sk = .96, kt = .133374; D23: sk = 1.68, kt = 2.76236; D24: sk = 2.40, kt = 6.60661. For Distributions D31-D34 all but the power method transformations have valid PDFs. D31: sk = 2.50, kt = 60; D32: sk =2.75, kt = 70; D33: sk = 3.00, kt = 80; D34: sk = 3.25, kt = 90.

Data Generation with the Power Methods

The third-order power method was used to generate data for the first category of six distributions with valid PDFs, and for the third category of four distributions where power methods do not have valid PDFs. For the second category of four distributions, the fifth-order power method was used. More specifically, for univariate distributions to be used in the between-subjects one-way ANOVA *F* tests and *KW* tests, the constants were obtained by simultaneously solving system of (B1)-(B4) in Headrick and Kowalchuk (2007) by setting the mean to zero, the variance to one, and the skewness and kurtosis to desired values. The Mathematica code to solve the constants was given in Table 3-13. The solved constants were placed back to equation in the form of (1.5) to generate desired data.

The second group of four distributions were selected from Table 2 in Headrick (2002, p. 698), and the fifth-order power method was used. Because constants were already given in the author's table, the univariate data generation only needs to place these constants into equation in the form of (2.33) or (2.34) to generate the data. The univariate PDFs and CDFs were plotted with the ParametricPlot command of Mathematica based on Equations (2.37), (2.38), (2.42), 2.44) and (2.45) (i.e., Equations 1, 2, 6a, 11, and 12 in Headrick & Kowalchuk, 2007), and the Mathematica program was listed in Table 3-14. Note that a separate program to plot the third-order power method was not listed. In order to plot PDFs and CDFs from the third-order power method, the corresponding constants and quantile function of the third-order power method in the program were replaced. The plotted PDFs and CDFs of the distributions generated by the third-order power method and the fifth-order power method were presented in Appendix A because they are lengthy.

Table 3-13

Solve Constants for the Fleishman Power Method

```
(* This program solves the constants for the Fleishman power method (see Headrick & Kowalchuk, 2007,
p248) given desired skewness and kurtosis. Example shows solution for a distribution with desired
skewness = 2.0, kurtosis = 8.0 *)

Skew = 2.0;
Kurt = 8.0;
FindRoot[ {
  c1+c3==0,
  c2^2+2*c3^2+6*c2*c4+15*c4^2==1,
   8*c3^3+6*c2^2*c3+72*c2*c3*c4+270*c3*c4^2==skew,
  24*(2*c3^4+c2^3*c4+180*c3^2*c4^2+405*c4^4
+2*c2^2*c3^2+18*c2^2*c4^2+36*c2*c3^2*c4+135*c2*c4^3)==kurt}, {{c1,-0.30}, {c2,.80},
{c3,0.30},{c4, 0.001}}]
Solution :
{c1→- 0.23336329782907406`,c2→0.7104365472481704`,
c3→0.23336329782907406`,c4→0.07226366557129085`}
```

Table 3-14

Plot PDF and CDF of Distribution Generated with the Power Method

```
(* This program plots the distribution based on the power method. Example shows plot of a distribution with skewness = 0.24, and Kurtosis = -1.209981 *)

(*power method constants*)
c11 = -0.147709;
c12 = 1.592592;
c13 = 0.153189;
c14 = -0.300343;
c15 = -0.001826;
c16 = 0.009490;

fz = (1/Sqrt[2*Pi])*Exp[-z^2/2];

y = c11 +c12*z +c13*z^2 +c14*z^3 + c15*z^4 +c16*z^5;

dy=D[y,z];

Fy = Integrate [fz,{z, -Infinity,z}];
SetOptions[ParametricPlot,AspectRatio→1/GoldenRatio];

ParametricPlot[{y, (fz/dy)},{z,-3,3}]
ParametricPlot[{y, Fy},{z,-3,3}]
```

In order to generate correlated data with the power methods for the repeated measures ANOVA *F* and *FR* tests, the first and third groups of distributions used third-order power transformation and Equation (7b) developed by Headrick and Sawilowsky (1999, p. 28) to solve the intermediate correlations (matrices). The second group of distributions used the fifth-order power method and Equation (27) developed by Headrick (2002, p. 694) to solve the intermediate correlation matrices. Mathematica programs for solving the intermediate correlation coefficients were listed in Tables 3-15 and 3-16, respectively, for the third-order power and fifth-order power transformations. The intermediate correlation coefficients were assembled into a matrix and subjected to a Cholesky factorization. The entries of the decomposed matrix were used to create standard normal deviates, which were then placed to the corresponding transformation equations in the form of (1.5) or (2.33) to generate desired data. The coefficients of the distributions associated with the third-order power method and the fifth-order power method are presented in Tables 3-17 and 3-18. The intermediate correlations associated with the distributions with the third-order power method and fifth-order power methods are presented in Tables 3-19 and 3-20. The values of coefficients and intermediate correlations (except the constants for the fifth-order power method) were rounded to 14 decimal places in actual implementation.

Table 3-15

Solve Intermediate Correlations for the Third-Order Power Method

---

```
(*This program calculate intermediate correlations Fleishman power method given two set of constants.
Example shows solution of  an intercorrelation of 0.259147044485012 of  two identical distributions
correlated at post correlation at 0.25*)

(*constants for distribution 1 with skewness = 1.0, kurtosis = 2.0*)
c11 = -0.14721081863342053;
c12 = 0.9047583031122518;
c13 = 0.14721081863342053;
c14 = 0.02386092280189755;
(*constants for distribution 2 with skewness = 1.0, kurtosis = 2.0*)
c21 = -0.14721081863342053;
c22 = 0.9047583031122518;
c23 = 0.14721081863342053;
c24 = 0.02386092280189755;

rho = 0.25; (* set the required post correlations*)

FindRoot[{rho==corr*(c12*c22+3*c22*c14+3*c12*c24+9*c14*c24+2*c11*c21*corr+6*c14*c24*corr^2)
},{corr, 0.25}]   (*Headrick & Sawilosky, 1999, Equation (7b). p. 28*)
Solution:

{corr→0.259147044485012}
```

Table 3-16

Solve Intermediate Correlations for the Headrick (2002) Fifth-Order Power Method

---

```
(*Example shows solution of intercorrelation 0.282476254679734 from two identical distributions with
skew = 0.24,Kurtosis = -1.209981, correlated at post correlation at 0.25*)

(*constants for distribution1*)
C11 = -0.147709;
C21 = 1.592592;
C31 = 0.153189;
C41 = -0.300343;
C51 = -0.001826;
C61 = 0.009490;

(*constants for distribution2*)
C12 = -0.147709;
C22 = 1.592592;
C32 = 0.153189;
C42 = -0.300343;
C52 = -0.001826;
C62 = 0.009490;

corr=0.25; (* set the required post correlations*)



(* Compute the intermediate correlation rij *)

FindRoot[{3 c12 c51+3 c32 c51+9 c51 c52+c11 (c12+c32+3 c52)+c21 c22 rij+3 c22 c41 rij+3 c21 c42
rij+9 c41 c42 rij+15 c22 c61 rij+45 c42 c61 rij+15 c21 c62 rij+45 c41 c62 rij+225 c61 c62 rij+12 c32 c51
rij^2+72 c51 c52 rij^2+6 c41 c42 rij^3+60 c42 c61 rij^3+60 c41 c62 rij^3+600 c61 c62 rij^3+24 c51 c52 rij^4+120
c61 c62 rij^5+c31 (c12+c32+3 c52+2 c32 rij^2+12 c52 rij^2)-corr==0}, {rij, 0.50}] (*Headrick, 2002, Eq.(26).
p,694*)

Solution:

{rij→0.282476254679734}
```

Table 3-17

Descriptive Parameters, c Constants of Distributions Generated with the Third-Order Power Transformation

Distribution	Skew	Kurtosis	c_1	c_2	c_3	c_4
D11	0.00	0.00	0.00000000	1.00000000	0.00000000	0.00000000
D12	0.00	1.00	0.00000000	0.90297660	0.00000000	0.03135645
D13	1.00	2.00	-0.14721082	0.90475830	0.14721082	0.02386092
D14	2.00	8.00	-0.23336330	0.71043655	0.23336330	0.07226367
D15	3.00	20.00	-0.26048168	0.44221691	0.26048168	0.14047819
D16	3.90	40.00	-0.26121314	0.12585812	0.26121314	0.21388197
D31	2.50	60.00	-0.13131504	-0.18606445	0.13131504	0.28909444
D32	2.75	70.00	-0.13923545	-0.30764127	0.13923545	0.30963706
D33	3.00	80.00	-0.14795528	-0.43590755	0.14795528	0.32942276
D34	3.25	90.00	-0.15807957	-0.58374772	0.15807957	0.34966114

Note. D11-D16 are distributions where all four transformations have valid PDFs. D31-D34 are distributions where the power method does not have valid PDFs, while other transformations have valid PDFs.

Table 3-18

Descriptive Parameters, c Constants of Distributions Generated with the Fifth-Order Power Transformation

Constant	D21 (sk= 0.24, KT =-1.209981)	D22 (sk= 0.96, KT =0.133374)	D23 (SK =1.68, KT =2.76236)	D24 (SK =2.40, KT =6.60661)
c_1	-0.147709	-0.446924	-0.542304	-0.498502
c_2	1.592592	1.242521	0.858518	0.577473
c_3	0.153189	0.500764	0.594187	0.548902
c_4	-0.300343	-0.184710	-0.039003	0.108152
c_5	-0.001826	-0.017947	-0.017294	-0.016800
c_6	0.009490	0.003159	-0.003255	-0.009363

Note. SK =skewness, KT =kurtosis. D21-D24 are the four distributions for which none of the transformation procedures have valid PDFs.

Table 3-19

Intermediate Correlation of Distributions Generated with the Third-Order Power Transformation at Different Post Correlation Levels

Distribution	0.25	0.40	0.55	0.70	0.85
D11	0.25000000	0.40000000	0.55000000	0.70000000	0.85000000
D12	0.25138931	0.40198826	0.55226433	0.70210019	0.85138197
D13	0.25914704	0.41166527	0.56198246	0.71012440	0.85611984
D14	0.28004618	0.43789856	0.58843056	0.73201200	0.86905838
D15	0.31254928	0.47750401	0.62680162	0.76238513	0.88620685
D16	0.36932960	0.53886627	0.68002876	0.80066113	0.90611574
D31	0.43542660	0.60079705	0.72831422	0.83278496	0.92188506
D32	0.47037554	0.62785766	0.74718243	0.84445747	0.92732845
D33	0.50606323	0.65366822	0.76465131	0.85509256	0.93223943
D34	0.54331171	0.67926799	0.78161719	0.86530698	0.93692407

Note. For distributions D11-D16 all four transformations have valid PDFs.

D11: skewness (SK) = 0.0, kurtosis (KT) = 0.0; D12: sk = .0, kt = 1.0; D13: sk = 1.0, kt = 2.0; D14: sk = 2.0, kt = 8.0; D15: sk = 3.0, kt = 20.0; D16: sk = 3.9, kt = 40.0. For Distributions D31-D34 all but the power method transformations have valid PDFs. D31: sk = 2.50, kt = 60; D32: sk =2.75, kt = 70; D33: sk = 3.00, kt = 80; D34: sk = 3.25, kt = 90.

Table 3-20

Intermediate Correlation of Distributions Generated with the Fifth-Order Power Transformation at Different Post Correlation Levels

Distribution	0.25	0.40	0.55	0.70	0.85
D21	0.33861850	0.51090126	0.65929326	0.78773345	0.90021876
D22	0.37013250	0.53468982	0.67391535	0.79511355	0.90281194
D23	0.37576526	0.53526376	0.67147487	0.79182960	0.90047159
D24	0.34726732	0.50882626	0.65057379	0.77777238	0.89355870

Note. For Distributions D21-D24 none of the transformations have valid PDFs. D21: sk = .24, kt = -1.209981; D22: sk = .96, kt = .133374; D23: sk = 1.68, kt = 2.76236; D24: sk = 2.40, kt = 6.60661.

Data Generation with the Burr Family Distributions

The univariate data generation followed the procedure based on the Burr Types III and XII family distributions proposed by Headrick and Pant (2010). Desired skewness (α_1) and kurtosis (α_2) values (note that notations are consistent with the original authors' again) were placed into Equations (2.51) and (2.52) or Equations (16) and (17) in Headrick and Pant (2010), to solve for the *c* and *k* parameters. The solved *c* and *k* values were then placed into Equations (2.53) and (2.54), or Equations (14) and (15) in Headrick and Pant (2010), to determine the means and standard deviations. The Mathematica program for solving the *c* and *k* parameters, mean, standard deviation, skewness and kurtosis were listed as Table 3-21. The generated distributions were plotted based on Equation (1.5) for Type III, and (1.6) for Type XII, and Equations (2.48) and (2.49) with Mathematica ParametricPlot command, and the source code for the plot was listed in Table 3-22. The descriptive parameters solved with Mathematica were presented in Table 3-23.

For generating multivariate distributions with the Burr transformations for the simple repeated measures *F* and *FR* tests, Equations (19)-(24) and Mathematica source code in Figure 4 in Headrick and Pant (2010, pp. 15-18) were used. The Mathematica code was reprinted in Table 3-24, and the intermediate correlations solved for the distributions were reported in Table 3-25. The plotted PDFs and CDFs of the distributions generated with the Burr transformations were reported in Appendix A. The values of *c, k, M, SD* and intermediate correlations were also kept with 14 places of decimals in actual implementation.

Table 3-21

Solve the c and k Parameters and the Mean, Standard Deviation, Skewness, and Kurtosis of the Burr Distributions

---

```
(*Part I. This program solves the values of k and c for the Burr distribution given desired skewness,
kurtosis, based on Headrick & Pant, 2010.  Example shows solution of a distribution with skewness = 3.90,
kurtosis = 40.0 *)

sk=3.9;
kt=40.0;

FindRoot[{(Gamma[(4+c)/c] Gamma[k]^3 Gamma[-(4/c)+k]-1/c^3 3 Gamma[-(1/c)+k] (4 c Gamma[1/c]
Gamma[3/c] Gamma[k]^2 Gamma[- (3/c)+k]-4 Gamma[1/c]^2 Gamma[2/c] Gamma[k] Gamma[-(2/c)+k]
Gamma[-(1/c)+k]+c^3 Gamma[1+1/c]^4 Gamma[-(1/c)+  k]^3))/(Gamma[(2+c)/c] Gamma[k] Gamma[-
(2/c)+k]-Gamma[1+1/c]^2 Gamma[-(1/c)+k]^2)^2-3 == kt,

 (1/(Gamma[(2+c)/c] Gamma[k] Gamma[-(2/c)+k]-Gamma[1+1/c]^2 Gamma[-(1/c)+k]^2))^(3/2)
(Gamma[(3+c)/c] Gamma[k]^2 Gamma[-(3/c)+k]-1/c^2 (6 Gamma[1/c] Gamma[2/c] Gamma[k] Gamma[-
(2/c)+ k] Gamma[-(1/c)+k])+2 Gamma[1+1/c]^3 Gamma[-(1/c)+k]^3) == sk},
{c,0.9},{k,5.8}]

{c→1.02943337515853,k→5.46427129719935}


(*Part II. This program calculate the parameters for the Burr distribution, given the values of k and c solved
from program in Part I *)

c = 1.0294333751585347;
k = 5.4642712971993515;

M1 = Gamma[1 + 1/c]*Gamma[k - 1/c]/Gamma[k]
SD = Sqrt[(Gamma[1 + 2/c]*Gamma[k - 2/c]/Gamma[k]) - M1^2]

skew = (1/(Gamma[(2 + c)/c] Gamma[k] Gamma[-(2/c) + k] - Gamma[1 + 1/c]^2 Gamma[-(1/c) +
k]^2))^(3/ 2) (Gamma[(3 + c)/c] Gamma[k]^2 Gamma[-(3/c) + k] - 1/c^2 (6 Gamma[1/c] Gamma[2/c]
Gamma[k] Gamma[-(2/c) +  k] Gamma[-(1/c) + k]) +  2 Gamma[1 + 1/c]^3 Gamma[-(1/c) + k]^3)

kurt = (Gamma[(4 + c)/c] Gamma[k]^3 Gamma[-(4/c) + k] -  1/c^3 3 Gamma[-(1/c) +
k] (4 c Gamma[1/c] Gamma[3/c]Gamma[k]^2 Gamma[-(3/c) + k] - 4 Gamma[1/c]^2 Gamma[2/c]
Gamma[ k] Gamma[-(2/c) + k] Gamma[-(1/c) +  k] +  c^3 Gamma[1 + 1/c]^4 Gamma[-(1/c) +
k]^3))/(Gamma[(2 + c)/ c] Gamma[k] Gamma[-(2/c) + k] - Gamma[1 + 1/c]^2 Gamma[-(1/c) + k]^2)^2 – 3
```

Table 3-22

Plot the Burr Distributions Given Mean, Standard Deviation, c and k Values

```
(*Part I. Plot the burr Type XII distribution.  Example plots the standard normal*)

c1 = 4.8737020673758416;
k1 = 6.157840848071374;
mu1 = 0.6447098211818171;
sig1 = 0.16198776712255047;


q1 = ((1 - u)^(-1/k1) - 1)^(1/ c1); (*c is positive -- Burr Type XII quantile function*)

x1 = (q1 - mu1)/sig1;

dx1 = D[x1, u];

SetOptions[ParametricPlot, AspectRatio -> 1/GoldenRatio];

ParametricPlot[{x1, 1/dx1}, {u, 0.000000001, 1}]

ParametricPlot[{x1, u}, {u, 0.00000001, 1}]

(*note the CDF for the uniform Dist. is u*)


(*Part II. Plot the burr Type III distribution, Example plots

a distribution  with skew = 2.50, kurtosis = 60*)

c1 = -4.283354087214409;
k1 = 0.32509150491110206;
mu1 = 0.6972710493080594;
sig1 = 0.47366545818620004;

q1 = ((u)^(-1/k1) - 1)^(1/c1); (*Burr Type III quantile function*)

x1 = (q1 - mu1)/sig1;

dx1 = D[x1, u];

SetOptions[ParametricPlot, AspectRatio -> 1/GoldenRatio];

ParametricPlot[{x1, 1/dx1}, {u, 0.001, 1}]

ParametricPlot[{x1, u}, {u, 0.001, 1}]
```

Table 3-23

Descriptive, c and k Parameters of Distributions Generated with the Burr Transformation

Distribution	Skew	Kurtosis	*c*	*k*	*M*	*SD*
D11	0.00	0.00	4.87370207	6.15784085	0.64470982	0.16198777
D12	0.00	1.00	27.07295344	1.32571082	0.98583266	0.05999091
D13	1.00	2.00	2.34709322	4.42864990	0.50604503	0.26238126
D14	2.00	8.00	1.44799614	5.45281901	0.31553940	0.25802410
D15	3.00	20.00	1.13506547	5.80606296	0.23695558	0.25094480
D16	3.90	40.00	1.02943338	5.46427130	0.23030022	0.27872995
D31	2.50	60.00	-4.28335409	0.32509150	0.69727105	0.47366546
D32	2.75	70.00	-4.30149907	0.47508691	0.82985874	0.48540662
D33	3.00	80.00	-4.30948801	0.62875544	0.92892310	0.49525483
D34	3.25	90.00	-4.31520386	0.81744275	1.02209252	0.50676115

Note. D11-D16 are distributions where all four transformations have valid PDFs. D31-D34 are distributions where the power method does not have valid PDFs.

Table 3-24

Solve the Intermediate Correlation of Two Burr Family Distributions

```
(*This program calculates the intermediate correlation of two Burr distributions.  Example shows solution
intermediate correlation 0.321612477088197 of two identical Burr Distribution with skewness = 3.9, and
kurtosis = 40.0, correlated at post correlation 0.25 *)

(* parameters for distribution 1 *)
c1 = 1.0294333751585347;
k1 = 5.4642712971993515;
m1 = 0.2303002225873409;
s1 = 0.27872994849253524;

(* parameters for distribution 1 *)
c2 = 1.0294333751585347;
k2 = 5.4642712971993515;
m2 = 0.2303002225873409;
s2 = 0.27872994849253524;

(*Manipulating the intermediate Correlations in order to get required post correlation*)
ρ12 = .321612477088197;

(*  Standard Normal CDFs*)
Φ1 = (Sqrt[2 * Pi])^(−1) * ∫_{−∞}^{z1} Exp[−u1^2/2] du1;

Φ2 = (Sqrt[2 * Pi])^(−1) * ∫_{−∞}^{z2} Exp[−u2^2/2] du2;

(* Quantile Functions *)

(* q1 = ((Φ1)^(-1/k1)-1)^(1/c1); *)  (* Type III quantile function,  since c is negative*)
q1 = ((1-Φ1)^(-1/k1)-1)^(1/c1);    (*Type XII since c is positive*)
q2 = ((1-Φ2)^(-1/k2)-1)^(1/c2);    (*Type XII since c is positive*)

(* Standardized Quantile Functions *)
x1 = (q1-m1)/s1;
x2 = (q2-m2)/s2;

(* Standard Normal Bivariate PDFs*)

f12 = (2*Pi*Sqrt[1-(ρ12^2)])^(-1)*Exp[(-1/(2*(1-ρ12^2)))*((z1^2)-2*ρ12*z1*z2+z2^2)];

(*Integrals to Compute the Specified Correlations. See Equation 22,
in Headrick and Pant,2010  *)
int1 = NIntegrate[x1*x2*f12, {z1, -8.0,8.0},{z2, -8.0,8.0}, Method→MultiDimensional]

0.25
```

Table 3-25

Intermediate Correlations of Distributions Generated with the Burr Transformation at Different Post Correlation Levels

Distribution	0.25	0.40	0.55	0.70	0.85
D11	0.25005772	0.40008788	0.55010776	0.70010834	0.85007763
D12	0.25144328	0.40206665	0.55235663	0.70218910	0.85144305
D13	0.25944594	0.41198152	0.56225187	0.71031430	0.85621685
D14	0.28113027	0.43838048	0.58820683	0.73135412	0.86844369
D15	0.30433415	0.46529045	0.61354824	0.75111780	0.87953420
D16	0.32216433	0.48810533	0.63864678	0.77654389	0.90383003
D31	0.27782958	0.43601709	0.58759094	0.73228005	0.86981872
D32	0.28041135	0.43947813	0.59123075	0.73535846	0.87164550
D33	0.28397926	0.44393765	0.59565816	0.73892950	0.87368399
D34	0.28795079	0.44875838	0.60032079	0.74260500	0.87574104

Note. For distributions D11-D16 all four transformations have valid PDFs.

D11: skewness (SK) = 0.0, kurtosis (KT) = 0.0; D12: sk = .0, kt = 1.0; D13: sk = 1.0, kt = 2.0; D14: sk = 2.0, kt = 8.0; D15: sk = 3.0, kt = 20.0; D16: sk = 3.9, kt = 40.0. For Distributions D31-D34 all but the power method transformations have valid PDFs. D31: sk = 2.50, kt = 60; D32: sk =2.75, kt = 70; D33: sk = 3.00, kt = 80; D34: sk = 3.25, kt = 90.

Statistical Tests

The fixed model ANOVA *F* tests were considered in two typical scenarios: one-way between-subjects and simple repeated measures (within-subjects) designs. The general linear models for the *F* tests were given in Chapter Two (i.e., Equation 2.61 for between-subjects design, Equation 2.64 for within-subjects design); the expected mean squares were given in Equations (2.62) and (2.63) for between-subjects ANOVA; and in Equations (2.65)-(2.67) for within-subjects design. The calculation followed Keppel and Wickens (2004, p. 32 and p. 352). It is worthy to note that the variances for the distributions in the between-subjects design were assumed to be equal, so the covariance matrices for the repeated measures design were assumed circular. In other words, homoskedasticity and sphericity were assumed due to the exploratory property of the study. The critical values for the *F* test were calculated with the FINV function in Microsoft Excel 2007.

For the nonparametric tests, the calculation for *KW* test was given in Equation (2.68), and that for *FR* test in Equation (2.69). Note that the data were generated in such a way that tied cases were eliminated and this will less likely happen in real situation. In addition, the *KW* test used exact critical values calculated based on a FORTRAN program developed by Headrick (2003) and on a replication of 5 million. The *FR* test used critical values calculated on the basis of a Visual Basic program developed by Bagui and Bagui (2005) replicated 1 million times. The critical values of ANOVA *F*, *KW* and *FR* tests were listed in Appendix B. Ideally the replications for the critical value of *FR* test would be consistent with that of the *KW* test. The Visual Basic program for calculating the critical Friedman test values, however, took a very long time for each of the four conditions. For instance, with a block size of 20, it took about 28 hours to run 1 million replications for one critical value. Thus, in this study, 5 million replications were not used in calculating critical values for the *FR* test.

Programming and Software Tools

As noted earlier, Mathematica 7.0 (Wolfram Research, 2008) was used to calculate the parameters, plot the PDFs and CDFs, and solve the intermediate correlations for the distributions to be generated. Microsoft Visual Basic Express 2008 (Microsoft, 2007) and Microsoft Excel 2007 were used to calculate the exact critical values of the *FR* test and *F* tests respectively. Microsoft FORTRAN PowerStation 4.0 (Microsoft, 1994) was used as the main programming language. The subroutines NORMB1, UNI1, and RANK from RANGEN (Blair, 1987) were used to generate pseudo-random normal and uniform deviates. SPSS 17.0 for Windows (SPSS Inc, 2007) was used to verify the calculations.

The implementation of the simulation study was coded into 11 programs for the between-subjects design, and another 11 programs for the within-subjects design, following the FORTRAN 77 protocol as the environment Microsoft FORTRAN PowerStation 4.0 (Microsoft, 1994) provided. More specifically, for each group of the considered distributions, one FORTRAN program was created for each of the data transformations and for each of the between-subjects and within-subjects designs. The four seed numbers (i.e., DSEED1 = 36976543215.D0, DSEED2 = 24678965412.D0, DSEED3 = 12345678985.D0, and DSEED4 = 73645658789.D0) were consistently used across all the data transformations to generate pseudo random variates in both between-subjects and within-subjects designs to ensure the fairness of comparison. The programs for the one-way ANOVA *F* tests and nonparametric (i.e., *KW* and *FR*) tests were more than 300 pages, thus not provided, but available upon request. The simulation was implemented at the Statistics Laboratory of Department of Educational Psychology and Special Education on a Dell OptiPlex 755 desktop computer operated with Microsoft Windows XP Professional Service Pack 3, and installed with 2.66 GHz Intel Core 2 Duo CPU, and 1.96 GB RAM.

CHAPTER 4

RESULTS

Validation of Results

Calculation of the statistics associated with the between-subjects ANOVA was validated with SPSS output for the sixth distribution (sk = 3.9, kt = 40.0) in the first category. Specifically, one data set with four groups with a cell size of 50 was generated from FORTRAN, and imported to SPSS. The calculated statistics (i.e., *F* and *KW*) from FORTRAN and SPSS were compared, and they matched well (to the six decimal place), with slight difference being assumed from the precision of data used. Similarly, for the within-subjects ANOVA *F* and *FR* tests, a sample drawn from the standard normal distribution of size 50 was used to validate the programming; similar results were obtained as with the between-subjects design. Thus, the results from the FORTRAN program matched well to the corresponding SPSS output for all of data transformations.

In addition, the Type I error rates for the *F* tests and *KW* tests, based on the standard normal distribution, were compared with Clinch and Keselman (1982) and Feir-Walsh and Toothaker (1974), the values all fell within their respective confidence intervals. Specifically, the Type I error rates of *F* tests associated with normal distribution generated with the third-order power method and *g*-and-*h* transformations (at cell size = 12 and replicated 2000 times, as in Clinch & Keselman, 1982) were all within the interval (.0404, .0596). Further, the Type I error rates under normal distribution generated with the third-order power method and *g*-and-*h* transformations (at cell sizes 7, 17, and 50, and replicated 1000 times, as in Feir-Walsh & Toothaker 1974) for both the *F* and *KW* tests were all within the interval (.0365, .0635). On the other hand, for the within-subjects design, Type I error rates for the *F* and *FR* tests were compared with Al-Subaihi (2000), and Harwell and Serlin (1994), also based on the standard normal distribution, and the values were also within the respective confidence intervals. More specifically, standard normal distributions were generated also with the third-order power method and *g*-and-*h* transformations, with cell sizes 10, 20 and 30, and replicated 30,000 times (set as in Al-Subaihi, 2000), the Type I error rates associated with the *F* and *FR* tests were all within the interval (.0475, .0525). Furthermore, when cell sizes were set at 10, 15, and 30, and replicated 2000 times as in Harwell and Serlin (1994), the Type I error rates associated with the *F* and *FR* tests were all within the interval (.0404, .0596).

Finally, inspection of Equation (1.3) indicates that an *h* distribution is analytically standard normal when *h* is zero. Thus, the results of Type I error and power rates of a standard normal distribution from the third-order power method should be exactly the same as the results of the standard normal distribution created with the *h* distribution. As predicted, the results of rejection rates (i.e., both the type I error and power rates) from the third-order power transformation were the same as their corresponding rates obtained from the *g*-and-*h* transformation in the between-subjects design, which is another indication of correct results.

Between-Subjects Design for the First Group of Distributions with Valid PDFs

Type I error rates for the ANOVA *F* and *KW* tests. The Type I error rates of the *F* and *KW* tests were reported in Table 4-1 for the six distributions with valid PDFs. Bold faced values (denoted the same hereafter) indicate that the Type I error rate was outside of the interval (.045, .055). As Table 4-1 indicates, the four data transformations all controlled the Type I error rates of the *F* tests within the interval (.045, .055) for the first three distributions (i.e., D11: standard normal; D12: sk = .0, kt = 1.0; and D13:sk =

1.0, kt = 2.0). In the fourth distribution (i.e., D14: sk = 2.0, kt = 8.0), the Type I error rates were robust except for the cell size = 10. In the fifth distribution (D15: sk = 3.0, kt = 20.0) the Type I error rates were robust at cell size = 50, conservative at cell sizes = 10, and 20, with a mixture of robust and conservative results at cell size 30. In the sixth distribution (D16: sk = 3.9, kt = 40.0), the Type I error rates of the *F* tests were conservative at the three smaller cell size levels (i.e., 10, 20, and 30) with mixed robust-conservative results at cell size = 50. Compared across the four data transformations, two conditions had the Type I error differences above .004 (cell sizes = 10 and 20, D16). Thus, two cases of mixed conservative-robust Type I errors and two cases of conservative Type I errors with extreme differences, made 4 out of 24 conditions where the Type I error of the *F* tests were different across the data transformations. For the *KW* tests, the four data transformations all controlled Type I errors within the interval (.045, .055), and thus, were similar in each of the conditions.

Table 4-1

Type I Error Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Six Distributions with Valid PDFs (%)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	5.07	5.07	5.07	5.08	5.04	5.04	5.04	5.04
20[a]	5.00	5.00	4.99	5.00	5.05	5.05	5.05	5.05
30[a]	5.00	5.00	5.00	4.99	5.00	5.00	5.00	5.00
50[a]	4.92	4.92	4.91	4.89	4.84	4.84	4.84	4.84
10[b]	4.87	4.88	4.86	4.87	4.90	4.90	4.90	4.90
20[b]	4.90	4.91	4.89	4.91	5.04	5.04	5.04	5.04
30[b]	5.05	5.05	5.06	5.04	5.10	5.10	5.10	5.10
50[b]	5.00	4.98	4.97	4.97	5.06	5.06	5.06	5.06
10[c]	4.83	4.83	4.80	4.86	5.16	5.16	5.16	5.16
20[c]	5.03	5.03	5.06	5.05	5.12	5.12	5.12	5.12
30[c]	5.01	5.03	5.01	5.07	5.03	5.03	5.03	5.03
50[c]	4.88	4.88	4.86	4.85	4.95	4.95	4.95	4.95
10[d]	**4.30**	**4.36**	**4.32**	**4.35**	5.02	5.02	5.02	5.02
20[d]	4.73	4.69	4.70	4.71	5.16	5.16	5.16	5.16
30[d]	4.61	4.69	4.66	4.69	4.91	4.91	4.91	4.91
50[d]	4.93	4.92	4.90	4.88	5.05	5.05	5.05	5.05
10[e]	**3.80**	**4.06**	**4.03**	**4.04**	4.97	4.97	4.97	4.97
20[e]	**4.09**	**4.22**	**4.24**	**4.26**	4.95	4.95	4.95	4.95
30[e]	**4.38**	4.50	**4.47**	**4.49**	4.93	4.93	4.93	4.93
50[e]	4.58	4.65	4.64	4.67	5.05	5.05	5.05	5.05
10[f]	**2.98**	**3.84**	**3.77**	**3.78**	4.80	4.80	4.80	4.80
20[f]	**3.71**	**4.29**	**4.23**	**4.23**	5.09	5.09	5.09	5.09
30[f]	**4.09**	**4.46**	**4.45**	**4.46**	4.91	4.91	4.91	4.91
50[f]	**4.32**	4.61	4.67	4.66	5.03	5.03	5.03	5.03

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Bold faced values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Power rates for the ANOVA *F* and *KW* tests. The power rates of the four data transformations for the between-subjects ANOVA were reported in Tables 4-2 to 4-4 for effect sizes of .25, .50 and .75, respectively. Bold faced (underscored) values denoted the maximum (minimum) power rates of a statistical test across the four transformations in a condition where the range of the power rates was above .05 (denoted the same hereafter). The power rates for effect sizes of 1.0 and above were similar (with the maximum range of .024) across the four data transformations and hence not reported. The power rates at effect size = .25 were summarized in Table 4-2. As indicated in Table 4-2, the four data transformations resulted in similar power rates for the *F* tests in each condition, with a maximum range within .014; in case of the *KW* tests, the four data transformations resulted in similar power rates within only the first four distributions. In the last two distributions (i.e., D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0). However, the four data transformations were different, with a range from .046 to .569 (see Table 4-2). The power rates at effect size = .50 were reported in Table 4-3. As the results in Table 4-3 show, the four data transformations obtained similar power rates for the *F* tests except in one condition (i.e., D16 with cell size = 10) where the range of power rates was above .05. The power rates for the *KW* tests at effect size = .50 had the similar trend as observed at effect size = .25, but the magnitude of difference (i.e., .065-.348) across the four data transformations in the last two distributions (i.e., D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9,kt = 40.0) were smaller, and the power rates were very close across the four data transformations at cell size = 50 in the last two distributions (see Table 4-3). The power rates at effect size = 0.75 were reported in Table 4-4. Results in Table 4-4 indicate that the four data transformations obtained similar power rates for the *F* tests in each of the conditions except in one condition (D16, cell size = 10) where the range of power was .055. For the *KW* tests, the four data transformations achieved similar power rates in each condition except two cases where sample size = 10 in the last two distributions with range of power rates at .101 and .173, respectively.

In summary, there were 2 out of 144 conditions where the power rates across the four data transformations were inconsistent for the *F* tests, with the extreme power difference at .055; while for the *KW* tests, 15 out of 144 conditions had inconsistent power rates across the four data transformations, with extreme power difference at .569. Because inconsistent power rates across the four data transformations for the *KW* tests were a concern, and the third-order power method contributed the most differences, it might be practically useful to compare the rest of the three data transformations. A comparison of the Burr, *g*-and-*h*, and GLD transformations resulted with only 7 out of 144 conditions inconsistent in power rates across the three data transformations (i.e., with the third-order power transformation excluded), with the extreme power differences down to .149 for the *KW* tests. Pairwise comparisons among the *g*-and-*h*, GLD, and the Burr transformation, indicate that, the difference of power rates across these three data transformations were within .15 for both the *F* and *KW* tests, and the results between the Burr and GLD were particularly consistent with only one case the power rate difference above .05 (i.e., .052, D14, cell size = 50, effect size = .25). Results also indicate that, in the six distributions where all four transformations have valid PDFs, the Type I error rates for the *F* tests were robust in the first three distributions, with a mixture of conservative and robust results in the last three distributions. They were similar in each condition compared across all the four data transformations except for four cases in the last two distributions. Type I error rates for the *KW* tests were more robust than the *F* tests in all conditions and they were similar across the four data transformations in each of the conditions. The power rates for the *F* tests were generally similar in each condition across the four data transformations. The power rates for the *KW* tests were similar in the first four distributions in each of the conditions, but dissimilar in the last two distributions. When the normality assumption was satisfied, the *F* tests were slightly more powerful with a difference within .035. In the five nonnormal distributions, however, the *KW* tests were systematically more powerful than or equally powerful to the *F* tests. More generally, the Type I error rates of the *F* tests and the power rates of the *KW* tests might be disparate when distributions were skew and heavy tailed and when they were close to their associated boundary conditions with valid PDFs (e.g., D15, sk = 3.0, kt = 20; and D15, sk = 3.9, kt = 40).

