

Southern Illinois University Carbondale

OpenSIUC

---

Music Research Papers

School of Music

---

Fall 9-7-2020

## **Pcset Networks Embedded in Tonal Syntax: Abstract Symmetry and Parallel Pathways in Dallapiccola's 'Sonatina Canonica II'**

Christopher Walczak  
walczak@siu.edu

Follow this and additional works at: [https://opensiuc.lib.siu.edu/music\\_papers](https://opensiuc.lib.siu.edu/music_papers)

---

### **Recommended Citation**

Walczak, Christopher. "Pcset Networks Embedded in Tonal Syntax: Abstract Symmetry and Parallel Pathways in Dallapiccola's 'Sonatina Canonica II'." (Fall 2020).

This Article is brought to you for free and open access by the School of Music at OpenSIUC. It has been accepted for inclusion in Music Research Papers by an authorized administrator of OpenSIUC. For more information, please contact [opensiuc@lib.siu.edu](mailto:opensiuc@lib.siu.edu).

**Pcset Networks Embedded in Tonal Syntax:  
Abstract Symmetry and Parallel Pathways in  
Dallapiccola's *Sonatina Canonica II***

Christopher Walczak



First system of musical notation, piano part, measures 1-4. The key signature has two flats (B-flat and E-flat). The music features a complex texture with many beamed sixteenth and thirty-second notes, creating a rapid, shimmering effect. The bass line is more rhythmic, with some triplets and slurs.

Second system of musical notation, piano part, measures 5-8. The texture continues with dense, beamed passages in both hands. There are some dynamic markings like *ff* and *f* visible. The bass line has some triplet markings.

Third system of musical notation, piano part, measures 9-12. The music continues with similar rapid, beamed passages. The bass line has some triplet markings and slurs.

Fourth system of musical notation, piano part, measures 13-16. The music continues with similar rapid, beamed passages. The bass line has some triplet markings and slurs. The system ends with a double bar line.

**Largo** (♩ = 36)

Fifth system of musical notation, piano part, measures 17-20. This system introduces a new texture with a *martell.* (martellato) effect in the right hand, indicated by a sharp accent and a series of slurs. The left hand has a *tr* (trill) marking. Dynamics include *ff*, *sf*, *f*, and *mf*. The system ends with a double bar line.

Sixth system of musical notation, piano part, measures 21-24. The music continues with a *pp* (pianissimo) dynamic. There are some triplet markings and slurs. The system ends with a double bar line.

The opening four measures appear on the surface to be composed of a descending arpeggio figure (alternating E major and E minor) juxtaposed against a rising trill figure. The arpeggio figure can be heard as a two measure "phrase" which repeats two octaves lower. Example 1.2 articulates this using the pitches of the music. Over mm. 5-6 the notion of rapid major-minor mode change is further established by the downward cascading arpeggios Eb major-minor in m. 5 and Db major-minor in m. 6. In the lower register we note the rising B major-minor across the barline.

Example 1.2. Descending major-minor arpeggio figure and ascending trill figure, mm. 1-4



Other tonal implications are immediately heard. The tonic-dominant relation between the initial E arpeggios (mm. 1-4) and the low B arpeggio in rhythmic diminution (across mm.5-6) parallels the tonic-dominant relation between the Eb arpeggio and the final Bb chord in m. 6. The pertinence of that semitonal connection is reinforced by other semitonal attributes of the music thus far. We have already observed the alteration of the third of each arpeggio during the rapid mode changes and even more impacting is the trill figure itself, consisting exclusively of (01) dyads.

The very end of the movement (mm. 31-36) is the exact same music as the first six measures of the movement (exempting the repetitions of the final chord) transposed a major third lower. The T8 transformation interacts very cogently with the semitonal aspects of the music and will assume greater importance later on, as we shall see. From this point on we shall refer to mm. 1-6 and mm. 31-37 as the "major-minor music." The stark dynamic contrast between the initial fortissimo and sudden pianissimo halfway through each segment allows the composer to "begin with a bang" yet end the piece softly after the reprise of the major-minor music.

