

STOCHASTIC DIFFERENTIAL SYSTEMS

WITH MEMORY*

Salah-Eldin A. Mohammed

Department of Mathematics
Southern Illinois University
Carbondale, IL 62901-4408
USA

Email: salah@sfde.math.siu.edu
Web page: <http://sfde.math.siu.edu>

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Abstract

This series of talks is intended as an introduction to certain aspects of stochastic differential systems, whose evolution depends on the history of the state. We shall frequently refer to such systems as *stochastic functional differential equations (sfde's)*. In the deterministic case, sfde's reduce to *retarded functional differential equations (rfde's)*. Such equations have received a great deal of attention by analysts during the last few decades. The reader may refer to fundamental works by J. Hale, Mallet-Paret, Mizel, etc.. The lectures will cover some of the following topics as much as time permits.

Part I: Existence.

Simple motivating examples: *the noisy feedback loop*, *the logistic time-lag model with Gaussian noise*, and *the classical "heat-bath" model of R. Kubo*, modeling the motion of a "large" molecule in a viscous fluid. These examples are embedded in a general class of stochastic functional differential equations (sfde's). Pathwise existence and uniqueness of solutions to these classes of sfde's under local Lipschitz and linear growth hypotheses on the coefficients. Existence of solutions under smooth constraints.

Part II: Markov Behavior.

The Markov (Feller) property holds for the trajectory random field of a sfde. The trajectory Markov semigroup is *not* strongly continuous for positive delays, and its domain of strong continuity does not contain tame (or cylinder) functions with evaluations away from 0. Quasitame functions. The weak infinitesimal generator.

Part III: Classification of SFDE's.

Non-existence of stochastic semiflows for SDDE's. Classification of sfde's into *regular* and *singular* types. Sufficient conditions for regularity of linear systems driven by white noise or semimartingales.

Part IV: Dynamics of Regular SFDE's.

Linear sfde's. Existence of a compacting stochastic semiflow. The multiplicative ergodic theory for regular linear sfde's. The saddle point property. Examples of one-dimensional linear sfde's: Estimates for the top Lyapunov exponent.

Nonlinear sfde's. Existence of semiflows. The local stable manifold theorem.

Part V: Miscellaneous Topics.

Existence of smooth densities for solutions of sfde's using the Malliavin calculus. Numerical solution. Small delays. Existence of stationary solutions. Applications to finance: The delayed Black-Scholes formula.