Table 4-2

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Effect Size = 0.25)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	7.68	7.68	7.68	7.65	7.53	7.53	7.52	7.55
20[a]	10.79	10.79	10.81	10.79	10.53	10.53	10.54	10.51
30[a]	14.26	14.26	14.26	14.21	13.66	13.66	13.69	13.67
50[a]	21.68	21.68	21.70	21.71	20.87	20.87	20.91	20.91
10[b]	7.82	7.81	7.81	7.84	8.00	7.97	8.03	8.01
20[b]	11.09	11.09	11.07	11.08	11.41	11.35	11.44	11.42
30[b]	13.96	13.97	13.97	13.96	14.69	14.60	14.79	14.75
50[b]	21.72	21.73	21.70	21.70	22.70	22.56	22.89	22.79
10[c]	7.63	7.61	7.61	7.66	8.15	8.09	8.25	8.00
20[c]	10.77	10.76	10.78	10.75	11.89	11.79	12.15	11.58
30[c]	14.34	14.37	14.38	14.34	16.10	15.90	16.47	15.58
50[c]	21.71	21.71	21.73	21.72	25.06	24.67	25.78	24.11
10[d]	7.48	7.46	7.45	7.49	9.98	9.44	9.56	9.62
20[d]	11.11	11.08	11.10	11.04	16.58	15.38	15.69	15.91
30[d]	14.14	14.15	14.13	14.10	23.01	21.24	21.67	22.12
50[d]	22.22	22.11	22.20	22.10	38.56	35.34	36.04	37.07
10[e]	7.61	7.45	7.43	7.36	16.01	11.44	11.82	12.54
20[e]	11.35	11.11	11.09	11.11	**30.88**	<u>20.31</u>	21.38	23.28
30[e]	15.29	15.08	15.10	15.03	**46.79**	<u>30.57</u>	32.26	35.79
50[e]	23.06	22.77	22.69	22.66	**71.39**	<u>49.90</u>	52.70	57.94
10[f]	8.42	7.61	7.58	7.58	**42.37**	<u>12.96</u>	15.12	15.57
20[f]	12.66	11.68	11.62	11.64	**76.51**	<u>23.81</u>	29.45	30.71
30[f]	16.62	15.55	15.57	15.57	**92.70**	<u>35.82</u>	44.88	46.90
50[f]	24.63	23.43	23.29	23.27	**99.58**	<u>58.62</u>	70.94	73.48

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-3

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Effect Size = 0.50)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	16.67	16.67	16.66	16.69	16.06	16.06	16.11	16.15
20[a]	32.21	32.21	32.22	32.19	30.79	30.79	30.89	30.82
30[a]	47.80	47.80	47.78	47.81	45.63	45.63	45.78	45.67
50[a]	72.25	72.25	72.22	72.26	69.69	69.69	69.82	69.66
10[b]	17.04	17.03	17.04	17.03	17.68	17.58	17.81	17.78
20[b]	32.47	32.46	32.47	32.46	33.86	33.66	34.11	34.00
30[b]	47.62	47.60	47.63	47.61	49.79	49.46	50.08	49.90
50[b]	71.93	71.94	71.91	71.91	74.61	74.27	74.99	74.77
10[c]	16.94	16.94	17.00	16.88	18.38	18.11	18.88	17.69
20[c]	32.60	32.60	32.67	32.59	37.46	36.87	38.42	36.07
30[c]	47.94	47.95	47.97	47.94	55.18	54.44	56.61	53.44
50[c]	72.42	72.36	72.43	72.35	80.50	79.81	81.72	78.76
10[d]	18.11	17.85	17.91	17.81	27.00	24.77	25.31	25.21
20[d]	34.11	33.80	33.89	33.77	55.27	51.64	52.49	52.86
30[d]	49.50	49.38	49.39	49.32	77.01	73.29	74.11	74.70
50[d]	72.84	72.96	72.91	72.96	95.50	93.99	94.37	94.72
10[e]	21.10	19.60	19.61	19.52	**46.63**	<u>33.56</u>	34.92	36.39
20[e]	37.31	36.01	36.00	35.84	**82.86**	<u>68.19</u>	70.27	72.62
30[e]	51.25	50.70	50.61	50.57	**95.95**	<u>88.24</u>	89.66	91.20
50[e]	73.92	73.86	73.85	73.82	99.88	98.95	99.22	99.47
10[f]	**26.56**	<u>21.40</u>	21.41	21.43	**74.46**	<u>39.66</u>	44.74	45.42
20[f]	41.59	38.09	38.02	37.95	**97.87**	<u>76.72</u>	82.79	83.49
30[f]	54.72	52.74	52.52	52.49	**99.88**	<u>93.41</u>	96.32	96.63
50[f]	74.75	74.24	74.24	74.19	100.00	99.70	99.91	99.92

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-4

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Effect Size = 0.75)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	34.09	34.09	34.09	34.06	32.32	32.32	32.44	32.37
20[a]	65.72	65.72	65.72	65.76	62.96	62.96	63.07	62.93
30[a]	84.99	84.99	84.99	84.98	82.87	82.87	82.96	82.82
50[a]	97.84	97.84	97.84	97.86	97.31	97.31	97.34	97.28
10[b]	34.86	34.83	34.90	34.88	35.65	35.43	35.87	35.78
20[b]	65.87	65.90	65.85	65.86	67.73	67.41	68.02	67.83
30[b]	84.85	84.84	84.82	84.82	86.44	86.23	86.67	86.49
50[b]	97.81	97.80	97.82	97.81	98.37	98.31	98.43	98.36
10[c]	35.11	35.04	35.16	34.95	38.55	38.04	39.45	37.21
20[c]	66.18	66.20	66.10	66.17	73.59	72.97	74.75	72.08
30[c]	84.99	85.00	84.93	84.97	91.31	90.90	91.98	90.34
50[c]	97.88	97.89	97.88	97.89	99.39	99.34	99.47	99.25
10[d]	38.33	37.84	37.96	37.68	54.82	51.58	52.35	51.83
20[d]	67.63	67.62	67.62	67.63	90.38	88.44	88.95	88.96
30[d]	85.09	85.17	85.15	85.14	98.68	98.26	98.35	98.39
50[d]	97.81	97.82	97.79	97.81	100.00	99.99	99.99	99.99
10[e]	43.28	41.39	41.38	41.29	**75.70**	<u>65.64</u>	66.75	67.51
20[e]	69.65	69.16	69.11	69.10	98.41	96.25	96.63	96.87
30[e]	85.27	85.50	85.50	85.52	99.93	99.75	99.80	99.83
50[e]	97.33	97.51	97.53	97.53	100.00	100.00	100.00	100.00
10[f]	**49.41**	<u>43.92</u>	43.87	43.89	**89.66**	<u>72.41</u>	75.70	76.04
20[f]	71.73	70.75	70.66	70.65	99.82	98.25	98.83	98.86
30[f]	85.21	86.03	86.00	85.99	100.00	99.96	99.97	99.98
50[f]	96.52	97.09	97.14	97.13	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Between-Subjects Design for the Second Group of Distributions without Valid PDFs

Type I error rates for the ANOVA *F* and *KW* tests. The Type I error rates of *F* and *KW* tests for the second group of four distributions without PDFs were reported in Table 4-5. As the results in Table 4-5 indicate, for the *F* tests, the three data transformations (i.e., the fifth-order power, *g*-and-*h*, and the GLD) achieved similar and robust Type I error rates in the first two distributions (i.e., D21: sk = .24, kt = -1.209981; and D22: sk = .96, kt = 0.133374). In the third distribution (i.e., D23: sk = 1.68, kt = 2.76236), the Type I error rates were robust except cell size = 10; the Type I error rates were conservative at cell size = 10 and robust at cell size = 50 in the last distribution (D24: sk = 2.40, kt = 6.606610). Mixed results of robust and conservative Type I errors were found at cell size = 10 in the third distribution (D23) and cell sizes = 20, and 30 in the fourth distribution (D24). Compared in each condition across data transformations, 3 out of 16 conditions had dissimilar Type I error for the *F* tests (Table 4-5). The Type I error rates of the *KW* tests were summarized on the right part in Table 4-5. As indicated in Table 4-5, Type I error rates of *KW* tests in each condition across all three data transformations were consistently within the interval (.045, .055).

Power rates for the ANOVA *F* and *KW* tests. The power rates of the second group of four distributions without valid PDFs were reported as Tables 4-6 to 4-9. As the results in Table 4-6 show, at effect size = .25, the power rates for the *F* tests were similar in each condition across the three data transformations, while the power rates for the *KW* tests were dissimilar at cell size = 50 in the second distribution (D22: sk = .96, kt = .133374,), and all cell sizes in the last distributions (D23: sk = 1.68, kt = 2.76236; D24: sk = 2.40, kt = 6.606610), with the differences ranging from .059 to .44 (Table 4-6).

Table 4-5

Type I Error Rates of F and Kruskal-Wallis Tests of Three Data Transformations in Four Distributions without Valid PDFs (%)

Cell Size	*F* Test			Kruskal-Wallis Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	5.08	5.09	5.12	4.95	5.03	5.03
20[a]	5.11	5.06	5.02	5.10	5.00	5.02
30[a]	5.19	5.11	5.11	5.07	5.01	5.02
50[a]	4.94	4.96	4.92	4.87	4.88	4.89
10[b]	4.84	4.90	4.88	4.93	4.89	4.92
20[b]	4.90	4.99	4.98	5.16	5.12	5.18
30[b]	5.11	5.17	5.15	5.16	5.12	5.10
50[b]	5.02	4.93	4.94	5.02	5.06	5.00
10[c]	**4.40**	4.53	**4.30**	4.92	5.16	4.94
20[c]	4.73	4.68	4.82	5.02	5.08	5.16
30[c]	5.02	5.03	4.93	5.01	5.04	5.08
50[c]	4.84	4.75	4.94	5.04	4.96	5.06
10[d]	**3.86**	**3.86**	**3.71**	4.98	4.97	4.95
20[d]	4.54	4.58	**4.41**	5.06	5.05	4.97
30[d]	4.60	4.54	**4.48**	4.85	4.91	4.99
50[d]	4.77	4.82	4.57	5.13	5.04	4.89

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of the interval (4.805, 5.195). None of four distributions have valid PDFs.

Table 4-6

Power Rates of F and Kruskal-Wallis Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Effect Size = 0.25)

Cell Size	*F* Test			Kruskal-Wallis Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	7.79	7.83	7.81	8.18	7.71	7.74
20[a]	10.58	10.76	10.75	11.98	11.13	11.19
30[a]	13.93	13.88	13.86	16.23	14.65	14.81
50[a]	21.19	21.22	21.23	25.60	22.95	23.21
10[b]	7.55	7.44	7.43	9.90	8.85	9.01
20[b]	10.87	10.74	10.81	16.78	14.84	15.30
30[b]	13.99	13.98	14.00	23.97	20.77	21.48
50[b]	21.72	21.81	21.78	**40.85**	34.93	36.28
10[c]	7.33	7.26	7.02	14.13	13.85	**30.64**
20[c]	10.69	10.68	10.58	27.77	27.05	**63.58**
30[c]	14.19	14.26	14.10	42.09	40.88	**84.83**
50[c]	21.42	21.62	21.29	67.61	66.24	**98.43**
10[d]	7.34	7.36	7.02	**24.72**	24.54	17.71
20[d]	11.09	11.15	10.75	**52.70**	51.76	35.28
30[d]	14.23	14.27	14.33	**74.85**	73.81	52.84
50[d]	21.60	21.73	21.70	**95.12**	94.65	78.64

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (bold faced) values are the minimum (maximum) of the condition. None distributions have valid PDFs.

As the results in Table 4-7 reveal, with an effect size = .50, the power rates for the *F* tests were similar in each condition across the three data transformations in all four distributions, while the power rates for the KW tests were dissimilar at three smaller cell sizes (i.e., 10, 20, 30) in the third distribution (D23: sk = 1.68, kt = 2.76236) with a maximum power difference ranging from .114 to .368. As Table 4-8 indicates, the power rates for the *F* tests at effect size = .75 were similar in each condition among the three data transformations, but for the *KW* tests they were dissimilar at small cell sizes in the last two distributions (cell sizes =10, and 20 in D23, and cell size = 10 in D24) with differences ranging .06 to .227. Also with an effect size = 1.00, the results reported in Table 4-9 indicate the power rates for the *F* tests were similar in each condition across data transformations as expected, but difference of power rates of the *KW* tests across data transformations in the conditions at cell size = 10 in the third and forth distributions were .088 and .082, respectively. The power rates at effect sizes greater than 1.0 were similar for each condition across data transformations, and hence not reported.

In summary, when the four distributions had no valid PDFs, the Type I error rates for the *F* tests were consistent in each condition across the three data transformations in the first two distributions, but inconsistent in three conditions in the last two distributions. Type I error rates for the *KW* tests were similar across the three data transformations. The power rates for the *F* tests were similar in each condition across the data transformations, while they were different for the *KW* tests in 17 out of 96 conditions mainly in the last two distributions. Because the GLD transformation was the most powerful and contributed most of the inconsistent conditions of power rates for the *KW* tests, it might be valuable practically to make a pairwise comparison of the fifth-order power method and *g*-and-*h* transformations. The results of the comparison indicate that the fifth-order power method and *g*-and-*h* transformations were relatively consistent, with only one case where the power difference was above .05. (i.e., .059, D22, cell size = 50, effect size = .25). More generally, the Type I error rates of the *F* tests and power rates of the *KW* tests might be very different when the distributions (i.e., D23: sk = 1.68, kt = 2.76236; and D24: sk = 2.40, kt = 6.606610) mildly departed from normality in this group of distributions without valid PDFs.

Table 4-7

Power Rates of F and Kruskal-Wallis Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Effect Size = 0.50)

Cell Size	*F* Test			Kruskal-Wallis Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	16.18	16.13	16.15	16.39	15.59	15.60
20[a]	31.45	31.36	31.38	32.40	30.46	30.66
30[a]	46.98	46.86	46.96	48.53	45.77	46.11
50[a]	72.21	72.03	72.04	74.28	71.16	71.52
10[b]	16.50	16.56	16.64	22.18	20.61	20.88
20[b]	32.20	32.14	32.14	46.86	43.33	43.91
30[b]	46.74	47.04	47.00	67.44	63.57	64.25
50[b]	72.55	72.28	72.24	91.41	88.63	89.15
10[c]	17.38	17.35	17.27	34.69	33.58	**70.41**
20[c]	32.64	32.61	32.69	69.90	68.18	**97.52**
30[c]	48.05	48.28	48.13	89.55	88.47	**99.91**
50[c]	72.60	72.41	72.54	99.29	99.06	100.00
10[d]	18.69	18.64	19.33	54.98	53.05	53.78
20[d]	33.95	33.93	33.88	91.07	89.79	89.45
30[d]	48.82	48.89	48.94	99.04	98.72	98.49
50[d]	73.00	72.90	72.78	99.99	99.99	99.98

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (bold faced) values are the minimum (maximum) of the condition. None distributions have valid PDFs.

Table 4-8

Power Rates of F and Kruskal-Wallis Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Effect Size = 0.75)

Cell Size	*F* Test			Kruskal-Wallis Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	32.99	32.93	32.94	30.92	30.16	30.14
20[a]	65.28	65.15	65.15	62.25	60.93	61.01
30[a]	85.41	85.34	85.40	83.00	81.98	82.09
50[a]	97.99	97.97	98.01	97.36	97.03	97.07
10[b]	33.92	33.87	33.83	41.28	40.19	40.18
20[b]	65.27	65.42	65.36	78.64	77.23	77.21
30[b]	84.91	84.88	84.92	94.48	93.81	93.79
50[b]	98.12	98.13	98.13	99.76	99.73	99.72
10[c]	35.75	35.93	35.80	59.69	<u>58.63</u>	**81.31**
20[c]	66.27	66.31	66.31	93.85	<u>93.41</u>	**99.40**
30[c]	85.46	85.46	85.69	99.45	99.35	99.99
50[c]	98.15	98.11	98.11	100.00	100.00	100.00
10[d]	38.68	38.68	38.89	77.85	<u>76.65</u>	**88.65**
20[d]	67.30	67.22	67.22	99.12	98.94	99.87
30[d]	85.36	85.52	85.27	99.98	99.98	100.00
50[d]	98.02	98.01	98.02	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (bold faced) values are the minimum (maximum) of the condition. None distributions have valid PDFs.

Table 4-9

Power Rates of F and Kruskal-Wallis Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Effect Size = 1.00)

Cell Size	*F* Test			Kruskal-Wallis Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	56.09	56.54	56.50	50.16	50.31	50.31
20[a]	90.49	90.47	90.52	86.01	85.98	85.99
30[a]	98.63	98.61	98.62	97.20	97.25	97.23
50[a]	99.97	99.98	99.98	99.92	99.93	99.93
10[b]	56.96	57.10	57.08	63.24	63.27	62.99
20[b]	90.61	90.58	90.57	95.19	95.37	95.23
30[b]	98.74	98.73	98.73	99.68	99.68	99.65
50[b]	99.99	99.99	99.99	100.00	100.00	100.00
10[c]	59.23	59.21	58.98	79.98	79.52	**88.32**
20[c]	90.71	90.67	90.77	99.33	99.28	99.85
30[c]	98.56	98.58	98.66	99.98	99.98	100.00
50[c]	99.99	99.99	99.98	100.00	100.00	100.00
10[d]	61.41	61.49	61.13	90.69	90.23	**98.40**
20[d]	90.23	90.22	90.07	99.93	99.92	100.00
30[d]	98.30	98.34	98.34	100.00	100.00	100.00
50[d]	99.98	99.99	99.97	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (bold faced) values are the minimum (maximum) of the condition. None distributions have valid PDFs.

Between-Subjects Design for the Third Group of Distributions Where All Transformations except the Power Method Have Valid PDFs

Type I error rates of *F* and *KW* tests. The Type I error rates of *F* and *KW* tests associated with the third-order power method, *g*-and-*h*, GLD, and the Burr transformations for the third group of distributions were reported in Table 4-10. In these distributions all the data transformation procedures except the third-order power method have valid PDFs. As results in Table 4-10 show, for the *F* tests the Type I error rates were all conservative at cell size = 10 in all distribution, at cell size = 20 in the second distribution, and at cell size = 30 in the third distribution. All other conditions had a mixture of conservative and robust Type I errors. The Burr and the third-order power transformations were all conservative. Compared in each condition across the four data transformations, all the 16 conditions either had mixed results of conservative-robust Type I errors or extreme Type I error differences (above .004). Thus, the 16 conditions were all inconsistent in Type I error rates for the *F* tests. For the *KW* test, the Type I error rates were all robust and consistent in each condition compared across data transformations as expected (Table 4-10).

Power rates of *F* and *KW* tests. The power rates at effect size = .25 level were reported in Table 4-11. Results in Table 4-11 indicate that for the *F* tests the power difference in each condition across the four data transformations were above .05 except three conditions (cell size = 10, and 20 in D31, and cell size = 10 in both D32 and D34) with differences ranging from .051 to .083, while the power differences for the *KW* tests in each condition across all data transformations were all above .26 (ranging from .269

to .835). The third-order power method was the most powerful with the Burr transformation the least powerful in both the *F* and *KW* tests (Table 4-11).

Table 4-10

Type I Error Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**2.26**	**4.37**	**4.35**	**3.84**	5.04	5.04	5.04	5.04
20[a]	**3.25**	4.54	4.54	**4.01**	5.01	5.05	5.05	5.05
30[a]	**3.70**	4.66	4.68	**4.03**	5.03	5.00	5.00	5.00
50[a]	**3.93**	4.61	4.60	**4.08**	4.86	4.84	4.84	4.84
10[b]	**2.17**	**4.30**	**4.28**	**3.80**	4.96	4.90	4.90	4.90
20[b]	**2.80**	**4.46**	**4.44**	**3.91**	4.96	5.04	5.04	5.04
30[b]	**3.39**	4.68	4.67	**4.14**	5.08	5.10	5.10	5.10
50[b]	**3.84**	4.65	4.63	**4.06**	4.82	5.06	5.06	5.06
10[c]	**2.13**	**4.45**	**4.41**	**3.87**	5.07	5.16	5.16	5.16
20[c]	**2.84**	4.67	4.63	**4.04**	5.06	5.12	5.12	5.12
30[c]	**3.46**	4.68	4.69	**4.06**	5.08	5.03	5.03	5.03
50[c]	**3.65**	4.63	4.60	**3.97**	5.00	4.95	4.95	4.95
10[d]	**2.31**	**4.37**	**4.31**	**3.75**	5.01	5.02	5.02	5.02
20[d]	**2.85**	4.53	4.52	**3.91**	4.88	5.16	5.16	5.16
30[d]	**3.14**	**4.48**	**4.45**	**3.89**	4.88	4.91	4.91	4.91
50[d]	**3.58**	4.67	4.69	**4.09**	4.89	5.05	5.05	5.05

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). In all four distributions all four data transformations except the third-order power method have valid PDFs.

Table 4-11

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 0.25)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	10.37	7.80	7.79	6.81	**67.50**	9.69	9.86	8.75
20[a]	14.70	11.35	11.33	9.89	**94.84**	15.39	15.77	13.28
30[a]	**18.50**	15.22	15.18	13.23	**99.41**	21.65	22.44	18.31
50[a]	**26.26**	22.82	22.78	19.82	**100.00**	34.63	35.85	29.15
10[b]	11.09	7.56	7.54	6.66	**74.73**	9.47	9.66	8.66
20[b]	**15.29**	11.56	11.56	10.16	**97.46**	15.78	16.23	13.97
30[b]	**19.23**	15.06	15.06	13.29	**99.86**	21.84	22.49	18.81
50[b]	**27.42**	23.46	23.42	20.58	**100.00**	35.82	37.10	31.07
10[c]	**12.26**	7.85	7.86	6.80	**54.54**	9.77	9.96	9.10
20[c]	**16.03**	11.51	11.49	9.97	**88.05**	16.09	16.52	14.54
30[c]	**20.05**	15.22	15.16	13.32	**97.93**	22.25	22.91	20.05
50[c]	**28.12**	23.25	23.19	20.23	**99.95**	36.71	37.91	32.75
10[d]	12.01	7.96	7.93	7.02	**36.66**	10.22	10.43	9.74
20[d]	**16.64**	11.66	11.66	10.30	**70.35**	16.27	16.72	15.23
30[d]	**20.43**	15.00	15.01	13.19	**88.65**	22.79	23.63	21.16
50[d]	**28.49**	23.09	23.06	20.20	**98.85**	37.39	38.82	34.70

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

The power rates at effect size = .50 were reported in Table 4-12. Results show that the power rate difference for the *F* tests were all above .098 (ranging from .098 to .194) in each condition across the four data transformations, while those for the *KW* tests ranged from .067 to .69. Again the third-order power method was the most powerful with Burr transformation the least powerful in both the *F* and *KW* tests (Table 4-12). The power rates at effect size = .75 were reported in Table 4-13. As the results in Table 4-13 show, the power rates for the *F* tests were different in each condition across the four data transformations except sample size = 50 in all distributions, with power differences ranging from .078 to .223. The power rates for the *KW* tests were inconsistent across the four data transformations only at the smaller cell size levels (i.e., at 10 and 20) in all distributions, with, however, a greater magnitude of difference (i.e., from .114 to .483). The power rates at effect size = 1.00 are reported in Table 4-14. Results in Table 4-14 indicate that the power rates for the *F* tests in each condition across the four data transformations are inconsistent (with difference from .066 to .159) at smaller cell size levels (i.e., at 10 and 20) in each distribution. The inconsistency (with difference from .188 to .238) for the *KW* tests occurred only at cell size = 10 in each distribution (Table 4-14). The power rates at effect sizes = 1.25 and 1.50 are reported in Tables 4-15 and 4-16 respectively. The results in Tables 4-15 and 4-16 indicate that the maximum difference of power rates for the *F* tests were still large (.054-.084) in each condition across data transformations when cell size = 10 in each distribution at both effect size levels. For the KW tests, however, the difference of power rates in each condition across the four data transformations were large (.056-.072) only at cell size = 10 and effect size = 1.25, and they were similar when effect sizes reached 1.50 level.

An overall inspection of the power rates of the four distributions where all of the four data transformation had valid PDFs except the power method, indicates that in more than half of the conditions (i.e., 56 out of 96) the range of power rates of the *F* tests were over .05 with an extreme value at .223, and the difference could maintain to effect size = 1.5 when cell size is small (i.e., 10). For the *KW* tests, the differences of power rates in each condition across the four data transformations were above .05 in half of the conditions (i.e., 48 out of 96), with the extreme value at .835. The third-order power transformation was in most cases the most powerful and the Burr transformation was the least powerful, and the trends were consistent in both the *F* and *KW* tests. The number of conditions with inconsistent power rates for the *F* tests reduced to 35 with extreme difference within .085 among the three data transformations with the third-order transformation excluded. And, those for the *KW* tests were 15, and within .096. The *KW* tests were consistently more powerful than the *F* tests in each data transformations compared same condition except when effect sizes were large, and/or sample sizes were large. An pairwise comparison of *g*-and-*h*, GLD, and the Burr transformations in terms of the power rate difference in each condition, indicates that these three transformations were relatively consistent, with the maximum difference within .09. The *g*-and-*h*, and GLD were particularly consistent, with the power difference in all conditions within .02. More concisely, the three procedures (i.e., *g*-and-*h*, GLD, and the Burr family distributions) that produce distributions with valid PDFs were more similar or consistent in Type-I errors for the ANOVA *F* tests and power rates in both ANOVA *F* tests and the nonparametric (*KW*) tests. The third-order power method was disparate from the other three transformations in this group of distributions.

Table 4-12

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 0.50)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**31.93**	19.46	19.54	16.57	**85.76**	25.39	26.11	21.27
20[a]	**45.44**	35.65	35.69	30.79	**99.44**	50.97	52.44	43.57
30[a]	**57.90**	51.26	51.22	44.28	**99.97**	71.56	73.14	63.59
50[a]	**75.34**	73.91	73.86	65.43	**100.00**	92.64	93.55	87.92
10[b]	**34.26**	19.67	19.75	16.92	**89.73**	26.11	26.87	22.62
20[b]	**46.83**	36.16	36.13	31.49	**99.70**	52.30	53.73	45.67
30[b]	**58.30**	50.91	50.84	44.39	**99.99**	72.33	73.96	65.68
50[b]	**75.49**	74.16	74.12	65.68	**100.00**	93.20	93.99	89.49
10[c]	**35.67**	19.57	19.60	17.13	**92.71**	26.51	27.33	23.75
20[c]	**48.62**	36.27	36.27	31.72	**99.91**	52.87	54.42	48.21
30[c]	**59.17**	51.77	51.67	45.44	**100.00**	73.88	75.45	69.03
50[c]	**76.13**	74.38	74.28	66.08	**100.00**	93.79	94.57	91.37
10[d]	**36.87**	20.17	20.19	17.48	**88.20**	27.22	28.00	25.19
20[d]	**49.31**	36.57	36.51	32.17	**99.74**	54.42	56.13	51.51
30[d]	**60.32**	51.65	51.58	45.33	**100.00**	74.90	76.80	72.02
50[d]	**76.13**	74.47	74.38	66.30	**100.00**	94.57	95.41	93.34

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

Table 4-13

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 0.75)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**54.07**	40.45	40.52	<u>34.29</u>	**93.18**	51.73	53.00	<u>44.92</u>
20[a]	**72.93**	69.26	69.16	<u>61.18</u>	**99.92**	87.01	87.99	<u>82.18</u>
30[a]	85.22	**86.16**	86.09	<u>78.43</u>	100.00	97.53	97.92	95.91
50[a]	95.60	97.08	97.10	92.84	100.00	99.96	99.97	99.87
10[b]	**55.51**	40.43	40.44	<u>34.97</u>	**94.97**	52.79	53.96	<u>47.30</u>
20[b]	**73.38**	69.45	69.41	<u>61.32</u>	**99.95**	87.76	88.70	<u>84.04</u>
30[b]	84.47	**85.52**	85.46	<u>77.66</u>	100.00	97.72	98.05	96.50
50[b]	95.47	97.16	97.16	92.93	100.00	99.95	99.96	99.89
10[c]	**57.51**	41.03	41.08	<u>36.02</u>	**96.42**	54.06	55.31	<u>50.09</u>
20[c]	**74.69**	69.92	69.86	<u>62.03</u>	**99.98**	88.83	89.84	<u>86.45</u>
30[c]	85.25	**86.05**	85.99	<u>78.55</u>	100.00	98.14	98.43	97.50
50[c]	95.38	97.21	97.22	92.79	100.00	99.97	99.98	99.95
10[d]	**58.60**	41.07	41.15	<u>36.35</u>	**97.66**	54.98	56.37	<u>52.50</u>
20[d]	**74.43**	69.79	69.70	<u>62.13</u>	**100.00**	89.52	90.72	<u>88.62</u>
30[d]	84.96	**86.22**	86.17	<u>78.46</u>	100.00	98.43	98.73	98.24
50[d]	95.16	97.31	97.32	92.98	100.00	99.99	99.99	99.98

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

Table 4-14

Power Rates of F and Kruskal-Wallis of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 1.00)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**71.06**	64.03	64.06	55.95	**96.63**	77.43	78.34	72.81
20[a]	88.60	**90.39**	90.34	83.49	99.99	98.49	98.66	97.94
30[a]	95.50	97.50	97.50	93.98	100.00	99.95	99.96	99.92
50[a]	99.45	99.78	99.79	99.16	100.00	100.00	100.00	100.00
10[b]	**72.18**	64.24	64.23	56.31	**97.49**	78.16	79.04	74.75
20[b]	88.38	**90.34**	90.28	83.47	100.00	98.77	98.94	98.40
30[b]	95.44	97.54	97.55	93.71	100.00	99.95	99.96	99.95
50[b]	99.31	99.72	99.74	99.00	100.00	100.00	100.00	100.00
10[c]	**72.62**	64.51	64.36	57.17	**97.98**	79.11	80.12	77.24
20[c]	88.46	**90.41**	90.32	83.80	100.00	98.94	99.12	98.84
30[c]	95.30	97.50	97.52	93.72	100.00	99.97	99.98	99.98
50[c]	99.21	99.74	99.76	98.92	100.00	100.00	100.00	100.00
10[d]	**73.51**	65.10	65.06	58.15	**98.64**	80.39	81.62	79.89
20[d]	87.95	90.22	**90.22**	83.51	100.00	99.12	99.30	99.22
30[d]	94.86	97.38	97.42	93.58	100.00	99.98	99.98	99.98
50[d]	99.08	99.70	99.73	98.92	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

Table 4-15

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 1.25)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**82.59**	81.76	81.66	74.26	**98.23**	91.91	92.29	91.04
20[a]	95.37	97.47	97.46	94.20	100.00	99.91	99.92	99.91
30[a]	98.72	99.53	99.54	98.49	100.00	100.00	100.00	100.00
50[a]	99.93	99.97	99.98	99.87	100.00	100.00	100.00	100.00
10[b]	**82.58**	81.71	81.64	74.34	**98.67**	92.33	92.78	92.02
20[b]	95.04	97.38	97.40	93.99	100.00	99.95	99.96	99.96
30[b]	98.60	99.51	99.52	98.44	100.00	100.00	100.00	100.00
50[b]	99.90	99.94	99.94	99.82	100.00	100.00	100.00	100.00
10[c]	**82.81**	81.87	81.72	74.45	**98.96**	93.04	93.49	93.19
20[c]	94.79	97.29	97.32	93.86	100.00	99.94	99.95	99.96
30[c]	98.45	99.46	99.49	98.34	100.00	100.00	100.00	100.00
50[c]	99.85	99.93	99.94	99.81	100.00	100.00	100.00	100.00
10[d]	**83.13**	81.92	81.86	74.76	99.27	93.63	94.18	94.41
20[d]	94.54	97.22	97.20	93.70	100.00	99.97	99.97	99.97
30[d]	98.24	99.39	99.41	98.11	100.00	100.00	100.00	100.00
50[d]	99.84	99.94	99.95	99.81	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

Table 4-16

Power Rates of F and Kruskal-Wallis Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Effect Size = 1.50)

	F Test				Kruskal-Wallis Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	89.33	**91.72**	91.63	86.25	99.09	97.73	97.80	98.02
20[a]	97.90	99.27	99.27	98.00	100.00	100.00	100.00	100.00
30[a]	99.69	99.91	99.91	99.69	100.00	100.00	100.00	100.00
50[a]	99.99	99.98	99.99	99.96	100.00	100.00	100.00	100.00
10[b]	88.98	**91.43**	91.34	85.77	99.20	97.93	98.00	98.41
20[b]	97.89	99.25	99.26	97.96	100.00	100.00	100.00	100.00
30[b]	99.55	99.84	99.86	99.52	100.00	100.00	100.00	100.00
50[b]	99.98	99.98	99.98	99.95	100.00	100.00	100.00	100.00
10[c]	89.15	**91.84**	91.72	86.13	99.46	98.31	98.40	98.79
20[c]	97.80	99.19	99.22	97.86	100.00	100.00	100.00	100.00
30[c]	99.52	99.84	99.85	99.51	100.00	100.00	100.00	100.00
50[c]	99.96	99.97	99.98	99.95	100.00	100.00	100.00	100.00
10[d]	89.36	**91.86**	91.71	86.14	99.58	98.46	98.66	99.01
20[d]	97.45	99.10	99.11	97.68	100.00	100.00	100.00	100.00
30[d]	99.38	99.83	99.83	99.50	100.00	100.00	100.00	100.00
50[d]	99.96	99.98	99.98	99.96	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk= 2.75, kt = 70.0; [c] D33: sk= 3.00, kt = 80.0; [d] D34: sk= 3.25, kt = 90.0. Underscored (bold faced) values are the minimum (maximum) of the condition. In all four distributions all four data transformations except the third-order power method have valid PDFs.

Within-Subjects Design for the First Group of Distributions with Valid PDFs

Type I error rates for the *F* and *FR* tests at correlation = .25 level. The Type I error rates of the *F* and *FR* tests at post correlation = .25 level in the first group of distribution with valid PDFs were reported in Table 4-17. Bold faced values denoted Type-I errors were outside the interval (.045, .055; denoted the same hereafter). As the results in Table 4-17 indicate, the Type I error rates for the *F* tests were all robust in the first three distributions (i.e., D11: sk = .0, kt = .0; D12: sk = .0, kt = 1.0; and D13: sk = 1.0, kt = 2.0). They were all robust in the fourth distribution except cell size = 10 (i.e., D14: sk = 2.0, kt = 8.0); but conservative in the last two distribution (i.e., D15: sk = 3.0, kt = 20.0, and D16: sk = 3.9, kt = 40.0) except cell size = 50. Compared in each condition across transformations, a mixture of robust-conservative Type I error rates was found at cell size = 50 in the last distribution (D16), in which two additional cases (cell sizes = 10 and 20) were all conservative but with extreme Type I error differences (above .004). Thus, Type I error rates were different in 3 out of 24 conditions for the *F* tests. For the *FR* tests, the Type I error rates were all robust except for a cell size of 10 in the third distribution. Compared in each condition across the four data transformations, there were no conditions with mixed conservative-robust Type I errors or conditions with extreme Type I error differences for the *FR* tests. Thus, the Type I error rates of the *FR* tests were similar in each conditions across the four data transformations at correlation = .25.

