ON SOME PHENOMENA ATTENDING THE FLIGHT OF PROJECTILES.[1]

BY PROF. ERNST MACH.

"*I have led my ragamuffins where they were peppered.*"—Falstaff.

TO SHOOT, in the shortest time possible, as many holes as possible in one another's bodies, and not always for exactly pardonable objects and ideals, seems to have risen to the dignity of a duty with modern men, who, by a singular inconsistency, and in subservience to a diametrically contrary ideal, are bound by the equally holy obligation of making these holes as small as possible, and, when made, of stopping them up and of healing them as speedily as possible. Since, then, shooting and all that appertains thereto, is a very important, if not the most important, affair of modern life, you will doubtless not be averse to giving your attention for an hour to some experiments which have been undertaken, not for advancing the ends of war, but for promoting the ends of science, and which throw some light on the phenomena attending the flight of projectiles.

Modern science strives to construct its picture of the world not from speculations but so far as possible from facts. It verifies its constructs by recourse to observation. Every newly observed fact completes its world-picture, and every divergence of a construct from observation points to some imperfection, to some lacuna in it. What is seen is put to the test of, and supplemented by, what is thought, which is again naught but the result of things previously seen. It is always peculiarly fascinating, therefore, to subject to direct verification by observation, that is, to render palpable to the senses, something which we have only theoretically excogitated or theoretically surmised.

In 1881, on hearing in Paris the lecture of the Belgian artiller-

[1] Translated from Professor Mach's MS. by T. J. McCormack.

ist Melsens, who hazarded the conjecture that projectiles travelling at a high rate of speed carry masses of compressed air before them which are instrumental in producing in bodies struck by the projectiles certain well-known facts of the nature of explosions, the desire arose in me of experimentally testing his conjecture and of rendering the phenomenon, if it really existed, perceptible. The desire was the stronger as I could say that all the means for realising it existed, and that I had in part already used and tested them for other purposes.

And first let us get clear regarding the difficulties which have to be surmounted. Our task is that of observing a bullet or other projectile which is rushing through space at a velocity of many hundred yards a second, together with the disturbances which the bullet causes in the surrounding atmosphere. Even the opaque solid body itself, the projectile, is only exceptionally visible under such circumstances—only when it is of considerable size and when we see its line of flight in strong perspective abridgement so that the velocity is apparently diminished. We see a large projectile quite clearly when we stand behind the cannon and look steadily along its line of flight or in the less pleasant case when the projectile is speeding towards us. There is, however, a very simple and effective method of observing swiftly moving bodies with as little trouble as if they were held at rest at some point in their path. The method is that of illumination by a brilliant electric spark of extremely short duration in a dark room. But since, for the full intellectual comprehension of a picture presented to the eye, a certain, not inconsiderable interval of time is necessary, the method of instantaneous photography will naturally also be employed. The pictures, which are of extremely minute duration, are thus permanently recorded and can be examined and analysed at one's convenience and leisure.

With the difficulty just mentioned is associated still another and greater difficulty which is due to the air. The atmosphere in its usual condition is generally not visible even when at rest. But the task presented to us is to render visible masses of air which in addition are moving with a high velocity.

To be visible, a body must either emit light itself, must shine, or must affect in some way the light which falls upon it, must take up that light entirely or partly, absorb it, or must have a deflective effect upon it, that is, reflect or refract it. We cannot see the air as we can a flame, for it shines only exceptionally, as in a Geissler's tube. The atmosphere is extremely transparent and colorless;

it cannot be seen, therefore, as a dark or colored body can, or as chlorine gas can, or vapor of bromine or iodine. Air, finally, has so small an index of refraction and so small a deflective influence upon light, that the refractive effect is commonly imperceptible altogether.