Much more is happening underneath the tonal surface of the first four measures. Example 1.3 gives a referential "configuration" for the pitches of mm. 1-4. The configuration comprises a progression of alternating (012) and (016) trichords. The fourth quarter note beat of m. 3 consists of

two imbricated (016) trichords, {Bb,Cb,E} and {F,Bb,Cb}. The first three beats of m. 4 can certainly be heard as another two imbricated (016) trichords - {G#,C#,D} and {C#,D,G} - as the G#5 and G2 sound simultaneously. On the last quarter note beats of both m. 2 and m. 4, a distinctive (013) trichord can be heard demarcating the end of each two measure phrase. In every case, the trills form the (01) subset of each trichord.

Example 1.3. Configurations of pitch-class sets, mm. 1-4.

(012) {B C Db} (016) {D Eb G#} (012) {D Eb E} (016) {E# F# B} (012) {E# F# G} (013) {E F# G} (016) {G Ab C#} (012) {G# A Bb} (016) {Bb Cb E} (016) {G# C# D} (016) {C# D G} (013) {C# D E}

(016) {F Bb Cb}

That understanding of m. 1-4 is the point of departure for the configuration of example 1.4., which presents the sequence of the exact pitch-class sets of both the (012) and (016) trichords as they progress across mm. 1-4 and mm. 31-34. The location of each trichord is labeled with a measure number followed by a beat number and separated by a comma. From this configuration, we note the aggregate produced by the combination of the four (012) trichords in both of the four-measure segments (example 1.4a). Even more arresting is that the progressions unfold so as to preserve a semitonally ordered chromatic collection.

Example 1.4. Sequence of (012) and (016) pitch-class sets across mm. 1-4 and mm. 31-34

a) (012)

T0: 1,1 {B C Db} 1,4 {D Eb E} 2,3 {F F# G} 3,3 {G# A Bb}

T8: 31,1 {G G# A} 31,4 {Bb B C} 32,3 {C# D Eb} 33,3 {E F F#}

b) (016)

T0: 1,2 {D Eb Ab} 2,1 {F F# B} 3,1 {G Ab Db} 3,4 {F Bb Cb} 4,1 {C# D G#}

T8: 31,2 {Bb Cb E} 32,1 {C# D G} 33,1 {Eb E A} 33,4 {C# F# G} 34,1 {A Bb E}

{F# G C} {C# D G}

The superset produced by the combination of the (016) trichords in example 1.4b is at first puzzling but ultimately fascinating. For each of the four-measure segments, a total of six (016) trichords can be partitioned, due largely to the aforementioned imbrications found in mm. 3-4 and mm. 33-34. Being quite possible to compose an aggregate consisting of the combination of four unique (016) trichords, we wonder why Dallapiccola hasn't architected that template as he did with the (012) trichords in example 1.4a.

The T8-relation of mm. 1-4 to mm. 31-34 illuminates a plausible explanation. The diagonal arrows in example 1.4b show that the imbricated (016) trichords {Bb,Cb,E} and {C#,D,G} from mm. 3-4 (see example 1.3) are identical to the first two (016) trichords of mm. 31-32. In that connection, the listener feels a certain momentum at the arrival of that music, a sense of "picking up where we left off."

The beginning of the piece demands further analysis. As the E major-minor "phrase" repeats over mm. 3-4, the texture thickens slightly and an explanation is needed for the added tones. Example 1.5 presents another series of trichords formed by the combination of the "new" pitches and those of the trills. At this point, the pitches of the music in mm. 1-4 have been accounted for entirely. The accumulation of (013) and (014) trichords embedded in and coexisting with other harmonic constructs is neither arbitrary nor haphazard. The prominence and isolated energy of the trill merits its role as a harmonic foundation from which other constructs of simultaneity are generated by its union with other more static pitch classes. These pitch classes "orbit" the trill figure through its ascent. Moreover, we have already heard the harmonic "progression" across mm. 1-2 and are familiar with the first layer of harmony. The "added tones" of mm. 3-4 are thus heard in addition to the extant (012) and (016) trichords.

Example 1.5. Configurations of embedded (013) and (014) pitch-class sets, mm. 3-4

The image displays four staves of musical notation. The top staff is a treble clef with two notes: A# and Bb. Above it are labels (014) {A Bb C#} and (014) {Bb Cb D}. The second staff is a treble clef with one note: E#. Below it is label (013) {E# G Ab}. The third staff is a bass clef with one note: C#. Below it is label (014) {C# D F}. The bottom staff is a bass clef with four notes: G, Ab, B, and D. Below it are labels (014) {G Ab B} and (013) {B C# D}. Vertical lines connect the notes across the staves, showing their relationships.