Table 4-17

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%, Correlation = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	4.99	4.99	5.01	5.01	5.40	5.40	5.40	5.40
20[a]	4.93	4.93	4.93	4.95	5.08	5.08	5.08	5.08
30[a]	5.02	5.02	5.02	5.01	5.21	5.21	5.21	5.21
50[a]	5.04	5.04	5.04	5.04	4.90	4.90	4.91	4.91
10[b]	4.78	4.79	4.78	4.78	5.13	5.13	5.13	5.13
20[b]	4.91	4.91	4.91	4.90	5.14	5.14	5.13	5.13
30[b]	5.02	5.02	5.02	5.02	5.27	5.27	5.27	5.27
50[b]	4.95	4.94	4.95	4.94	5.00	5.00	5.00	5.00
10[c]	4.78	4.79	4.77	4.74	**5.57**	**5.57**	**5.57**	**5.57**
20[c]	4.96	4.99	4.96	4.95	5.10	5.10	5.10	5.10
30[c]	4.95	4.93	4.95	4.92	5.25	5.24	5.25	5.25
50[c]	4.90	4.90	4.89	4.91	4.97	4.97	4.98	4.98
10[d]	**4.30**	**4.34**	**4.34**	**4.31**	5.42	5.42	5.42	5.42
20[d]	4.65	4.74	4.70	4.69	5.04	5.03	5.04	5.05
30[d]	4.53	4.60	4.58	4.57	5.23	5.21	5.22	5.23
50[d]	4.83	4.80	4.80	4.81	5.05	5.04	5.03	5.03
10[e]	**3.57**	**3.88**	**3.87**	**3.88**	5.37	5.33	5.34	5.34
20[e]	**4.10**	**4.17**	**4.16**	**4.13**	5.03	5.06	5.05	5.05
30[e]	**4.29**	**4.39**	**4.38**	**4.35**	5.15	5.14	5.14	5.14
50[e]	4.58	4.55	4.57	4.55	5.02	5.03	5.04	5.05
10[f]	**2.79**	**3.59**	**3.54**	**3.55**	5.09	5.09	5.08	5.09
20[f]	**3.49**	**4.07**	**4.01**	**4.00**	5.06	5.06	5.05	5.05
30[f]	**3.99**	**4.36**	**4.34**	**4.31**	5.19	5.15	5.15	5.16
50[f]	**4.19**	4.53	4.54	4.54	5.00	5.03	5.01	5.01

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Bold faced values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .40 level. The Type I error rates of the *F* and *FR* tests at a post correlation = .40 for the first group of six distributions were reported in Table 4-18. As the results in Table 4-18 show, the Type I error rates of the *F* tests were all robust in the first and third distributions (D11: sk = .0, kt = .0 and D13: sk = 1.0, kt = 2.0) and robust at two lager cell sizes (i.e., 30 and 50) in the fourth distribution (D14: sk = 2.0, kt = 8.0). In the last two distributions (D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0) the Type I error rates were conservative except two cases (cell size = 50 in D15, and cell size = 20 in D16) with a mixture of robust-conservative results. In the second distribution, the Burr transformation were all liberal while the other three transformations were robust, which made the four conditions inconsistent. There were three conditions (i.e., cell size = 20, in D15, and cell sizes = 30 and 50 in D16) where the results were all conservative but with Type I error differences above 0.004. Thus, an overall comparison in each condition across data transformations results in 9 out of 24 conditions where type I error rates were inconsistent. For the *FR* tests, two conditions (cell sizes = 10 and 50) in the second distribution (D12) and another (cell size = 10) in the last distribution were found with mixed robust-liberal results while the rest conditions were all robust. Thus, 3 out of 24 conditions were classified as inconsistent in Type I error rates for the *FR* tests by comparison across data transformations in each condition.

Table 4-18

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	5.03	5.03	5.03	5.04	5.39	5.39	5.39	5.39
20[a]	4.98	4.98	4.98	5.00	5.04	5.04	5.04	5.04
30[a]	5.08	5.08	5.06	5.05	5.19	5.19	5.19	5.18
50[a]	4.83	4.83	4.82	4.84	4.84	4.84	4.84	4.84
10[b]	5.07	5.07	5.07	**6.44**	5.37	5.37	5.37	**6.00**
20[b]	4.79	4.78	4.79	**6.32**	5.04	5.04	5.05	5.42
30[b]	4.93	4.93	4.92	**6.43**	5.06	5.06	5.06	5.50
50[b]	5.10	5.10	5.11	**6.27**	5.01	5.01	5.01	**5.56**
10[c]	4.81	4.82	4.81	4.84	5.35	5.35	5.34	5.34
20[c]	4.85	4.86	4.88	4.85	5.03	5.02	5.03	5.03
30[c]	4.79	4.80	4.80	4.78	5.17	5.17	5.17	5.17
50[c]	4.78	4.78	4.78	4.79	4.87	4.87	4.87	4.87
10[d]	**4.14**	**4.19**	**4.17**	**4.16**	5.36	5.37	5.36	5.36
20[d]	**4.38**	**4.38**	**4.38**	**4.37**	4.97	4.98	4.97	4.97
30[d]	4.86	4.83	4.87	4.84	5.20	5.20	5.20	5.19
50[d]	4.72	4.69	4.72	4.68	4.89	4.89	4.89	4.89
10[e]	**3.44**	**3.82**	**3.77**	**3.74**	5.36	5.38	5.38	5.37
20[e]	**3.97**	**4.33**	**4.33**	**4.38**	5.03	5.00	5.00	4.99
30[e]	**4.11**	**4.29**	**4.26**	**4.30**	5.26	5.25	5.26	5.27
50[e]	**4.49**	4.61	4.60	4.58	5.03	5.07	5.06	5.08
10[f]	**3.28**	**3.56**	**3.43**	**3.43**	**5.52**	5.14	5.14	5.14
20[f]	5.13	**3.81**	**3.83**	**3.82**	5.27	4.92	4.94	4.92
30[f]	**7.16**	**4.24**	**4.19**	**4.20**	5.38	5.23	5.22	5.23
50[f]	**11.28**	**4.49**	**4.44**	**4.45**	5.26	5.15	5.17	5.19

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Bold faced values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .55 level. The results of Type I error rates for the six distributions with valid PDFs at post correlation = .55 were reported in Table 4-19. As Table 4-19 indicates, the Type I error rates of the *F* tests were all robust in the first three distributions (D11: sk = .0, kt = .0; D12: sk = .0, kt = 1.0; and D13: sk = 1.0, kt = 2.0), while they were all conservative in the last distribution (D16: sk = 3.9, kt = 40.0), and conservative in the fifth distribution (D15, D15: sk = 3.0, kt = 20.0) except cell size at 50. In the fourth distribution, the Type I error rates were all conservative at cell size = 10, and robust at cell sizes = 30 and 50 with mixed robust-conservative Type I errors at one condition (cell size = 20). Among the all conservative conditions in the last two distributions (i.e., D15, D16), five of them were with differences above .004. Thus a comparison of Type I error rates of the *F* test in each condition across the four data transformations results in six inconsistent conditions (out of 24). For the *FR* tests, the Type I error rates were all robust except in one condition (cell size = 10 in D16) where they were all liberal, but with the difference within .004. Thus the Type I error rates of the *FR* tests were all consistent compared in each condition across the four data transformations at the .55 post correlation level.

Table 4-19

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .55)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	4.88	4.88	4.87	4.89	5.13	5.13	5.13	5.13
20[a]	4.96	4.96	4.97	4.95	5.04	5.04	5.04	5.04
30[a]	5.14	5.14	5.14	5.13	5.34	5.34	5.34	5.34
50[a]	5.16	5.16	5.14	5.14	5.09	5.09	5.09	5.09
10[b]	4.86	4.86	4.84	4.84	5.36	5.37	5.36	5.36
20[b]	5.06	5.07	5.04	5.06	5.14	5.14	5.14	5.14
30[b]	4.88	4.88	4.89	4.87	5.14	5.14	5.14	5.14
50[b]	4.81	4.81	4.83	4.80	4.88	4.88	4.87	4.87
10[c]	4.65	4.66	4.62	4.65	5.27	5.22	5.26	5.27
20[c]	4.66	4.69	4.67	4.68	4.91	4.93	4.91	4.91
30[c]	4.89	4.87	4.88	4.82	5.19	5.21	5.19	5.18
50[c]	4.90	4.90	4.90	4.94	4.88	4.85	4.87	4.88
10[d]	**3.92**	**4.00**	**3.99**	**4.01**	5.36	5.36	5.36	5.36
20[d]	4.51	4.54	4.52	**4.50**	4.90	4.90	4.90	4.91
30[d]	4.71	4.72	4.70	4.71	5.30	5.29	5.30	5.30
50[d]	4.76	4.74	4.76	4.73	5.17	5.17	5.17	5.17
10[e]	**3.28**	**3.71**	**3.68**	**3.64**	5.22	5.19	5.19	5.18
20[e]	**3.86**	**4.18**	**4.16**	**4.15**	5.04	5.08	5.08	5.05
30[e]	**4.28**	**4.40**	**4.42**	**4.40**	5.39	5.36	5.36	5.37
50[e]	4.52	4.64	4.63	4.67	5.17	5.16	5.16	5.16
10[f]	**2.40**	**3.49**	**3.37**	**3.33**	**5.56**	**5.54**	**5.54**	**5.53**
20[f]	**3.10**	**3.69**	**3.63**	**3.63**	4.92	4.92	4.94	4.95
30[f]	**3.67**	**4.38**	**4.30**	**4.28**	5.42	5.33	5.35	5.36
50[f]	**3.89**	**4.35**	**4.35**	**4.33**	4.93	4.95	4.93	4.93

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Light colored values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .70 level. The Type I error rates of *F* and *FR* tests at the post correlation .70 in the six distributions with valid PDFs were reported in Table 4-20. As the results in Table 4-20 show, Type I error rates of the *F* tests were all robust in the first three distributions (i.e., D11: sk = .0, kt = .0; D12: sk = .0, kt = 1.0; and D13: sk = 1.0, kt = 2.0), and all conservative in the last distribution (i.e., D16: sk = 3.9, kt = 40.0), while they were conservative except cell size = 50 in the other two distributions (i.e., D14: sk = 2.0, kt = 8.0; and D15: sk = 3.0, kt = 20.0). Mixed robust-conservative Type I errors occurred at cell size = 50 in the fifth distribution(D15) , while the Type I error differences were above .004 at the three smaller cell sizes (10, 20, and 30) in the sixth distribution (D16), and at cell size = 10 in the fifth distribution (D15). Thus the Type I error rates of the *F* tests were classified as inconsistent in five (out of 24) conditions compared across the four data transformations in each condition. For the *FR* tests, the Type I error rates were all robust in the 24 conditions except cell size = 10 in the third distribution (D13), where the *g*-and-*h* and GLD transformations were liberal and the Burr and the third-order power transformations were robust, hence the Type I error rates of the *FR* tests were found inconsistent in 1 out of 24 conditions compared across data transformations by each condition.

Table 4-20

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .70)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	5.05	5.05	5.05	5.06	5.33	5.33	5.33	5.33
20[a]	5.01	5.01	5.01	4.99	5.09	5.09	5.09	5.09
30[a]	4.84	4.84	4.83	4.85	5.18	5.18	5.18	5.18
50[a]	4.99	4.99	4.98	4.99	4.95	4.95	4.95	4.95
10[b]	4.72	4.73	4.72	4.73	5.23	5.24	5.23	5.23
20[b]	5.19	5.20	5.20	5.17	5.14	5.14	5.15	5.15
30[b]	4.88	4.88	4.88	4.90	5.07	5.07	5.07	5.07
50[b]	5.01	5.02	4.99	4.99	4.99	4.99	5.00	5.00
10[c]	4.72	4.74	4.68	4.72	5.15	5.15	5.15	5.15
20[c]	4.84	4.83	4.84	4.84	5.25	5.24	5.25	5.25
30[c]	4.84	4.85	4.82	4.87	5.17	5.17	5.18	5.18
50[c]	4.99	4.98	5.02	4.99	4.96	4.96	4.96	4.96
10[d]	**3.89**	**3.91**	**3.93**	**3.89**	5.49	**5.51**	**5.51**	5.50
20[d]	**4.35**	**4.35**	**4.36**	**4.36**	5.03	5.03	5.03	5.03
30[d]	**4.32**	**4.39**	**4.37**	**4.39**	5.22	5.21	5.20	5.21
50[d]	4.75	4.68	4.70	4.66	5.03	5.00	5.00	5.02
10[e]	**3.17**	**3.60**	**3.54**	**3.50**	5.29	5.29	5.29	5.30
20[e]	**3.78**	**4.04**	**4.01**	**4.04**	5.07	5.09	5.09	5.08
30[e]	**4.15**	**4.27**	**4.29**	**4.36**	5.23	5.20	5.22	5.23
50[e]	**4.36**	4.63	4.63	4.62	5.03	5.03	5.04	5.04
10[f]	**2.24**	**3.41**	**3.32**	**3.25**	5.20	5.20	5.20	5.20
20[f]	**3.16**	**3.96**	**3.92**	**3.89**	5.12	5.12	5.12	5.11
30[f]	**3.34**	**4.01**	**4.03**	**4.02**	5.06	5.06	5.06	5.07
50[f]	**3.88**	**4.17**	**4.15**	**4.16**	4.97	5.00	4.97	4.96

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Bold faced values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .85 level. The Type I error rates of *F* and *FR* tests at the post correlation .85 in the six distributions with valid PDFs were reported in Table 4-21. As the results in Table 4-20 show, the Type I error rates of the *F* tests were all robust in the first three distributions (i.e., D11: sk = .0, kt = .0; D12: sk = .0, kt = 1.0; and D13: sk = 1.0, kt = 2.0), all conservative in the last distribution (D16: sk = 3.9, kt = 40.0), while they were conservative at cell size = 10 but robust at cell sizes = 30 and 50 in the fourth distribution (D14: sk = 2.0, kt = 8.0), and conservative at the three smaller cell size levels (10, 20, and 30) in the fifth distribution (D15: sk = 3.0, kt = 20.0). Conservative-robust mixed results were found in two conditions (cell size = 20 in D14, and cell size = 50 in D15), while extreme type I error differences were found in the four all-conservative conditions (cell size = 10 in D14, cell sizes = 10, 20, and 30 in D15). Thus the Type I error rates were classified as inconsistent in six (out of 24) conditions by a comparison across the four data transformations in each condition. For the *FR* tests, the Type I error rates were liberal but similar across the four data transformations at cell size = 10 in the second distribution, while robust-liberal mixed Type I error rates across data transformations were found in one condition (cell size = 10) in the fourth distribution (D14). As such, the Type I error rates of the *FR* tests were considered inconsistent in only one out of the 24 conditions.

An overall inspection on the five post correlation levels of the six distributions with valid PDFs indicates that that there were 29 conditions where the Type error rates of the *F* tests were inconsistent across the four data transformations, of which the majority occurred in the last two distribution (i.e., D15 and D16) with the third-order power transformation to the most conservative side departed from the other three. If the third-order power transformation was ignored, the number of conditions with inconsistent Type I errors reduced to five. For the *FR* tests, there were only five conditions where the Type I error rates were different across the four data transformations. The number of inconsistent type I errors reduced to four across the three data transformations if the third-order power transformation was excluded. More generally, the Type I error rates of the *F* tests might be dissimilar or inconsistent when the generated distributions were skewed and heavy tailed and close to associated boundary conditions with valid PDFs (e.g., D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0). This trend was consistent with results of the same distributions in the one-way between-subjects design.

Table 4-21

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .85)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	5.02	5.02	5.02	5.03	5.46	5.46	5.32	5.46
20[a]	5.01	5.01	5.14	5.00	4.99	4.99	5.00	5.00
30[a]	5.12	5.12	5.19	5.13	5.17	5.17	5.24	5.16
50[a]	4.98	4.98	5.04	5.04	4.92	4.92	4.91	4.92
10[b]	5.01	5.01	4.99	4.99	**5.51**	**5.51**	**5.51**	**5.51**
20[b]	4.93	4.93	4.92	4.93	5.07	5.07	5.07	5.07
30[b]	4.95	4.97	4.95	4.94	5.24	5.24	5.24	5.24
50[b]	4.95	4.96	4.94	4.94	4.94	4.94	4.94	4.94
10[c]	4.82	4.81	4.81	4.85	5.42	5.42	5.42	5.42
20[c]	4.71	4.71	4.74	4.72	4.99	4.99	5.00	4.99
30[c]	5.00	5.00	4.97	4.97	5.16	5.16	5.15	5.16
50[c]	4.72	4.73	4.72	4.74	4.87	4.87	4.87	4.87
10[d]	**4.03**	**4.07**	**4.05**	**4.10**	**5.50**	5.50	5.50	5.50
20[d]	**4.46**	4.57	4.51	4.60	5.06	5.06	5.06	5.06
30[d]	4.58	4.58	4.58	4.61	5.24	5.25	5.25	5.25
50[d]	4.56	4.57	4.60	4.59	4.94	4.93	4.93	4.93
10[e]	**3.09**	**3.51**	**3.48**	**3.42**	5.32	5.34	5.33	5.33
20[e]	**3.87**	**4.12**	**4.12**	**4.10**	5.20	5.24	5.23	5.23
30[e]	**4.09**	**4.32**	**4.31**	**4.32**	5.06	5.08	5.09	5.08
50[e]	**4.37**	4.52	4.53	4.58	4.91	4.91	4.90	4.89
10[f]	**2.03**	**3.34**	**3.20**	**3.12**	5.30	5.28	5.29	5.30
20[f]	**2.88**	**3.72**	**3.67**	**3.66**	5.00	4.99	5.01	5.01
30[f]	**3.57**	**3.99**	**3.98**	**3.99**	5.08	5.13	5.12	5.08
50[f]	**3.97**	**4.25**	**4.30**	**4.29**	4.90	4.90	4.89	4.89

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Bold faced values are out of interval (4.50, 5.50). All six distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .25 level. The Power Rates of the *F* and *FR* tests in the six distributions with valid PDFs at effect sizes = .25, .50, and .75 were reported in Tables 4-22 to 4-23, respectively for the .25 post correlation. The bold faced (underscored) values were maximum (minimum) power rate among the four data transformation in a condition where the maximum power rate difference (i.e. range) among the four data transformations was above .05 (denoted the same hereafter). As the results in Table 4-22 show, the power rates of the *F* tests were similar in all the six distributions, while the power rates of the *FR* tests were similar in all except the last two distributions (i.e., D15: sk = 3.0, kt = 20.0; D16: sk = 3.9, kt = 40.0). More specifically, the power rates of the *FR* tests were different at the larger cell size levels (i.e., 20, 30, and 50) in the fifth distribution, and they were all different at all conditions in the sixth distribution. In all the conditions where the power rates of the *FR* tests were different, the third order power transformation was the most, and the *g*-and-*h* transformation was the least powerful, with the extreme difference at .492. At effect size = 0.50, as the results in Table 4-23 indicate, compared across data transformations in each condition, the power rates of the *F* tests were similar in all distributions except one condition (cell size = 10, in D16) with a difference of .063. The power rates of the *FR* tests were also different in the last two distributions, but at smaller cell size levels (cell sizes = 10, 20, and 30 in D15; and cell sizes = 10, and 20 in D16), with the extreme difference at .275. At effect size = 0.75, as the results in Table 4-24 demonstrate, the power rates of the *F* test were also similar compared across the four data transformations. The power rates of the *FR* tests were similar in all conditions except two conditions (cell size = 10) in the last two distributions, with the largest difference at .131. An overall inspection of power rates of the six distributions at post correlation .25, the power rates of the *F* tests were different in only one condition, while the power rates of the *FR* tests were different in 14 conditions in the last two distributions, in which the third order power transformation is the most powerful and the *g*-and-*h* transformation is the least powerful. The Power rates for effect sizes greater than .75 were similar in each condition for both the *F* and *FR* tests, as compared across the four data transformations. Therefore, they were not reported.

Table 4-22

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .25; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	8.53	8.53	8.52	8.58	8.27	8.27	8.28	8.28
20[a]	12.86	12.86	12.89	12.89	11.13	11.13	11.13	11.15
30[a]	17.49	17.49	17.47	17.50	14.70	14.70	14.74	14.69
50[a]	27.84	27.84	27.85	27.86	22.09	22.09	22.10	22.03
10[b]	8.80	8.79	8.81	8.77	8.79	8.76	8.82	8.81
20[b]	13.01	13.02	13.03	13.01	11.74	11.70	11.79	11.75
30[b]	17.38	17.39	17.39	17.37	15.82	15.77	15.86	15.84
50[b]	27.85	27.85	27.88	27.87	23.84	23.70	23.97	23.96
10[c]	8.44	8.43	8.43	8.45	8.92	8.85	9.00	8.82
20[c]	12.72	12.68	12.77	12.67	12.49	12.40	12.73	12.29
30[c]	17.86	17.82	17.82	17.78	17.39	17.21	17.72	17.13
50[c]	28.03	28.06	28.04	28.07	26.81	26.46	27.45	26.21
10[d]	8.50	8.52	8.49	8.47	11.05	10.76	10.68	11.18
20[d]	13.50	13.55	13.49	13.55	17.94	17.15	17.13	18.33
30[d]	17.93	17.93	17.97	17.94	25.81	24.75	24.63	26.71
50[d]	28.91	28.83	28.80	28.79	41.84	40.19	40.07	43.69
10[e]	8.95	8.58	8.59	8.68	17.51	13.62	14.12	15.53
20[e]	14.54	13.92	13.88	13.82	**32.89**	23.75	24.94	28.20
30[e]	19.79	19.31	19.29	19.23	**49.83**	36.73	38.63	43.93
50[e]	30.27	29.72	29.70	29.65	**73.92**	57.34	60.03	67.17
10[f]	11.04	9.11	9.13	9.12	**42.27**	15.72	19.03	19.79
20[f]	16.68	14.84	14.86	14.88	**75.41**	28.48	35.89	37.57
30[f]	22.11	20.17	20.19	20.16	**92.24**	43.09	54.31	56.62
50[f]	32.35	30.89	30.82	30.82	**99.45**	66.97	79.66	81.88

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-23

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .25; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	20.55	20.55	20.55	20.56	17.68	17.68	17.71	17.70
20[a]	41.40	41.40	41.43	41.41	32.70	32.70	32.76	32.69
30[a]	60.72	60.72	60.75	60.70	48.97	48.97	49.10	49.08
50[a]	84.55	84.55	84.56	84.55	72.79	72.79	72.87	72.78
10[b]	20.97	20.96	20.96	20.98	19.14	19.06	19.22	19.18
20[b]	41.40	41.46	41.41	41.45	35.43	35.16	35.59	35.51
30[b]	60.00	60.02	60.02	60.05	52.19	51.99	52.46	52.32
50[b]	84.32	84.31	84.31	84.33	76.72	76.46	77.03	76.86
10[c]	21.32	21.29	21.43	21.19	20.58	20.38	20.93	20.16
20[c]	42.04	42.02	42.00	41.95	39.96	39.55	40.74	39.17
30[c]	60.69	60.72	60.65	60.74	58.97	58.37	60.02	58.06
50[c]	84.77	84.72	84.80	84.71	83.14	82.72	83.99	82.38
10[d]	23.15	23.00	23.06	22.88	29.82	28.59	28.72	29.41
20[d]	44.17	44.01	44.05	43.90	58.29	56.33	56.28	58.03
30[d]	62.17	62.19	62.22	62.17	79.75	77.82	77.89	79.34
50[d]	85.00	85.12	85.13	85.09	96.31	95.57	95.68	96.37
10[e]	27.91	25.81	25.83	25.79	**48.33**	<u>38.82</u>	40.14	41.42
20[e]	48.13	46.90	46.83	46.74	**83.32**	<u>72.97</u>	74.62	76.54
30[e]	64.10	63.77	63.71	63.63	**96.22**	<u>91.11</u>	92.14	93.24
50[e]	85.17	85.36	85.38	85.33	99.88	99.35	99.52	99.65
10[f]	**34.97**	<u>28.67</u>	28.82	28.79	**72.64**	<u>45.15</u>	49.64	50.17
20[f]	53.00	49.85	49.75	49.74	**97.28**	<u>80.86</u>	85.53	85.98
30[f]	66.95	66.02	65.69	65.70	99.86	95.32	97.21	97.37
50[f]	84.93	85.23	85.21	85.24	100.00	99.85	99.94	99.95

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-24

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%, Correlation = .25; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	42.97	42.97	42.94	42.99	35.25	35.25	35.29	35.32
20[a]	78.43	78.43	78.44	78.40	65.82	65.82	65.91	65.80
30[a]	93.75	93.75	93.74	93.74	85.47	85.47	85.55	85.42
50[a]	99.64	99.64	99.64	99.64	97.93	97.93	97.97	97.89
10[b]	43.86	43.87	43.90	43.91	37.61	37.46	37.79	37.73
20[b]	78.49	78.50	78.49	78.44	69.49	69.25	69.76	69.63
30[b]	93.49	93.49	93.47	93.49	88.12	87.97	88.27	88.21
50[b]	99.62	99.61	99.62	99.61	98.67	98.64	98.70	98.67
10[c]	44.62	44.59	44.63	44.47	41.73	41.37	42.43	40.89
20[c]	78.63	78.68	78.64	78.70	76.00	75.61	76.81	75.14
30[c]	93.65	93.64	93.68	93.62	92.71	92.45	93.17	92.19
50[c]	99.63	99.63	99.63	99.62	99.52	99.49	99.58	99.45
10[d]	48.42	48.16	48.29	48.09	56.86	55.14	55.44	55.49
20[d]	79.51	79.57	79.55	79.58	90.98	90.18	90.27	90.34
30[d]	93.18	93.26	93.27	93.35	98.86	98.65	98.71	98.73
50[d]	99.49	99.49	99.50	99.48	100.00	100.00	99.99	99.99
10[e]	53.88	52.46	52.40	52.26	**75.21**	<u>68.13</u>	68.99	69.24
20[e]	80.40	80.57	80.53	80.48	98.21	96.53	96.80	96.87
30[e]	92.50	92.93	92.95	92.94	99.93	99.77	99.82	99.83
50[e]	99.12	99.16	99.17	99.19	100.00	100.00	100.00	100.00
10[f]	59.48	55.27	55.17	55.14	**87.54**	<u>74.42</u>	76.32	76.45
20[f]	81.05	81.31	81.25	81.23	99.73	98.27	98.59	98.60
30[f]	91.56	92.48	92.52	92.54	99.99	99.94	99.97	99.97
50[f]	98.61	98.86	98.92	98.93	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .40 level. The power rates of the *F* and *FR* tests in the six distributions at post correlation .40 were reported in Tables 4-25 to 4-30, respectively for effect sizes at .25, .50, .70, 1.0, 1.25 and 1.50. As the results in Table 4-25 show, at effect size = .25 level, the power rates of the *F* tests were similar, except in two conditions (cell sizes = 30 and 50) in the sixth distribution (D16: sk = 3.9, kt = 40.0) with the largest difference at .148. Power rates of the *FR* tests were similar in all conditions in the first four distributions and at cell size = 10 in the fifth distribution (D15: sk = 3.0, kt = 20.0). In the seven inconsistent conditions in the last two distribution (D15 and D16), the difference of power rates of the *FR* tests ranged from .10 to .422 across the data transformations (Table 4-25). At effect size = 0.50 level (Table 4-26), inconsistent power rates of the *F* tests occurred in six conditions at cell sizes = 20, 30, and 50 in the second and last distributions (D12: sk = .0, kt = 1.0; and D16: sk = 3.9, kt = 40.0) with power differences .088 -.19. Inconsistent power rates for the *FR* tests occurred in seven conditions associated with three distributions (i.e., cell sizes = 20, 30, and 50 in D12; and cell sizes = 10 and 20 in both D15 and D16), with power differences .053-.196 (Table 4-26). At effect size = .75 level (Table 4-27), inconsistent power rates of the *F* tests across the data transformations occurred in six conditions (cell sizes = 10, 20, and 30) in both the second and six distributions with power differences from .051-.148, while the inconsistent power rates of *FR* tests occurred in four conditions (cell sizes = 10, and 20 in the D12; and cell size = 10 in both D15 and D16) with power difference .052 -.076 (Table 4-27). At effect size = 1.0 (Table 4-28), inconsistent power rates of the *F* tests were found in three conditions (cell size = 10 in D12, and cell sizes = 10 and 20 in D16) with differences from .086 -.121, while the power rates of the *FR* tests were inconsistent in one condition (cell size = 10 in D12) with a difference at .086 (Table 4-28). At effect size = 1.25 level (Table 4-29), the power rates of the *F* tests were inconsistent across the data transformations in two conditions (cell size = 10 in both D12 and D16) with a difference at .076 and .102, respectively, while in one condition (cell size = 10 in D12) the power rates were inconsistent with the difference at .081 (Table 4-29). At effect size = 1.50 level (Table 4-30), the power rates of the *F* tests were inconsistent across the data transformations in one condition (cell size = 10 in D16) with the difference at .077, while the power rates of the *FR* tests were similar in all conditions of the sixth distribution. An overall inspection of the power rates across data transformations at post correlation = .40 level reveals that there were 20 conditions associated with the second and last distributions, in which the power rates of the *F* tests were inconsistent, and the third-order power transformation was the least powerful while the *g*-and-*h* or Burr transformation was the most powerful when these conditions were generally associated with the sixth distribution (D16). For the *FR* tests, there were also 20 conditions with inconsistent power rates across data transformations. The inconsistent conditions, however, were often associated with the last two distributions (D15 and D16). In the inconsistent cases associated with the fifth and sixth distributions, the *g*-and- *h* transformation was the least powerful while the third-order power method was generally the most powerful.

Table 4-25

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	9.42	9.42	9.44	9.40	8.74	8.74	8.73	8.74
20[a]	15.20	15.20	15.18	15.12	12.72	12.72	12.72	12.69
30[a]	20.99	20.99	21.00	21.02	17.45	17.45	17.47	17.52
50[a]	34.09	34.09	34.07	34.11	26.60	26.60	26.63	26.61
10[b]	9.27	9.25	9.27	9.96	9.12	9.09	9.13	9.16
20[b]	15.09	15.07	15.08	14.25	13.22	13.15	13.27	12.47
30[b]	21.24	21.23	21.26	18.88	18.67	18.58	18.74	17.06
50[b]	34.09	34.09	34.06	28.18	28.95	28.79	29.11	25.43
10[c]	9.43	9.39	9.42	9.39	9.91	9.88	10.00	9.85
20[c]	15.10	15.10	15.07	15.08	14.92	14.82	15.20	14.91
30[c]	21.64	21.63	21.65	21.67	21.41	21.21	21.82	21.34
50[c]	34.27	34.30	34.23	34.23	33.17	32.81	33.89	32.99
10[d]	9.65	9.56	9.57	9.50	12.76	12.58	12.39	13.21
20[d]	16.16	16.02	16.08	16.01	21.70	21.33	21.01	23.26
30[d]	22.84	22.76	22.79	22.75	32.57	32.02	31.48	34.88
50[d]	35.35	35.32	35.32	35.27	51.64	50.86	50.07	55.45
10[e]	10.42	10.13	10.14	10.13	20.86	16.36	16.99	19.06
20[e]	17.88	17.28	17.26	17.23	**40.83**	30.82	32.21	36.78
30[e]	24.44	23.79	23.71	23.63	**59.59**	46.57	48.57	55.08
50[e]	37.39	37.12	37.07	36.98	**83.90**	70.82	73.41	79.91
10[f]	11.36	11.02	11.09	11.13	**44.87**	18.97	23.24	24.22
20[f]	14.05	18.18	18.17	18.29	**79.09**	36.89	46.11	48.14
30[f]	17.18	**25.44**	25.23	25.32	**94.15**	55.24	66.92	69.19
50[f]	24.11	38.82	38.72	**38.94**	**99.69**	80.39	89.87	91.24

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-26

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	25.12	25.12	25.16	25.15	21.15	21.15	21.15	21.16
20[a]	50.42	50.42	50.43	50.41	40.25	40.25	40.26	40.28
30[a]	70.71	70.71	70.69	70.69	58.64	58.64	58.72	58.56
50[a]	92.10	92.10	92.11	92.11	82.57	82.57	82.59	82.61
10[b]	25.55	25.54	25.57	22.85	22.55	22.44	22.64	20.34
20[b]	51.14	51.11	**51.17**	<u>41.97</u>	43.14	42.97	**43.29**	<u>37.96</u>
30[b]	**71.18**	71.17	71.17	<u>59.05</u>	62.08	61.85	**62.35**	<u>55.50</u>
50[b]	92.05	92.04	**92.06**	<u>83.26</u>	85.75	85.47	**85.93**	<u>80.14</u>
10[c]	26.29	26.27	26.31	26.15	25.23	25.01	25.61	24.96
20[c]	51.39	51.42	51.39	51.45	49.25	48.81	49.89	48.72
30[c]	71.30	71.30	71.25	71.32	69.48	69.17	70.35	69.25
50[c]	92.27	92.24	92.24	92.23	91.29	91.07	91.88	91.04
10[d]	29.03	28.69	28.73	28.53	36.63	35.51	35.42	36.46
20[d]	53.65	53.57	53.64	53.57	69.32	67.95	67.80	69.66
30[d]	72.48	72.47	72.51	72.46	88.42	87.57	87.44	88.87
50[d]	91.85	91.82	91.84	91.88	98.74	98.60	98.55	98.88
10[e]	34.66	32.63	32.62	32.51	**57.27**	<u>47.76</u>	49.12	50.46
20[e]	57.18	56.64	56.48	56.41	**90.16**	<u>83.17</u>	84.40	85.58
30[e]	74.11	74.03	74.00	73.89	98.54	96.36	96.83	97.21
50[e]	91.73	92.06	92.05	92.06	99.98	99.89	99.92	99.93
10[f]	33.20	36.12	36.25	36.43	**74.38**	<u>54.83</u>	58.90	59.49
20[f]	<u>45.17</u>	**59.59**	59.37	59.53	**97.89**	<u>89.33</u>	92.02	92.34
30[f]	<u>56.50</u>	75.41	75.30	**75.48**	99.90	98.36	98.95	99.04
50[f]	<u>75.71</u>	91.50	91.59	**91.70**	100.00	99.98	99.99	99.99

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-27

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	52.36	52.36	52.38	52.41	42.72	42.72	42.79	42.82
20[a]	87.30	87.30	87.30	87.34	76.32	76.32	76.28	76.22
30[a]	97.65	97.65	97.64	97.64	92.45	92.45	92.48	92.47
50[a]	99.95	99.95	99.95	99.96	99.45	99.45	99.45	99.45
10[b]	**52.99**	52.97	52.98	44.27	45.02	44.86	**45.22**	39.02
20[b]	**87.33**	87.33	87.31	76.56	79.04	78.92	**79.27**	71.64
30[b]	**97.55**	97.55	97.54	92.42	94.09	94.00	94.19	90.07
50[b]	99.94	99.94	99.94	99.47	99.66	99.66	99.68	99.09
10[c]	54.56	54.61	54.59	54.53	51.20	50.84	51.83	50.56
20[c]	87.79	87.80	87.74	87.76	85.77	85.51	86.18	85.28
30[c]	97.51	97.51	97.48	97.51	97.14	97.05	97.33	96.96
50[c]	99.95	99.95	99.95	99.95	99.93	99.94	99.95	99.93
10[d]	57.97	57.73	57.85	57.67	66.72	65.40	65.71	65.45
20[d]	87.43	87.56	87.52	87.56	95.75	95.24	95.38	95.33
30[d]	96.95	96.98	96.97	97.02	99.68	99.64	99.64	99.66
50[d]	99.88	99.88	99.88	99.87	100.00	100.00	100.00	100.00
10[e]	62.94	61.76	61.72	61.43	**82.45**	77.24	77.79	77.70
20[e]	87.41	87.94	87.93	87.87	99.35	98.63	98.73	98.71
30[e]	95.88	96.20	96.23	96.25	99.98	99.95	99.97	99.96
50[e]	99.71	99.70	99.72	99.74	100.00	100.00	100.00	100.00
10[f]	55.06	**65.41**	65.05	65.18	**88.76**	82.90	83.80	83.90
20[f]	72.96	**87.73**	87.61	87.73	99.81	99.39	99.48	99.48
30[f]	85.09	95.83	95.89	**95.95**	100.00	99.99	99.99	99.99
50[f]	96.23	99.46	99.48	99.50	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-28

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = 1.00)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	79.50	79.50	79.52	79.49	67.25	67.25	67.32	67.30
20[a]	98.93	98.93	98.93	98.93	95.38	95.38	95.39	95.41
30[a]	99.98	99.98	99.98	99.98	99.66	99.66	99.65	99.65
50[a]	100.00	100.00	100.00	100.00	99.99	99.99	99.99	99.99
10[b]	**79.43**	79.41	79.41	68.81	69.67	69.53	**69.79**	61.17
20[b]	98.83	98.82	98.84	95.49	96.39	96.34	96.42	93.01
30[b]	99.96	99.96	99.96	99.64	99.75	99.75	99.75	99.20
50[b]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	80.10	80.15	80.08	80.21	76.03	75.83	76.31	75.47
20[c]	98.81	98.81	98.81	98.80	98.36	98.37	98.40	98.27
30[c]	99.95	99.95	99.96	99.95	99.94	99.94	99.94	99.93
50[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[d]	81.27	81.29	81.25	81.28	87.19	86.24	86.59	85.79
20[d]	98.15	98.18	98.16	98.18	99.79	99.76	99.76	99.72
30[d]	99.91	99.89	99.89	99.89	100.00	100.00	100.00	100.00
50[d]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[e]	82.03	82.31	82.24	82.14	93.63	92.02	92.12	91.80
20[e]	97.00	97.30	97.35	97.39	99.96	99.94	99.93	99.93
30[e]	99.62	99.62	99.62	99.64	100.00	100.00	100.00	100.00
50[e]	100.00	99.98	99.98	99.98	100.00	100.00	100.00	100.00
10[f]	71.62	**83.74**	83.41	83.53	94.96	94.53	94.35	94.36
20[f]	88.42	96.92	96.95	**97.01**	99.99	99.99	99.98	99.99
30[f]	95.56	99.22	99.27	99.29	100.00	100.00	100.00	100.00
50[f]	99.44	99.92	99.93	99.93	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-29

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = 1.25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	94.59	94.59	94.60	94.58	86.18	86.18	86.20	86.18
20[a]	99.98	99.98	99.98	99.97	99.64	99.64	99.64	99.65
30[a]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[a]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[b]	94.38	**94.41**	94.35	86.81	87.63	87.63	**87.71**	79.63
20[b]	99.98	99.97	99.98	99.60	99.70	99.71	99.70	98.89
30[b]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98
50[b]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	94.24	94.26	94.16	94.26	91.31	91.26	91.39	91.08
20[c]	99.96	99.96	99.95	99.95	99.94	99.94	99.94	99.94
30[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[d]	93.09	93.22	93.20	93.26	95.97	95.55	95.75	95.18
20[d]	99.81	99.79	99.79	99.80	99.99	99.98	99.99	99.98
30[d]	100.00	99.99	99.99	99.99	100.00	100.00	100.00	100.00
50[d]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[e]	91.89	92.59	92.55	92.56	97.67	97.51	97.47	97.32
20[e]	99.47	99.45	99.47	99.49	100.00	100.00	100.00	100.00
30[e]	99.95	99.94	99.94	99.94	100.00	100.00	100.00	100.00
50[e]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[f]	82.21	92.35	92.31	**92.39**	97.64	98.43	98.24	98.22
20[f]	94.93	99.04	99.11	99.12	100.00	100.00	100.00	100.00
30[f]	98.82	99.84	99.86	99.86	100.00	100.00	100.00	100.00
50[f]	99.92	99.99	99.99	99.99	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-30

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .40; Effect Size = 1.50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	99.20	99.20	99.20	99.19	95.61	95.61	95.62	95.57
20[a]	100.00	100.00	100.00	100.00	99.99	99.99	99.99	99.99
30[a]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[a]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[b]	98.97	98.97	98.97	95.70	96.01	96.04	96.00	90.78
20[b]	100.00	100.00	100.00	99.99	99.98	99.99	99.99	99.89
30[b]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[b]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	98.90	98.91	98.92	98.92	97.77	97.76	97.83	97.72
20[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
30[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[c]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[d]	97.65	97.74	97.72	97.80	98.91	98.88	98.91	98.77
20[d]	99.98	99.97	99.97	99.97	100.00	100.00	100.00	100.00
30[d]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
50[d]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[e]	96.40	96.79	96.82	96.87	99.16	99.22	99.17	99.09
20[e]	99.86	99.84	99.84	99.84	100.00	100.00	100.00	100.00
30[e]	100.00	99.99	99.99	99.99	100.00	100.00	100.00	100.00
50[e]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10[f]	<u>88.84</u>	96.45	96.48	**96.52**	98.89	99.56	99.48	99.46
20[f]	97.99	99.72	99.75	99.75	100.00	100.00	100.00	100.00
30[f]	99.59	99.95	99.95	99.95	100.00	100.00	100.00	100.00
50[f]	99.98	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .55 level. The Power rates of the *F* and *FR* tests in the six distributions with valid PDFs at the post correlation .55 were reported in Tables 4-31 and 4-32, respectively for the effect size at .25 and .50. The power rates at other effect size levels at correlation = .55 were similar across data transformations, hence not reported. As the results in Table 4-31 indicate, the power rates of the *F* tests at effect size = .25, were similar across the four data transformations in all conditions, while the power rates of the *FR* tests at effect size = .25, were similar except in five conditions in the last two distributions (i.e., cell size = 10 in D15; and all conditions in D16) with power differences from .055 -.403 (Table 4-31). At effect size = .50 level, as the results in Table 4-32 show, the power rates of the *F* tests were similar except in one condition (cell size = 10) in the last distribution (D16), where the difference was .052. The power rates of the *FR* tests at effect size = .50 level were inconsistent across data transformations at cell size = 10 in the last two distributions (D15 and D16), with power differences .084 and .17 (Table 4-32). At this post correlation level, in conditions with inconsistency, the third-order power method was the most powerful with *g*-and-*h* transformation mostly as the least powerful data transformation.

