MISCELLANEOUS.

EXPERIMENTAL MATHEMATICS.

While the best mathematical minds have always gathered their knowledge as well as their power from observational and experiential contact with the forms of reality, the empirical method has not until recently been applied so systematically to instruction in mathematics as it has been to instruction in the natural sciences. Advancement is always more rapid by the lecture method, or by that of continuous exposition from a text-book. The truth is here presented ready made, the pupil absorbs it easily and is not put to the trouble of seeking it anew. Hence, these methods have always been preferred by educational machinists, and students have, as a rule, been left to their own resources in acquiring solid and enduring knowledge.

But we know from history that geometry was originally a body of empirical knowledge; that it began, in the case of the Egyptians and the Greeks, with the observation of the forms of real things and of individual relations; that the empirical knowledge, thus observationally discovered, was systematised and classified; and that by induction the empirical facts were subsequently organised into a science.

The induction in the case of geometrical discovery is, it is true, of an entirely different character from what it is in the case of discoveries in the classificatory natural sciences. The induction, the guess, the divination, in geometry, usually proceeds from a *small* body of suggestive hints gathered from a *narrow* and *thoroughly determined* field of experience. But, so far as the *development* of the science is concerned, it is induction nevertheless, and it would seem that a sound method of instruction should require that the development of this knowledge in the individual should proceed pretty much along the same lines as the development in the race. Not that the pupil should retrace all the tortuous steps through which the science has been gradually and laboriously brought to its present stage of perfection; as a matter of fact, the instructional development will have to depart in many respects widely from the actual development; but it will always receive its natural support, its guidance, and its general trend from that development. Short-cuts, abridgements, and all the devices which economy of mental effort may suggest are permissible, and all will lead in the end, not to a method of re-discovery, but to a method of genetic and logical reconstruction. Results will always be preceded by investigation, always be provoked by actual inquiry.

To quote the words of Dr. Paul H. Hanus, Assistant Professor of Teaching in Harvard University, and the author of the little pamphlet entitled *Geometry in the Grammar School*, which we are now considering, "To present the 'net product of an inquiry without the inquiry which led to it,' is to cultivate a reliance upon the verbal memory to the neglect of the power of overcoming difficulties and of assimilating experiences; moreover, the accumulation of such unrelated mental stores is merely transitory,—they are soon forgotten; there is no permanent gain of either knowledge or power. A method of continuous exposition is productive only with minds already developed, not with those *to be developed*. By its exclusive use with minds at all degrees of maturity the best results can never follow. Self-activity, interest, self-reliance, the power to be useful, these will never follow a method of instruction by which mental stores are imparted as so many free gifts. Fortunately such gifts are really impossible; there is no inheritance of knowledge and power. Only capabilities are inherited. There is but one universal inheritance,—*ignorance*; one universal means provided by nature of rising above this inheritance,—*self-exertion*. Who does not employ the means Nature has provided remains unlearned and helpless, though he may for a time simulate attainment and intellectual strength."[1]

Professor Hanus is merely repeating what hundreds of thinkers and educators have time and again insisted upon, when he says that principles of conduct and rules of procedure, whether in life or in science, do not become real possessions until experience has verified them and shown their efficacy; that it is a well-known mental law that intelligence always proceeds from thing to name and symbol, from facts to principles and rules, and that in conformity with this law the facts of geometry must enter the learner's mind *through experience*. The learner must see and feel material bodies, their surfaces, their bounding lines, their corners; he must see that two vertical angles or two triangles under certain conditions are equal, by actually superposing them; must see by placing them side by side with their vertices coinciding that the three angles of any triangle together form two right angles, etc., etc. The generalisations will then follow. Above all, it is essential to develop the questioning attitude, and then to satisfy the inquiring mind by furnishing it with the *opportunity* of reaching the truth. "The attitude of a boy who has measured heights and distances by using the propositions concerning the equality of triangles, toward those propositions themselves, is very different from the attitude of the boy whose first experience with those propositions is drawn from a text-book or from a formal presentation by the teacher. The formal statement of the propositions and their logical proofs are to be introduced gradually and after the facts have been presented empirically."