A more detailed analysis of mm. 3-4 supports an inclination toward partitioning embedded (013) and (014) trichords. Compare the first beat of m. 3 with that of m. 4 on example 1.5. In each case, the added pitch classes are F and B: E#4 and the octaves B3/B4 in m.3, and F3 and B2 in m.4. The interchange of the set-theoretical role of these two pitch classes seems deliberate. F completes both the (013) trichord {E#,G,Ab} in m. 3 and, conversely, the (014) trichord {F,C#,D} in m. 4. B completes the (014) trichord {B,G,Ab} in m.3 and the (014) trichord {F,C#,D} in m. 4. In this observation, we note that the embedded (013) trichord {E#,G,Ab} on the downbeat of m. 3 is in T1-relation to the “end-demarcating” (013) trichord {E,F#,G} immediately preceding it on the last beat of m.2, creating a sense of continuity and acceleration across the barline and into the next “phrase.”

The last two remaining “added tones” C#5 and D4 on beats 3 and 4 (respectively) of m. 3 complete the (014) trichords {C#,A,Bb} and {D,Bb,Cb}. These two pitch classes are neither part of the trill figure nor the arpeggio and for that reason stand out, making it easy to justify their association with the apex of the trill figure in m. 4, the dyad {C#6,D6}. All of these relations apply, of course, to the T8 transformation of mm. 33-34.

The very tonal “canon,” as we shall now call it, spans mm. 7-29 and is nestled between the major-minor music, separated from it by rests on both sides. Although clearly in Bb major, Dallapiccola has refrained from using a key signature. The lively and lighthearted canon can be heard in two distinct “passes,” the first one beginning at m. 7 and the second one virtually restarting at m. 17. Modulation to the subdominant Eb major occurs at m. 11 during the first pass and at m. 21 during the second. The modulations afford the composer some ingenious structural maneuvering.

The downbeat of m. 9 piques the interest of the ear. The Ab5 (note ‘8va’ in the upper staff) - the “seventh” in a “V/IV” secondary dominant - sounds a single eighth note before the A4 “lower neighbor” from the canonic figure in the bottom staff. Both sound simultaneously with the pitch class Bb. At the “Vivacissimo” tempo, the perception of verticality becomes somewhat blurred and the sonority leaps out of the racing musical torrent. It is no coincidence that this sonority forms the (012) trichord {Ab,A,Bb}. The phenomenon confirms its functional purpose as it occurs again on the downbeat of the next measure.

On the downbeat of m. 11, as the canonic figure modulates to Eb in the top staff, we hear the flicker of another familiar sonority. The lower neighbor Bb-A-Bb in the bottom staff sounds simultaneously with the pitch class Eb in the upper staff to form the (016) trichord {A,Bb,Eb}. In the same manner, the downbeat of m. 12 consists of the (016) trichord {D,Eb,Ab}. The deliberate and strong metric positioning of these (012) and (016) trichords leads us to understand their



significance and the clear association with the trichords of mm. 1-4 and mm. 31-34. Example 1.6 highlights the embedded pitch-class sets that occur in the first pass of the canon and correspond to the first four measures of the major-minor music.

Example 1.6. Embedded occurrences of (012), (016), (014), and (013) trichords, mm. 7-16

Example 1.6 displays musical notation for measures 9 through 15, illustrating embedded trichords. The notation is presented in two staves. Measures 9-12 are on the top staff, and measures 14-15 are on the bottom staff. Measure 13 is omitted. Each measure is labeled with a measure number and a trichord set.

Measure	Trichord Set
m. 9	(012) {Ab A Bb}
m. 10	(012) {Ab A Bb}
m. 11	(016) {Ab A Eb}
m. 12	(016) {D Eb Ab}
m. 14	(014) {Eb Gb G}
m. 15	(014) {Gb A Bb}
m. 16	(013) {A Bb C}

The remainder of the first pass of the canon confirms the suspicion of a parallel pathway shared by mm.1-4 and mm. 33-34. The occurrence of the (016) trichord {A,D,Eb} in m. 14 and its relation to the {D,Eb,Ab} from m. 12 by a single semitonal exchange between pitch classes A and Ab recalls the transformation of {G#,C#,D} to {C#,D,G} from m. 4. The two embedded (014) trichords in m. 14-15 associate with those of m. 3. And perhaps the most substantial parallel is the "end-demarcating" (013) trichord {A,Bb,C} in m. 13 which signals the end of the first pass while recalling the (013) trichords that punctuated the endings of the arpeggio phrases from the final beats of m. 2 and m. 4.

