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SINR-based Channel Assignment for Dense Wireless LANs

Xiangping Qin, Xingang Guo, Randall Berry

Abstract—The biggest challenge in channel assignment for dense, multi-cell/AP wireless LANs is to arrange co-channel cells so as to maximize the aggregate network throughput. Most previous work models this problem as a vertex coloring problem. In this paper, we model it as a non-linear optimization problem to maximize overall network throughput. We prove that the new optimization problem is NP-hard and vertex-coloring is a simplified case. We then propose a polynomial time heuristic algorithm called MIF (Most-Interfered-First) for channel assignment. The performance for a line topology is analyzed. Simulations for random topologies show that MIF consistently produces better network throughput than vertex-coloring based heuristic algorithms with less computation cost.

I. INTRODUCTION

Over the past few years we have witnessed the rapid proliferation of wireless LANs in various network environments. The need for higher throughput and improved coverage has led to the deployment of multi-cell networks in places such as enterprises and hotspots. Meanwhile, similar multi-cell networks have also started to appear in residential areas, where cells are often formed of WLANs in homes or apartment units within close proximity. In both cases, each cell is managed and serviced by its own access point (AP). The AP communicates in a selected frequency channel and every client in that cell must use the same channel.

Partitioning radio spectrum into channels is an engineered design choice that enables concurrent communications using different channels. As cells quickly outnumber available channels, channels need to be carefully allocated to each cell so as to reduce the interference and maximize the overall performance of the entire network. Good channel allocation has proven to be one of the biggest challenges in deploying and managing wireless LANs. For example, average consumers normally don’t coordinate with each other in selecting channels, nor do they often possess the knowledge of how to select the best channel in their residential neighborhood. Even in a managed network such as the enterprise or hotspot network, channel configuration relies on IT personnel for site surveying and manual configuration. Good understanding to the channel allocation problem and an effective method for autonomous channel allocation method are critical to the scalability of the WLAN technology, and ultimately, good end-user experience.

Channel allocation schemes have been studied extensively for cellular networks, where the base stations are often deployed in pre-selected locations to form hexagon-shape cells, see for example [5]–[8], [15]–[17]. However, due to the low-cost and flexibility of WLAN devices, APs in multi-cell WLANs often form a random topology. Furthermore, the multiplexing approaches used in WLANs are typically quite different from the cellular environment. For example, in 802.11 wireless LANs the same channel is used by all clients to communicate to a given AP and this channel is used for both up-link and downlink traffic. On the other hand, in many cellular systems each user may be assigned a different channel, and different channels are used for the up-link and downlink. Another important consideration is that in current wireless LANs, it is desirable for the channel assignments to be relatively static, i.e., it is not feasible to consider dynamic assignment schemes that vary on the time-scale of the traffic (see e.g., [8]). Traditionally, channel allocation for WLANs has been abstracted into various graph theoretic vertex-coloring problems (e.g., [9], [10], [12]–[14]), where each AP or cell is represented by a vertex and channels are represented by different colors. One widely used example is the minimum distance-2 vertex coloring [11]. Two APs within a certain distance are consider connected, and an edge, therefore, is created between the two corresponding vertices. The goal is to find a coloring scheme such that no neighboring nodes will have the same color. Hence, neighboring cells will use different channels and avoid interfering each other.

In this paper, instead of relying on the vertex-coloring approach, we consider a model that optimizes channel allocation for aggregate network throughput, which is
determined by the SINR (Signal to Interference and Noise Ratio) at each AP. Our contribution in this paper lays in two aspects: First, we will present a model for channel allocation as a non-linear integer optimization problem. Furthermore, we prove that channel allocation for maximal network throughput is NP-hard, and vertex-coloring is a simplified case of our model. Second, we propose a MIF (Most-Interfered-First) algorithm – a heuristic algorithm. The performance for a line topology is analyzed. When compared against a heuristic algorithm based on vertex-coloring, simulations show channel allocations produced by MIF are consistently better with less computational cost.

It is worth pointing out that in this paper, we consider the downlink model where the traffic is heavy and the interference from other APs is considered as noise in the physical layer. We do not consider a model where an AP may defer transmissions to avoid interference from a co-channel AP. Such issues with a random traffic model and MAC layer design are left as topics of future work.