Table 4-31

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .55; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	11.05	11.05	11.06	11.09	10.18	10.18	10.18	10.18
20[a]	18.66	18.66	18.67	18.67	15.30	15.30	15.30	15.31
30[a]	27.81	27.81	27.81	27.79	22.47	22.47	22.52	22.50
50[a]	44.70	44.70	44.72	44.73	34.95	34.95	34.92	34.91
10[b]	10.75	10.77	10.77	10.76	10.44	10.40	10.47	10.46
20[b]	18.97	19.00	18.99	19.00	16.05	15.97	16.11	16.09
30[b]	27.65	27.64	27.64	27.67	23.68	23.58	23.77	23.72
50[b]	44.65	44.66	44.66	44.68	37.15	36.96	37.39	37.29
10[c]	11.07	10.93	11.09	11.09	11.45	11.26	11.51	11.43
20[c]	19.48	19.11	19.52	19.47	18.69	18.36	18.94	18.82
30[c]	27.88	27.40	27.87	27.95	27.33	26.65	27.78	27.55
50[c]	44.99	44.13	45.00	45.19	43.68	42.65	44.31	43.91
10[d]	11.78	11.67	11.67	11.68	15.48	15.54	15.19	16.68
20[d]	20.57	20.45	20.51	20.49	28.09	28.05	27.33	30.73
30[d]	29.24	29.01	29.08	28.93	42.04	42.07	40.85	46.14
50[d]	46.24	46.08	46.17	46.04	65.63	65.81	64.41	70.75
10[e]	13.84	12.92	12.93	12.89	**26.97**	<u>21.37</u>	22.24	24.63
20[e]	23.65	22.46	22.41	22.42	52.58	41.62	43.37	48.37
30[e]	32.20	31.31	31.31	31.25	73.30	61.01	63.31	69.14
50[e]	48.72	48.30	48.21	48.09	92.91	84.81	86.65	90.65
10[f]	18.73	14.15	14.37	14.60	**56.65**	<u>24.97</u>	30.35	31.62
20[f]	28.21	24.63	24.55	24.95	**89.56**	<u>49.25</u>	59.47	61.64
30[f]	36.39	33.55	33.49	33.98	**98.27**	<u>70.35</u>	80.52	82.47
50[f]	51.51	50.22	49.87	50.60	**99.97**	<u>91.50</u>	96.35	97.08

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-32

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .55; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	32.96	32.96	32.97	32.99	27.43	27.43	27.42	27.43
20[a]	64.01	64.01	64.02	64.03	51.79	51.79	51.83	51.96
30[a]	83.92	83.92	83.95	83.94	72.70	72.70	72.68	72.68
50[a]	97.64	97.64	97.64	97.65	92.55	92.55	92.53	92.51
10[b]	33.44	33.40	33.44	33.48	28.71	28.57	28.83	28.81
20[b]	64.03	64.05	64.04	64.05	54.25	54.07	54.50	54.36
30[b]	83.85	83.83	83.87	83.91	75.34	75.15	75.55	75.37
50[b]	97.67	97.68	97.68	97.68	94.12	94.02	94.24	94.13
10[c]	34.28	33.62	34.34	34.29	32.71	31.95	33.03	32.60
20[c]	64.98	63.95	65.01	64.91	62.65	61.20	63.28	62.39
30[c]	83.93	83.06	83.92	83.98	82.94	81.80	83.38	82.68
50[c]	97.62	97.37	97.63	97.65	97.37	96.98	97.59	97.28
10[d]	38.39	37.90	38.00	37.84	47.14	46.26	46.05	47.28
20[d]	66.89	66.83	66.92	66.76	81.65	80.96	80.71	82.03
30[d]	84.13	84.20	84.20	84.20	95.51	95.25	95.17	95.75
50[d]	97.16	97.13	97.12	97.18	99.85	99.83	99.80	99.87
10[e]	45.50	43.21	43.29	43.11	**67.87**	59.45	60.65	61.08
20[e]	69.43	69.27	69.14	68.96	95.91	92.10	92.79	93.03
30[e]	84.43	84.71	84.66	84.67	99.65	99.03	99.15	99.21
50[e]	96.34	96.55	96.59	96.61	100.00	99.99	100.00	99.99
10[f]	**52.37**	47.21	47.13	47.75	**84.39**	67.40	69.95	70.67
20[f]	72.17	71.52	71.24	71.94	99.39	95.50	96.45	96.65
30[f]	84.05	85.05	84.97	85.44	99.99	99.67	99.79	99.82
50[f]	95.72	96.19	96.28	96.46	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .70 level. The Power rates of the *F* and *FR* tests in the six distributions with valid PDFs at the post correlation .70 were reported in Tables 4-33 and 4-34, respectively for the effect size at .25 and .50. They were similar across the four data transformations in each condition for other effect size levels at correlation = .70, hence not reported. As the results in Table 4-33 indicate, at effect size = .25, the power rates of the *F* tests were similar except one condition (cell size = 10) in the sixth distribution (D16: sk = 3.9, kt = 40.0) with the power difference at .067, while the power rates of the *FR* tests were inconsistent across the data transformations in seven conditions in the last three distributions (cell size = 10 in D14: sk = 2.0, kt = 8.0; and cell sizes = 10, 20 and 30 in both D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0), with power differences .057 - .311 (Table 4-33). At effect size = .50, as the results in Table 4-34 indicate, the power rates of the *F* tests were all consistent across data transformations, while there were two conditions (cell size = 10) in the last two distributions (D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0),where the power rates of the *FR* tests were inconsistent with the differences at .064 and .091, respectively (Table 4-34).

Table 4-33

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .70; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	14.06	14.06	14.07	14.05	12.65	12.65	12.68	12.71
20[a]	26.60	26.60	26.60	26.65	21.19	21.19	21.20	21.21
30[a]	39.99	39.99	39.98	39.96	31.95	31.95	31.94	31.95
50[a]	63.02	63.02	63.02	63.00	50.74	50.74	50.74	50.77
10[b]	14.25	14.28	14.27	14.23	13.37	13.34	13.38	13.34
20[b]	26.79	26.80	26.81	26.81	22.47	22.38	22.61	22.50
30[b]	40.02	40.04	40.04	40.02	33.64	33.53	33.77	33.75
50[b]	63.32	63.31	63.31	63.29	53.64	53.42	53.88	53.73
10[c]	14.98	14.96	14.93	14.93	14.89	14.87	14.95	15.10
20[c]	27.31	27.28	27.32	27.33	26.31	26.28	26.66	26.72
30[c]	40.49	40.57	40.48	40.60	39.78	39.73	40.23	40.42
50[c]	63.33	63.27	63.31	63.30	62.30	62.25	62.90	63.12
10[d]	16.42	16.24	16.29	16.19	21.61	21.73	21.17	23.25
20[d]	30.15	29.88	29.95	29.74	40.98	41.34	40.21	44.73
30[d]	43.02	43.00	43.04	42.87	60.40	61.02	<u>59.51</u>	**65.20**
50[d]	64.48	64.50	64.57	64.46	84.01	84.51	83.17	87.82
10[e]	20.59	19.03	19.09	19.02	**37.81**	<u>30.93</u>	32.18	34.66
20[e]	34.85	33.23	33.16	33.05	**70.10**	<u>59.27</u>	61.38	65.13
30[e]	46.60	45.70	45.62	45.48	**88.57**	<u>79.76</u>	81.67	84.91
50[e]	66.57	66.64	66.65	66.55	98.71	96.20	96.93	97.91
10[f]	**27.68**	<u>21.00</u>	21.29	22.27	**67.37**	<u>36.33</u>	41.97	44.09
20[f]	39.90	35.83	35.74	37.18	**95.02**	<u>67.87</u>	75.68	78.03
30[f]	50.95	48.73	48.53	50.34	**99.56**	<u>87.48</u>	92.50	93.78
50[f]	68.04	67.73	67.52	69.51	100.00	98.52	99.44	99.62

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Table 4-34

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .70; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	47.65	47.65	47.66	47.65	38.71	38.71	38.73	38.78
20[a]	82.97	82.97	82.99	82.99	70.81	70.81	70.83	70.92
30[a]	95.83	95.83	95.82	95.86	89.29	89.29	89.30	89.35
50[a]	99.89	99.89	99.89	99.88	98.86	98.86	98.88	98.88
10[b]	47.53	47.52	47.55	47.53	40.35	40.26	40.47	40.42
20[b]	83.04	83.03	83.06	83.05	73.59	73.48	73.70	73.69
30[b]	95.70	95.68	95.69	95.68	90.59	90.47	90.71	90.57
50[b]	99.85	99.84	99.85	99.85	99.18	99.18	99.21	99.19
10[c]	49.10	49.06	49.07	48.98	46.04	45.84	46.57	45.79
20[c]	83.01	83.05	83.01	83.05	81.04	80.86	81.57	80.91
30[c]	95.57	95.57	95.53	95.60	95.12	95.08	95.32	95.04
50[c]	99.84	99.84	99.84	99.84	99.79	99.77	99.79	99.78
10[d]	54.05	53.73	53.90	53.48	63.17	61.97	62.19	62.33
20[d]	83.50	83.57	83.53	83.55	93.89	93.47	93.50	93.58
30[d]	94.86	94.92	94.93	94.92	99.39	99.25	99.28	99.31
50[d]	99.60	99.59	99.59	99.60	100.00	100.00	100.00	100.00
10[e]	60.62	59.15	59.03	58.66	**81.03**	<u>74.63</u>	75.45	75.11
20[e]	83.66	84.15	84.10	84.05	98.97	97.87	98.01	97.93
30[e]	93.54	93.89	93.96	94.03	99.99	99.91	99.93	99.94
50[e]	99.22	99.23	99.25	99.27	100.00	100.00	100.00	100.00
10[f]	65.18	62.95	62.58	64.10	**90.37**	<u>81.23</u>	82.03	83.06
20[f]	83.70	84.96	84.74	85.94	99.79	99.05	99.15	99.26
30[f]	92.50	93.68	93.68	94.43	100.00	99.98	99.98	99.99
50[f]	98.77	98.91	99.01	99.16	100.00	100.00	100.00	100.00

Note. [a] D11: skewness (sk) = .0, kurtosis (kt) = .0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .85 level. The Power rates of the *F* and *FR* tests in the six distributions with valid PDFs at the post correlation .85 were reported in Tables 4-35 for the effect size at .25 level. They were similar across the four data transformations in each condition for other effect size levels, hence not reported. As the results in Table 4-35 indicate, for the *F* tests at effect size = .25, three conditions (cell sizes = 10, 20, and 30) in the sixth distribution (D16: sk = 3.9, kt = 40.0) had inconsistent power rates with power differences .056 – .06. For the *FR* tests at effect size = .25, however, three conditions of inconsistent power rates were associated with the fifth and sixth distributions (cell size = 10 in D15: sk = 3.0, kt = 20.0; and cell sizes =10 and 20 in D16: sk = 3.9, kt = 40.0) with differences at .073 – .211.

An overall inspection of the power rates of the *F* and *FR* tests in the six distribution with valid PDFs at all the considered correlation levels reveals that, there were 27 out of 720 conditions where the power rates for the *F* tests were inconsistent (i.e., maximum power rates difference in each condition above 5.0 percent points) with the extreme difference at .19. Among the inconsistent conditions, 17 conditions were associated with the sixth distribution (D16: sk = 3.9, kt = 40.0), of which the majority were with the third-order power transformation as least powerful. For the *FR* tests, there were 61 out of 720 conditions under which the power rates were inconsistent across the four data transformations with the extreme value at .492. Among the 61 inconsistent conditions, 24 were associated with the fifth distribution (D15: sk = 3.0, kt = 20.0) and 25 with the sixth distribution (D16: sk = 3.9, kt = 40.0). The third-order power method were generally the most powerful and the *g*-and-*h* transformation the least powerful but close to the rest

Table 4-35

Power Rates of F and Friedman Tests of Four Data Transformations in Six Distributions with Valid PDFs (%; Correlation = .85; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	25.06	25.06	23.75	25.08	21.21	21.21	20.69	21.26
20[a]	50.56	50.56	48.46	50.62	40.47	40.47	39.62	40.54
30[a]	70.77	70.77	68.67	70.85	58.74	58.74	57.85	58.92
50[a]	91.94	91.94	91.09	91.97	82.48	82.48	81.78	82.56
10[b]	25.10	25.07	25.12	25.10	21.76	21.71	21.81	21.81
20[b]	50.52	50.57	50.50	50.54	41.65	41.54	41.81	41.71
30[b]	71.12	71.12	71.19	71.14	61.11	60.98	61.33	61.24
50[b]	91.76	91.78	91.78	91.78	84.46	84.35	84.64	84.56
10[c]	26.55	26.58	26.61	26.59	26.03	26.03	26.24	26.38
20[c]	51.87	51.81	51.92	51.78	49.98	49.99	50.49	50.78
30[c]	71.27	71.26	71.36	71.28	70.26	70.31	70.62	71.16
50[c]	92.06	92.08	92.07	92.04	91.31	91.37	91.52	91.88
10[d]	30.98	30.58	30.71	30.43	38.37	38.47	37.91	40.23
20[d]	55.16	54.85	55.04	54.69	71.20	71.62	70.67	73.57
30[d]	73.01	73.00	73.07	72.90	89.29	89.33	88.86	90.65
50[d]	91.53	91.62	91.63	91.66	98.90	98.95	98.83	99.20
10[e]	37.89	35.10	35.19	34.89	**61.25**	52.06	53.71	54.42
20[e]	59.92	59.04	59.00	58.72	92.36	86.26	87.56	88.23
30[e]	74.11	74.21	74.19	74.00	99.02	97.14	97.64	97.82
50[e]	90.95	91.26	91.38	91.30	100.00	99.93	99.96	99.97
10[f]	**46.02**	39.99	40.06	44.89	**80.84**	59.77	63.16	67.59
20[f]	63.26	62.11	61.76	**67.49**	**98.83**	91.56	93.48	95.45
30[f]	75.63	76.27	76.13	**81.20**	99.96	98.78	99.23	99.61
50[f]	90.06	90.97	91.12	93.90	100.00	99.98	99.99	100.00

Note. [a] D11: skewness (sk) = 0.0, kurtosis (kt) = 0.0; [b] D12: sk = .0, kt = 1.0; [c] D13: sk = 1.0, kt = 2.0; [d] D14: sk = 2.0, kt = 8.0; [e] D15: sk = 3.0, kt = 20.0; [f] D16: sk = 3.9, kt = 40.0. Underscored (bold faced) values are the minimum (maximum) of the condition. All six distributions have valid PDFs.

two transformations. It is worth to note that, as compared across the Burr, *g*-and-*h*, GLD three transformations with the third-order power transformation excluded, the number of inconsistent conditions reduced to 11, with largest power difference at .121 for the *F* tests. For the *FR* tests inconsistent conditions reduced to 39, with the largest power difference at .149. Further, an overall comparison of the *FR* and *F* tests in each condition indicates that, in the first three distributions (D11: sk = .0, kt = .0; D12: sk = .0, kt = 1.0; and D13: sk = 1.0, kt = 2.0), the *F* tests were more or equally powerful for each of the transformations except cell size = 10, at effect size = .25 for the third distribution for the lower three correlation levels (i.e., .25, .40 and .55), the maximum power advantage over the *FR* tests in those three distributions were .126 , .126, .125, and .126, respectively for the Burr, *g*-and-*h*, GLD, and the third-order power transformations. In the last three distributions (D14: sk = 2.0, kt = 8.0; D15: sk = 3.0, kt = 20.0; and D16: sk = 3.9, kt = 40.0), however, the *FR* tests were more or equally powerful with extreme power advantage over the *F* tests at .523, .416, .512, and .77, respectively for the Burr, *g*-and-*h*, GLD, and the third-order power transformations.

Within-Subjects Design for the Second Group of Distributions without Valid PDFs

Type I error rates for the *F* and *FR* tests at correlation = .25 level. The Type I error rates of the *F* and *FR* tests of the four distributions in the second group without valid PDFs at post correlation .25 were reported in Table 4-36. Bold faced values were outside of the predefined interval (.045, .055, denoted the same hereafter). As the results in Table 4-36 indicate, the Type I error rates of the *F* test were all robust in the first two distributions (D21: sk = .24, kt = -1.209981; and D22: sk = .96, kt = .133374), and robust in the third distribution (D23: sk = 1.68, kt = 2.76236) except the cell size = 10 level, while they were conservative in the last distribution (D24: D24: sk = 2.40, kt = 6.606610) except the cell size = 50 level. There was one condition (cell size =50 in D24) with mixed robust-conservative Type I errors, and in the three all-conservative conditions (cell size = 10 in D23, and cell sizes = 10, and 20 in D24) Type I error differences were above the .004 cut-off value. Thus 4 out of 16 conditions were considered inconsistent in Type I errors across the three data transformations for the *F* tests. The Type I error rates of the *FR* tests at this correlation level, however, were all robust and consistent across the data transformations in all conditions (Table 4-36).

Table 4-36

Type I Error Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	4.98	4.99	4.99	5.31	5.41	5.42
20[a]	4.99	5.03	5.04	5.10	5.08	5.10
30[a]	5.09	5.15	5.17	5.29	5.31	5.27
50[a]	4.92	4.94	4.96	4.85	4.98	4.92
10[b]	4.73	4.74	4.72	5.24	5.15	5.12
20[b]	4.95	4.89	4.89	5.05	5.13	5.24
30[b]	5.00	5.22	5.19	5.24	5.20	5.13
50[b]	5.01	4.89	4.92	4.98	5.15	5.16
10[c]	**4.34**	**4.18**	**3.64**	5.47	5.41	5.42
20[c]	4.61	4.64	4.52	5.13	4.99	5.03
30[c]	4.73	4.83	4.79	5.20	5.14	5.33
50[c]	4.81	4.79	4.71	4.99	5.01	5.05
10[d]	**3.56**	**3.50**	**2.78**	5.28	5.31	5.39
20[d]	**4.43**	**4.40**	**4.02**	5.17	5.04	5.19
30[d]	**4.47**	**4.44**	**4.35**	5.05	5.14	5.35
50[d]	4.80	4.78	**4.49**	5.12	5.15	4.73

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of bound (4.50, 5.50). None of the distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .40 level. The Type I error rates of the *F* and *FR* tests in the second group of the four distributions without valid PDFs at post correlation .40 were reported in Table 4-37. Results indicate that Type I error rates of the *F* tests were all consistent, compared across the three transformations in each condition in the first two distributions (D21: sk = .24, kt = -1.209981; and D22: sk = .96, kt = .133374) , and in three conditions (cell size = 30, and 50 in D23: sk = 1.68, kt = 2.76236 , and cell size = 50 in D24: sk = 2.40, kt = 6.606610) of the last two distributions. The Type I error rates of the *F* tests, however, were conservative in three conditions (cell size = 10 in D23, and cell sizes = 10, and 20 in D24), with mixed robust-conservative results in two conditions (cell sizes = 20 in D23, and cell size = 30 in D24) in the last two distributions. All the three all-conservative conditions had Type I error differences above .004. As such, there were 5 out of 16 conditions where the Type I errors were classified as inconsistent across the three data transformations for the *F* test. For the *FR* test, The Type I error rates were all robust and consistent across the three data transformations.

Table 4-37

Type I Error Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .40)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	4.95	4.94	5.00	5.40	5.32	5.36
20[a]	4.91	4.87	4.88	4.88	5.10	5.04
30[a]	4.84	5.00	5.01	5.01	5.13	5.07
50[a]	5.02	4.84	4.88	5.16	5.02	5.07
10[b]	4.63	4.69	4.70	5.27	5.35	5.37
20[b]	5.02	4.93	4.92	4.99	5.11	5.10
30[b]	4.96	4.97	4.95	5.33	5.17	5.17
50[b]	4.92	5.02	5.04	5.08	4.96	4.98
10[c]	**4.16**	**4.17**	**3.37**	5.22	5.37	5.26
20[c]	4.70	4.69	**4.41**	4.94	4.87	5.01
30[c]	4.81	4.98	4.70	5.33	5.47	4.98
50[c]	4.98	5.02	4.78	5.16	5.13	5.06
10[d]	**3.64**	**3.55**	**2.53**	5.38	5.44	5.31
20[d]	**4.11**	**4.10**	**3.64**	5.12	5.03	5.01
30[d]	**4.47**	4.50	**4.07**	5.08	5.08	5.37
50[d]	4.79	4.75	4.64	4.93	4.86	4.92

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of bound (4.50, 5.50). None of the distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .55 level. The Type I error rates of the *F* and *FR* tests in the second group of the four distributions without valid PDFs at the post correlation .55 were reported in Table 4-38. As the results in Table 4-38 indicate, the Type I error rates of the *F* tests were all robust and consistent in the first two distributions (D21: sk = .24, kt = -1.209981; and D22: sk = .96, kt = .133374) and at cell size = 50 in the last two distributions (D23: sk = 1.68, kt = 2.76236; and D24: sk = 2.40, kt = 6.606610). They were all conservative at cell size = 10 in the third distribution (D23) and at the three smaller cell sizes (10, 20, and 30) in the last distribution (D24). two cell sizes (20, and 30) in the third distribution had mixed robust-conservative Type I errors, while the four all-conservative conditions had Type I error differences above .004. Thus six conditions were classified as inconsistent or dissimilar in Type I error rates for the *F* tests across the three data transformations. As for the *FR* test, all conditions were consistent and robust except cell size = 10 in the last two distributions (D23: sk = 1.68, kt = 2.76236; and D24: sk = 2.40, kt = 6.606610), where mixed robust-liberal Type I errors occurred. As a result, 2 out of 16 conditions were considered inconsistent in Type I errors across the three data transformations for the *FR* tests (Table 4-38).

Table 4-38

Type I Error Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = 0.55)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	4.83	4.99	5.03	5.30	5.35	5.40
20[a]	4.97	5.20	5.12	5.09	5.18	5.16
30[a]	4.74	5.04	5.01	5.07	4.99	4.99
50[a]	4.97	5.14	5.07	4.99	5.01	5.06
10[b]	4.52	4.53	4.52	5.17	5.28	5.19
20[b]	4.96	4.91	4.97	5.09	5.13	5.08
30[b]	4.84	4.80	4.83	5.09	5.08	5.02
50[b]	4.83	4.83	4.79	5.12	5.02	4.95
10[c]	**4.17**	**3.90**	**2.91**	5.29	**5.53**	5.18
20[c]	4.60	**4.46**	**4.20**	5.00	5.05	5.00
30[c]	4.75	4.58	**4.46**	5.04	5.19	5.28
50[c]	4.82	4.80	4.71	4.92	4.85	4.95
10[d]	**3.26**	**3.29**	**2.20**	5.36	5.33	**5.54**
20[d]	**4.01**	**4.09**	**3.42**	5.04	5.00	4.96
30[d]	**4.46**	**4.45**	**4.01**	5.13	5.23	5.23
50[d]	4.61	4.60	4.54	4.99	4.98	5.12

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of bound (4.50, 5.50). None of the distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .70 level. The Type I error rates of the *F* and *FR* tests in the second group of the four distributions without valid PDFs at the post correlation level .70 were reported in Table 4-39. As the results in Table 4-39 indicate, the Type I error rates for the *F* tests were robust in the first distributions (D21: sk = .24, kt = -1.209981), while they were all conservative in five conditions: cell size = 10 in the middle two distributions (D22: sk = .96, kt = .133374; and D23: sk = 1.68, kt = 2.76236), and three smaller cell sizes (10, 20, and 30) in the last distribution (D24: sk = 2.40, kt = 6.606610). Four of the all-conservative conditions in the last two distributions had Type I error difference above .004, three conditions (cell sizes = 20, and 30 in D23, and cell size = 50 in D24) had mixed robust-conservative results. As such, seven conditions were classified as inconsistent in Type I error rates across the three data transformations for the *F* tests. For the *FR* tests, however, all the conditions were robust and consistent in Type I errors across the three data transformations.

Table 4-39

Type I Error Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .70)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	4.68	4.69	4.68	5.17	5.24	5.20
20[a]	4.96	4.97	5.00	5.06	5.09	5.08
30[a]	5.12	5.07	5.07	5.28	5.34	5.35
50[a]	4.93	5.07	5.08	5.18	5.02	4.97
10[b]	**4.46**	**4.40**	**4.38**	5.42	5.32	5.35
20[b]	4.71	4.87	4.90	5.00	5.09	5.14
30[b]	4.90	4.84	4.83	5.11	5.17	5.14
50[b]	5.12	4.98	4.98	5.08	4.92	4.97
10[c]	**4.07**	**3.80**	**2.63**	5.28	5.35	5.25
20[c]	4.62	**4.39**	**4.11**	4.97	4.77	5.06
30[c]	4.65	4.63	**4.35**	5.19	5.19	5.25
50[c]	4.95	4.78	4.85	4.93	4.81	5.08
10[d]	**3.03**	**2.89**	**2.13**	5.28	5.27	5.29
20[d]	**4.19**	**4.15**	**3.21**	5.02	4.92	4.62
30[d]	**4.31**	**4.31**	**3.72**	5.13	5.33	5.25
50[d]	4.67	4.65	**4.16**	4.90	5.10	4.93

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of bound (4.50, 5.50). None of the distributions have valid PDFs.

Type I error rates for the *F* and *FR* tests at correlation = .85 level. The Type I error rates of the *F* and *FR* tests of the four distributions in the second group of distributions without valid PDFs at the post correlation level .85 were reported in Table 4-40. Results indicate that the Type I error rates of the *F* tests were all consistent across the three transformations in each condition in the first distribution (D21: sk = .24, kt = -1.209981) and at the three lager sample sizes (20, 30, and 50) in the second distribution (D22: sk = .96, kt = .133374), while they were conservative in four conditions (cell size = 10 in D23: sk = 1.68, kt = 2.76236; and cell sizes = 10, 20, and 30 in D24: sk = 2.40, kt = 6.606610). Mixed robust-conservative Type I errors occurred in the remaining five conditions (cell size = 10 in D22; cell sizes = 20, 30, and 40 in D23, and cell size = 50 in D24) in the last three distributions, while the four all-conservative conditions had differences above .004. Thus nine conditions (out of 16) were considered inconsistent or different in Type I errors across the three data transformation for the *F* tests. For the *FR* tests, only one condition (cell size = 10 in D23) had mixed robust-liberal Type I errors and all other conditions were robust and consistent in Type I errors across the three data transformations.

An overall inspection of the Type I error rates of *F* and *FR* tests in the four distributions without valid PDFs reveals that 31 out of 80 conditions had inconsistent Type I errors across the three data transformations for the *F* tests at all the five levels (.25, .40, .55, .70, and .85) of post correlation, of which the GLD transformation were most conservative and contributed most inconsistent conditions. For the *FR* tests, however, inconsistent Type I errors occurred in three out of the 80 conditions, thus

Table 4-40

Type I Error Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .85)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	4.52	4.62	4.65	5.35	5.30	5.38
20[a]	4.90	4.94	4.90	5.00	4.97	5.06
30[a]	4.81	4.92	4.90	5.22	5.33	5.36
50[a]	5.04	4.99	4.97	4.77	4.87	5.12
10[b]	**4.44**	4.54	4.55	5.37	5.34	5.36
20[b]	4.60	4.71	4.68	4.88	5.14	5.16
30[b]	4.97	4.79	4.81	5.31	5.22	5.25
50[b]	5.08	4.96	4.96	5.10	5.09	5.09
10[c]	**4.06**	**3.79**	**2.35**	5.35	**5.51**	5.42
20[c]	4.74	4.70	**3.87**	5.02	5.18	4.99
30[c]	4.79	4.73	**4.28**	5.22	5.17	5.09
50[c]	4.81	4.88	**4.34**	5.00	4.86	4.81
10[d]	**3.08**	**2.98**	**2.06**	5.27	5.34	5.19
20[d]	**4.00**	**3.90**	**2.94**	4.90	5.04	5.02
30[d]	**4.39**	**4.30**	**3.56**	5.37	5.24	5.09
50[d]	4.60	4.63	**4.21**	5.00	5.02	5.08

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Bold faced values are out of bound (4.50, 5.50). None of the distributions have valid PDFs.

did not seem a concern. A comparison between the fifth-order power method and the *g*-and-*h* transformation (i.e., with the GLD transformation excluded), the number of conditions with inconsistent Type I errors for the *F* tests reduced to four, although the number of inconsistency of Type I errors for the *FR* tests only reduced to two conditions. More generally, Type I error rates for the *F* tests might be dissimilar or inconsistent in this group of distributions (e.g., D23: sk = 1.68, kt = 2.76236; and D24: sk = 2.40, kt = 6.606610) generated with the three data transformations without valid PDFs even if the distributions departed mildly from normality.

Power rates for the *F* and *FR* tests at correlation = .25 level. The power rates of the *F* and *FR* tests of the three data transformations in the second group of distributions without PDFs were reported in Tables 4-41 to 4-44, respectively for effect sizes = .25, .50, .75 and 1.0 at post correlation .25. The bold faced (underscored) values were the maximum (minimum) value of the three data transformations (denoted the same hereafter) in a condition where the maximum difference across the three data transformations were above 0.05. As results in Table 4-41 show, at effect size = .25, the power rates of the *F* tests were consistent in each condition in the four distributions and the differences across the three data transformations were within .01. The power rates for the *FR* tests were all consistent and similar in the first two and the last distributions, while they were inconsistent in all conditions in the third distribution with maximal differences across data transformations ranging from .207 to .435, all with the GLD transformation at the most powerful extreme. At effect size .50, as Table 4-42 reveals, the power rates of the *F* tests again were consistent across the three data transformations. The power rates for the *FR* tests were consistent in all the first two distributions and three conditions in the last two distributions (cell size = 50 in D23, and cell sizes = 30, and 50 in D24). The remaining five conditions (cell sizes = 10, 20, 30 in D23, and cell sizes = 10, and 20 in D24) had inconsistent power rates for the *FR* tests, with the power differences ranging from .057 to .125. For effect size = .75 (Table 4-43), the power rates of the *F* tests were consistent in all conditions as expected, while the power rates for the *FR* tests were inconsistent at cell size=10 in the third and fourth distributions with differences .197 and .146, respectively. At effect size 1.0, as Table 4-44 indicates, the power rates of the *F* tests were consistent in all conditions, but the power rates for the *FR* tests were inconsistent at cell size=10 in the third and fourth distributions with differences at .083 and .081 respectively. The power rates of the *F* and *FR* tests were similar at other effect size levels, hence were not reported.