Example 1.7 configures a similar pathway for the second pass of the canon. Note that the entrance in the bottom staff at m. 17 is offset, a quarter note late when compared to the first pass. The progression of embedded trichord forms remains consistent, however, and after we hear the second pass come to a close on yet another (013) trichord, it becomes useful to note the symmetry of these "end-demarcating" sonorities. The trichord {E,F#,G} that ends the phrase in m. 2 is related to the trichord {C#,D,E} that ends the phrase in m. 4 by inversion around the invariant pitch class E (see example 1.3). Similarly, the trichord {A,Bb,C} that ends the first pass of the canon (m. 15, ex. 1.6) is related to the trichord {G,A,Bb}, which ends the second pass (m. 29, ex. 1.7), by inversion around the invariant pitch classes A and Bb. It will also be noted that the trichord {A,Bb,C} "wraps

up" the end-demarcating phenomena near the end of the piece (the last beat of m. 24) as well as the first pass of the canon (m.15, ex. 1.6).

Example 1.7. Embedded occurrences of (012), (016), (014), and (013) trichords, mm. 17-29

Example 1.7 displays musical notation for measures 19 through 29, illustrating the embedded occurrences of specific trichords. The notation is presented in two systems, each with a treble and bass staff. Trichord labels are provided below the notes in each measure.

Measure	Trichord Label
m. 19	(012) {Ab A Bb}
m. 20	(012) {Ab A Bb}
m. 21	(016) {A Bb Eb}
m. 22	(016) {D Eb Ab}
m. 23	(016) {Eb Ab A}
m. 23	(014) {Gb G Bb}
m. 23	(014) {D Gb G}
m. 29	(013) {G A Bb}

Example 1.8. Parallel transformational pathway shared by the four segments of the music.

(012) / (016) trichords → imbricated (016) trichords → additive (014) trichords → concluding (013) trichord

m. 1-2  
m. 9-11  
m. 19  
m. 31-32

m. 3-4  
m. 12-14  
m. 21-22  
m. 33-34

m. 3-4  
m. 14  
m. 22-23  
m. 33-34

m. (2) 4  
m. 15  
m. 29  
m. (32) 34

The visual motif of example 1.8 represents the transformational pathway of the embedded pitch-class sets which unfolds across the four distinct segments of the movement. The common gestural profile that links the segments as variations of one another can be described as follows. After establishing a progression of (012) and (016) trichords, the music intensifies by the emergence of imbricated (016) trichords and the addition of embedded (014) trichords. The conclusion of each segment is demarcated by a final (013) trichord which is inversionally related to at least one other end-demarcating (013) trichord.

Let us now turn to other observations pertinent to understanding the work. The T8-relation of the opening and closing major-minor music allows the piece to cohere powerfully and on many levels. Firstly, this transformation places the root of the major-minor arpeggio figure (E in mm. 1-4) on C in m. 31. The arrival of C major-minor is calculated and definitive. After the initial C5 (of

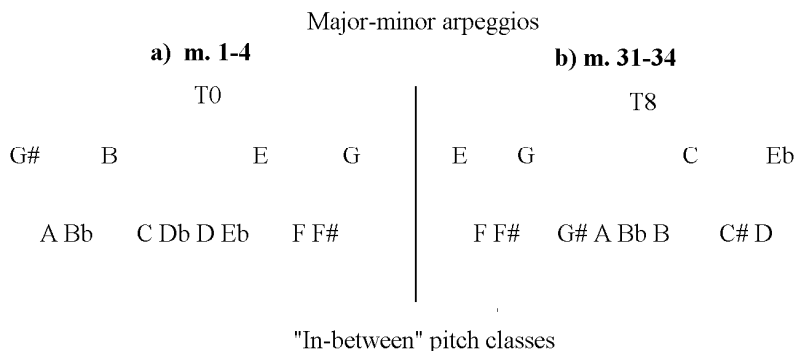
the trill figure) launches the music from the downbeat of the entire piece, we do not hear that pitch class during the remainder of the major-minor music from mm. 1-6. Pitch class C is "held out" and mysteriously "passed over" during the ascent of the trill figure between mm. 3-4, almost tauntingly so as its vacancy is surrounded on both sides by Cb6 and C#6.