A glass rod is visible in air or in water, but it is almost invisible in a mixture of benzol and bisulphuret of carbon, which has the same mean index of refraction as the glass. Powdered glass in the same mixture has a vivid coloring, because owing to the decomposition of the colors the indices are the same for only one color which traverses the mixture unimpeded, whilst the other colors undergo repeated reflexions.[1]

Water is invisible in water, alcohol in alcohol. But if alcohol be mixed with water the flocculent streaks of the alcohol in the water will be seen at once and *vice versa*. And in like manner the air, too, under favorable circumstances, may be seen. Over a roof heated by the burning sun, a tremulous wavering of objects is noticeable, as there is also over red-hot stoves, radiators, and registers. In all these cases tiny flocculent masses of hot and cold air, of slightly differing refrangibility, are mingled together.

In like manner the more highly refracting parts of non-homogeneous masses of glass, the so-called striæ or imperfections of the glass, are readily detectible among the less refracting parts which constitute the bulk of the same. Such glasses are unserviceable for optical purposes, and special attention has been devoted to the investigation of the methods for eliminating or avoiding these defects. The result has been the development of an extremely delicate method for detecting optical faults—the so-called method of Foucault and Toepler—which is suitable also for our present purpose.

Even Huygens when trying to detect the presence of striæ in polished glasses viewed them under oblique illumination, usually at a considerable distance, so as to give full scope to the aberrations, and had recourse for greater exactitude to a telescope. But the method was carried to its highest pitch of perfection in 1867 by Toepler who employed the following procedure: A small luminous source a (Fig. 1) illuminates a lens L which throws an image b of the luminous source. If the eye be so placed that the image falls on the pupil, the entire lens, if perfect, will appear equally illuminated, for the reason that all points of it send out rays to the eye. Coarse imperfections of form or of homogeneity are rendered visi-

[1] Christiansen, ***Wiedemann's Annalen***, XXIII. S. 298, XXIV., p. 439 (1884, 1885).

ble only in case the aberrations are so large that the light from many spots passes by the pupil of the eye. But if the image *b* be partly intercepted by the edge of a small slide, then those spots in the lens as thus partly darkened will appear brighter whose light by its greater aberrations still reaches the eye in spite of the intercepting slide, while those spots will appear darker which in consequence of aberration in the other direction throw their light entirely upon the slide. This artifice of the intercepting slide which had previously been employed by Foucault for the investigation of the optical imperfections of mirrors enhances enormously the delicacy

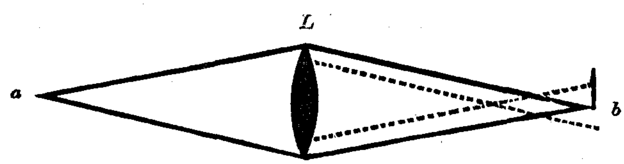

Fig. 1.

of the method, which is still further augmented by Toepler's employment of a telescope behind the slide. Toepler's method, accordingly, enjoys all the advantages of the Huygens and the Foucault procedure combined. It is so delicate that the minutest irregularities in the air surrounding the lens can be rendered distinctly visible, as I shall show by an example. I place a candle

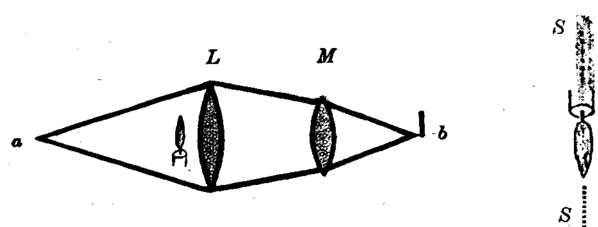

Fig. 2.

before the lens *L* (Fig. 2) and so arrange a second lens *M* that the flame of the candle is imaged upon the screen *S*. As soon as the intercepting slide is pushed into the focus, *b*, of the light issuing from *a*, you see the images of the changes of density and the images of the movements induced in the air by the flame quite distinctly upon the screen. The distinctness of the phenomenon as a whole depends upon the position of the intercepting slide *b*. The removal of *b* increases the illumination but decreases the distinctness. If the luminous source *a* be removed, we see the image of the candle flame only upon the screen *S*. If we remove the flame and allow *a* to continue shining, the screen *S* will appear uniformly illuminated.