Table 4-41

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .25; Effect Size = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	8.27	8.47	8.51	9.96	9.42	9.35
20[a]	12.65	12.80	12.85	15.21	13.62	13.74
30[a]	17.16	17.32	17.32	21.84	19.50	19.54
50[a]	27.52	27.41	27.45	34.39	30.05	29.87
10[b]	8.28	8.38	8.36	11.51	10.90	11.00
20[b]	13.06	12.93	12.96	19.26	17.84	18.17
30[b]	17.33	17.20	17.22	28.07	26.02	26.58
50[b]	28.26	28.24	28.29	46.81	42.82	43.85
10[c]	8.23	8.21	8.49	<u>15.68</u>	16.76	**36.37**
20[c]	13.05	12.99	13.32	<u>28.69</u>	31.91	**68.73**
30[c]	17.82	17.80	18.03	<u>44.61</u>	48.33	**88.13**
50[c]	28.22	28.19	28.32	<u>69.16</u>	73.63	**98.82**
10[d]	8.58	8.67	9.23	27.19	28.82	27.36
20[d]	13.67	13.74	14.18	53.36	55.73	52.85
30[d]	18.21	18.33	19.13	75.31	77.93	73.71
50[d]	28.17	28.37	29.10	94.77	95.73	93.12

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-42

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .25; Effect Size = .50)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	20.49	20.33	20.32	21.58	20.15	20.21
20[a]	41.01	40.78	40.78	40.95	37.53	37.62
30[a]	59.86	59.73	59.75	60.43	56.43	56.43
50[a]	84.80	84.77	84.73	84.64	80.97	81.11
10[b]	20.97	20.97	20.90	26.23	24.68	24.90
20[b]	41.62	41.45	41.44	51.91	49.40	49.68
30[b]	60.21	59.94	59.96	73.16	70.51	70.90
50[b]	84.91	84.91	84.98	93.70	92.07	92.35
10[c]	22.46	22.60	24.05	36.47	37.61	**68.96**
20[c]	42.64	42.68	43.89	69.85	71.19	**96.41**
30[c]	60.99	60.95	61.50	89.28	90.31	**99.81**
50[c]	84.92	84.84	84.86	99.07	99.23	100.00
10[d]	25.17	25.23	28.74	54.37	54.35	**67.02**
20[d]	44.10	44.24	45.97	89.70	89.38	**95.10**
30[d]	61.94	62.05	62.63	98.58	98.50	99.57
50[d]	85.06	84.92	84.66	99.99	99.99	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-43

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .2; Effect Size = .75)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	42.65	42.46	42.50	38.73	36.70	36.92
20[a]	78.39	78.40	78.32	71.74	69.13	69.10
30[a]	93.99	94.02	94.06	90.08	88.12	88.21
50[a]	99.66	99.68	99.68	99.03	98.57	98.52
10[b]	43.96	43.59	43.57	46.57	44.91	45.05
20[b]	78.43	78.40	78.38	81.49	80.28	80.24
30[b]	93.65	93.76	93.71	95.78	95.18	95.16
50[b]	99.69	99.69	99.68	99.85	99.80	99.80
10[c]	46.32	46.31	47.72	59.41	60.12	**79.10**
20[c]	79.01	78.92	79.11	92.64	93.10	**98.92**
30[c]	93.69	93.73	93.56	99.21	99.35	99.97
50[c]	99.65	99.64	99.50	100.00	100.00	100.00
10[d]	49.10	49.17	50.72	75.01	74.78	**89.33**
20[d]	79.03	79.03	78.91	98.36	98.30	99.85
30[d]	93.31	93.29	92.79	99.97	99.96	100.00
50[d]	99.58	99.58	99.35	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-44

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .25; Effect Size = 1.00)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	69.18	69.14	69.18	58.27	57.03	56.92
20[a]	96.67	96.71	96.73	91.08	90.03	89.92
30[a]	99.80	99.78	99.80	98.78	98.47	98.45
50[a]	100.00	100.00	100.00	99.98	99.99	99.97
10[b]	69.73	69.56	69.61	66.44	66.07	65.73
20[b]	96.57	96.79	96.79	95.83	95.55	95.49
30[b]	99.80	99.79	99.79	99.73	99.65	99.64
50[b]	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	70.88	70.90	71.16	77.72	78.06	**86.00**
20[c]	96.37	96.32	96.15	98.86	98.89	99.64
30[c]	99.73	99.70	99.51	99.97	99.98	100.00
50[c]	100.00	100.00	99.99	100.00	100.00	100.00
10[d]	72.12	72.19	72.03	87.55	87.53	**95.64**
20[d]	95.64	95.66	95.11	99.80	99.78	99.98
30[d]	99.59	99.60	99.33	100.00	100.00	100.00
50[d]	100.00	100.00	99.98	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .40 level. The power rates of the *F* and *FR* tests of the three data transformations in the second group of distributions without PDFs were reported in Tables 4-45 to 4-48, respectively for effect sizes = .25, .50, .75, and 1.0 at the post correlation .40. The power rates at other levels of effect sizes were similar across the three data transformations in each condition, thus not reported. As results in Table 4-45 reveal, at effect size = .25, the power rates of the *F* tests were all similar across data transformations in all conditions of the distributions, while the power rates of the *FR* tests were inconsistent in all conditions of the third distribution (D23: sk = 1.68, kt = 2.76236) and one condition in the fourth distribution (cell size = 20, in D24: sk = 2.40, kt = 6.606610) with differences ranging from .058 to .444. For effect size = .50 as the results in Table 4-46 indicate, power rates of the *F* tests were similar in each condition in all distributions across the three data transformations, while the power rates of the *FR* tests were inconsistent or dissimilar in the three smaller cell sizes (10, 20, and 30) in the third distribution (D23), with the differences ranging from .061 to .315. For effect size = .75 as results in Table 4-47 show, the power rates of the *F* tests were again similar in each condition in all the distributions across the three data transformations, while the power rates of the *FR* tests were dissimilar in the cell size = 10 conditions in both the third and fourth distributions, with the power rate differences at .167 and .123, respectively. For effect size = 1.0 (Table 4-48), power rates of the *F* tests were similar in each condition in all the distributions across the three data transformations, while the inconsistent power rates of the *FR* tests occurred in the cell size = 10 in both the third and forth distributions with differences at .062 and .055, respectively.

Table 4-45

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .40; Effect Size = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	9.60	9.84	9.74	12.09	11.26	11.25
20[a]	15.07	15.02	14.99	19.60	17.47	17.32
30[a]	20.94	20.90	21.00	28.49	25.41	25.27
50[a]	34.01	33.88	33.87	45.71	40.67	40.61
10[b]	9.33	9.35	9.32	13.69	13.03	13.34
20[b]	15.29	15.03	15.06	23.62	22.60	22.86
30[b]	21.24	21.31	21.28	35.53	33.69	34.30
50[b]	34.62	34.70	34.65	57.08	54.51	55.21
10[c]	9.28	9.34	10.09	<u>17.93</u>	20.62	**44.42**
20[c]	15.21	15.33	15.83	<u>34.29</u>	39.63	**78.69**
30[c]	21.76	21.95	22.52	<u>52.28</u>	59.17	**94.05**
50[c]	34.67	34.60	35.30	<u>77.39</u>	83.29	**99.67**
10[d]	9.99	10.06	12.12	31.44	33.98	35.93
20[d]	16.55	16.68	17.90	<u>61.01</u>	64.81	**66.82**
30[d]	22.57	22.59	23.93	82.17	85.28	86.09
50[d]	35.12	35.19	36.61	97.36	98.12	98.10

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-46

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .40; Effect Size = .50)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	25.31	24.76	24.79	27.31	25.27	25.35
20[a]	51.06	50.53	50.52	53.31	49.38	49.44
30[a]	70.95	70.70	70.70	73.92	69.77	69.86
50[a]	92.16	92.19	92.22	93.46	91.28	91.33
10[b]	26.19	25.89	25.85	31.85	30.96	30.89
20[b]	51.31	50.67	50.64	62.33	60.13	60.28
30[b]	71.60	71.48	71.47	83.33	81.43	81.59
50[b]	91.96	92.19	92.15	97.46	96.94	96.97
10[c]	28.10	28.22	30.60	42.48	45.15	**73.98**
20[c]	52.31	52.45	53.70	77.94	80.16	**97.75**
30[c]	72.06	71.92	72.44	93.81	94.93	**99.88**
50[c]	92.21	92.20	91.72	99.74	99.81	100.00
10[d]	31.24	31.39	35.94	61.25	61.73	**76.35**
20[d]	54.50	54.66	55.99	93.37	93.44	98.19
30[d]	72.39	72.51	72.99	99.32	99.34	99.94
50[d]	91.86	91.94	91.48	99.99	99.99	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-47

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .40; Effect Size = .75)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	52.72	52.28	52.37	48.91	46.59	46.51
20[a]	87.57	87.57	87.57	83.55	81.29	81.28
30[a]	97.56	97.75	97.78	96.09	95.07	95.09
50[a]	99.94	99.96	99.96	99.86	99.79	99.77
10[b]	53.91	53.36	53.36	55.33	54.32	54.17
20[b]	87.36	87.36	87.38	89.66	88.62	88.42
30[b]	97.51	97.52	97.50	98.37	98.13	98.10
50[b]	99.96	99.95	99.95	99.99	99.97	99.97
10[c]	56.36	56.55	57.82	<u>67.24</u>	68.81	**83.96**
20[c]	87.42	87.37	87.04	95.88	96.35	99.32
30[c]	97.43	97.39	96.93	99.71	99.74	99.98
50[c]	99.94	99.93	99.84	100.00	100.00	100.00
10[d]	58.62	58.55	60.47	<u>80.48</u>	80.60	**92.76**
20[d]	87.05	87.03	86.32	99.10	99.11	99.93
30[d]	97.05	97.07	96.54	99.98	99.98	100.00
50[d]	99.91	99.92	99.82	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-48

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .40; Effect Size = 1.00)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	79.73	79.55	79.59	70.08	68.29	68.21
20[a]	98.99	99.05	99.06	96.66	96.12	96.09
30[a]	99.97	99.98	99.97	99.79	99.71	99.70
50[a]	100.00	100.00	100.00	100.00	100.00	100.00
10[b]	80.00	79.76	79.76	76.26	75.77	75.48
20[b]	98.82	98.93	98.93	98.43	98.23	98.19
30[b]	99.96	99.98	99.98	99.97	99.95	99.95
50[b]	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	79.92	79.81	79.88	83.56	84.39	**89.75**
20[c]	98.62	98.64	98.18	99.53	99.54	99.84
30[c]	99.94	99.95	99.83	99.99	100.00	100.00
50[c]	100.00	100.00	100.00	100.00	100.00	100.00
10[d]	80.03	80.11	79.47	90.94	91.07	**96.48**
20[d]	98.15	98.11	97.53	99.91	99.90	100.00
30[d]	99.87	99.88	99.75	100.00	100.00	100.00
50[d]	100.00	100.00	100.00	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .55 level. The power rates of the *F* and *FR* tests of the three data transformations in the second group of distributions without PDFs were reported in Tables 4-49 to 4-51, respectively for effect sizes = .25, .50, and .75 at the post correlation .40. The power rates at other effect size levels were similar in each condition across the three data transformations, thus were not reported. At effect size = .25, as the results in Table 4-49 indicate, the power rates for the *F* tests were consistent in each condition across the three data transformations for all the distributions, while the power rates for the *FR* tests were inconsistent all conditions in the third distribution (D23: sk = 1.68, kt = 2.76236) and in the three smaller cell size conditions (10, 20, and 30) in the forth distribution (D24: sk = 2.40, kt = 6.606610), with differences ranging from .054 to .449. At effect size = .50 (Table 4-50), the power rates for the *F* tests were inconsistent in one condition (cell size = 10, in D24) with a difference at .051, while those for the *FR* tests were inconsistent in three conditions (cell sizes = 10, and 20 in D23, and cell size = 10 in D24) with the power differences ranging from .127 to .282. As the results in Table 4-51 show, at effect size = .75, power rates for the *F* tests were consistent in all distributions, while for the *FR* test, they were in consistent at cell sizes =10 in the third and fourth distributions (D23, and D24), with differences at .119 and .088, respectively.

Table 4-49

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .55; Effect Size = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	11.26	11.01	10.88	14.91	13.91	13.52
20[a]	19.37	19.41	19.05	25.96	23.78	23.07
30[a]	27.81	27.69	27.18	38.69	35.62	34.83
50[a]	44.60	44.39	43.52	60.24	55.20	54.26
10[b]	11.10	11.16	11.14	16.16	16.14	16.09
20[b]	18.92	18.79	18.75	30.02	29.18	29.26
30[b]	28.08	27.91	27.91	45.99	45.24	45.47
50[b]	44.95	44.52	44.47	70.09	68.58	68.91
10[c]	11.73	11.89	13.76	21.70	25.48	**53.93**
20[c]	19.79	19.82	21.28	42.39	50.35	**87.27**
30[c]	28.28	28.26	30.01	62.75	70.68	**97.64**
50[c]	45.57	45.52	46.34	86.72	91.95	**99.94**
10[d]	12.79	13.06	16.27	37.23	40.49	**45.88**
20[d]	21.43	21.39	24.15	69.90	73.70	**80.29**
30[d]	29.80	30.01	31.88	89.00	91.31	**94.37**
50[d]	46.00	46.11	47.75	98.91	99.34	99.67

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-50

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .55; Effect Size = .50)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	33.48	32.97	32.35	36.09	34.02	33.21
20[a]	64.41	64.24	63.22	67.62	64.25	63.21
30[a]	84.03	83.84	83.02	86.68	84.22	83.26
50[a]	97.67	97.61	97.33	98.40	97.55	97.26
10[b]	34.87	34.31	34.33	40.86	39.87	39.98
20[b]	64.47	64.08	64.11	74.47	72.98	72.89
30[b]	83.88	83.86	83.84	91.79	90.90	90.76
50[b]	97.56	97.61	97.63	99.41	99.26	99.22
10[c]	36.49	36.55	40.32	51.16	54.04	**79.31**
20[c]	65.66	65.92	66.62	85.94	88.29	**98.65**
30[c]	84.08	83.76	83.91	97.21	97.99	99.96
50[c]	97.55	97.50	97.02	99.94	99.97	100.00
10[d]	40.31	40.57	**45.36**	68.64	69.25	**84.66**
20[d]	66.23	66.26	67.44	96.33	96.57	99.47
30[d]	83.93	83.90	83.55	99.75	99.78	99.99
50[d]	97.19	97.08	96.48	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-51

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .55; Effect Size = .75)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	66.54	66.08	65.00	62.12	59.94	58.88
20[a]	95.18	95.23	94.81	92.91	91.48	90.89
30[a]	99.54	99.60	99.53	99.11	98.82	98.68
50[a]	100.00	100.00	100.00	100.00	99.98	99.98
10[b]	67.65	66.97	66.97	67.20	65.99	65.68
20[b]	95.11	95.43	95.42	95.75	95.30	95.19
30[b]	99.53	99.64	99.64	99.71	99.67	99.66
50[b]	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	68.44	68.34	69.01	<u>75.95</u>	77.28	**87.73**
20[c]	94.95	94.79	94.11	98.25	98.58	99.70
30[c]	99.41	99.39	98.97	99.96	99.97	100.00
50[c]	100.00	100.00	99.97	100.00	100.00	100.00
10[d]	70.28	70.25	70.78	<u>86.29</u>	86.46	**95.09**
20[d]	94.06	94.03	92.83	99.62	99.65	99.97
30[d]	99.11	99.09	98.77	99.99	99.99	100.00
50[d]	99.98	99.99	99.95	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .70 level. The power rates of the *F* and *FR* tests of the three data transformations in the second group of distributions without PDFs were reported in Tables 4-52 to 4-54, respectively for effect sizes = .25, .50, and .75 at the post correlation .70. The power rates at other effect size levels were similar in each condition across the three data transformations, hence not reported. As the results indicate in Table 4-52, the power rates of the *F* tests were inconsistent at cell size = 10 in the fourth distribution (D24: sk = 2.40, kt = 6.606610) with a difference at .064, while the power rates of the *FR* tests were different in six conditions (cell sizes = 10, 20, 30, and 50 in D23: sk = 1.68, kt = 2.76236; and cell sizes = 10, and 20 in D24: sk = 2.40, kt = 6.606610) with differences ranging from .051 to .385. At effect size = .50 (Table 4-53), the power rates of the *F* tests were similar in all four distributions, while in the three conditions (cell sizes = 10, and 20 in D23; and cell sizes = 10 in D24) power rates of the *FR* tests were inconsistent across the three data transformations with differences ranging from .054 to .215. As the results at effect size = .75 in Table 4-54 reveal, power rates of the *F* tests were all consistent in each condition among the three data transformations, while in only one case the power rates of the *FR* tests were dissimilar with a difference at .062.

Table 4-52

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .70; Effect Size = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	14.37	14.22	14.21	19.52	18.59	18.47
20[a]	27.22	26.99	26.92	37.07	34.53	34.64
30[a]	40.42	40.19	40.16	54.89	51.56	51.80
50[a]	63.76	63.38	63.39	79.18	76.01	76.20
10[b]	14.92	14.87	14.90	21.92	21.77	21.65
20[b]	27.65	27.47	27.45	41.82	41.97	41.93
30[b]	40.93	40.61	40.65	61.41	61.51	61.24
50[b]	63.67	63.33	63.34	85.64	85.19	85.12
10[c]	15.64	16.23	20.04	28.22	33.56	**65.30**
20[c]	28.19	28.50	31.68	55.52	63.31	**94.06**
30[c]	41.49	41.76	43.85	76.58	83.74	**99.33**
50[c]	63.62	63.78	64.60	94.89	97.44	**100.00**
10[d]	18.16	18.46	**24.59**	46.50	49.60	**60.53**
20[d]	30.81	30.99	35.20	80.88	83.46	**91.84**
30[d]	42.95	43.03	45.99	94.93	96.05	98.81
50[d]	64.41	64.35	65.28	99.75	99.84	99.99

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-53

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .70; Effect Size = .50)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	49.08	48.32	48.31	50.36	48.23	48.34
20[a]	83.13	83.29	83.27	84.30	82.22	82.30
30[a]	95.87	95.96	95.94	96.45	95.50	95.62
50[a]	99.83	99.86	99.86	99.88	99.82	99.83
10[b]	50.04	49.24	49.22	54.43	53.58	53.38
20[b]	83.21	82.96	82.99	88.17	87.43	87.22
30[b]	95.32	95.60	95.56	97.79	97.59	97.49
50[b]	99.82	99.84	99.83	99.97	99.95	99.94
10[c]	51.36	51.92	55.36	<u>63.18</u>	66.30	**84.42**
20[c]	83.01	83.02	82.90	<u>94.08</u>	95.29	**99.43**
30[c]	95.20	95.17	94.49	99.38	99.61	99.99
50[c]	99.80	99.81	99.56	100.00	100.00	100.00
10[d]	55.07	55.31	59.05	<u>78.18</u>	78.58	**91.64**
20[d]	82.88	82.77	82.44	98.58	98.68	99.92
30[d]	94.81	94.77	93.64	99.95	99.95	100.00
50[d]	99.67	99.68	99.41	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-54

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .70; Effect Size = .75)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	84.20	84.32	84.33	78.48	76.93	76.94
20[a]	99.38	99.50	99.51	98.70	98.39	98.35
30[a]	99.99	99.98	99.98	99.96	99.93	99.92
50[a]	100.00	100.00	100.00	100.00	100.00	100.00
10[b]	84.48	84.58	84.55	81.87	80.81	80.57
20[b]	99.29	99.41	99.42	99.21	99.08	99.01
30[b]	99.98	99.98	99.98	99.98	99.98	99.98
50[b]	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	84.28	83.91	83.47	86.13	87.12	**92.37**
20[c]	99.10	99.07	98.53	99.64	99.71	99.90
30[c]	99.98	99.98	99.85	100.00	100.00	100.00
50[c]	100.00	100.00	100.00	100.00	100.00	100.00
10[d]	83.72	83.66	82.71	92.23	92.28	96.96
20[d]	98.65	98.64	97.86	99.92	99.92	99.99
30[d]	99.92	99.92	99.80	100.00	100.00	100.00
50[d]	100.00	100.00	100.00	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Power rates for the *F* and *FR* tests at correlation = .85 level. The power rates of the *F* and *FR* tests of the three data transformations in the second group of distributions without PDFs were reported in Tables 4-55 and 4-56, respectively for effect sizes = .25, and .50 at the post correlation .85 level. The power rates at other effect size levels were similar in each condition across the three data transformations, hence not reported. As the results at effect size = .25 in Table 4-55 indicate, power rates of the *F* tests were inconsistent at cell sizes = 10 in both the third and fourth distributions (D23: sk = 1.68, kt = 2.76236; and D24: sk = 2.40, kt = 6.606610) with differences at .072 and .087 respectively, while the power rates of the *FR* tests were dissimilar in four conditions in the last two distributions (cell sizes = 10, 20, 30 in D23; and cell size = 10 in D24) with differences ranging from .065 to .336.

An overall inspection of the power performances of the three data transformations in the four distributions without PDFs indicates four important results: (a) Less than 1% (i.e., 4 out of 480) of the conditions had inconsistent power rates across data transformations for the *F* tests with the extreme difference at .087, thus, power rate inconsistency did not seem a great concern for the *F* tests; (b) In about 12% (i.e., 58 out of 480) of the conditions inconsistent power rates occurred across data transformations for the *FR* tests with the extreme difference at .449, which made the inconsistent power rates of the nonparametric tests a great concern; (c) The GLD transformation was consistently the most powerful and departed far from the rest two transformations (i.e., the *g*-and-*h*, and fifth-order power method) among these inconsistent conditions for the *FR* tests, thus excluding the GLD transformation (i.e., compare the *g*-and-*h*, and fifth-order power transformations only) reducing the number of inconsistencies to 13, with the extreme difference down to .08; and (e) The relative power advantages for the *FR* tests were complicated in that they were more powerful than the associated *F* tests in a distribution at some effect size level(s), but not at other effect size levels with same condition in the same distribution. In the first distribution (D21: sk = .24, kt = -1.209981), for example, the *FR* tests were more powerful at size .25, but the *FR* tests became less powerful at effect size .75 for all the three data transformations at all cell sizes. This change of power for the *FR* tests from advantage to disadvantage within each transformation at different effect sizes occurred in all the four distributions but with different magnitudes, although in the fourth distribution the *FR* tests was generally more powerful than or equally powerful to the *F* tests (cf., results from the first group of six distributions with valid PDFs).

More generally, in the second group of four distributions departing mildly from normality and generated with the three data transformations (the fifth-order power method, *g*-and-*h*, and the GLD transformations) without valid PDFs, the Type I error rates for the repeated measures *F* tests might be dissimilar, and the power rates for the nonparametric *FR* tests might be disparate especially when correlations and effect sizes were small to medium. The relative power advantage of nonparametric *FR* tests over the parametric *F* tests under same condition depended on effect sizes. More specifically, at some effect size levels the *FR* tests were more powerful, whereas at other effect size levels the *F* tests were more powerful.

Table 4-55

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .85; Effect Size = .25)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	26.78	26.31	26.42	32.77	31.63	31.68
20[a]	51.93	51.72	51.68	62.20	59.88	60.21
30[a]	71.42	71.10	71.16	82.28	79.94	80.47
50[a]	91.84	92.07	92.02	96.78	96.00	96.14
10[b]	27.10	26.82	26.67	35.36	35.62	35.45
20[b]	52.18	51.75	51.72	66.43	66.54	66.06
30[b]	71.77	71.67	71.57	86.24	86.16	85.79
50[b]	91.96	92.03	92.05	98.21	98.11	98.04
10[c]	29.04	29.55	**36.22**	43.85	48.98	**77.48**
20[c]	53.25	53.67	56.38	77.90	83.44	**98.17**
30[c]	71.98	71.87	72.79	93.42	96.00	**99.89**
50[c]	91.94	91.69	90.98	99.55	99.82	100.00
10[d]	33.34	33.86	**42.06**	62.99	64.84	**80.46**
20[d]	55.29	55.37	58.63	93.09	93.94	98.88
30[d]	72.51	72.42	73.26	99.12	99.30	99.96
50[d]	91.51	91.45	90.87	99.99	99.99	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Table 4-56

Power Rates of F and Friedman Tests of Three Data Transformations in Four Distributions without Valid PDFs (%; Correlation = .85; Effect Size = .50)

Cell Size	*F* Test			Friedman Test		
	Fifth Power	*g*-and-*h*	The GLD	Fifth Power	*g*-and-*h*	The GLD
10[a]	80.15	79.74	79.63	76.46	74.67	74.88
20[a]	98.53	98.69	98.68	98.09	97.64	97.71
30[a]	99.92	99.97	99.96	99.91	99.91	99.90
50[a]	100.00	100.00	100.00	100.00	100.00	100.00
10[b]	79.87	79.57	79.60	78.50	77.66	77.39
20[b]	98.44	98.55	98.59	98.66	98.52	98.46
30[b]	99.94	99.95	99.95	99.96	99.94	99.95
50[b]	100.00	100.00	100.00	100.00	100.00	100.00
10[c]	80.01	79.98	79.79	83.33	84.80	**92.14**
20[c]	98.34	98.13	97.16	99.36	99.51	99.90
30[c]	99.91	99.93	99.69	99.99	99.99	100.00
50[c]	100.00	100.00	99.99	100.00	100.00	100.00
10[d]	79.73	79.78	79.68	90.21	90.44	**96.44**
20[d]	97.55	97.51	96.33	99.84	99.86	99.99
30[d]	99.76	99.78	99.53	100.00	100.00	100.00
50[d]	99.99	100.00	99.99	100.00	100.00	100.00

Note. [a] D21: skewness (sk) = .24, kurtosis (kt) = -1.209981; [b] D22: sk = .96, kt = .133374; [c] D23: sk = 1.68, kt = 2.76236; [d] D24: sk = 2.40, kt = 6.606610. Underscored (Bold faced) values are minimum (maximum) of the condition. None of the distributions have valid PDFs.

Within-Subjects Design for the Third Group of Distributions where All Transformations except the Power Method have valid PDFs

Type I error rates for the *F* and *FR* tests at correlation = .25 level. The Type I error rates for the *F* and *FR* tests in the third group of distributions where all four transformations except the third-order power method have valid PDFs at post correlation .25 were reported in Table 4-57. The bold faced values were outside of the (.045, .055) interval in a simulated condition (denoted the same hereafter). As the results in Table 4-57 show, for the *F* tests, the Type I error rates were all conservative in six conditions (cell size = 10 in all distributions, cell size = 20 in D32: sk = 2.75, kt = 70.0; and cell size = 30 in D34: sk = 3.25, kt = 90.0), where all the six conditions had differences above the .004, while the other ten conditions had mixed conservative-robust Type I errors. Thus all 16 conditions were classified as inconsistent in Type I errors as compared in each condition across the four data transformations. In addition, the third-order power transformation was the most conservative and far apart from the other three data transformations. For the *FR* tests, only one condition had mixed robust-liberal Type I errors (cell size = 10 in D33: sk = 3.00, kt = 80.0), while in all other conditions the *FR* tests were robust and similar across the four transformations.

Type I error rates for the *F* and *FR* tests at correlation = .40 level. The Type I error rates for the *F* and *FR* tests in the third group of distributions where all four transformations except the third-order power method have valid PDFs at post correlation .40 were reported in Table 4-58. As the results in Table 4-58 indicate, for the *F* tests, nine conditions (cell sizes = 10, and 20 in D31: sk = 2.50, kt = 60; in D33: sk = 3.00, kt = 80.0; and in D34: sk = 3.25, kt = 90.0; cell sizes = 10, 20, and 30 in D32: sk =

2.75, kt = 70.0) were all conservative across the four data transformations, with the differences all above .004, while the other seven conditions had mixed conservative-robust Type I errors. Again, all 16 conditions were considered inconsistent in Type I errors for the *F* tests, and the third-order power transformation was also the most conservative and far apart from the other three data transformations. For the *FR* tests, mixed robust-conservative Type I errors occurred in two conditions (cell size = 30 in D33: sk = 3.00, kt = 80.0; and cell size = 10 in D34: sk = 3.25, kt = 90.0), thus two conditions were counted with inconsistent Type I errors, while all other conditions were similar or consistent in type I errors for the *FR* tests as compared in each condition across the four data transformations.

Type I error rates for the *F* and *FR* tests at correlation = .55 level. The Type I error rates for the *F* and *FR* tests in the third group of distributions where all four transformations except the third-order power method have valid PDFs at post correlation .55 were reported in Table 4-59. As the results in Table 4-59 indicate, for the *F* tests, eight conditions (cell sizes = 10 in D31: sk = 2.50, kt = 60; cell sizes = 10, and 30 in D32: sk = 2.75, kt = 70.0; cell sizes = 10, 20, and 30 in D33: sk = 3.00, kt = 80.0; and cell sizes = 10, and 20 in D34: sk = 3.25, kt = 90.0) had all conservative Type I errors with all differences in each condition above .004, while the other eight conditions had conservative-robust mixed Type I errors. Thus similar as in the previous two correlation levels, all 16 conditions were inconsistent in Type I errors for the *F* tests across the three data transformations, with the third-order power transformation as the most conservative and far apart from the other three data transformations. For the *FR* tests, two conditions (cell sizes =10 in both D31: sk = 2.60, kt = 60; and D33, sk = 3.00, kt = 80.0) had mixed robust-liberal Type I errors, which engendered two inconsistent conditions, while all the other conditions were robust and consistent in Type I errors for the *FR* tests.

Type I error rates for the *F* and *FR* tests at correlation = .70 level. The Type I error rates for the *F* and *FR* tests in the third group of distributions where all four transformations except the third-order power method have valid PDFs at post correlation .70 were reported in Table 4-60. As the results in Table 4-60 indicated, for the *F* tests, six conditions (cell sizes = 20, 30, and 50 in D31: sk = 2.50, kt = 60.0; cell sizes = 20, and 30 in D32: sk = 2.75, kt = 70.0; and cell size = 50 in D33: sk = 3.00, kt = 80.0) had conservative-robust mixed Type I errors, while the other ten conditions were all conservative but with differences above .004 across the four data transformations. Thus all 16 conditions were considered inconsistent in Type I errors, with again, the third-order power transformation as the most conservative and far apart from the other three data transformations for the *F* test. For the *FR* tests, however, all conditions were consistent and similar, thus no conditions were inconsistent in Type I errors across the four data transformations.

Type **I error rates for the *F* and *FR* tests at correlation = .85 level.** The Type I error rates for the *F* and *FR* tests in the third group of distributions where all four transformations except the third-order power method have valid PDFs at post correlation .85 were reported in Table 4-61. As the results in Table 4-61 indicate, for the *F* tests, five conditions had conservative-robust mixed Type I errors, while the remaining 11 conditions were all conservative with differences all above the .004. Thus all 16 conditions were considered inconsistent across the four data transformations in Type I errors for the *F* tests, with the third-order power transformation as the most conservative and far apart from the other three data transformations, for the *FR* tests, one condition (cell sizes =10 in D33: sk = 3.00, kt = 80.0) had mixed robust-liberal Type I errors, which engendered one inconsistent condition, while the other conditions were robust and consistent in Type I errors for the *FR* tests.

A scrutiny of the Type I error rates of the *F* and *FR* tests at the five post correlation levels reveal that inconsistent Type I errors of the *F* tests across the four data transformations were a great concern in the third group of distributions where all four transformations except the third-order power method have valid PDFs, and that the *FR* tests were more robust and consistent than the *F* tests in this group of distributions. It is important to note that inconsistent Type I errors across data transformations did occur even for nonparametric *FR* tests in this group of distributions with more extreme skewness and kurtosis. The results strongly indicated the importance of data transformation with valid PDFs. More specifically, the third-order power transformation (without valid PDFs) was the most conservative and attributable to most of the inconsistencies, the number of inconsistent conditions for the *F* tests reduced from 80 to nine by a comparison among the three data transformations (i.e., the Burr, *g*-and-*h*, and GLD transformations, with valid PDFs) with the third-order power method excluded, while the number of inconsistent Type I errors for the *FR* tests reduced from six to one.

Table 4-57

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**2.04**	**4.27**	**4.24**	**4.42**	5.34	5.38	5.39	5.38
20[a]	**2.97**	**4.43**	**4.42**	4.55	5.04	5.13	5.13	5.10
30[a]	**3.41**	4.57	4.57	4.69	5.15	5.31	5.30	5.29
50[a]	**3.78**	4.66	4.64	4.62	4.89	4.94	4.93	4.93
10[b]	**1.91**	**4.24**	**4.20**	**4.30**	5.44	5.14	5.15	5.14
20[b]	**2.73**	**4.41**	**4.37**	**4.41**	5.02	5.17	5.16	5.11
30[b]	**3.22**	4.67	4.66	4.79	5.37	5.24	5.24	5.23
50[b]	**3.73**	4.55	4.55	4.71	5.16	5.02	5.00	5.03
10[c]	**1.95**	**4.34**	**4.30**	**4.30**	5.29	**5.57**	**5.59**	**5.57**
20[c]	**2.68**	4.54	4.52	4.61	5.05	5.11	5.13	5.11
30[c]	**3.11**	4.56	4.56	4.63	5.17	5.23	5.21	5.22
50[c]	**3.52**	4.61	4.61	4.57	4.70	5.06	5.04	5.05
10[d]	**2.17**	**4.33**	**4.28**	**4.32**	5.22	5.43	5.46	5.44
20[d]	**2.66**	**4.44**	**4.39**	4.51	4.93	5.05	5.06	5.06
30[d]	**3.14**	**4.37**	**4.35**	**4.41**	5.04	5.23	5.26	5.25
50[d]	**3.63**	4.69	4.69	4.70	4.75	5.03	5.05	5.08

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-58

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .40)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**1.83**	**4.33**	**4.30**	**4.40**	5.17	5.32	5.32	5.30
20[a]	**2.78**	**4.47**	**4.47**	**4.47**	4.99	5.08	5.08	5.05
30[a]	**3.28**	4.53	4.52	4.49	5.29	5.18	5.19	5.17
50[a]	**3.61**	4.57	4.53	4.66	4.77	5.03	5.02	4.98
10[b]	**1.84**	**4.34**	**4.27**	**4.33**	5.19	5.41	5.41	5.42
20[b]	**2.61**	**4.39**	**4.38**	**4.48**	4.96	5.06	5.06	5.03
30[b]	**2.98**	**4.40**	**4.38**	**4.47**	5.11	5.16	5.18	5.15
50[b]	**3.70**	4.63	4.64	4.74	4.99	4.92	4.93	4.92
10[c]	**1.90**	**4.14**	**4.13**	**4.26**	5.15	5.39	5.37	5.38
20[c]	**2.47**	**4.46**	**4.44**	4.52	4.87	5.07	5.08	5.08
30[c]	**2.96**	4.66	4.60	4.70	5.26	**5.54**	**5.54**	**5.53**
50[c]	**3.65**	4.67	4.67	4.75	4.97	5.09	5.10	5.10
10[d]	**2.25**	**4.38**	**4.34**	**4.30**	5.49	**5.55**	**5.55**	**5.55**
20[d]	**2.57**	**4.27**	**4.22**	**4.33**	5.10	5.08	5.07	5.07
30[d]	**2.99**	**4.48**	**4.48**	4.52	5.20	5.25	5.26	5.25
50[d]	**3.67**	4.69	4.65	4.68	5.03	4.98	4.94	4.97

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-59

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .55)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**1.74**	**4.29**	**4.23**	**4.36**	5.47	**5.53**	**5.52**	**5.53**
20[a]	**2.67**	4.58	4.57	4.57	5.00	5.06	5.06	5.04
30[a]	**3.12**	**4.38**	**4.36**	4.51	5.04	5.01	5.00	5.02
50[a]	**3.65**	4.66	4.68	4.68	5.12	5.12	5.11	5.10
10[b]	**1.63**	**4.11**	**4.08**	**4.19**	5.20	5.23	5.23	5.20
20[b]	**2.53**	4.55	4.52	4.69	4.96	5.03	5.02	5.05
30[b]	**2.99**	**4.29**	**4.26**	**4.35**	5.24	5.12	5.12	5.10
50[b]	**3.67**	4.70	4.70	4.76	4.96	4.92	4.93	4.97
10[c]	**1.91**	**4.04**	**4.00**	**4.11**	5.45	5.50	**5.51**	5.49
20[c]	**2.52**	**4.39**	**4.36**	**4.38**	5.22	4.97	4.95	4.96
30[c]	**2.96**	**4.32**	**4.29**	**4.25**	5.13	5.14	5.12	5.13
50[c]	**3.54**	4.56	4.55	4.61	4.96	4.95	4.92	4.94
10[d]	**2.06**	**4.17**	**4.10**	**4.17**	5.33	5.34	5.37	5.34
20[d]	**2.77**	**4.48**	**4.47**	**4.43**	5.02	5.00	4.98	4.99
30[d]	**3.01**	4.54	**4.48**	**4.43**	5.13	5.22	5.26	5.22
50[d]	**3.56**	4.63	4.58	4.57	5.06	5.06	5.04	5.07

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-60

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .70)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**1.64**	**4.19**	**4.14**	**4.21**	5.24	5.18	5.18	5.17
20[a]	**2.44**	**4.33**	**4.31**	**4.54**	5.12	5.00	5.01	5.02
30[a]	**3.24**	4.62	4.61	4.71	5.19	5.37	5.37	5.37
50[a]	**3.65**	4.73	4.72	4.77	5.07	5.12	5.12	5.11
10[b]	**1.75**	**4.24**	**4.22**	**4.32**	5.39	5.37	5.39	5.36
20[b]	**2.62**	**4.48**	**4.42**	4.54	5.22	5.17	5.18	5.13
30[b]	**3.01**	**4.48**	**4.45**	4.56	4.96	5.24	5.24	5.24
50[b]	**3.55**	**4.31**	**4.31**	**4.43**	4.93	4.79	4.80	4.80
10[c]	**1.80**	**4.04**	**3.96**	**4.04**	5.08	5.29	5.29	5.30
20[c]	**2.45**	**4.21**	**4.17**	**4.26**	5.13	4.79	4.80	4.78
30[c]	**2.99**	**4.49**	**4.46**	**4.44**	5.31	5.12	5.12	5.13
50[c]	**3.55**	**4.45**	**4.46**	4.51	4.74	4.87	4.86	4.86
10[d]	**2.09**	**3.90**	**3.85**	**3.80**	5.29	5.31	5.29	5.31
20[d]	**2.66**	**4.46**	**4.40**	**4.39**	5.04	4.93	4.91	4.92
30[d]	**3.01**	**4.45**	**4.42**	**4.47**	5.15	5.35	5.35	5.34
50[d]	**3.35**	**4.43**	**4.40**	**4.49**	4.58	5.14	5.17	5.15

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-61

Type I Error Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .85)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**1.67**	**4.17**	**4.09**	**4.17**	5.42	5.38	5.39	5.42
20[a]	**2.57**	**4.26**	**4.23**	**4.37**	4.97	5.06	5.04	5.07
30[a]	**3.06**	4.66	4.62	4.78	5.02	5.30	5.30	5.33
50[a]	**3.63**	4.54	4.54	4.62	4.91	4.95	4.96	4.95
10[b]	**1.74**	**4.29**	**4.22**	**4.33**	5.50	5.35	5.34	5.36
20[b]	**2.41**	**4.35**	**4.33**	**4.43**	5.11	5.07	5.07	5.08
30[b]	**3.05**	4.51	4.50	4.61	5.11	5.15	5.14	5.14
50[b]	**3.62**	**4.46**	**4.44**	**4.46**	5.01	5.06	5.05	5.03
10[c]	**1.79**	**4.17**	**4.08**	**4.17**	5.34	**5.61**	**5.61**	**5.60**
20[c]	**2.59**	**4.44**	**4.38**	**4.45**	5.00	5.03	5.01	5.01
30[c]	**2.93**	**4.37**	**4.32**	**4.42**	5.24	5.26	5.26	5.26
50[c]	**3.71**	**4.47**	**4.46**	4.59	4.99	5.01	5.00	5.02
10[d]	**2.11**	**4.12**	**4.01**	**4.10**	5.41	5.38	5.38	5.39
20[d]	**2.66**	**4.22**	**4.15**	**4.18**	5.09	4.91	4.97	4.91
30[d]	**2.96**	**4.29**	**4.29**	**4.27**	5.17	5.30	5.28	5.29
50[d]	**3.44**	**4.48**	**4.46**	**4.46**	5.02	4.92	4.94	4.92

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Bold faced values are out of bound (4.50, 5.50). All the four transformations except the third-order power have valid PDFs for the four distributions.