In that vein, occurrences of pitch class C during the canon are extremely scarce considering its role as supertonic. Apart from the blatant incision of C4 on the downbeat of m. 24 and its echo, C6 on beat two, pitch class C is for the most part cunningly relegated to the grace note of the canonic figure. By m. 31, we have become desperate to hear it and that craving is amplified by the dominant G7 chord at the end of m. 29, ceasing, and trailing off into oblivion.

The voice leading across the whole note rest at m. 30 lends itself to a convincing arrival at m. 31. The octaves G3/G4 in m. 29 transfer to G5/G6 in m. 31. The G4 in m. 29 proceeds smoothly, up a semitone to the Ab4 of the trill in m. 31. The scalar descent of G-F-Eb-D over the final two beats of m. 29 in the highest voice, each note marked with a tenuto, unmistakably resolves on the C5 of m. 31.

In light of the T8 transformation of the major-minor music, the remainder of the C major triad (the pitch-class dyad {E,G}) at m. 31 assumes structural significance. Example 1.9 gives referential configurations that combine the major-minor "tonic" chords found at both T0 (E major-minor, mm. 1-4) and T8 (C major-minor, mm. 31-34) as well as configurations for the "in-between" pitch classes, i.e. the eight complementary pitch classes that are in-between the major-minor chords at both T0 and T8. Tidily, the ascending trill figure happens to be composed almost entirely of the "in-between" pitch classes. The partitioning of the pitch classes in example 1.9 reveals the symmetry of the abstracted structure. The "third" and "fifth" of the C major chord, pitch classes E and G, it will be noted, are the only two pitch classes to be both doubled in the major-minor "tonic" chords and absent from the "in-between" pitch classes.

Example 1.9. Referential configurations combining "tonic" major-minor arpeggios from m. 1 and 31



That observation brings to light the striking properties of the (0347) tetrachord that comprises each arpeggio in the major-minor music; the overarching logic of the T8 transformation crystallizes: combining the pitch classes of each corresponding pair of major-minor (0347) tetrachords from the opening and closing major-minor music produces respective instances of (014589) hexachords. In other words, combining the pitch classes of the arpeggios in mm. 1-6 (T0) that logistically “match” the arpeggios in mm. 31-36 (T8) will yield the symmetric “hexatonic collection” (014589). Example 2.0 configures the corresponding (0347) tetrachords and the resultant hexachords.

Example 2.0. Hexachords generated by combining corresponding T0 and T8 major-minor arpeggios

- a)** Four possible collections of hexachord (014589)

$$\begin{array}{lll} \{A \ Bb \ C\# \ D \ F \ F\# \} \\ \{Bb \ B \ D \ D\# \ F\# \ G \} \\ \{B \ C \ Eb \ E \ G \ G\# \} \\ \{C \ C\# \ E \ F \ G\# \ A \} \end{array}$$

- b)** Resultant (014589) hexachord produced by combining E major-minor (mm. 1-4, T0) with C major-minor (mm. 31-34, T8)

$$\begin{array}{ll} \{E \ G \ G\# \ B \} \\ \{C \ Eb \ E \ G \} \end{array} \longrightarrow \begin{array}{lll} \{B \ C \ Eb \ E \ G \ G\# \} \\ (0 \ 1 \ 4 \ 5 \ 8 \ 9) \end{array}$$

- c)** Resultant (014589) hexachord produced by combining Eb major-minor (m. 5, T0) with B major-minor (m. 35, T8)

$$\begin{array}{ll} \{Eb \ Gb \ G \ Bb \} \\ \{B \ D \ D\# \ F\# \} \end{array} \longrightarrow \begin{array}{lll} \{Bb \ B \ D \ D\# \ F\# \ G \} \\ (0 \ 1 \ 4 \ 5 \ 8 \ 9) \end{array}$$