II. SYSTEM MODEL

In a multi-cell wireless network, assume there are \( N \) APs, \( AP_n, n = 1, \ldots, N \) and \( K \) channels, \( f_k, k = 1, \ldots, K \). We focus on a backlogged downlink model in this paper, where each AP always has a packet to send to some client. A \( K \times N \) allocation matrix \( A \) is used to indicate how the channels are allocated to APs. It is a \( \{0, 1\} \) matrix, where

\[
A_{kn} = \begin{cases} 
1, & \text{if } f_k \text{ is allocated to } AP_n, \\
0, & \text{otherwise}.
\end{cases}
\]

(1)

We focus on the case where each AP can only transmit on one channel, i.e., \( \sum_{k=1}^{K} A_{kn} = 1 \) for all \( n = 1, \ldots, N \). This is commonly the case, for example in 802.11 wireless LANs. We assume that constant power is used, i.e., \( P_n = P \) for all \( n = 1, \ldots, N \); again, this is commonly the case, though some slow time-scale power allocation may be possible. Let \( H_{mn} \) denote the channel gain between \( AP_m \) and \( AP_n \). We say that the channel gains are symmetric if \( H_{mn} = H_{nm} \) for all \( m, n \); this will be the case, for example, when these gains represent the channel gains between the two APs’ (assuming reciprocity).

We consider maximizing the total “throughput” given by the following optimization problem,

\[
\max_A \sum_n \sum_k W \log \left( 1 + \frac{A_{kn} PH_n}{\sum_m PA_{km} H_{mn} + \sigma^2} \right)
\]

s.t. \( \sum_{k=1}^{K} A_{kn} = 1 \) for \( n = 1, \ldots, N \).

(2)

where \( W \) is the bandwidth of one channel, \( H_n \) is the channel gain of \( AP_n \) to a reference point in the cell and \( \sigma^2 \) is background noise power. The interference at the reference point in a cell is measured by the corresponding AP. Clearly this is a simplification; however, we will show in an example later, that because of the convexity of the model, this is a good approximation. Also in practice, it is desirable to do channel assignment on a relatively slow time-scale. During the time a channel is assigned, the actual client locations will likely vary and not be known \textit{a priori}. In particular, when an access point must choose a channel and there are no clients present.

In much of the previous work, the channel allocation problem has been modelled as a graph coloring problem. The coloring approach can be viewed as a special case of (2), where the channel gain matrix is simplified by setting

\[
H_{ij} = \begin{cases} 
H_0 & \text{for all } H_{ij} > H_{th} \\
0 & \text{Otherwise}
\end{cases}
\]

(3)

where \( H_{th} \) is a given threshold. This binary quantization simplifies the model so that APs either interfere with each other or not. The APs who interfere with each other are considered to be neighbors. A graph is created, where APs are represented as vertices and APs who are neighbors are connected by edges. The vertex coloring problem is to assign colors so that vertices that are connected have different colors, i.e., no two neighbors can be assigned to the same channel. The coloring problem has two formulations. One is to find a feasible allocation for a given number of channels. The other to find the minimal number of channels for a given topology. In most applications, the first formulation is used. It is well-known that the vertex coloring problem is NP-hard [18]. The following proposition and its proof further illustrates the relation between the coloring problem and the problem in (2).

Proposition 1: The optimization problem in (2) is NP-hard.

Proof: As noted above, a vertex coloring problem can be modelled as a special case of (2). Specifically given any graph, let the set of nodes correspond to a set of AP’s in (2) and set \( H_{ij} = 1 \) if node \( i \) and node \( j \) have an edge between them and set \( H_{ij} = 0 \) otherwise. Suppose we can efficiently solve (2) for any given number of channels. The solution to (2) must be less than or equal to \( N^{s_0} \), where \( s_0 = W \log(1 + \frac{PH_n}{\sigma^2}) \), and this will be achieved if there are enough channels available so that no two nodes \( i,j \) with \( H_{i,j} = 1 \) share the same channel. But such an assignment corresponds to a vertex coloring of the original graph. Thus for a given number of channels, \( K \), if the solution to (2) is less than
Let \( N_{s0} \), then the original graph cannot be colored with \( K \) colors. Therefore, if we can solve (2) in polynomial time, then we can also solve the vertex coloring problem in polynomial time. So (2) must be NP-hard.

In this paper, instead of modeling the channel assignment problem as a coloring problem, we propose heuristic algorithms to solve (2) directly. In the following section, we show that this approach achieves better performance than the coloring approach and simplifies the procedure of graph generation and solving the problem for difficult topologies.