Power rates for the *F* and *FR* tests at correlation = .25 level. The power rates of the *F* and *FR* tests at the post correlation .25 were reported in Tables 4-62 to 4-66 for effect sizes at .25, .50, .75, 1.00 and 1.25, respectively, in the four distributions where all the four transformations except the third-order power method have valid PDFs. Bold faced (underscored) values were maximum (minimum) across the four data transformations in a condition where the maximum difference of power rates across the four data transformations was above .05 (denoted the same hereafter). As the results at effect size .25 in Table 4-62 indicate, in all conditions of the four distributions the power rate differences of the *F* and *FR* tests were above .05, ranging from .053 to.079 for the *F* tests, and from .291 to .807 for the *FR* tests. At effect size .50 as results in Table 4-63 showed, the power rate differences of the *F* tests were above the .05, ranging from .054 to .198 except five conditions (cell size = 10 in D31: sk = 2.50, kt = 60.0; D32: sk = 2.75, kt = 70; and D34: sk = 3.25, kt = 80.0; cell sizes = 30, and 50 in D33: sk = 3.00, kt = 90.0), while the power rate differences of the *FR* tests were above the .05, ranging from .064 to .614 except cell size = 50 in the last distribution (D34). At effect size .75 as results in Table 4-64 reveal, the power rates for the *F* tests were different (i.e. with differences above the .05 value) only at cell size = 10 in all the four distributions, with differences ranging from .134 to .142, while those for the *FR* tests were different at cell sizes = 10, and 20 in the four distributions with differences ranging from .096 to .417. At effect sizes 1.00 as results in Tables 4-65 and 4-66 indicate, power rates for the *F* tests were similar (i.e. with differences within the .05), those for the *FR* tests, however, were dissimilar at cell size = 10 in all four distributions, with differences ranging from .054 to .205. Apparently the power rates differed more frequently and with greater magnitude for the *FR* tests than for the *F* tests at this correlation level. The power rates at effect size 1.50 at correlation .25 were similar, thus not reported.

Table 4-62

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25, Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**13.96**	8.75	8.77	8.62	**63.49**	10.48	10.65	10.00
20[a]	**19.48**	14.04	14.06	13.69	**93.04**	16.20	16.63	14.88
30[a]	**24.64**	19.22	19.23	18.80	**99.10**	23.42	24.16	21.26
50[a]	**34.60**	29.71	29.69	28.89	**99.99**	36.56	37.86	33.34
10[b]	**15.34**	8.72	8.76	8.60	**69.49**	10.23	10.39	9.66
20[b]	**20.42**	14.15	14.18	13.94	**95.88**	16.62	17.04	15.19
30[b]	**25.83**	19.06	19.05	18.80	**99.62**	23.68	24.40	21.48
50[b]	**35.83**	30.48	30.43	30.14	**100.00**	38.13	39.26	34.49
10[c]	**15.84**	8.91	8.90	8.82	**51.70**	10.79	11.00	10.38
20[c]	**20.68**	14.13	14.10	13.94	**85.53**	16.79	17.30	15.70
30[c]	**25.54**	19.31	19.30	18.94	**96.96**	24.24	25.08	22.66
50[c]	**35.31**	30.25	30.32	30.12	**99.89**	38.98	40.36	36.32
10[d]	**15.86**	9.17	9.20	9.11	**40.03**	11.16	11.47	10.97
20[d]	**21.67**	14.37	14.39	14.28	**72.90**	17.11	17.76	16.67
30[d]	**27.11**	19.26	19.40	19.27	**90.54**	24.91	26.02	24.09
50[d]	**36.66**	29.95	30.14	29.99	**99.23**	39.26	41.15	37.94

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-63

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**40.35**	24.81	24.87	23.72	**82.57**	27.44	28.10	24.87
20[a]	**56.21**	46.45	46.43	45.30	**99.16**	53.13	54.57	48.44
30[a]	**68.71**	64.36	64.29	63.19	**99.98**	74.46	75.89	69.68
50[a]	84.58	85.63	85.55	85.41	100.00	93.72	94.47	91.36
10[b]	**42.13**	25.00	25.09	24.12	**86.02**	28.10	28.86	25.71
20[b]	**57.03**	46.99	46.88	45.94	**99.51**	54.47	55.72	50.45
30[b]	**68.83**	64.05	63.95	63.46	**100.00**	75.42	76.71	71.05
50[b]	84.57	85.56	85.48	85.56	100.00	94.29	94.91	92.31
10[c]	**42.94**	25.25	25.42	24.72	**88.52**	28.96	29.91	27.12
20[c]	**56.80**	46.97	47.04	46.44	**99.69**	55.24	56.85	52.46
30[c]	68.06	64.56	64.61	64.12	100.00	76.69	78.33	74.12
50[c]	83.72	85.57	85.58	85.56	100.00	94.68	95.43	93.56
10[d]	**45.10**	25.84	26.10	25.34	**84.92**	29.59	30.79	28.78
20[d]	**58.84**	47.25	47.53	46.86	**99.37**	56.83	58.97	55.67
30[d]	**69.92**	64.57	64.75	64.47	**99.99**	77.74	79.84	77.05
50[d]	84.68	85.44	85.54	85.59	100.00	95.52	96.36	95.28

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-64

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**62.84**	50.90	51.02	49.22	**91.30**	54.16	55.31	49.63
20[a]	81.29	80.78	80.70	80.47	**99.87**	88.01	88.91	84.88
30[a]	90.92	93.47	93.41	93.65	100.00	97.96	98.20	97.10
50[a]	97.91	98.94	98.96	99.14	100.00	99.96	99.97	99.92
10[b]	**63.92**	51.16	51.17	49.81	**93.01**	55.36	56.35	51.76
20[b]	81.35	81.13	80.98	80.91	**99.93**	88.89	89.63	86.59
30[b]	90.51	92.89	92.82	93.07	100.00	98.25	98.50	97.64
50[b]	97.72	98.96	98.98	99.08	100.00	99.97	99.98	99.95
10[c]	**64.28**	51.62	51.85	50.93	**94.17**	56.31	57.63	54.12
20[c]	81.02	81.57	81.56	81.34	**99.94**	89.84	90.78	88.90
30[c]	89.82	93.21	93.20	93.32	100.00	98.49	98.80	98.28
50[c]	97.34	98.97	99.01	99.09	100.00	99.98	99.98	99.97
10[d]	**65.81**	52.11	52.39	51.65	**95.39**	57.34	58.98	56.81
20[d]	81.51	80.95	81.02	81.02	**99.97**	90.34	91.41	90.43
30[d]	90.65	93.15	93.19	93.22	100.00	98.61	98.92	98.69
50[d]	97.67	98.85	98.93	98.97	100.00	99.98	99.99	99.99

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-65

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25; Effect Size = 1.00)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	78.25	75.13	75.06	74.29	**95.73**	78.51	79.27	75.23
20[a]	92.84	95.40	95.37	95.73	99.98	98.67	98.79	98.19
30[a]	97.60	99.02	99.05	99.22	100.00	99.93	99.95	99.93
50[a]	99.78	99.87	99.88	99.91	100.00	100.00	100.00	100.00
10[b]	78.51	75.30	75.13	74.63	**96.42**	79.24	79.93	77.09
20[b]	92.52	95.34	95.29	95.53	99.99	98.84	98.95	98.67
30[b]	97.47	99.01	99.01	99.15	100.00	99.96	99.96	99.96
50[b]	99.70	99.88	99.91	99.91	100.00	100.00	100.00	100.00
10[c]	77.96	75.28	75.29	74.94	**96.69**	80.04	80.95	79.33
20[c]	91.88	95.37	95.40	95.56	100.00	99.01	99.11	98.99
30[c]	97.11	98.92	98.95	99.04	100.00	99.98	99.98	99.97
50[c]	99.58	99.85	99.86	99.88	100.00	100.00	100.00	100.00
10[d]	79.36	75.94	76.04	75.79	**97.46**	80.94	82.16	81.48
20[d]	92.25	95.20	95.20	95.33	100.00	99.13	99.30	99.32
30[d]	97.11	98.79	98.85	98.93	100.00	99.98	99.98	99.99
50[d]	99.61	99.82	99.85	99.86	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-66

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .25; Effect Size = 1.25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	87.24	89.18	89.07	89.34	**97.63**	91.71	92.05	91.06
20[a]	97.21	98.89	98.90	99.09	100.00	99.91	99.92	99.92
30[a]	99.37	99.76	99.77	99.83	100.00	100.00	100.00	100.00
50[a]	99.96	99.98	99.98	99.99	100.00	100.00	100.00	100.00
10[b]	87.29	89.29	89.18	89.25	**97.93**	92.38	92.63	92.12
20[b]	96.86	98.73	98.76	98.94	100.00	99.95	99.95	99.96
30[b]	99.34	99.74	99.77	99.80	100.00	100.00	100.00	100.00
50[b]	99.95	99.96	99.97	99.98	100.00	100.00	100.00	100.00
10[c]	86.60	89.17	89.10	89.35	**98.17**	92.79	93.27	92.89
20[c]	96.63	98.71	98.74	98.86	100.00	99.94	99.95	99.96
30[c]	99.10	99.74	99.77	99.81	100.00	100.00	100.00	100.00
50[c]	99.93	99.94	99.96	99.97	100.00	100.00	100.00	100.00
10[d]	87.41	89.12	89.04	89.19	**98.65**	93.28	93.80	93.90
20[d]	96.80	98.64	98.69	98.76	100.00	99.97	99.97	99.98
30[d]	99.05	99.64	99.67	99.70	100.00	100.00	100.00	100.00
50[d]	99.94	99.96	99.97	99.98	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Power rates for the *F* and *FR* tests at correlation = .40 level. The power rates of the *F* and *FR* tests at the post correlation .40 were reported in Tables 4-67 to 4-70 for effect sizes at .25, .50, .75 and 1.0, respectively, in the four distributions where all the four transformations except the third-order power method have valid PDFs. As the results at effect size .25 in Table 4-67 indicate, in all conditions of the four distributions the power rate differences of the *F* and *FR* tests across the four data distributions were all above .05, ranging from .052 to .097 for the *F* tests, and from .348 to .783 for the *FR* tests. At effect size .50 (Table 4-68), the power differences of the *F* tests across the four data distributions were above .05 at cell sizes = 10, and 20 in all distributions, ranging from .071 to .191, while those of the *FR* tests across the four data distributions were above .05 at cell sizes = 10, 20, and 30 in all distributions, ranging from .13 to .565. As the results at effect size .75 in Table 4-69 show, the power differences of the *F* tests across the four data distributions were above 5.0 at cell sizes = 10 in all distributions, ranging from .084 to .108, while those for the *FR* tests were above .05 in six conditions (cell sizes = 10, and 20 in D31: sk = 2.50, kt = 60; and D32: sk = 2.75, kt = 70.0; and cell size = 10 in D33: sk = 3.00, kt = 80.0; and D34: sk = 3.25, kt = 90.0), ranging from .062 to .334. At effect size 1.00 as results in Table 4-70 reveal, the power differences of the *F* tests across the four data distributions were all within .05, while those for the *FR* tests were above 5.0 at cell size = 10 in all distributions, ranging from .097 to .127. Again the power rates differed more frequently and with greater magnitude for the *FR* tests than for the *F* tests at this correlation level. The power rates at effect size 1.25 and 1.50 levels at correlation .40 were similar, hence not reported.

Table 4-67

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .40; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**17.88**	10.25	10.25	10.00	**67.80**	12.06	12.32	11.57
20[a]	**24.30**	16.70	16.78	16.19	**94.96**	19.01	19.56	18.03
30[a]	**30.89**	23.72	23.71	23.06	**99.52**	28.29	29.18	26.60
50[a]	**42.67**	37.02	37.00	36.34	**100.00**	45.15	46.47	43.10
10[b]	**18.99**	9.97	9.99	9.86	**72.40**	12.00	12.31	11.33
20[b]	**25.39**	16.71	16.67	16.36	**96.68**	19.70	20.26	18.35
30[b]	**31.71**	23.70	23.71	23.19	**99.82**	29.10	30.08	27.10
50[b]	**43.84**	37.80	37.72	37.07	**100.00**	47.11	48.44	43.91
10[c]	**19.61**	10.13	10.19	10.01	**57.49**	12.17	12.46	11.65
20[c]	**25.07**	17.16	17.24	16.76	**89.81**	20.32	21.00	19.44
30[c]	**31.52**	24.23	24.29	24.00	**98.27**	30.00	31.14	28.48
50[c]	**42.18**	37.37	37.43	36.97	**99.96**	47.31	49.08	45.32
10[d]	**19.82**	10.22	10.34	10.08	**46.87**	12.28	12.66	12.11
20[d]	**26.68**	17.36	17.42	17.28	**80.81**	20.65	21.57	20.32
30[d]	**33.09**	24.06	24.31	23.84	**94.93**	30.58	32.13	30.24
50[d]	**44.65**	37.68	37.93	37.50	**99.71**	48.46	50.70	47.90

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-68

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .40; Effect Size = .50)

Cell Size	*F* Test: Third Power	*F* Test: *g*-and-*h*	*F* Test: The GLD	*F* Test: The Burr	Friedman Test: Third Power	Friedman Test: *g*-and-*h*	Friedman Test: The GLD	Friedman Test: The Burr
10[a]	**47.21**	30.44	30.57	29.22	**85.88**	33.27	34.19	30.73
20[a]	**63.91**	56.60	56.61	55.51	**99.50**	63.49	64.99	60.22
30[a]	76.18	74.61	74.54	73.78	**99.99**	83.84	84.90	81.15
50[a]	90.13	92.06	92.04	92.18	100.00	97.57	97.88	96.85
10[b]	**48.78**	31.19	31.31	30.09	**88.59**	34.71	35.58	32.14
20[b]	**64.73**	57.07	56.96	55.85	**99.68**	65.35	66.73	61.82
30[b]	76.38	74.78	74.65	74.39	**100.00**	85.00	86.16	82.53
50[b]	90.01	91.93	91.90	92.03	100.00	97.92	98.25	97.19
10[c]	**48.81**	31.76	32.01	31.26	**89.93**	34.93	36.02	33.59
20[c]	**63.89**	57.36	57.52	56.80	**99.81**	66.72	68.36	64.80
30[c]	75.17	74.81	74.82	74.51	**100.00**	85.90	87.30	84.70
50[c]	88.88	91.96	92.02	92.10	100.00	98.24	98.61	97.98
10[d]	**50.86**	32.09	32.49	31.82	**88.02**	36.26	37.71	35.78
20[d]	**66.04**	57.98	58.31	57.74	**99.72**	67.78	70.14	67.91
30[d]	76.78	75.15	75.34	75.01	**100.00**	87.04	88.70	87.35
50[d]	90.04	91.76	91.94	91.96	100.00	98.43	98.84	98.50

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-69

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .40; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**70.00**	60.83	60.84	59.18	**93.67**	63.83	64.79	60.28
20[a]	86.43	88.40	88.32	88.34	**99.93**	94.21	94.68	92.79
30[a]	94.13	96.46	96.44	96.76	100.00	99.42	99.50	99.16
50[a]	99.00	99.56	99.58	99.67	100.00	100.00	100.00	99.99
10[b]	**70.50**	61.67	61.61	60.44	**94.37**	65.16	66.12	62.42
20[b]	86.34	88.29	88.23	88.38	**99.95**	94.69	95.10	93.72
30[b]	93.82	96.24	96.22	96.45	100.00	99.48	99.56	99.36
50[b]	98.96	99.58	99.61	99.65	100.00	100.00	100.00	99.99
10[c]	**70.22**	62.41	62.57	61.81	**95.12**	66.50	67.72	65.21
20[c]	85.69	88.58	88.53	88.53	99.97	95.18	95.81	95.00
30[c]	92.97	96.38	96.44	96.62	100.00	99.61	99.70	99.57
50[c]	98.68	99.45	99.48	99.54	100.00	99.99	99.99	100.00
10[d]	**71.81**	62.46	62.84	62.25	**96.17**	67.33	69.22	67.61
20[d]	86.71	88.12	88.20	88.31	99.98	95.64	96.31	95.95
30[d]	93.67	96.22	96.27	96.35	100.00	99.74	99.81	99.78
50[d]	98.64	99.44	99.48	99.53	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-70

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = 0.40; Effect Size = 1.00)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	82.88	83.53	83.34	82.86	**96.76**	86.13	86.65	84.11
20[a]	94.99	97.45	97.43	97.80	99.99	99.69	99.72	99.61
30[a]	98.64	99.42	99.44	99.57	100.00	99.99	99.99	99.99
50[a]	99.87	99.91	99.92	99.94	100.00	100.00	100.00	100.00
10[b]	83.37	83.60	83.46	83.12	**97.24**	87.09	87.56	86.11
20[b]	94.98	97.46	97.47	97.73	100.00	99.72	99.76	99.65
30[b]	98.38	99.31	99.35	99.46	100.00	100.00	100.00	99.99
50[b]	99.87	99.91	99.92	99.93	100.00	100.00	100.00	100.00
10[c]	82.55	83.65	83.63	83.43	**97.32**	87.80	88.54	87.60
20[c]	94.55	97.40	97.43	97.59	99.99	99.73	99.79	99.73
30[c]	98.21	99.33	99.38	99.45	100.00	100.00	100.00	100.00
50[c]	99.80	99.88	99.90	99.90	100.00	100.00	100.00	100.00
10[d]	83.49	83.61	83.76	83.70	**97.87**	88.37	89.24	89.10
20[d]	94.83	97.22	97.29	97.44	100.00	99.80	99.85	99.87
30[d]	98.34	99.26	99.30	99.38	100.00	100.00	100.00	100.00
50[d]	99.83	99.89	99.91	99.91	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Power rates for the *F* and *FR* tests at correlation = .55 level. The power rates of the *F* and *FR* tests at the post correlation .55 were reported in Tables 4-71 to 4-74 for effect sizes at .25, .50, .75 and 1.00, respectively, in the four distributions where all the four transformations except the third-order power method have valid PDFs. As the results at effect size .25 in Table 4-71 indicate, the differences of power rates of the F test across the four data transformations were above the .05 in all the four distributions except cell size = 50 in the third distribution (D33: sk = 3.00, kt = 90.0), ranging from .06 to .128, while those for the FR tests were above the .05 value in all distributions, ranging from .379 to .739. At effect size .50 (see results in Table 4-72), the power differences of the *F* tests were above the cut-off value (.050) at cell size = 10 in all distributions, ranging from .157 to .174, while those for the *FR* tests were above .05 in nine conditions (cell sizes = 10, 20, and 30) in the first three distributions (D31: sk = 2.50, kt = 60; D32: sk = 2.75, kt = 70.0; and D33: sk = 3.00, kt = 80.0) and two conditions (cell sizes = 10, and 20) in the last distribution (D34: sk = 3.25, kt = 90.0), ranging .059 to .487. At effect size .75 and 1.00 as results in (Tables 4-73 and 4-74) showed, the power differences of the *F* tests were all small (i.e., within .05), while those for the *FR* tests were above .05 at cell size = 10 in all distributions at effect size .75, and at cell size = 10 in the first distribution (D31) at effect size 1.00, with the differences ranging .056 to .218. Similar to the previous two correlation levels the power rates differed more frequently and with greater magnitude for the *FR* tests than for the *F* tests at this correlation level. The power rates at effect size 1.25 and 1.50 levels at correlation .55 were similar, hence not reported.

Table 4-71

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .55; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**23.32**	12.52	12.52	11.95	**72.30**	14.22	14.54	13.96
20[a]	**31.56**	22.05	22.09	21.48	**96.74**	25.03	25.81	24.38
30[a]	**39.23**	31.31	31.31	30.23	**99.75**	37.40	38.58	36.47
50[a]	**52.79**	48.41	48.35	47.24	**100.00**	58.07	59.65	57.01
10[b]	**25.07**	12.61	12.64	12.29	**76.26**	14.56	14.91	13.87
20[b]	**32.64**	21.82	21.85	21.28	**97.82**	25.24	26.05	23.89
30[b]	**40.62**	32.05	32.01	31.26	**99.88**	38.65	39.79	36.80
50[b]	**54.25**	49.02	48.89	48.03	**100.00**	59.56	61.19	56.92
10[c]	**24.70**	12.77	12.82	12.48	**65.07**	14.94	15.42	14.43
20[c]	**32.12**	22.35	22.50	21.91	**93.81**	26.28	27.27	25.55
30[c]	**39.19**	31.94	32.14	31.31	**99.31**	39.39	41.06	38.16
50[c]	52.91	49.39	49.53	48.84	**99.99**	61.32	63.48	59.86
10[d]	**25.27**	12.72	12.89	12.73	**54.98**	15.41	16.00	15.33
20[d]	**34.00**	22.57	22.83	22.42	**88.34**	26.93	28.28	26.88
30[d]	**41.67**	32.11	32.49	31.96	**97.89**	40.35	42.50	40.54
50[d]	**55.01**	49.11	49.60	49.06	**99.95**	62.08	64.96	62.49

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-72

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .55; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**55.76**	40.76	40.92	38.99	**89.02**	43.06	44.13	40.85
20[a]	73.36	70.25	70.16	69.07	**99.76**	77.32	78.53	74.94
30[a]	83.89	85.33	85.25	85.13	**100.00**	93.35	93.97	91.91
50[a]	94.89	96.69	96.68	96.97	100.00	99.57	99.66	99.47
10[b]	**57.83**	41.69	41.87	40.39	**91.01**	44.58	45.66	42.31
20[b]	73.31	70.09	69.98	69.31	**99.86**	78.26	79.42	76.11
30[b]	84.47	85.68	85.56	85.47	**100.00**	93.90	94.48	92.70
50[b]	94.70	96.66	96.71	96.81	100.00	99.64	99.71	99.53
10[c]	**56.65**	41.63	42.02	40.91	**92.04**	45.27	46.84	44.26
20[c]	72.63	71.04	71.11	70.52	**99.88**	79.91	81.52	79.06
30[c]	82.57	85.41	85.49	85.45	**100.00**	94.50	95.28	94.15
50[c]	93.66	96.51	96.60	96.68	100.00	99.75	99.80	99.71
10[d]	**58.48**	42.15	42.83	41.93	**91.03**	46.43	48.29	46.67
20[d]	74.05	70.23	70.70	70.29	**99.87**	81.05	83.18	81.75
30[d]	84.78	85.66	85.88	85.77	100.00	95.06	95.95	95.48
50[d]	94.42	96.27	96.43	96.52	100.00	99.79	99.85	99.85

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-73

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .55; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	76.96	73.60	73.56	72.21	**95.31**	76.51	77.33	73.54
20[a]	91.40	94.17	94.11	94.51	99.98	98.37	98.52	97.88
30[a]	97.00	98.48	98.50	98.77	100.00	99.93	99.94	99.90
50[a]	99.66	99.82	99.84	99.88	100.00	100.00	100.00	100.00
10[b]	77.58	74.20	74.11	73.23	**96.12**	77.75	78.64	75.67
20[b]	91.42	94.38	94.33	94.54	99.98	98.60	98.76	98.35
30[b]	96.90	98.40	98.42	98.64	100.00	99.96	99.97	99.94
50[b]	99.54	99.74	99.76	99.79	100.00	100.00	100.00	100.00
10[c]	76.78	74.30	74.47	73.78	**96.29**	78.87	80.09	78.31
20[c]	90.37	94.13	94.13	94.38	99.99	98.71	98.89	98.75
30[c]	96.17	98.33	98.40	98.50	100.00	99.96	99.97	99.97
50[c]	99.40	99.66	99.70	99.74	100.00	100.00	100.00	100.00
10[d]	78.28	74.91	75.33	74.72	**97.12**	79.71	81.26	80.31
20[d]	91.42	93.99	94.09	94.19	100.00	98.80	99.09	99.09
30[d]	96.55	98.13	98.22	98.31	100.00	99.96	99.98	99.97
50[d]	99.45	99.67	99.70	99.73	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-74

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .55; Effect Size = 1.00)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	88.25	90.93	90.85	90.91	**98.02**	93.53	93.84	<u>92.38</u>
20[a]	97.52	98.88	98.90	99.11	100.00	99.96	99.97	99.94
30[a]	99.48	99.75	99.76	99.81	100.00	100.00	100.00	100.00
50[a]	99.97	99.95	99.96	99.97	100.00	100.00	100.00	100.00
10[b]	88.03	90.84	90.67	90.79	98.15	94.16	94.43	93.69
20[b]	97.09	98.56	98.59	98.77	100.00	99.98	99.98	99.97
30[b]	99.27	99.61	99.64	99.68	100.00	100.00	100.00	100.00
50[b]	99.96	99.93	99.93	99.94	100.00	100.00	100.00	100.00
10[c]	87.00	90.82	90.78	90.85	98.10	94.34	94.78	94.56
20[c]	96.86	98.64	98.68	98.85	100.00	99.99	99.99	99.98
30[c]	99.20	99.63	99.67	99.69	100.00	100.00	100.00	100.00
50[c]	99.93	99.93	99.94	99.94	100.00	100.00	100.00	100.00
10[d]	88.22	90.79	90.85	90.79	98.50	94.92	95.42	95.33
20[d]	97.12	98.60	98.64	98.72	100.00	99.97	99.98	99.99
30[d]	99.19	99.58	99.65	99.69	100.00	100.00	100.00	100.00
50[d]	99.95	99.93	99.94	99.95	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Power rates for the *F* and *FR* tests at correlation = .70 level. The power rates of the *F* and *FR* tests at the post correlation .70 were reported in Tables 4-75 to 4-77 for effect sizes at .25, .50, and .75, respectively, in the four distributions where all the four transformations except the third-order power method have valid PDFs. As the results at effect size .25 in Table 4-75 indicate, the differences of power rates of the *F* tests were above .05 across the four data transformation except cell size = 50 in all the four distributions, ranging from .062 to .164, while those for the *FR* tests were all above .05, ranging from .184 to .63. At effect size .50 (see results in Table 4-76), the differences of power rates of the *F* tests were above .05 across the four data transformation only at cell size = 10 in all the four distributions, ranging from .091 to .113, while those for the *FR* tests were above .05 across the four data transformations at only cell sizes = 10, and 20 in all the four distributions, ranging from .057 to .354. At effect size .75 as the results in Table 4-77 reveal, the differences of power rates of the *F* tests were small (i.e., within .05), while those for the *FR* tests were above .05 across the four data transformation at only cell sizes = 10 in all the four distributions, ranging from .059 to .088. Similar to the previous correlation levels the power rates differed more frequently and with greater magnitude for the *FR* tests than for the *F* tests at this correlation level. The power rates of other effect size levels were similar across the four data transformations, hence not reported for this correlation level.

Table 4-75

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .70; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**32.55**	17.43	17.54	16.58	**78.70**	19.20	19.71	18.86
20[a]	**43.43**	32.28	32.43	31.05	**98.17**	36.05	37.13	35.81
30[a]	**52.86**	46.05	46.05	44.52	**99.90**	53.65	55.16	53.63
50[a]	69.33	67.49	67.51	66.37	**100.00**	78.24	79.73	78.33
10[b]	**33.26**	17.72	17.79	17.19	**81.35**	19.82	20.31	19.17
20[b]	**43.89**	32.52	32.53	31.55	**98.88**	37.36	38.52	35.84
30[b]	**53.64**	46.11	46.06	45.16	**99.96**	55.19	56.74	53.37
50[b]	69.39	67.86	67.80	67.18	**100.00**	79.45	81.00	78.15
10[c]	**33.14**	18.12	18.32	17.68	**74.38**	20.30	21.10	20.05
20[c]	**42.93**	32.76	32.99	32.29	**97.53**	37.94	39.66	37.40
30[c]	**52.46**	46.82	47.20	46.31	**99.82**	56.85	58.92	56.17
50[c]	67.49	67.34	67.56	67.13	**100.00**	80.43	82.42	80.00
10[d]	**34.32**	17.91	18.41	17.88	**67.68**	20.67	21.69	21.06
20[d]	**45.37**	33.23	33.86	33.19	**95.17**	39.40	41.62	39.91
30[d]	**54.50**	46.80	47.32	46.59	**99.56**	57.89	60.78	59.16
50[d]	69.70	67.31	67.96	67.47	**100.00**	81.57	84.07	82.78

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-76

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .70; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**67.22**	58.06	58.18	55.96	**92.79**	60.56	61.77	57.41
20[a]	83.65	85.68	85.62	85.40	**99.90**	92.11	92.77	90.77
30[a]	92.22	94.86	94.84	95.20	100.00	98.99	99.14	98.72
50[a]	98.31	99.05	99.09	99.23	100.00	99.99	99.99	99.99
10[b]	**67.85**	58.66	58.75	57.18	**93.74**	61.62	62.78	59.42
20[b]	83.91	85.53	85.40	85.15	**99.93**	92.78	93.38	91.57
30[b]	92.12	94.59	94.57	94.83	100.00	99.15	99.31	98.89
50[b]	98.15	98.96	99.01	99.16	100.00	99.99	100.00	99.99
10[c]	**67.30**	58.94	59.38	58.25	**94.23**	62.34	64.11	61.57
20[c]	82.91	85.67	85.80	85.55	**99.97**	93.54	94.31	93.23
30[c]	90.83	94.39	94.48	94.61	100.00	99.25	99.42	99.18
50[c]	97.83	98.92	98.99	99.06	100.00	100.00	100.00	100.00
10[d]	**69.03**	59.36	60.16	59.04	**94.74**	63.54	65.83	64.45
20[d]	84.03	85.45	85.66	85.51	**99.98**	94.23	95.22	94.82
30[d]	92.06	94.33	94.55	94.66	100.00	99.42	99.56	99.50
50[d]	98.06	98.79	98.91	98.96	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-77

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .70; Effect Size = .75)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	85.45	87.46	87.33	87.01	**97.35**	90.34	90.75	88.55
20[a]	96.14	97.94	97.95	98.24	99.99	99.90	99.90	99.79
30[a]	98.92	99.44	99.48	99.57	100.00	100.00	100.00	100.00
50[a]	99.93	99.87	99.89	99.91	100.00	100.00	100.00	100.00
10[b]	85.25	87.50	87.39	87.05	**97.56**	91.12	91.58	90.05
20[b]	95.76	97.72	97.72	97.97	100.00	99.89	99.90	99.89
30[b]	98.81	99.41	99.45	99.53	100.00	100.00	100.00	100.00
50[b]	99.92	99.86	99.87	99.89	100.00	100.00	100.00	100.00
10[c]	84.47	87.22	87.18	87.03	**97.71**	91.49	92.07	91.33
20[c]	95.35	97.69	97.75	97.87	99.99	99.91	99.92	99.92
30[c]	98.52	99.33	99.38	99.45	100.00	100.00	100.00	100.00
50[c]	99.87	99.87	99.89	99.90	100.00	100.00	100.00	100.00
10[d]	85.59	87.23	87.41	87.30	**98.11**	92.22	93.09	92.76
20[d]	95.59	97.59	97.66	97.82	100.00	99.94	99.94	99.93
30[d]	98.66	99.23	99.31	99.36	100.00	100.00	100.00	100.00
50[d]	99.88	99.82	99.84	99.86	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Power rates for the *F* and *FR* tests at correlation = .85 level. The power rates of the *F* and *FR* tests at post correlation .85 were reported in Tables 4-78 to 4-79 for effect sizes at .25 and .50, respectively, in the four distributions where all the four transformations except the third-order power method have valid PDFs. As the results at effect size .25 in Table 4-78 indicate, the power differences of the *F* tests across the four data transformations were large (above .05) in nine conditions (cell sizes at 10, and 20 in both D31: sk = 2.50, kt = 60.0; and D32: sk = 2.75, kt = 70.0; cell size = 10 in D33: sk = 3.00, kt = 80.0; and cell sizes = 10, 20, 30, and 50 in D34: sk = 3.25, kt = 90.0), ranging from .07 to .185, while those for *FR* tests were large at cell sizes = 10, 20, and 30 in all four distributions, ranging from .115 to .535. At effect size .50 (see results in Table 4-79), the power differences of the *F* tests across the four data transformations were above .05 in one condition (cell size = 10 in D34) with difference at .092, while those for the *FR* tests were large at cell size = 10 in all four distributions, ranging from .078 to .117. the power rates of the *F* and *FR* tests for other effect size levels at post correlation .85 were similar across the four data transformations, thus not reported.

A careful examination of the power rates of the *F* and *FR* tests across the four data transformations at all the effect size and correlation levels in the third group of distributions (where all transformations except the third-order power method have valid PDFs) indicates that (a) over one fifth (i.e., 104 out of 480) of the conditions had large power differences (i.e., above .05) for the *F* tests, (b) more than one third (i.e., 161 out of 480) of the conditions had large power differences for the *FR* tests, (c) the magnitude of power difference for the *FR* tests was larger than that for the *F* tests (with extreme values .807 and .198, respectively), (d) the third-order power transformation was the most powerful and apart from the other three procedures in most of the cases for both the *F* and *FR* tests, (e) the power differences associated with the three transformations (i.e., the Burr, *g*-and-*h,* and GLD) were reduced in terms of frequency and magnitude for both the *F* and *FR* tests (i.e., 5 out of 480, .18 for the *F* tests vs. 9 out of 480, .194 for the *FR* tests), and (f) the power difference between the Burr and *g*-and-*h* transformations (i.e., pairwise comparison) in each condition were very small (i.e., within .004 and .016 respectively for the *F* and *FR* tests).

More generally, in the group of four distributions with more skewness and kurtosis generated with the four transformations where the third-order power method had invalid PDFs and the *g*-and-*h*, GLD and the Burr transformations had valid PDFs, the Type I error and power rates of both the *F* and *FR* tests, might be disparate. The inconsistencies of Type I error and power rates were frequently between the third-order power transformation (without valid PDFs) and the rest three transformation procedures with valid PDFs (i.e., the Burr, *g*-and-*h,* and GLD). Type I error and power rates for both the *F* and *FR* tests were generally relatively similar across the three transformation procedures (i.e., the Burr, *g*-and-*h,* and GLD) with valid PDFs.