After Toepler had sought long and in vain to render the irregularities produced in air by sound-waves visible by this principle, he was at last conducted to his goal by the favorable circumstances attending the production of electric sparks. The waves generated in the air by electric sparks and accompanying the explosive snapping of the same, are of sufficiently short period and sufficiently powerful to be rendered visible by these methods. Thus we see how by a careful regard for the merest and most shadowy indications of a phenomenon and by slight progressive and appropriate alterations of the circumstances and the methods, ultimately the most astounding results can be attained. Consider, for example, two such phenomena as the rubbing of amber and the electric lighting of modern streets. A person ignorant of the myriad minute links that join these two things together, will be absolutely nonplussed at their connexion, and will comprehend it no more than the ordinary observer who is unacquainted with embryology, anatomy, and paleontology will understand the connexion between a saurian and a bird. The high value and significance of the co-operation of inquirers through centuries, where each has but to take up the thread of work of his predecessors and spin it onwards, is rendered forcibly evident by such examples. And such knowledge destroys, too, in the clearest manner imaginable that impression of the marvellous which the spectator may receive from science, and at the same time is a most salutary admonishment to the worker in science against superciliousness. I have also to add the sobering remark that all our art would be in vain did not nature herself afford at least some slight guiding threads leading from a hidden phenomenon into the domain of the observable. And so it need not surprise us that once under particularly favorable circumstances an extremely powerful sound-wave which had been caused by the explosion of several hundred pounds of dynamite threw a directly visible shadow in the sunlight, as Boys has recently told us. If the sound-waves were absolutely without influence upon the light, this could not have occurred, and all our artifices would then, too, be in vain. And so, similarly, the phenomenon accompanying projectiles which I am about to show you was once in a very imperfect manner incidentally seen by a French artillerist, Journée, while that observer was simply following the line of flight of a projectile with a telescope, just as also the undulations produced by candle flames are in a weak degree directly visible and in the bright sunlight are imaged in shadowy waves upon a uniform white background.

Instantaneous illumination by the electric spark, the method of rendering visible small optical differences or striæ, which may hence be called the *striate*, or *differential*, method,[1] invented by Foucault and Toepler, and finally the *recording* of the image by a *photographic* plate, these therefore are the chief means which are to lead us to our goal.

I instituted my first experiments in the summer of 1884 with a target-pistol, shooting the bullet through a striate field as described above, and taking care that the projectile whilst in the field should disengage an illuminating electric spark from a Leyden jar or Franklin's pane, which spark produced a photographic impression of the projectile upon a plate, especially arranged for the purpose. I obtained the image of the projectile at once and without difficulty. I also readily obtained, with the still rather defective dry plate which I was using, exceedingly delicate images of the sound-waves (spark-waves). But no atmospheric condensation produced by the projectile was visible. I now determined the velocity of my projectile and found it to be only 240 metres per second, or considerably less than the velocity of sound (which is 340 metres per second). I saw immediately that under such circumstances no noticeable compression of the air could be produced, for any atmospheric compression must of necessity travel forward at the same speed with sound (340 metres per second) and consequently would be always ahead of and speeding away from the projectile.

I was so thoroughly convinced, however, of the existence of the supposed phenomenon at a velocity exceeding 340 metres per second, that I requested Professor Salcher, of Fiume, an Austrian port on the Gulf of Quarnero, to undertake the experiment with projectiles travelling at a high rate of speed. In the summer of 1886 Salcher in conjunction with Professor Riegler conducted in a spacious and suitable apartment placed at their disposal by the Directors of the Royal Imperial Naval Academy, experiments of the

[1] The German phrase is *Schlierenmethode*, by which term the method is known even by American physicists. It is also called in English the "shadow-method." But a term is necessary which will cover all the derivatives, and so we have employed alternatively the words *striate* and *differential*. The etymology of *schlieren*, it would seem, is uncertain. Its present use is derived from its technological signification in glass-manufacturing, where by *die Schlieren* are meant the wavy streaks and imperfections in glass. Hence its application to the method for detecting small optical *differences* and faults generally. Professor Crew of Evanston suggests to the translator that *schlieren* may be related to our *slur* (L. G., *slüren*, to trail, to draggle), a conjecture which is doubtless correct and agrees both with the meaning of *schlieren* as given in the large German dictionaries and with the intransitive use of our own verb *slur*, the faults in question being conceived as "trailings," "streakings," etc.—*T. J. McC.*

kind indicated and conforming in method exactly to those which I had instituted, with the precise results expected. The phenomenon, in fact, accorded perfectly with the *a priori* sketch of it which I had drafted previously to the experiment. As the experimenting was continued, new and unforeseen features made their appearance.