As an aside, we now briefly discuss the validity of the approximation in (2). Consider a model where the channel gain is given by a path loss model, \textit{i.e.} the received power at distance \( d \) is given by \( P_r(d) = \frac{P_r}{d^\gamma} \). Here \( \gamma \) is the path loss exponent, and \( P_r \) is the received power at reference distance \( \bar{d} \). Typically, \( \gamma \) is between 2 to 4. We choose \( \gamma = 2.4 \) in this example. Equivalently, the channel gain between \( AP_i \) and the reference point of \( AP_j \) is given by

\[
H_{ij} = H(\bar{d}) \left( \frac{\bar{d}}{d_{ij}} \right)^\gamma,
\]

where \( d_{ij} \) is the distance between the \( AP_i \) and the reference point \( AP_j \), and \( H(\bar{d}) \) is the channel gain at the reference distance \( \bar{d} \). We set \( \bar{d} = 1 \) and assume that \( H(1) = 1 \) so that \( H_{ij} = \left( \frac{1}{d_{ij}} \right)^\gamma \). In one cell of the AP, clients that are closer to the interfering AP have a lower throughput while clients that are farther away have a higher throughput. Consider a simple case where there is only one interfering \( AP_i \) and \( M \) clients for \( AP_j \). The total throughput is

\[
S = \frac{1}{M} \sum_{m=1}^{M} W \log \left( 1 + \frac{PH}{d_{im} + \sigma^2} \right),
\]

where \( d_{im} \) is the distance of \( AP_i \) to client \( m \). Let \( f(x) = W \log \left( 1 + \frac{PH}{x + \sigma^2} \right) \). With commonly used parameters, \( f(x) \) is a concave function. So we have

\[
S \leq W \log \left( 1 + \frac{PH}{\left( \sum_{m=1}^{M} d_{im} \right) + \sigma^2} \right).
\]

Let \( M \) clients uniformly distributed on a circle of radius, then \( \sum_{m=1}^{M} d_{im} = d_{ij} \), where \( d_{ij} \) is the distance between \( AP_i \) and \( AP_j \). Then the throughput is upper bounded by

\[
W \log \left( 1 + \frac{PH}{d_{ij} + \sigma^2} \right),
\]

where interference is measured at the AP. Simulation shows that in this simple model, with commonly used parameters, there is less than 1 percent of a throughput difference. Therefore, it is reasonable to use (2) as a throughput metric. The downlink throughput is then given by

\[
\sum_{m} \sum_{k} W \log(1 + \frac{PH_{km}d_{km}^{\gamma}}{\sum_{m} P_{km}H_{km} + \sigma^2}).
\]

III. THE MOST-INTERFERED-FIRST ALGORITHM AND ANALYSIS FOR A LINE TOPOLOGY

To solve (2), we propose the following heuristic algorithm. This algorithm is essentially a greedy procedure that attempts to assign each AP to the channel that generates the least interference.

\textbf{Most-Interfered-First (MIF) algorithm:}

\textit{initialize:} Randomly choose an initial \( AP_M \), and assign it to any channel \( f_l \); set \( N = \{ m = 1, \ldots, N ; m \neq M \} \).

\textbf{while} \( N \neq \phi \) \textbf{do}

\begin{itemize}
  \item Measure interference \( I_n = \sum_k I_{kn} \) for all \( n \in N \).
  \item set \( M = \{ m \mid m = \arg \max_{j \in N} I_j \} \).
  \item randomly choose \( m \in M \).
  \item Assign \( AP_m \) to channel \( f_l \) where \( l = \arg \min_{k \in \{1, \ldots, K\}} I_{km} \).
  \item set \( N = N \setminus \{m\} \).
\end{itemize}

\textbf{end while}

In this algorithm, channels are assigned to APs sequentially; the set \( N \) contains those access points that are not yet assigned a channel. The initial AP is chosen randomly and randomly assigned some channel. After this AP is assigned to that channel, the other APs re-measure the interference on each channel and compute the sum of the interference they see across all channels \( I_j = \sum_{i} I_{ij} \). The algorithm then chooses the most interfered AP, \textit{i.e.} access point \( AP_m \) such that \( m = \arg \max_{j \in N} I_j \); this AP is assigned its least interfered channel \( f_l \). Once assigned a channel an AP is removed from the set \( N \); the algorithm continues until there are no more APs in this set. Note that at each iteration, the interference measured includes that generated by all the APs which have previously been assigned channels.