Table 4-78

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .85; Effect Size = .25)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	**49.66**	33.35	33.74	31.42	**86.14**	35.14	36.32	34.24
20[a]	**65.19**	59.70	59.98	57.73	**99.55**	65.71	67.59	64.82
30[a]	76.16	76.04	76.17	75.03	**99.99**	85.27	86.71	84.45
50[a]	89.86	91.87	92.01	91.97	**100.00**	97.94	98.32	97.76
10[b]	**50.78**	33.92	34.40	32.25	**88.28**	35.97	37.16	34.80
20[b]	**65.59**	60.10	60.32	58.55	**99.68**	67.30	69.13	65.71
30[b]	76.57	76.54	76.72	75.71	**99.99**	86.70	88.11	85.69
50[b]	89.64	91.94	92.05	92.12	**100.00**	98.32	98.63	98.02
10[c]	**49.82**	34.49	35.22	33.50	**86.87**	36.63	38.43	36.28
20[c]	**64.04**	60.50	61.13	59.70	**99.56**	68.63	71.05	68.37
30[c]	74.68	76.77	77.25	76.54	**99.99**	87.61	89.35	87.64
50[c]	88.11	91.63	91.96	91.81	100.00	98.40	98.79	98.41
10[d]	**51.49**	34.64	51.44	34.61	**84.37**	38.02	56.20	38.99
20[d]	66.24	60.15	**78.04**	60.05	**99.34**	69.75	89.15	71.33
30[d]	76.46	76.10	**89.48**	76.34	**99.98**	88.44	98.15	89.58
50[d]	89.74	91.43	**97.22**	91.82	100.00	98.75	99.97	98.97

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

Table 4-79

Power Rates of F and Friedman Tests of Four Data Transformations in Four Distributions All except Power Method Have Valid PDFs (%; Correlation = .85; Effect Size = .50)

	F Test				Friedman Test			
Cell Size	Third Power	*g*-and-*h*	The GLD	The Burr	Third Power	*g*-and-*h*	The GLD	The Burr
10[a]	83.07	84.69	84.70	84.01	**96.69**	87.58	88.28	84.98
20[a]	95.01	97.08	97.12	97.51	100.00	99.74	99.78	99.51
30[a]	98.47	99.11	99.15	99.30	100.00	100.00	100.00	100.00
50[a]	99.82	99.81	99.82	99.86	100.00	100.00	100.00	100.00
10[b]	83.00	84.77	84.77	84.19	**97.00**	88.56	89.26	86.95
20[b]	94.68	96.87	96.93	97.14	99.99	99.75	99.80	99.71
30[b]	98.35	99.01	99.06	99.18	100.00	100.00	100.00	100.00
50[b]	99.86	99.80	99.82	99.84	100.00	100.00	100.00	100.00
10[c]	82.20	84.79	84.97	84.52	**97.17**	89.12	90.03	88.71
20[c]	94.00	96.80	96.89	96.98	99.99	99.79	99.84	99.76
30[c]	97.97	98.96	99.05	99.13	100.00	100.00	100.00	100.00
50[c]	99.79	99.77	99.81	99.83	100.00	100.00	100.00	100.00
10[d]	83.52	84.74	**92.70**	84.58	**97.69**	89.86	97.29	89.96
20[d]	94.74	96.68	98.67	96.89	100.00	99.83	99.99	99.87
30[d]	98.21	98.82	99.58	99.03	100.00	100.00	100.00	100.00
50[d]	99.82	99.78	99.92	99.82	100.00	100.00	100.00	100.00

Note. [a] D31: skewness (sk) = 2.50, kurtosis (kt) = 60.0; [b] D32: sk = 2.75, kt = 70.0; [c] D33: sk = 3.00, kt = 80.0; [d] D34: sk = 3.25, kt = 90.0. Underscored (Bold faced) values are the minimum (maximum) of the condition. All the four transformations except the third-order power have valid PDFs for the four distributions.

CHAPTER 5

CONCLUSIONS

A Monte Carlo simulation study was conducted to investigate the Type I error and power properties associated with the four data transformations: the Burr, *g*-and-*h,* GLD, and the power method. Both between-subjects and within-subjects ANOVA *F* tests and their nonparametric competitors, the *KW* and *FR* tests under a four group/occasion design were replicated 50,000 times. The simulated factors considered were 14 distributions of three categories, seven levels of effect sizes, and four levels of sample sizes for the between-subjects design, while for the within-subjects design a five-level correlation was added in addition to the factors considered in the between-subjects design.

First Group of Distributions

In the first group of six distributions all four data transformations have valid PDFs; but the last two distributions (i.e., D15 and D16) are close to their bounadary conditions for the third-order power transformation to produce valid PDFs. Results of this simulation study suggested the following conclusions:

1. Inconsistent Type I error rates for the parametric *F* tests was a concern. More specifically, about 17% (i.e., 4/24) of the conditions for the between-subjects *F* tests, and about 24 % (i.e., 29/120) for the within-subjects *F* tests, had inconsistent Type I errors. The inconsistent Type I errors across the four data transformations for the parametric *F* tests frequently occurred in the last two distributions that were more apart from normality, while the Type I errors were generally robust and consistent in the first four distributions. The third-order power method was systematically more conservative in the last two distributions and contributed most of the inconsistency of type I errors for the parametric *F* tests.

2. The Type I error rates for the *KW* tests were all robust and consistent across the four data transformations, whereas those for the within-subjects *FR* tests were inconsistent in about 4% (i.e., 5/120) of the conditions across the four data transformations. Both the *KW* and *FR* tests were systematically more robust than their respective parametric competitors, particularly when distribution assumptions were violated. Type I error rates of the nonparametric tests were similar or the inconsistency occurred with low frequency in the cases where they were dissimilar; thus, did not seem an important concern.

3. About 1% (i.e., 2/144) of the conditions for the between-subjects *F* tests had inconsistent power rates with the extreme difference at .055, while about 4% (28/720) of the conditions for within-subjects *F* tests had inconsistent power rates with the extreme difference at .19. The magnitude of power differences for the within-subjects *F* tests could be a concern.

4. About 10% (i.e., 15/144) of the conditions had inconsistent power rates for the between-subjects *KW* tests with the extreme difference at .569, while about 9% (61/720) of the conditions for the within-subject *FR* tests had inconsistent power rates with the extreme difference at .492. Inconsistent power rates of the nonparametric tests across the four data transformations frequently occurred in the more skewed and heavy-tailed distributions close to the associated boundary conditions with valid PDFs (e.g., D15 and D16), and thus, were an important concern. The third-order power transformation was mostly more powerful and contributed the majority of the inconsistency of the power rates for the nonparametric tests.

5. ANOVA *F* tests were more or equally powerful in contrast to their respective nonparametric competitors when the distribution assumption was satisfied (in the case of between-subjects design) or not severely violated (in the case of within-subjects design); the nonparametric tests were more powerful than or equally powerful to their parametric counterparts when normality was severely violated. This result is consistent with Feir-Walsh and Toothaker (1974); Harwell and Serlin (1994); and Iman, Hora, and Canover (1984).

6. The Burr, *g*-and-*h* and GLD three data transformations were more consistent in both Type I error and power rates for both the parametric and nonparametric tests considered in this study in all conditions of the six distributions. The occurrence of inconsistent Type I errors for the between-subjects (within-subjects) *F* tests reduced to 1/24 (5/120). The occurrence of inconsistent power rates for the within-subjects *F* tests reduced to 11/720, with the extreme difference reduced to .121. And, the occurrence of inconsistent power rates for the *KW* (*FR*) tests reduced to 7/144 (39/720), with both extreme power differences reduced to .149.

7. A pairwise comparison between the *g*-and-*h* and GLD transformations further narrowed down the power differences to .127 for the nonparametric tests with excellent consistency of power rates for the parametric *F* tests (power differences within .022. The Burr and GLD pair transformations further narrowed the power differences of the nonparametric tests down to .087, but remained a power difference at .122 for the parametric *F* tests.

More concisely, inconsistent or dissimilar Type I error rates of the parametric *F* tests happened more frequently in the context of skewed and heavy-tailed distributions close to their associated boundary conditions with valid PDFs (e.g., D15 and D16). In those more skewed and heavy-tailed distributions, power rates of the nonparametric (*KW* and *FR*) tests might be disparate across the four data transformations. Type I error and power rates were consistent and similar across the four data transformations in all conditions of the rest of the distributions (i.e., D11, D12, D13, and D14) for both parametric and nonparametric tests. The third-order power transformation seemed dissimilar to the other three procedures (i.e., the Burr, *g*-and-*h*, and GLD) in Type I error and power rates in most of the inconsistent cases, particularly in the last two distributions (D15, and D16).

Second Group of Distributions

For the second group of four distributions, three transformation procedures (i.e., the *g*-and-*h*, GLD and the fifth power method) were used to generate data, and none of the three data transformations were able to gnerate distributions with valid PDFs. Results of this study suggested the following conclusions:

1. About 19% (i.e., 3/16) of the conditions for the between-subjects design (39% or 31/80 conditions for the within-subjects design) had inconsistent Type I errors for the *F* tests across the three data transformations. Inconsistent Type I errors of the parametric *F* tests in both the between-subjects and within-subjects were an important concern, particularly in the last two distributions. The GLD transformation was conservative in the last distribution (i.e., D24) for the between-subjects *F* tests, and mostly conservative in the last two distributions (i.e., D23, and D24) for the within-subjects *F* tests, which contributed the majority of the inconsistent occurrence.

2. The Type I error rates for the *KW* tests were all consistent across the three data transformations, while those for the *FR* tests were inconsistent in about 4% (i.e., 3/80) of the conditions. As such, Type I errors of the nonparametric tests were generally robust and consistent.

3. Inconsistent power rates for the parametric tests were not of concern. Power rates of the *F* tests in the between-subjects design were all consistent with extreme difference at .007, while inconsistent power rates of the *F* tests in the within-subjects design occurred in only less than 1% (i.e., 4/480) of the conditions with the extreme deterrence at .087.

4. Inconsistent power rates of the nonparametric tests might be a great concern. About 18% (i.e., 17/96) of the conditions had inconsistent power rates for the *KW* tests with the extreme power difference at .44, while 12 % (58/480) of the conditions had inconsistent power rates with the extreme difference at .449 for the *FR* tests. The GLD transformation, again contributed most of the inconsistencies, frequently in the last two distributions (D23, and D24).

5. The relative power advantage of nonparametric tests over their corresponding parametric tests was complicated in that the former was more powerful at some effect size level(s) in a distribution but less powerful at other effect size levels under same condition. This phenomenon occurred in the first distribution (i.e., D21) for the between-subjects design, and in the first three distributions (i.e., D21, D22, and D23) for the within-subjects design.

6. The *g*-and-*h* and the fifth-order power transformations were more consistent for Type I error and power rates for both parametric and nonparametric tests in this group of

distributions. With the GLD transformation excluded from the comparison, the occurrence of inconsistent Type I errors reduced to 1/16 (4/80) for the between-subjects (within-subjects) *F* tests, whereas for the *KW* (*FR*) tests, the occurrence of inconsistent power rates reduced to 1/96 (13/480) with the extreme power difference at .06 (.08) for the *KW* (*FR*) tests.

More generally, in this group of four distributions generated with the three transformations without valid PDFs, Type I error rates of the parametric *F* tests, and power rates of the nonparametric tests were frequently dissimilar across the three data transformation procedures, especially in the last two distributions (i.e., D23 and D24). The fifth-order power method and *g*-and-*h* transformations were, however, relatively consistent. The relative power advantages between the nonparametric and parametric tests maybe dependent on effect size levels when other conditions were held the same.

Third Group of Distributions

For the third group of four more skewed and kurtotic distributions, the three data transformations (i.e., the Burr, *g*-and-*h*, and GLD) had valid PDFs; but the third-order power method could not produce valid PDFs. Results of this study suggested the following conclusions:

1. Inconsistent Type I errors for the parametric *F* tests might be of concern. Specifically, the Type I error rates were inconsistent among the four data transformations within all conditions of the four distributions for both the between-subjects and within-subjects *F* tests. Even if compared the remaining three data transformations with the third-order power transformation excluded, the inconsistency still remained. But pairwise

comparison between the *g*-and -*h* and GLD transformations, Type I error rates were relatively consistent.

2. The Type I error rates for the *KW* tests were all robust and consistent across the four data transformations within all the conditions in the four distributions, while about 8% (i.e., 6/80) of the conditions had inconsistent Type I errors for the *FR* tests. Although inconsistent Type I errors for the *FR* tests seemed a concern. Nonparametric tests were generally more robust than their parametric alternatives. Pairwise comparison between the *g*-and -*h* and GLD transformations revealed consistent Type I errors for both *KW* and *FR* tests.

3. Inconsistent power rates of the parametric *F* tests across the four data transformations might be a great concern. Specifically, inconsistent power rates across the four data transformations occurred in about 58% (i.e., 56/96) of the conditions in the four distributions for the between-subjects *F* test, with the extreme difference at .223, while the inconsistency for the within-subjects *F* tests occurred in about 22% (i.e., 104/480) of the conditions, with the extreme difference at .198. The power rates of the *F* tests were more consistent, however, when compared between the *g*-and -*h* and GLD transformations (with zero occurrences, and extreme difference at .002 for the between-subjects *F* test, and 5/480, and .18 for the within-subjects *F* tests).

4. Inconsistent power rates for the nonparametric tests might be an important concern. More specifically, inconsistent power rates for the *KW* tests across the four data transformations occurred in 50% (i.e., 48/96) of the conditions, with the extreme difference at .835, while for the *FR* tests the inconsistent power rates occurred in about 34% (i.e., 161/480) of the conditions with the extreme difference at .807. The power rates

were more consistent, however, when compared between the *g*-and -*h* and GLD transformations (with zero occurrences, and the extreme difference at .02 for *KW* tests, and 5/480, and .194 for *FR* tests).

5. Nonparametric tests were generally more powerful than or equally powerful to their parametric competitors. This result is consistent with Feir-Walsh and Toothaker (1974); Iman et al. (1984); and Harwell and Serlin (1994).

In summary, for the first group of six distributions, Type I error and power rates were generally consistent across the four data transformations in the first four distributions. In the last two distributions (i.e., D15 and D16) departing more from normality and close to their boundaries of transformations with valid PDFs, Type I error rates of the parametric *F* tests and power rates of the nonparametric tests might be disparate. Considering the six distributions all together, Type I error rates for the nonparametric tests and the power rates of the parametric tests were fairly consistent because of the low frequency of occurrence of inconsistency. Inconsistent Type I error rates for the parametric tests and inconsistent power rates for the nonparametric tests were a concern. For the second group of four distributions, where the three data transformations had invalid PDFs, similar trends were found as in the first group of six distributions, but the frequency of inconsistent Type I errors for the parametric tests and the frequency of inconsistent power rates for the nonparametric tests were slightly larger than they were in the first group of distributions. In the third group of four distributions, Type I errors for *KW* test were similar and consistent across the four data transformations, whereas both Type I errors and power rates for all other tests were inconsistent across the four data distributions. The frequency and magnitude of the inconsistency were the largest among the three groups of distributions. As for the relative robustness and power advantages, the parametric tests were robust and slightly more powerful, when the distribution assumptions were satisfied or mildly violated. When the assumptions were moderately or severely violated, the nonparametric tests were robust regardless of the normality assumption, and more powerful than or equally powerful to their parametric counterparts.

Recommendations

An implicit assumption for a simulation study is that the data generating systems are independent of the empirical results such as Type I error and power rates in this study, so that the results achieved are valid and a methodologist can make unbiased interpretations and/or comparisons regardless of the procedures by which the data were generated. The preliminary results of this study suggested that methodologists should be cautious about data generation, because how the data were generated might have a strong influence on the results of a simulation study. In general, data transformation procedures able to generate valid PDFs were preferred. Mixed use of data transformation procedures that generate distributions with and without valid PDFs should be generally avoided. It would be safe to use one single data transformation procedure in a study to generate all the data if the results were to be compared among different statistical tests. Unfortunately, one single data transformation generally cannot serve all the requirements of distributions generated in a Monte Carlo study. When several data transformations have to be used to generate the data distribution(s), researchers face a situation to make decision(s) about data transformations.

The primary recommendation of this study is that researchers conducting Monte Carlo studies in the context described herein should use data transformation procedures that produce valid PDFs. This recommendation is important to the extent that researchers using transformations that produce invalid PDFs increase the likelihood of limiting their study to the data generating procedure being used i.e. Type I error and power results may be substantially disparate between different procedures. Further, it also recommended that *g*-and-*h*, GLD, Burr, and fifth-order power method transformations be flexibly used if it is desired to generate distributions with extreme skew and/or heavy-tails, whereas third-order polynomials should be avoided in this context. More specifically, In situations similar to the first group of six distributions and consistency of Type I error and power rates are a concern, methodologists are recommended to use more consistent data transformations such as the *g*-and-*h*, GLD, and the Burr transformations; in other situations similar to the second group of distributions, the *g*-and-*h* and fifth-order power transformations were more consistent; in situations similar to the third group of distributions, the *g*-and-*h*, and GLD transformations were more consistent. The third-order power method transformation tended more conservative in the parametric tests and more powerful in the nonparametric tests, for the extremely skewed and kurtotic distributions such as the last two distributions in the first group, and all the distributions in the third group of distributions. Thus, with extremely skewed and kurtotic distributions, the third-order power transformation should be generally avoided. Although the nonparametric tests considered in this study are generally more consistent and/or robust across the data transformation procedures and are oftentimes more powerful than or equally powerful to their parametric counterparts within each data transformation when the normality assumption is violated, researchers should keep in mind that a parametric test and its nonparametric counterpart are not testing the same hypothesis. In the real studies where assumptions are violated, robust procedures, and resampling techniques are also available alternatives for researchers to select.

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APPENDICES

Apendix A PDFs and CDFs of Distributions Plotted with Different Transformations

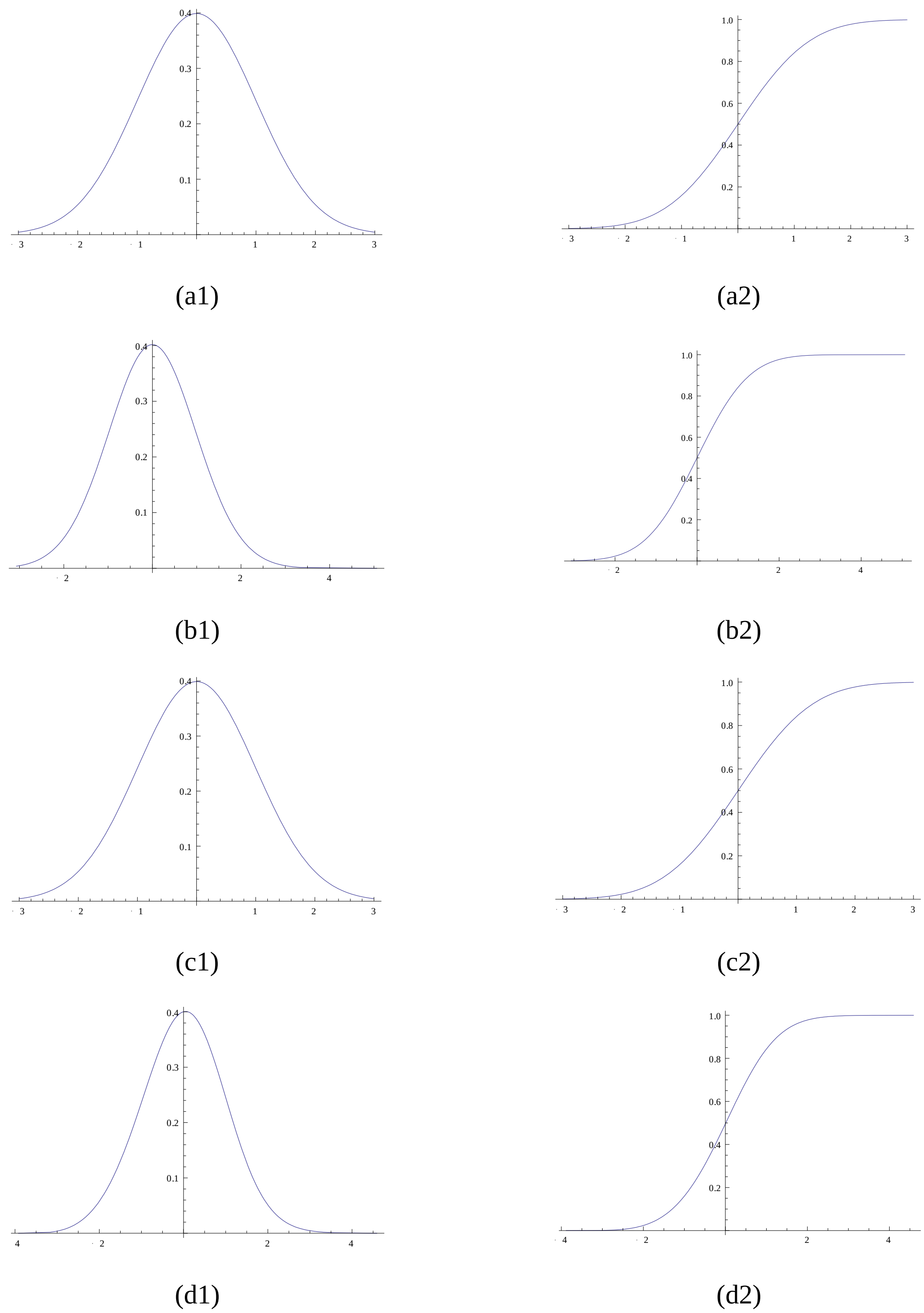


Figure A-1. D11 (standard normal) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

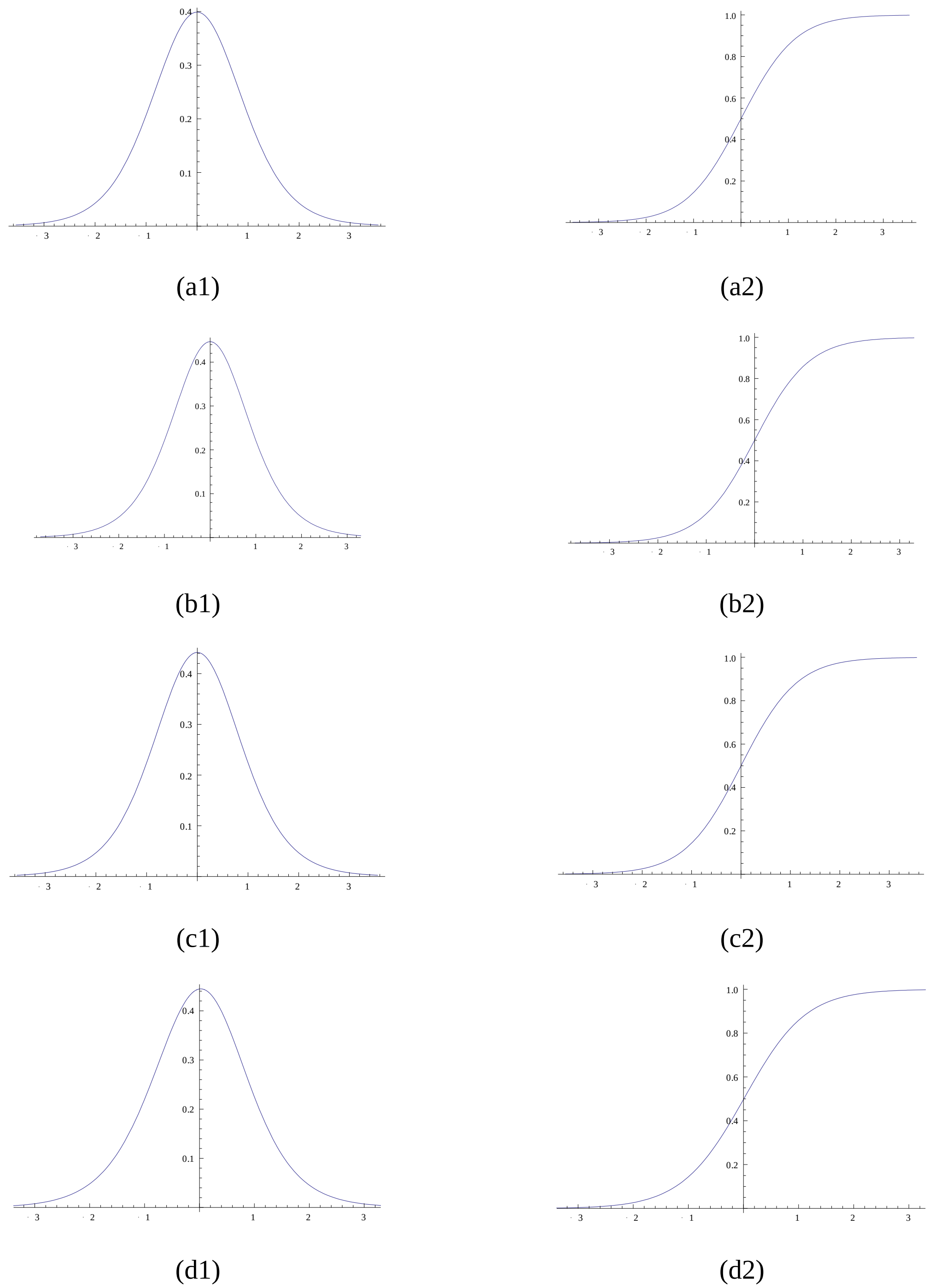


Figure A-2. D12 (skewness = 0.0, kurtosis = 1.0) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

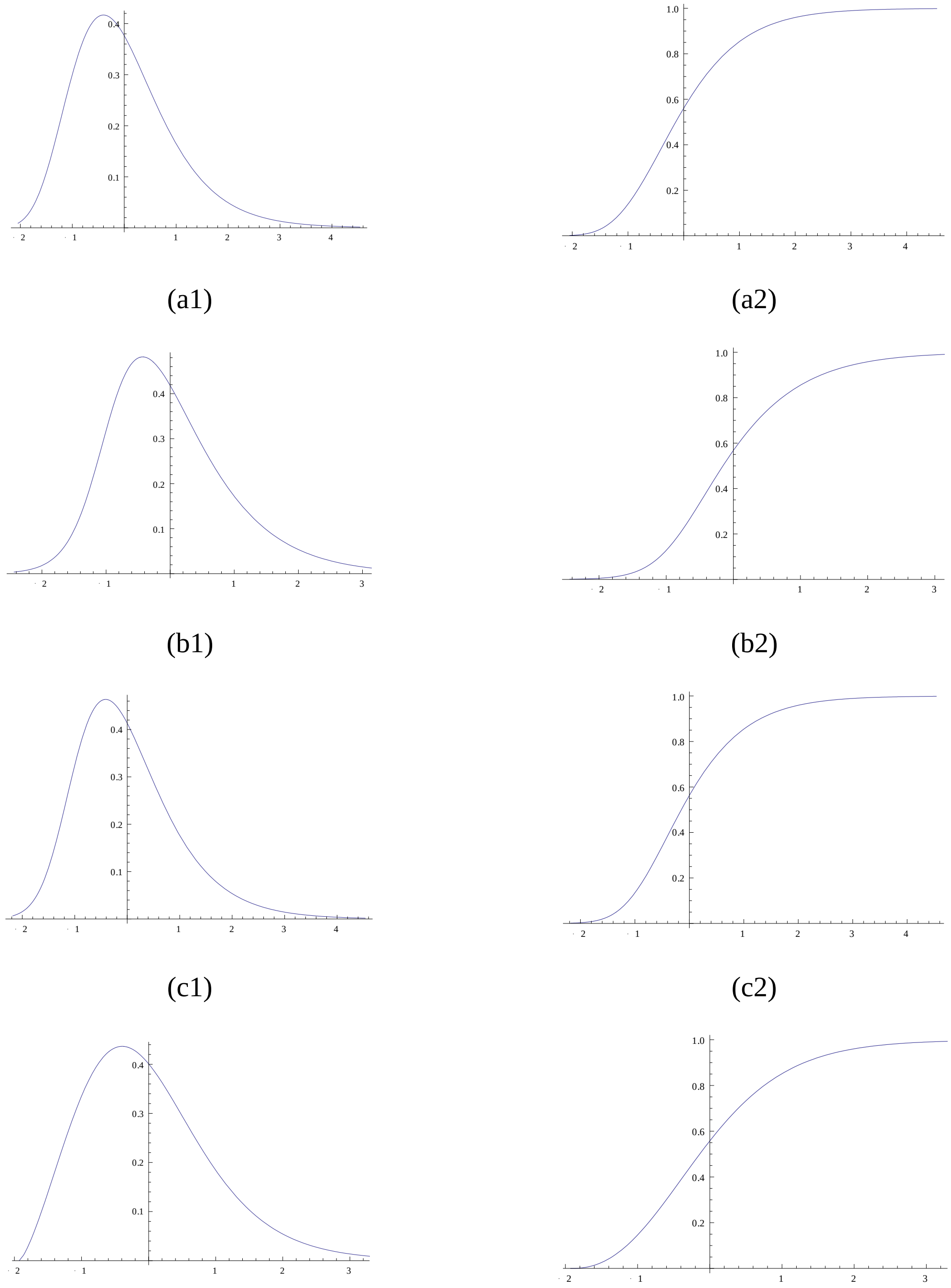


Figure A-3. D13 (skewness = 1.0, kurtosis = 2.0) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

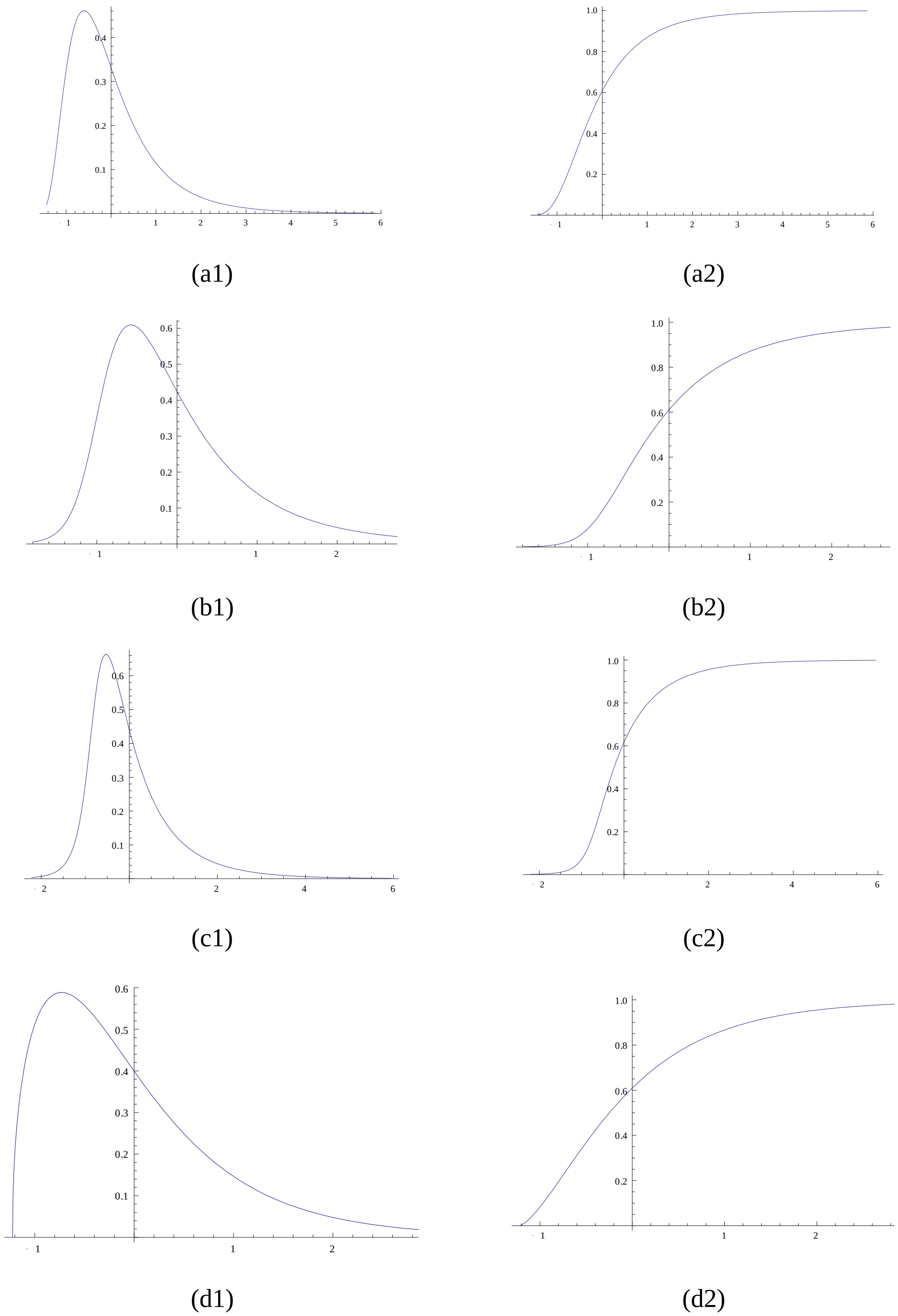


Figure A-4. D14 (skewness = 2.0, kurtosis = 8.0) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

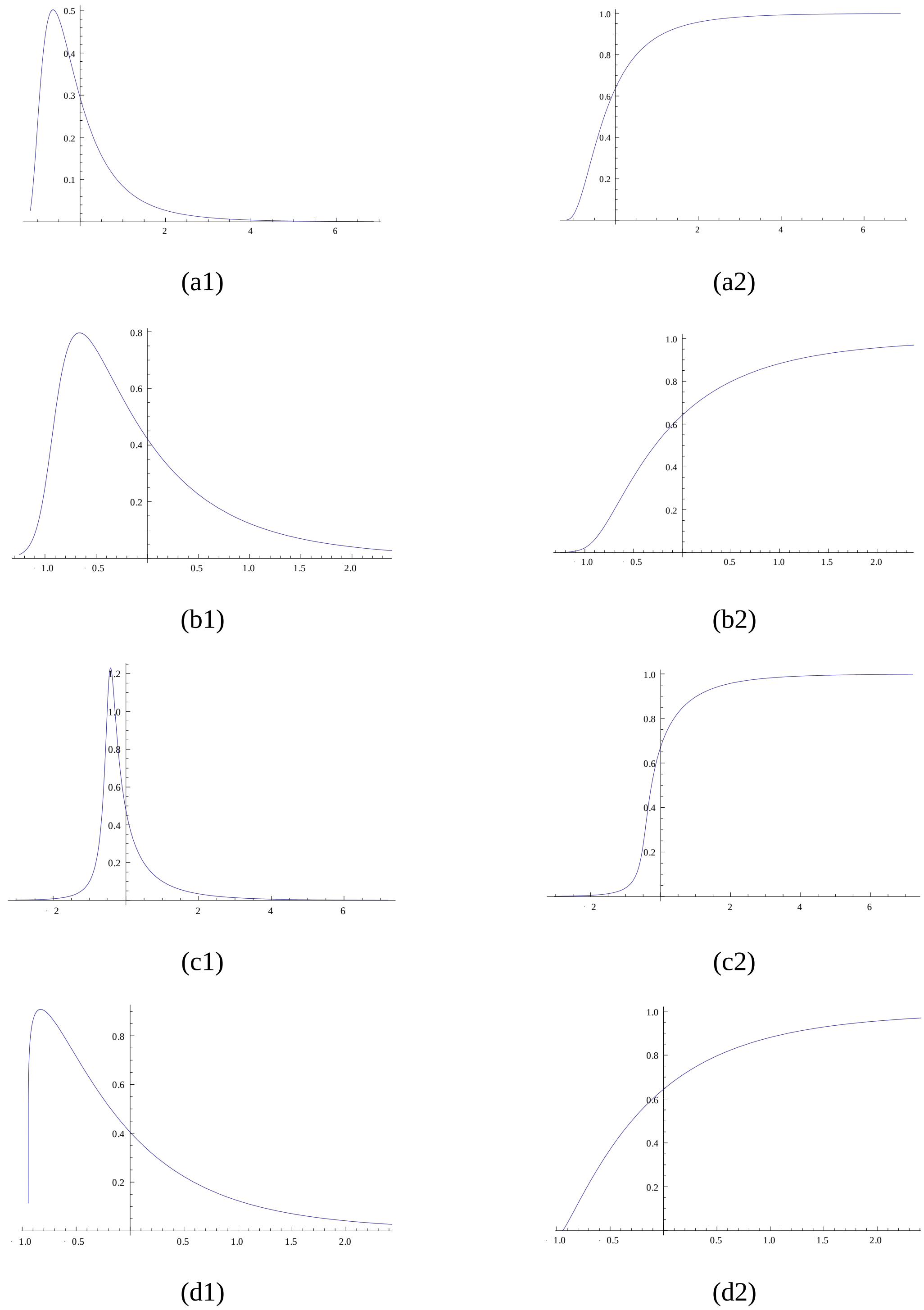


Figure A-5. D15 (skewness = 3.0, kurtosis = 20.0) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

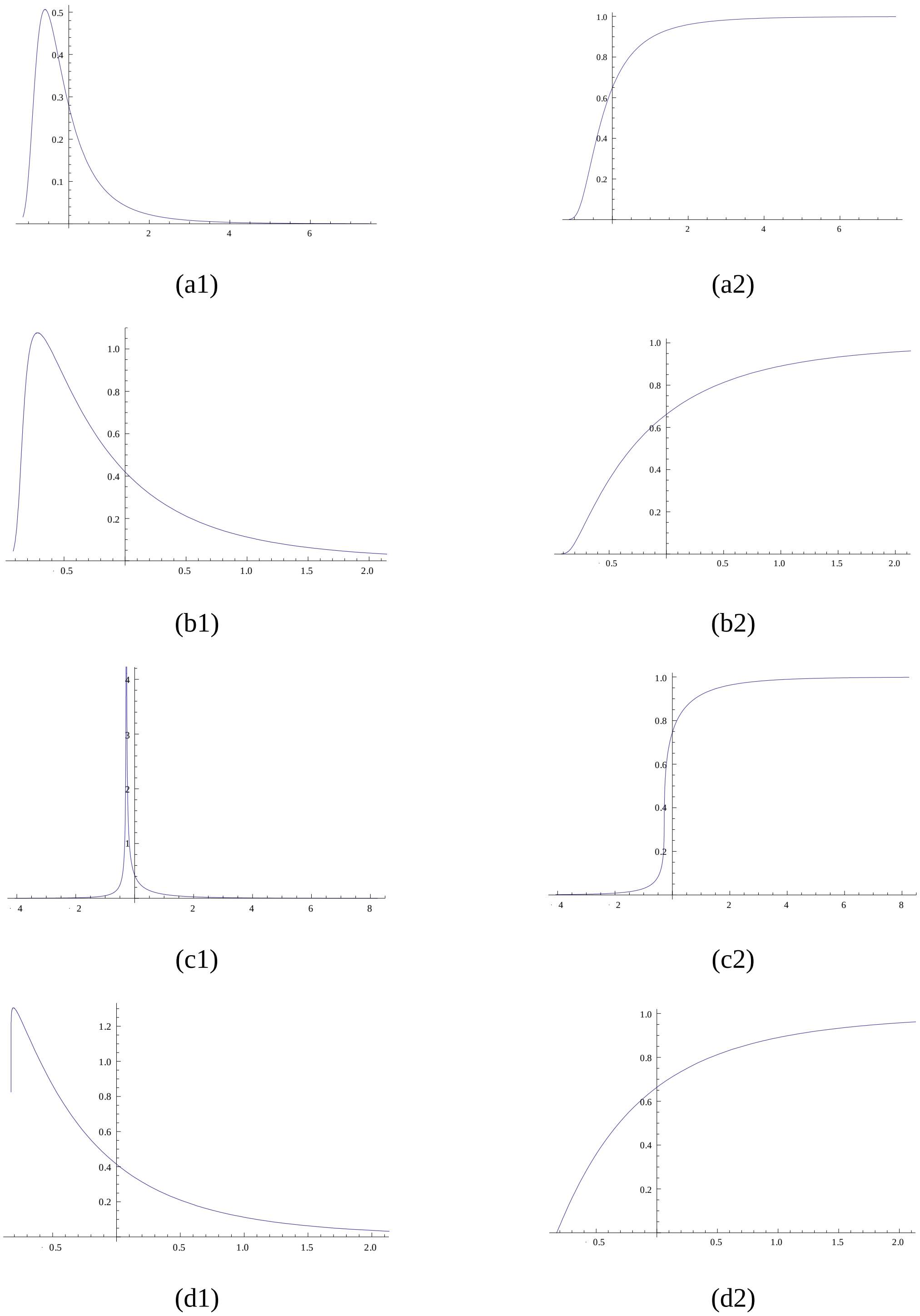


Figure A-6. D16 (skewness = 3.9, kurtosis = 40.0) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type XII distribution.