It would be unfair, of course, to expect from the very first experiments faultless and highly distinct photographs. It was sufficient that success was secured and that I had convinced myself that further labor and expenditure would not be vain. And on this score I am greatly indebted to the two gentlemen above mentioned.

The Austrian Naval Department subsequently placed a cannon at Salcher's disposal in Pola, an Adriatic seaport, and I myself, together with my son, then a student of medicine, having received and accepted a courteous invitation from Krupp, repaired to Meppen, a town in Hanover, where we conducted with only the necessary apparatus several experiments on the open artillery range. All these experiments furnished tolerably good and complete pictures. Some little progress, too, was made. The outcome of our experience on both artillery ranges, however, was the settled conviction that really good results could be obtained only by the most careful conduct of the experiments in a laboratory especially adapted to the purpose. The expensiveness of the experiments on a large scale was not the determining consideration here, for the size of the projectile is indifferent. Given the same velocity and the results are quite similar, whether the projectiles are large or small. On the other hand, in a laboratory the experimenter has perfect control over the initial velocity, which, provided the proper equipment is at hand, can be altered at will simply by altering the charge and the weight of the projectile. The requisite experiments were accordingly conducted by me in my laboratory at Prague, partly in conjunction with my son and partly afterwards by him alone. The latter are the most perfect and I shall accordingly speak in detail here of these only.

Picture to yourself an apparatus for detecting optical striæ set up in a dark room. In order not to make the description too complicated, I shall give the essential features only of the apparatus, leaving out of account altogether the minuter details which are rather of consequence for the technical performance of the experiment than for its understanding. We suppose the projectile speeding on its path, accordingly, through the field of our differential optical apparatus. On reaching the centre of the field (Fig. 3) the projectile disengages an illuminating electric spark *a*, and the image of the projectile, so produced, is photographically impressed upon the plate of the camera behind the intercepting slide *b*. In the last and best experiments the lens *L* was replaced by a spherical silvered-glass mirror made by K. Fritsch (formerly Prokesch) of Vienna, whereby the apparatus was naturally more complicated than it appears in our diagram. The projectile having been carefully aimed passes in crossing the differential field between two vertical isolated wires which are connected with the two coatings of a Leyden jar, and completely filling the space between the wires discharges the jar. In the axis of the differential apparatus the circuit has a second gap *a* which furnishes the illuminating spark, the image of which falls on the intercepting slide *b*. The wires in the differential field having occasioned manifold disturbances were sub-

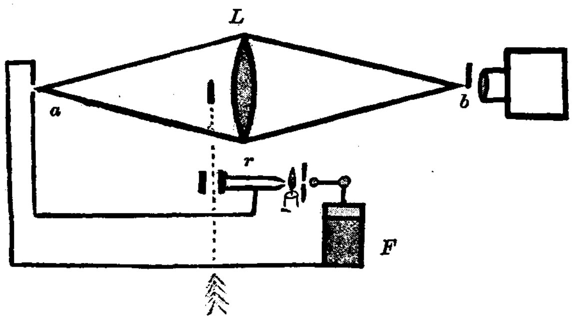

Fig. 3.