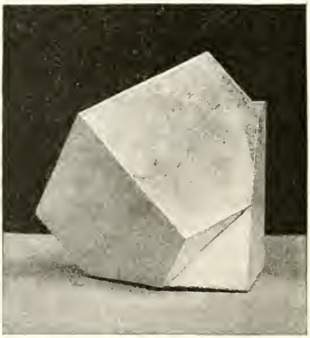
Affirming the psychological truth that "clear mental perception can only follow clear physical perception." Professor Hanus then proceeds to indicate in large outlines the course and methods of instruction which are to be followed in the teaching of elementary geometry in the grammar school, and he has appended to his little pamphlet a synopsis of the simple experimental work which might be done in geometry in the last three years of the grammar school course. The nature and subject of the work only are indicated; the development is left almost entirely to the teacher. The method is altogether object teaching. Records of observations are kept by the pupil, who is led to express himself by drawing, by construction, and in words, and to convince himself of geometrical truths primarily through measurement, drawing, cutting, superposition, and construction. Every

[1] *Geometry in the Grammar School.* An Essay together with illustrative class exercises, and an outline of the work for the last three years of the grammar school. By Paul H. Hanus, Assistant Professor of the History and Art of Teaching, Harvard University. 1898. Boston: D. C. Heath & Co., Publishers. Pages, ix, 52.

opportunity is to be taken of making the pupil's geometrical knowledge bear directly upon life. The use of the foot-rule, tape or chain, with some form of the goniometer, is recommended for obtaining data and for imparting to theoretical results body and significance. "Nothing can exemplify the value of class-room instruction like a practical application to construction in the shop, or measurement in the field. *For geometry as for geography 'field work' is well-nigh indispensable.*"

* * *

The methods which have been roughly indicated in the preceding paragraphs have been carried out, so far as the elementary features of the subject are concerned, with almost superfluous detail, in an attractive and profusely illustrated book entitled *Observational Geometry*,[1] by William T. Campbell, A. M., instructor in mathematics in the Boston Latin School,—a work which forms part of the Phillips-Loomis Mathematical Series. The elementary "laboratory work" and "field-work of geometry," noted above, are here developed to the utmost extent, and even carried out in cases where with average pupils it would seem almost un-



Card-Board Model of a Twin-Crystal of Calcite.
(From Campbell's *Observational Geometry*.)

necessary. Aiming to give to the hand dexterity and skill in making drawings and models of geometrical figures, it devotes 125 pages to the consideration of elementary forms and the construction of models. The uses of the main geometrical and mechanical instruments are taught, and directions given for the construction from thin cardboard of all the principal geometrical solids. In the annexed cut will be found a representation of a model of a twin crystal of calcite consisting of two interpenetrating cubes, made from a single piece of cardboard so outlined, cut, and folded as to take the shape seen in the figure. The second part of the book is devoted to plane geometrical construction (lines, angles, polygons, and circles), the measurement of areas, similar figures, and surveying.

Dr. A. W. Phillips, the editor of the series in which this book appears, remarks that the revolt against the old arithmetic problems, which resulted in the substitution of nature studies for arithmetic drill, was due to a want of careful and

[1] Published by The American Book Co., New York. Pages, ix, 240.

systematic development of the subject as a means of cultivating the faculties of observation. Now this want, he contends, is supplied by observational geometry, which "combines the training of the nature studies, so far as these educate the eye to keen and intelligent perception, with the training which the more valuable problems of the old arithmetics furnish, and so gives a mental discipline at once rigorous and entirely free from that one-sidedness which either of these systems fosters when taken alone." The truth of this may be readily gathered from the exercises and problems of the second part of Mr. Campbell's book. The measurements are all actually carried out here in connexion with real objects, and the instruction thus takes on the character of a serious and intrinsically interesting investigation, as contrasted with that of a purely theoretical study. The second illustration, showing the method of determining the height of a tree by means of surveying instruments and the theory of similar triangles, has been taken from Mr. Campbell's book, and is typical of the character of the work there outlined. It is one only of a large number of similar illustrations.

* * *

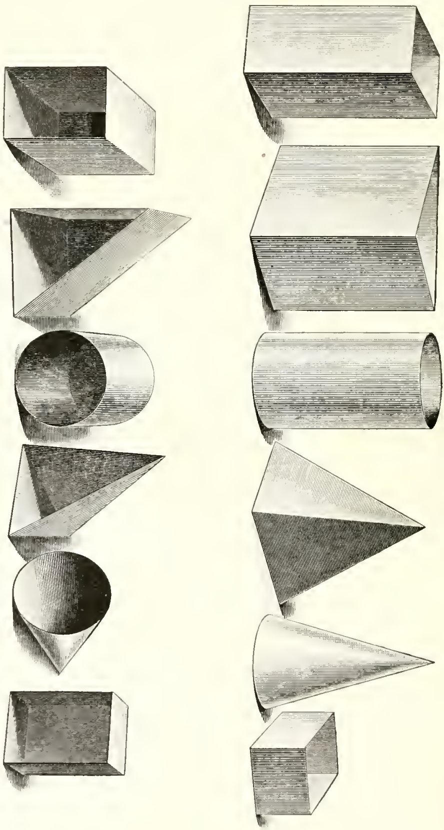
In his *Advanced Arithmetic*,[1] Mr. William W. Speer, District Superintendent of Schools, Chicago, has extended to the general theory of fractions, proportions