IV. ANALYSIS FOR LINEAR TOPOLOGIES

We next look at the performance of the MIF algorithm for a simple line topology as in Fig. 1, where neighboring APs are equal distance apart. For this topology, it can be seen that MIF algorithm will always result a periodic channel assignment as shown in Fig. 1. Intuitively, this is a desireable assignment for this setting. We next make this precise by showing that this assignment minimizes the total interference when there are a large number of APs.\(^1\) Note we are not proving that this assignment is the optimal solution to (2) - for a symmetric model, we conjecture that this is the case, but we have not been able to prove this. We proceed by first giving two lemmas.

\(^1\)The reason for considering a large number of APs is to avoid "edge effects".
The first lemma says if $N$ APs share one channel, then, for the same total length, APs that are equal distance apart will have a smaller total interference than any other allocation, for a large number of APs. Given $N$ such APs on a line, let the distance between two adjacent APs be $d_1, d_2, \ldots, d_{N-1}$ and let $I(d_1, d_2, \ldots, d_{N-1})$ denote the total interference summed over all the APs, where we assume that the interference for AP is measured at the AP.

**Lemma 1:** For large enough $N$, 
\[
I(d_1, d_2, \ldots, d_{N-1}) / I(d_0, d_0, \ldots, d_0) > 1,
\]
where $d_0 = \frac{1}{N-1} \sum_{i=1}^{N-1} d_i$.

**Proof:** Omitted due to space limitation.

Now assume we have $N$ AP's located on a line of length $D = cN$ and a total of $K$ channels available, where $D$ is a multiple of $N$. Let $N_k$ denote the number of APs assigned to each channel $k = 1, \ldots, K$, and assume that all AP sharing a given channel are equal distance apart (as suggested by lemma 1).\(^2\) The next lemma says that, we should divide the $N$ APs evenly among the $K$ channels to minimize the total interference. Specifically, let $I(N_1, \ldots, N_K)$ denote the total interference as a function of the number of AP assigned to each AP. Then we have:

**Lemma 2:** For large enough values of $N$, if $N/K$ is an integer,
\[
I(N_1, N_2, \ldots, N_K) / I(N_0, \ldots, N_0) > 1,
\]
where $N_0 = N/K = \frac{1}{K} \sum_{k=1}^{K} N_k$.

**Proof:** Omitted due to space limitation.

In the above lemmas, we did not consider the fact that the locations of the APs are fixed. The next proposition applies to the case where the AP have a fixed location and are all equal distance from their neighbors.

**Proposition 2:** Consider a network with $N$ APs located equal distance $d_0$ apart on a line, and $K$ channels, where $M = \frac{N}{K}$ is an integer. Let $I^*$ denote the minimum total interference achievable by any channel allocation, and let $I_{MIF}$ be the total interference of from the MIF algorithm. For a fixed $K$, then
\[
\lim_{N \to \infty} I^* / I_{MIF} = 1
\]

**Proof:** Omit due to space limitation.

\(^2\)Note that this assumption implies that if we change the number AP assigned to a channel, we can also change the location of the AP.

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**V. TWO DIMENSIONAL RANDOM TOPOLOGY: COMPARISON TO THE COLORING APPROACH**

In this section, we consider a $L$ meters by $L$ meters area with $N$ APs randomly located in it, *i.e.* each AP's $x$ coordinate and $y$ coordinate is uniformly distributed in $(0, L)$. We use the same path loss model as in (4). First, consider a common coloring approach. A graph is created taking APs as vertices. The edges are drawn in the way described in (3). Setting a threshold on the channel gain $H_{ij}$ is the same as setting a threshold on the distance between two APs $D_{ij}$. So we will first set a threshold $d_{th}$. If the distance between two APs is less than $d_{th}$, an edge is drawn between them and these two APs are considered as neighbors. After a graph is generated, we will color the vertices so that no two neighboring vertices are assigned the same color. The number of colors is the same as the number of channels. There are many coloring heuristics. In our simulation, we use one of the most commonly used coloring heuristics, saturated degree algorithm [1] [2] [3], as an example to color the graph. The coloring result can be written as a channel allocation matrix $A$ as defined in (1) and a total throughput $s(A)$ is computed as in (2), where
\[
s(A) = \sum_n \sum_k W \log(1 + \frac{A_{kn} PH}{\sum_m PAT_{km} H_{mn} + \sigma^2}).
\]