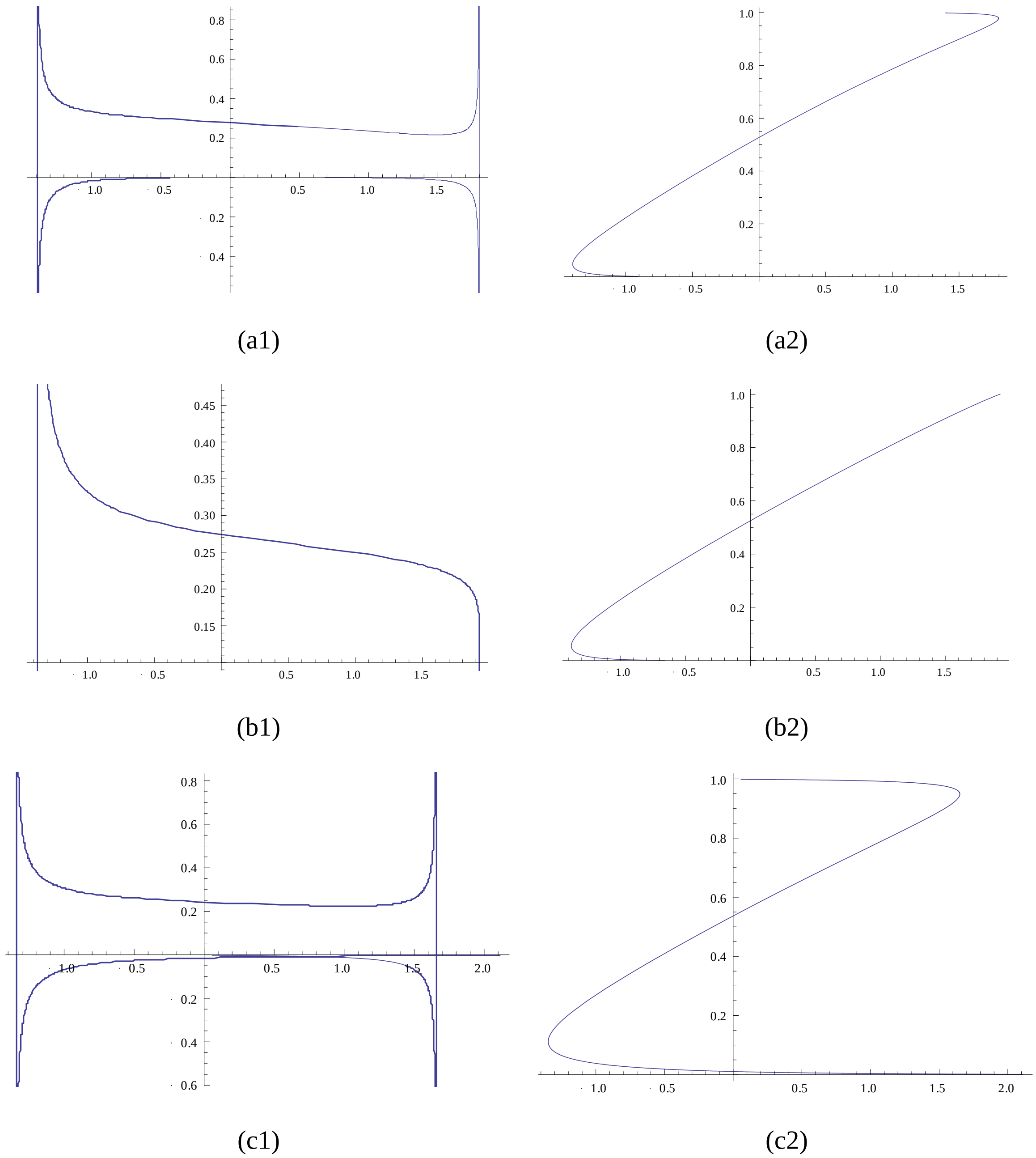


Figure A-7. D21 (skewness = 0.24, kurtosis = -1.209981) generated and plotted with the three data transformations: a1 - c1 = PDFs; a2 - c2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the fifth-order power method; the Burr family could not generate this distribution.

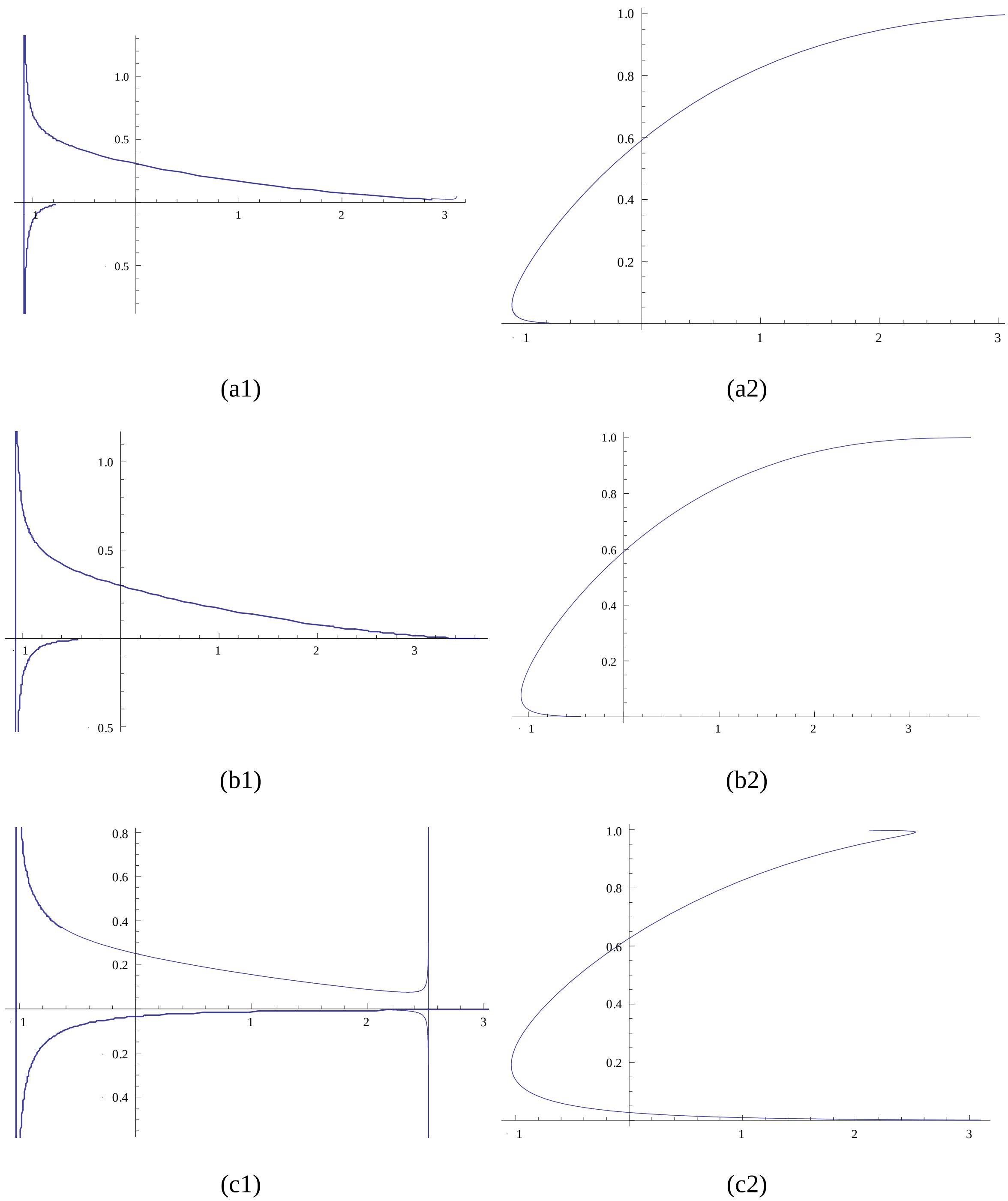
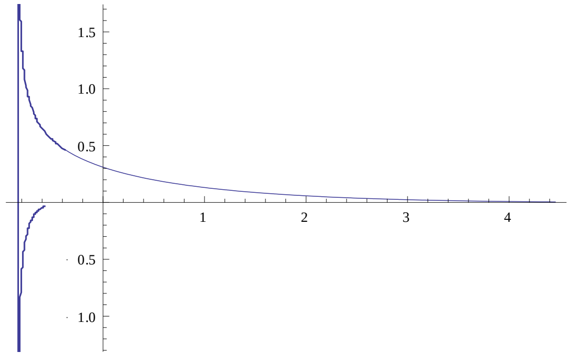
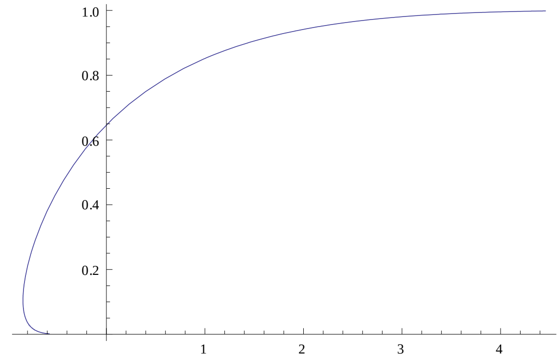


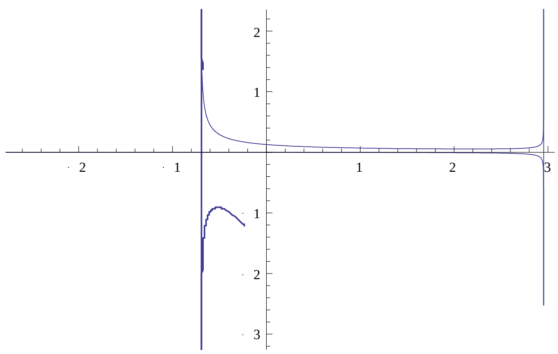
Figure A-8. D22 (skewness = 0.96, kurtosis = 0.133374) generated and plotted with the three data transformations: a1 - c1 = PDFs; a2 - c2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the fifth-order power method; the Burr family could not generate this distribution.



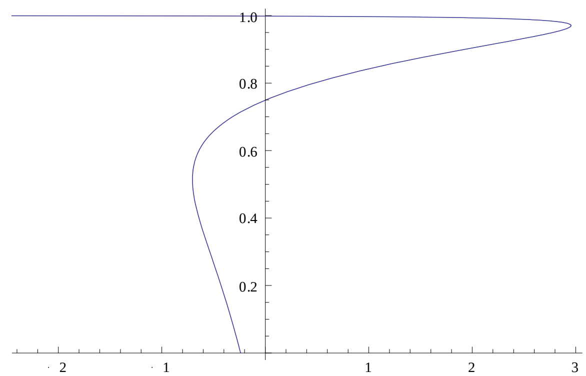
(a1)



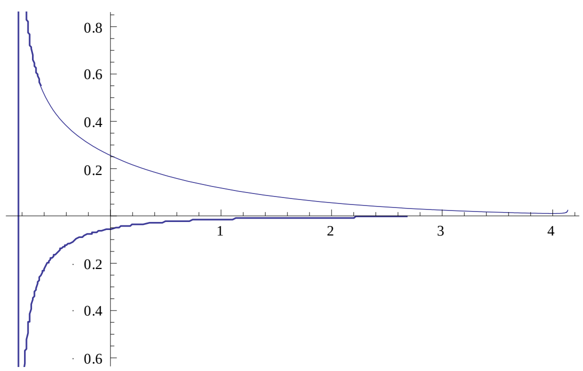
(a2)



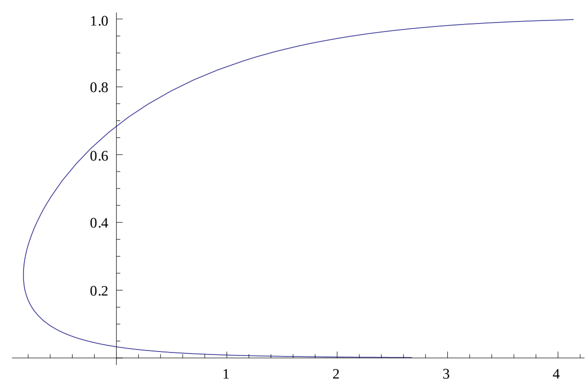
(b1)



(b2)

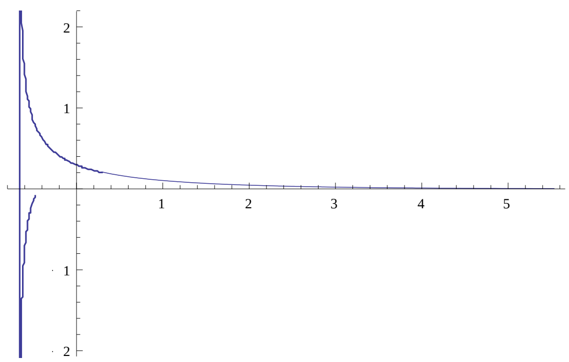


(c1)

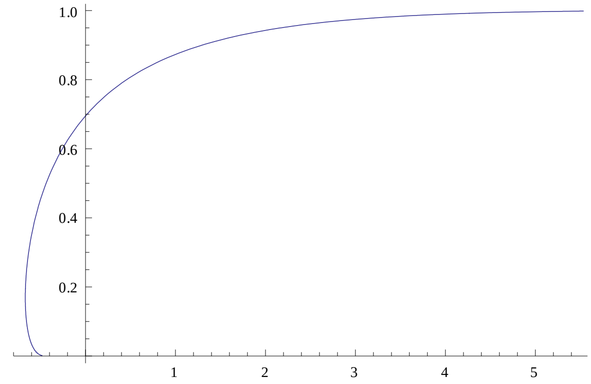


(c2)

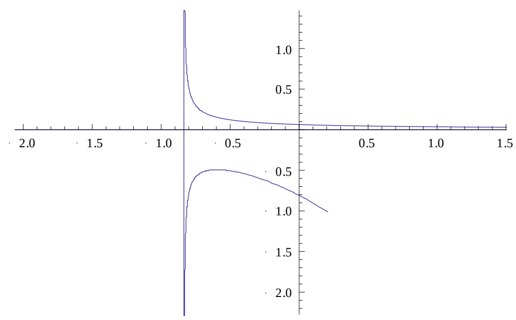
Figure A-9. D23 (skewness = 1.68, kurtosis = 2.76236) generated and plotted with the three data transformations: a1 - c1 = PDFs; a2 - c2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the fifth-order power method; the Burr family could not generate this distribution.



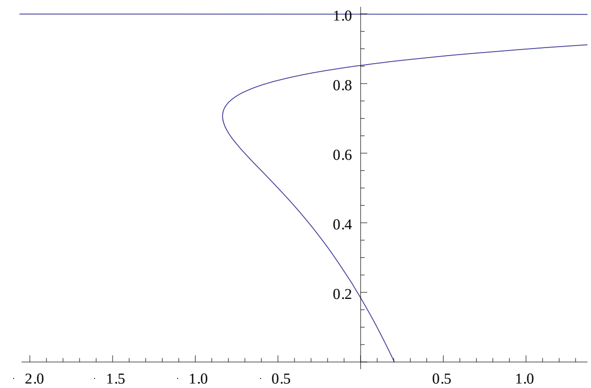
(a1)



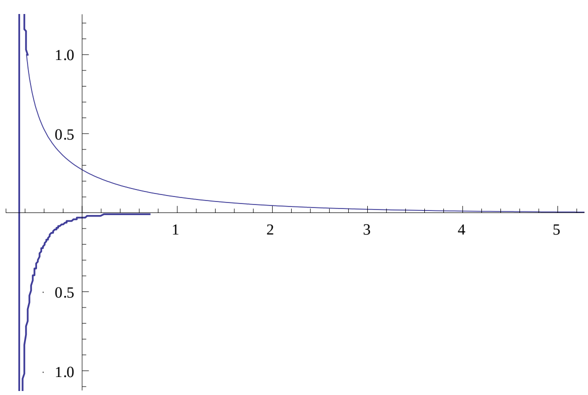
(a2)



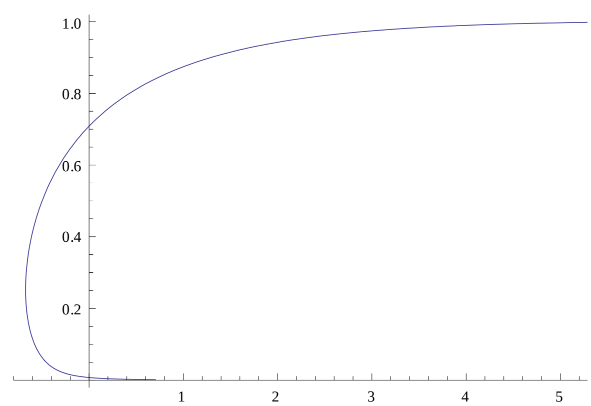
(b1)



(b2)



(c1)



(c2)

Figure A-10. D24 (skewness = 2.40, kurtosis = 6.606610) generated and plotted with the three data transformations: a1 - c1 = PDFs; a2 - c2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the fifth power method; the Burr family could not generate this distribution.

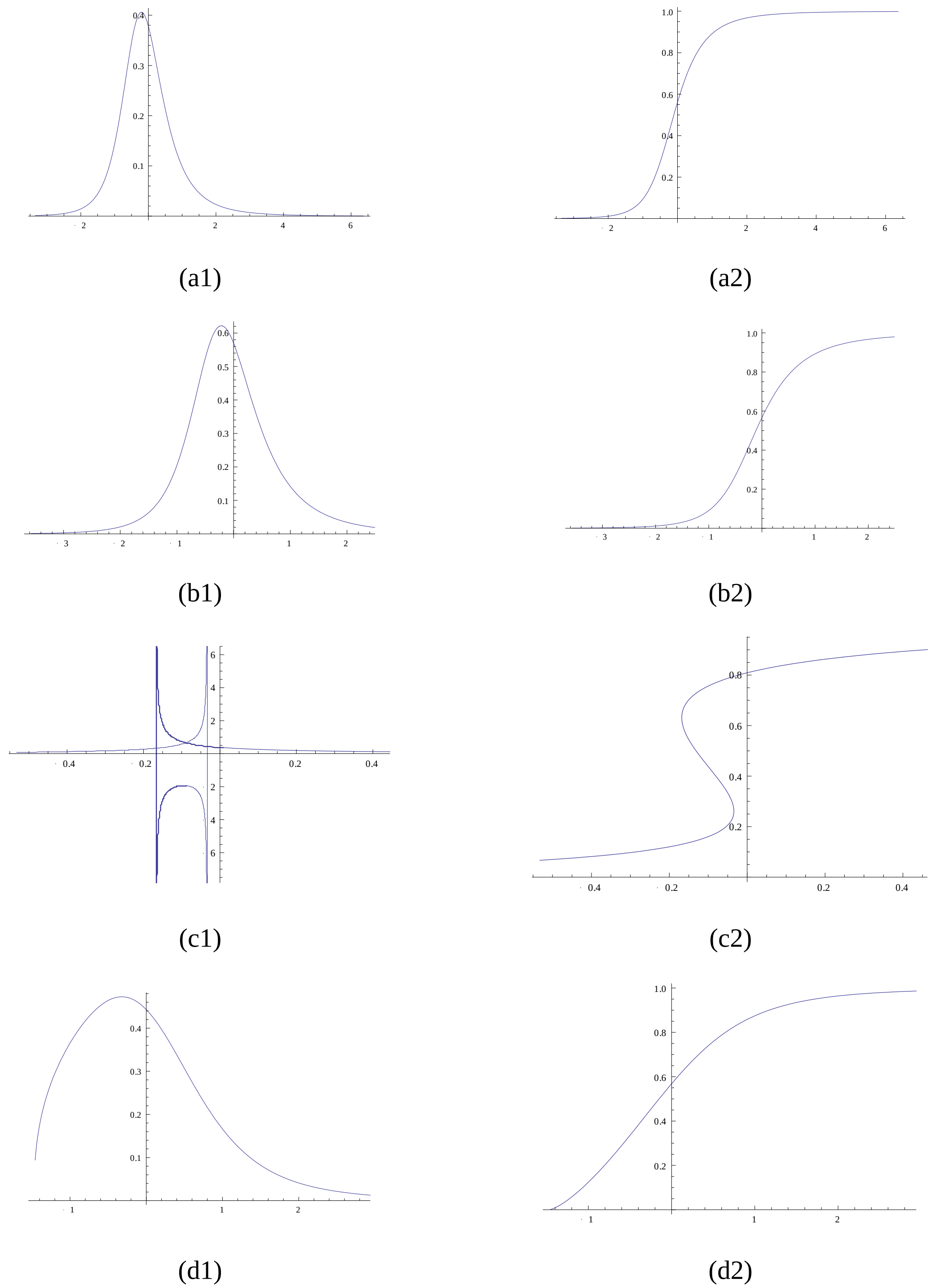


Figure A-11. D31 (skewness = 2.50, kurtosis = 60) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type III distribution.

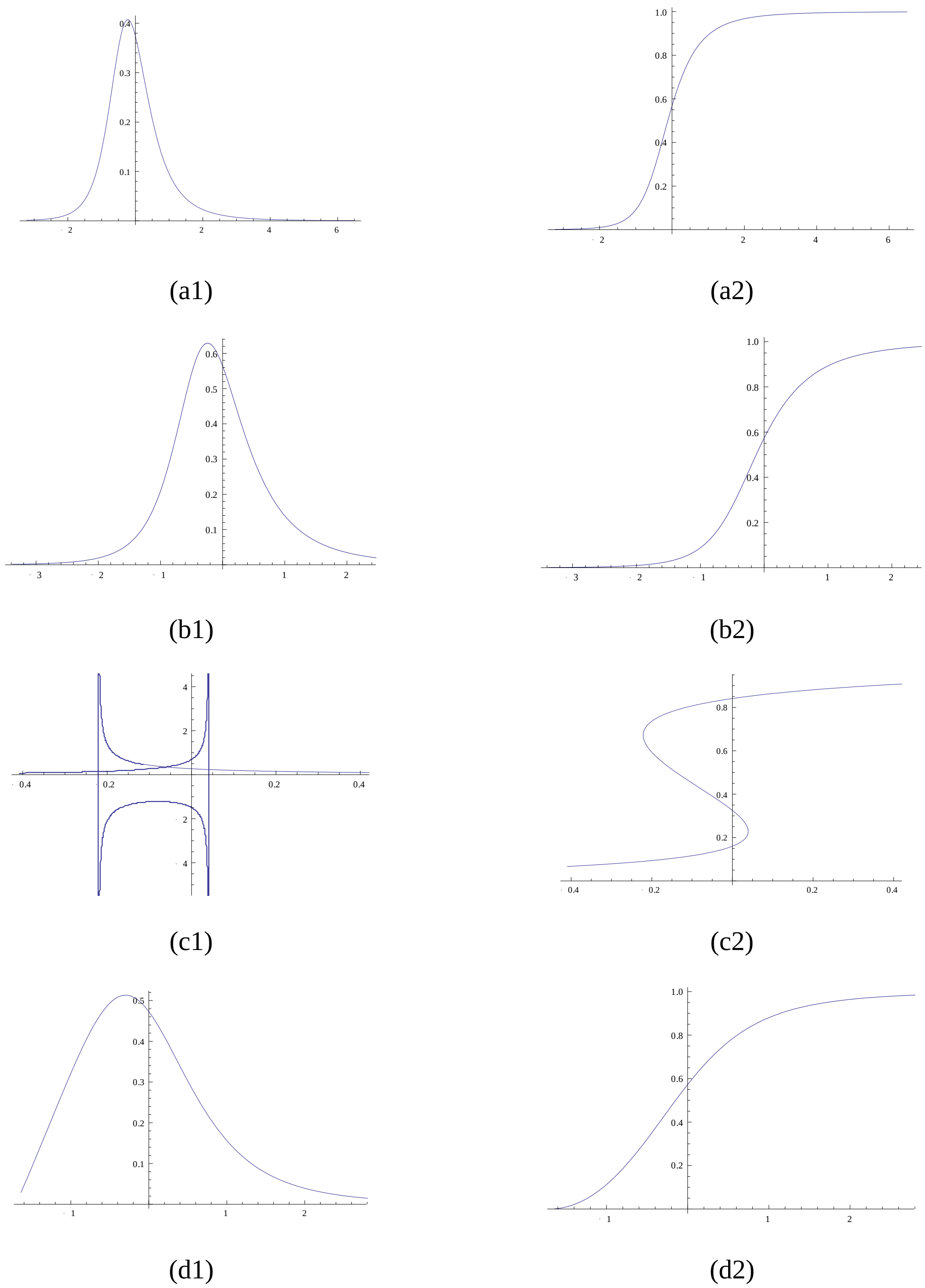


Figure A-12. D32 (skewness = 2.75, kurtosis = 70) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type III distribution.

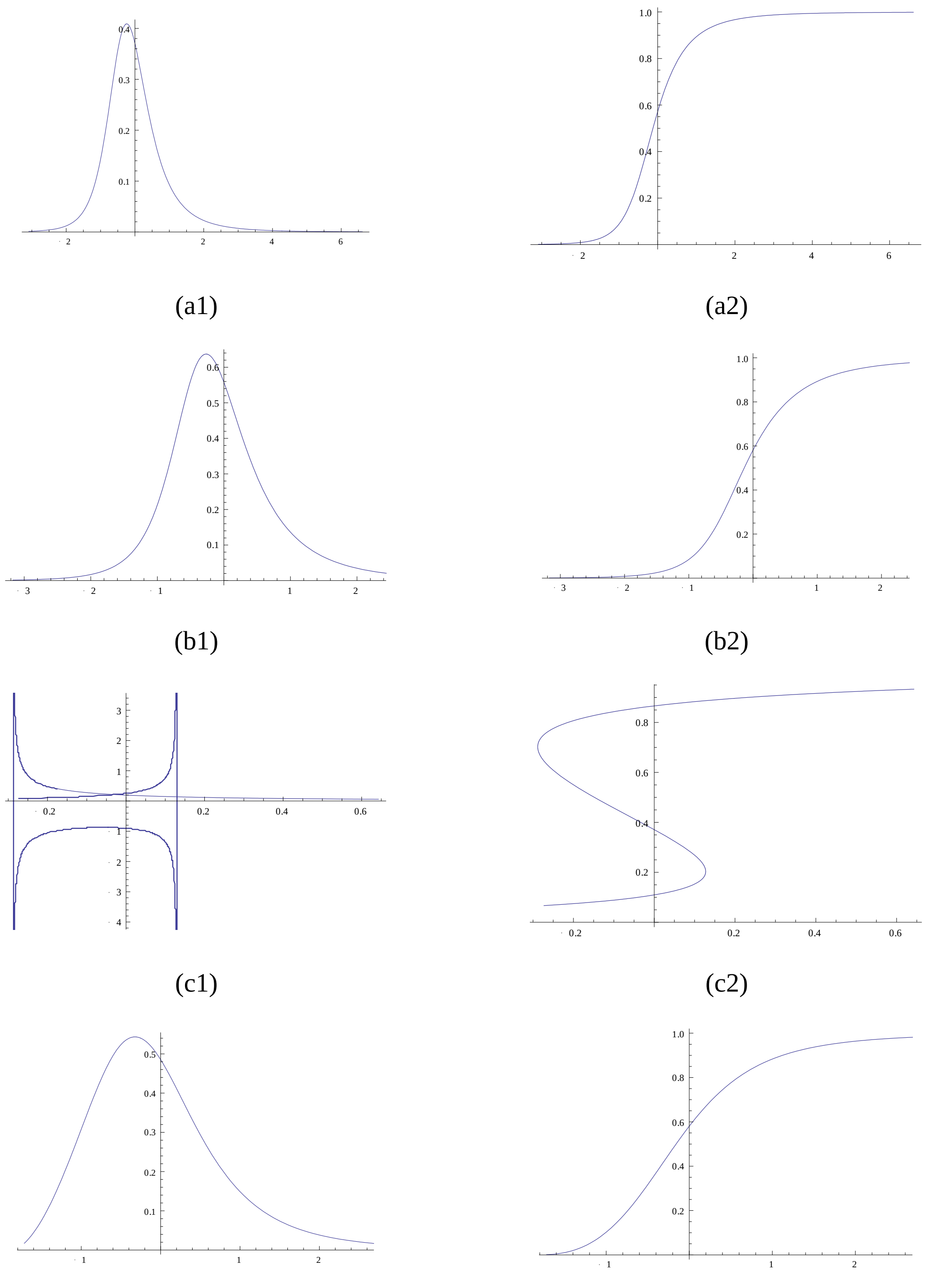


Figure A-13. D33 (skewness = 3.00, kurtosis = 80) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type III distribution.

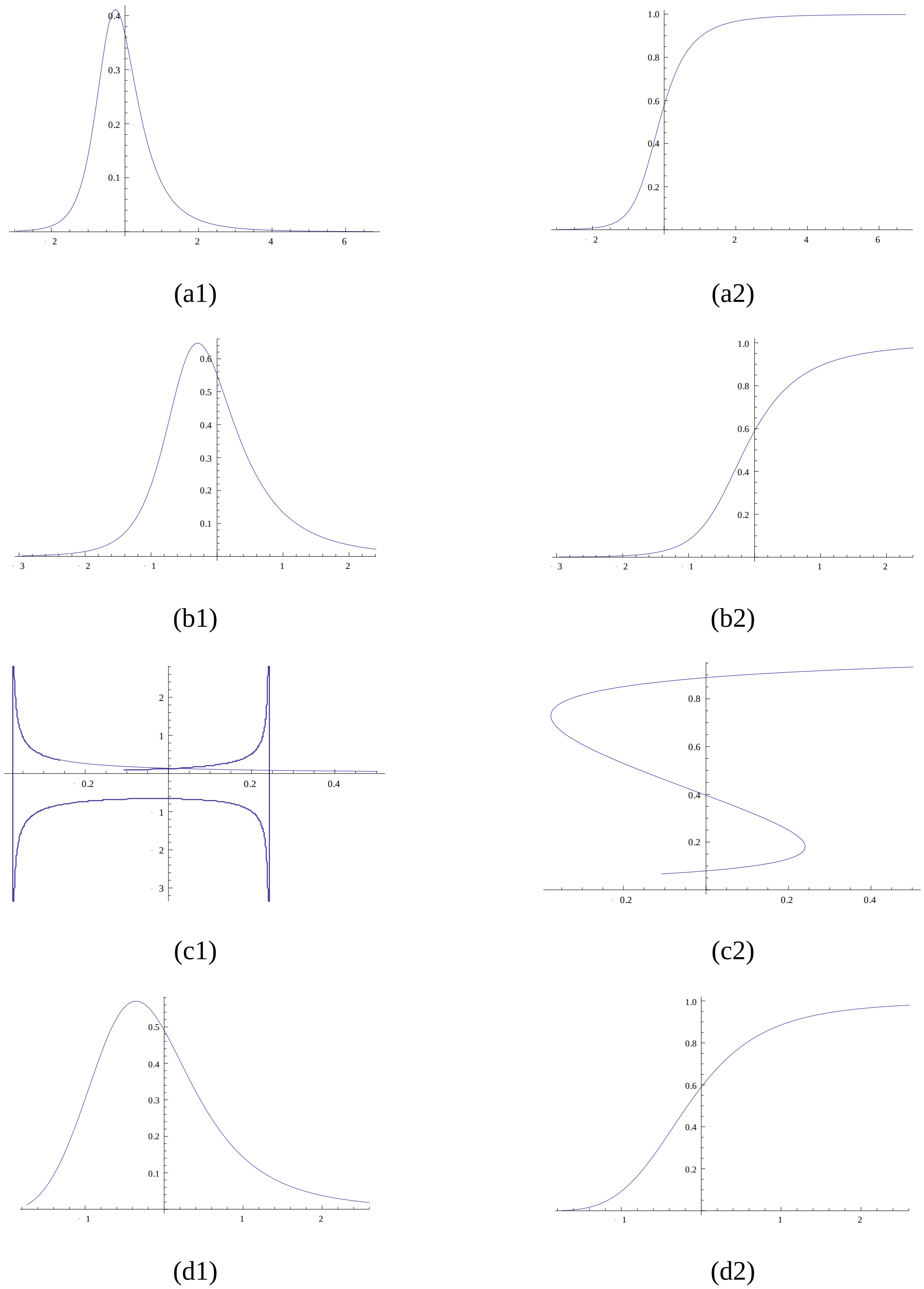


Figure A-14. D34 (skewness = 3.25, kurtosis = 90) generated and plotted with the four data transformations: a1 - d1 = PDFs; a2 - d2 = CDFs. a = *g*-and-*h* distribution, b = the GLD distributions, c = the third-order power method; d = the Burr Type III distribution.

Apendix B Critical Values of the Statistical Tests

Table B

Critical Values of the Statistical Tests

Cell/Block Size	10	20	30	50
BT Design				
df Numerator	3	3	3	3
df Denominator	36	76	116	196
Critical F^a	2.86626556	2.72494395	2.68280941	2.65067652
Critical KW[b]	7.590734	7.707957	7.743802	7.76888
WT Design				
df Numerator	3	3	3	3
df Denominator	27	57	87	147
Critical F^a	2.96035132	2.76643794	2.70940218	2.66614879
Critical Friedman[c]	7.56	7.74	7.72	7.8

Note. [a] Calculated with Microsoft Excel (2007) INV function; [b] Calculate with FORTRAN program (Headrick, 2003) based on 5 million replications; [c] Based on Visual Basic program (Bagui & Bagui, 2005) with 1 million replications.

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