- d)** Resultant (014589) hexachord produced by combining Db major-minor (m. 6, T0) with A major-minor (m. 36, T8)

$$\begin{array}{ll} \{Db \ Fb \ F \ Ab \} \\ \{A \ C \ C\# \ E \} \end{array} \longrightarrow \begin{array}{lll} \{C \ C\# \ E \ F \ G\# \ A \} \\ (0 \ 1 \ 4 \ 5 \ 8 \ 9) \end{array}$$

- e)** Resultant (014589) hexachord produced by combining B major-minor (m. 5-6, T0) with G major-minor (m. 35-36, T8)

$$\begin{array}{ll} \{B \ D \ D\# \ F\# \} \\ \{G \ Bb \ B \ D \} \end{array} \longrightarrow \begin{array}{lll} \{Bb \ B \ D \ D\# \ F\# \ G \} \\ (0 \ 1 \ 4 \ 5 \ 8 \ 9) \end{array}$$

- f)** "Missing" (014589) hexachord produced by combining "missing" F# major-minor and Bb major-minor

$$\begin{array}{ll} \{Bb \ Db \ D \ F \} \\ \{F\# \ A \ A\# \ C\# \} \end{array} \longrightarrow \begin{array}{lll} \{A \ Bb \ C\# \ D \ F \ F\# \} \\ (0 \ 1 \ 4 \ 5 \ 8 \ 9) \end{array}$$

In that configuration, we are reminded of the special properties – those relevant to our analysis - belonging to set class (014589). Firstly, there are precisely four unique collections of this set. These are included as a reference in example 2.0a. Secondly, T2 or T10 transformation of the

collection (transposition by a whole step up or down) produces the complementary hexachord belonging to the same set class, thus forming a combinatorial aggregate. Finally, T4 or T8 transformation produces a redundancy, identical pitch classes that “map on” to the original hexachord.

Given the semitonal attributes of the majority of the musical structure, the T2-relation between the “cascading” Eb and Db major-minor arpeggios (in m. 5 and m.6 respectively) sounds odd. It sounds less odd after the conclusion of the movement, and we can attribute this to the resultant hexachords produced by the combination of these arpeggios with their T8 correspondents in mm. 35-36. Observe from example 2.0c and 2.0d how the two resultant hexachords {Bb,B,D,D#,F#,G} and {C,C#,E,F,G#,A} “interlock” to form the aggregate.

The complementary nature of the hexachords from example 2.0c and 2.0d leads us toward the expectation of finding in the music the pair of major-minor arpeggios that produces the complement of the hexachord {B,C,Eb,E,G,Gb} which was generated by the combination of the original E major-minor arpeggio from mm. 1-4 and its T8 correspondent C major-minor from mm. 31-34. But we do not find it. In fact, when the lower register “rising B major-minor arpeggio” from m. 5 combines with its G major-minor correspondent in m. 35 (see example 2.0e), we get the redundant hexachord {Bb,B,D,D#,F#,G}, also found in example 2.0c. The “missing” hexachord shown in example 2.0f {A,Bb,C#,D,F,F#} cannot be identified.

The necessary (0347) tetrachords needed to combine in order to produce the “missing” hexachord are Bb major-minor and F# major-minor as shown in example 2.0f. We might daringly assert that the Bb chord has already been realized through recurring saturation as the tonic triad of the canon. The inexplicable {F#,C#} dyad that ends the movement now makes sense within this abstracted configuration, especially in connection to the major-minor “thirds” {A,A#} unearthed in the Bb2 on the downbeat of m. 36 and the multiple occurrences of pitch class A found throughout the music in the top staff of the same measure. Regardless of these shadows, the missing hexachord induces a sense of incompleteness; the piece recedes, holding back a degree of conclusion, impelling us toward the next movement.

---

#### Notes on Terminology:

{x,y,z} means the unordered set comprising three things x, y, and z. x-y-z means the three-element series whose successive members are x, y, and z, in that order.

References to specific pitches are made according to the notation suggested by the Acoustical Society of America: The pitch class is symbolized by an upper-case letter and its specific octave placement by a number following the letter. An octave number refers to pitches from a given C through the B a major seventh above it. Cello C is C2, viola C is C3, middle C is C4, and so on. Any B# gets the same octave number as the B just below it; thus, B#3 is enharmonically C4. Likewise, any Cb gets the same octave number as the C just above it; thus Cb4 is enharmonically B3.