If there is no feasible coloring solution, we let the throughput be zero to be easy to capture. The type of the graphs generated from the same topology depends on the choice of $d_{th}$. In the following simulation, we do an exhaustive search for an optimal $d_{th}$. In a 100 meter by 100 meter area, we randomly generate networks with 25 APs or 50 APs. We simulated for 100 random realizations. For each realization, $d_{th}$ is set to the value of 5 meters, 10 meters, 15 meters, ..., 100 meters, accordingly. With different $d_{th}$, different graphs are generated. For each graph, we use the saturated degree algorithm to color the graph and a throughput is calculated. Parameters used are $\sigma^2 = 10^{-12}$, $H_0 = 1$, $W = 1 M H z$, $\gamma = 2.4$. Compare the throughput for all these graphs with different $d_{th}$, the optimal $d_{th}$ is found. Fig. 2 shows the optimal $d_{th}$ with 25 APs and 50 APs and four channels. It can be seen that the optimal threshold is different for different random graphs, for 25 APs, the optimal $d_{th}$ varies from 5 to 30. The optimal $d_{th}$ also varies with different parameters, such as the number of APs, the dimension of the area and the number of channels. The exhaustive search has a high computational cost. Therefore in real applications, it is not realistic to assume for any topology, the optimal threshold $d_{th}$ can be found efficiently.

If an optimal threshold $d_{th}$ is provided, the maximal throughput achieved by coloring is compared to the

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Fig. 1. Channel Assignment of the MIF algorithm in a linear network.
Fig. 2. Optimal threshold to maximize the throughput of coloring

throughput of the MIF algorithm. Fig. 3 shows the throughput ratio of these two. On average, for 25 APs the throughput ratio of coloring with the optimal threshold to MIF is 0.9816. For some random graphs with 25 APs, coloring does slightly better than MIF. For 50 APs, the average throughput ratio of coloring to MIF is 0.9017. Therefore the coloring approach has no better performance in terms of total throughput compared to the MIF algorithm, but with a much higher computational cost.

Besides the total throughput, another important performance criterion is the fairness criterion. Here we use the minimum throughput among all the APs as a fairness criterion. The minimum throughput gives a worst case throughput guarantee. Fig. 4 shows the ratio of the minimum throughput of the coloring with the optimal threshold to the MIF. For 25 APs, the average ratio is 0.8655. The coloring has a worse fairness guarantee. For 50 APs, the average ratio is 0.9625. For certain topologies, coloring has a high minimal throughput. Overall, coloring does not provide a better fairness guarantee.

Since the computation cost is high to find the optimal $d_{th}$, we simulate when $d_{th}$ is fixed for all the random graphs. Fig. 5 shows the throughput for random topologies if we set a fixed threshold $d_{th} = 20$ for 25 APs. It can be seen that for certain graphs, no feasible solution exists, (throughput is shown as zero). In order to avoid these graphs, we set $d_{th} = 5$ for 50 APs in the Fig. 6. It shows if a low distance threshold is used, throughput of the coloring approach is much lower than the MIF. On average, the throughput ratio is only 0.7436, because no maximal spatial reuse is guaranteed.

Therefore, we conclude that the MIF algorithm has a better performance than the common coloring approach in terms of total throughput and worst case minimum throughput guarantee. MIF algorithm has a much lower computational complexity. With $N$ APs, the computational cost of the MIF is only $\log(N!)$, which is less than $N \log(N)$. The original saturation degree coloring algorithm for a given graph has a computational cost of $O(N^2)$ [1], with special programming techniques, it can be reduced to $M \log(N)$ where $M$ is the number of edges [4]. For a random topology, if each node has two or more than two neighbors in the coloring graph, then $M \geq N$. Moreover, the search for $h_{th}$ will multiply the computational cost for the coloring approach. Further more, MIF is easy to implement. The only input parameter is the total interference which can be easily measured at each AP.
minimizing the SINR for aggregate throughput gain is a promising method for research and algorithm design in channel assignment for WLANs.

REFERENCES


VI. CONCLUSION

In this paper, we investigated the problem of assigning wireless channels to each cell (AP) so as to maximize the aggregate network throughput. We have modeled this problem as a non-linear optimization problem to maximize network throughput. We have proved that this channel assignment problem is NP-hard, and vertex-coloring is a simplified case. Hence, we proposed a heuristic MIF (Most-Interfered-First) algorithm that produces channel assignment in polynomial time. When compared with the most commonly used heuristic algorithm based on vertex-coloring, simulations have shown that MIF consistently results in better network throughput. It also produced better fairness with less computational overhead. Autonomous channel assignment is a key enabler for the future scalability and ease-of-management of wireless networks. We believe examining approaches based upon