MEASURING THE HEIGHT OF A TREE.
(From Campbell's *Observational Geometry*.)

and elementary mensuration the same principles which guided him in the preparation of his *Primary* and *Elementary Arithmetic*. We noticed Mr. Speer's books at length in No. 504 of *The Open Court* (May, 1898), and little remains to be said here upon his latest work. In his system the great body of arithmetical truths is not differentiated and split up into arbitrary chapters and divisions, as it is in the common run of arithmetics, but is developed genetically as a connected organic whole. Sense-training is throughout made the basis of the development of arithmetic thought, and *concrete* relations of magnitude are made the foundation of all mathematical inferences. Every possible variety of quantitative relation in nature, industry, science, and art, is employed for this purpose. Sets of blocks, bundles of fagots, and sets of geometrical solids accompany Mr. Speer's books, and are indispensable for the concrete instruction which they require. Coins, clock dials, liquid and dry measures, and metric forms of every conceivable kind, are also em-

[1] *Advanced Arithmetic*. By William W. Speer. 1899. Boston: Ginn & Co. Pages, xx, 261.

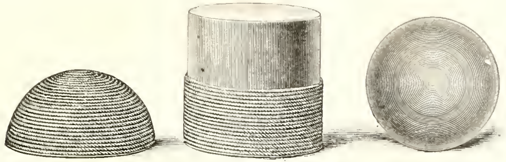
ployed or recommended. The comparison of volumes is made by the actual measurement of contents, as shown in the appended illustrations of prisms, cylinders, pyramids, and cones taken from Mr. Speer's book. Paper-cutting and modelling are also extensively used in the present book. The two illustrations which we have reproduced on page 639 are instances of the determination of rela-



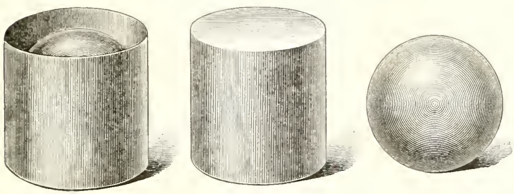
HOLLOW PRISMS, CYLINDERS, PYRAMIDS, AND CONES.
Used for the comparison of volumes. (From Speer's *Advanced Arithmetic*.)

tive volumes and surfaces by experimental measurements. In each case the cylinder is the circumscribing cylinder of the sphere represented in the figure. The lateral surface of the cylinder is shown to be equal to the surface of the sphere by comparing the length of the cord which covers the curved surface of the hemisphere with that which covers one half of the lateral surface of the cylinder. In the other figure, the volumes of the sphere and the circumscribing cylinder may be compared by filling them with water, and the volume of the sphere shown to be two thirds of the volume of the cylinder.

Mr. Speer's system is being used with great and merited success in the schools of Chicago and elsewhere. There is but one serious criticism which suggests itself in connexion with it, and that is that the introductions and the directions to teachers which are psychologically sound in the main, are put in too abstract and disconnected a form for readers of general training, and that for this reason many teachers who have not had the advantage of personal initiation into the method might



Experimental Comparison of the Surface of a Sphere and the Lateral Surface of the Circumscribing Cylinder.
(From Speer's *Advanced Arithmetic.*)



Experimental Comparison of the Volumes of a Sphere and Its Circumscribed Cylinder by Measuring with Water.
(From Speer's *Advanced Arithmetic.*)

find the books difficult to use and perhaps fail therefore to appreciate the power of the system to its full extent. If the exposition of the subject were as concrete and continuous as the system itself aims to be, we believe that nothing could stand in the way of its widespread introduction. T. J. McC.

IMMORTALITY.

BY SOLOMON SOLIS-COHEN.

I dreamed my spirit broke the bars of sense
That hold the gates of consciousness shut fast,
Threw off the prison garb of self, and passed
Into the wonder of Omniscience.

As mists that rise from ocean and condense
In clouds, in million rain-drops melt, at last