sequently done away with. In the new arrangement the projectile passes through a ring (see dotted line, Fig. 3), to the air in which it imparts a sharp impulse which travels forward in the tube *r* as a sound-wave having the approximate velocity of 340 metres per second, topples over through the aperture of an electric screen the flame of a candle situated at the other opening of the tube, and so discharges the jar. The length of the tube *r* is so adjusted that the discharge occurs the moment the projectile enters the centre of the now fully clear and free field of vision. We will also leave out of account the fact that to secure fully the success of the experiment, a large jar is first discharged by the flame, and that by the agency of this first discharge the discharge of a second small jar having a spark of very short period which furnishes the spark really illuminating the projectile is effected. Sparks from large jars have an appreciable duration, and owing to the great velocity of the projectiles furnish blurred photographs only. By carefully husbanding the light of the differential apparatus, and owing to the fact that much more light reaches the photographic plate in this way than would otherwise reach it, we can obtain beautiful, strong, and sharp photographs with incredibly small sparks. The contours of the pictures appear as very delicate and very sharp, closely adjacent double lines. From their distance from one another, and from the velocity of the projectile, the duration of the illumination, or of the spark, is found to be $\frac{1}{800000}$ of a second. It is evident, therefore, that experiments with mechanical snap slides can furnish no results worthy of the name.

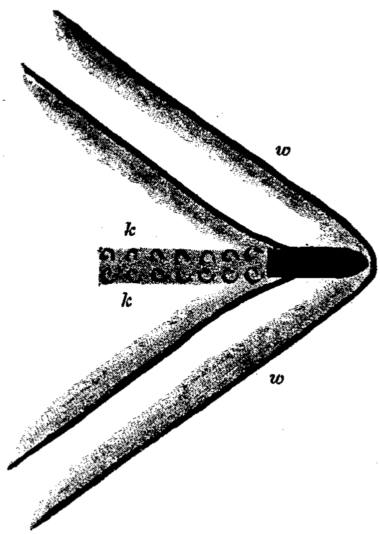

Fig. 4.

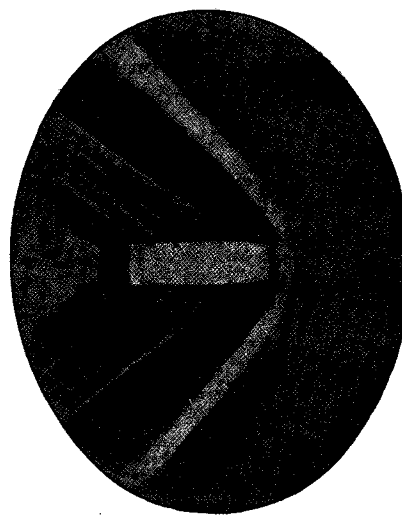

Fig. 5. Photograph of a blunted projectile.

Let us consider now first the picture of a projectile in the rough, as represented in Figure 4, and then let us examine it in its photographic form as seen in Figure 5. The latter picture is of a shot from an Austrian Mannlicher rifle. If I were not to tell you what the picture represented you would very likely imagine it to be a bird's eye view of a boat *b* moving swiftly through the water. In front you see the bow-wave and behind the body a phenomenon *k* which closely resembles the eddies formed in the wake of a ship. And as a matter of fact the dark hyperboloid arc which streams from the tip of the projectile really is a compressed wave of air exactly analogous to the bow-wave produced by a ship moving through the water, with the exception that the wave of air is not a surface-wave. The air-wave is produced in atmospheric space and encompasses the projectile in the form of a shell on all sides. The wave is visible for the same reason that the heated shell of air surrounding the candle flame of our former experiments is visible. And the cylinder of friction-heated air which the projectile throws off in the form of vortex rings really does answer to the water in the wake of a vessel.

Now just as a slowly moving boat produces no bow-wave, but the bow-wave is seen only when the boat moves with a speed which is greater than the velocity of propagation of surface-waves in water, so, in like manner, no wave of compression is visible in front of a projectile so long as the speed of the projectile is less than the velocity of sound. But if the speed of the projectile reaches and exceeds the velocity of sound, then the head-wave, as we shall call it, augments noticeably in power, and is more and more extended, that is, the angle made by the contours of the

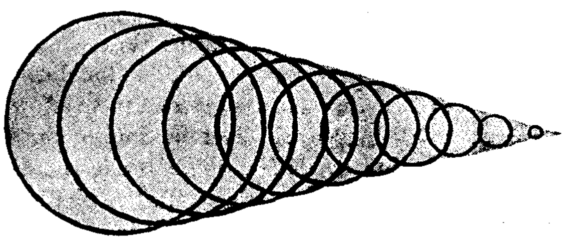

Fig. 6.

wave with the direction of flight is more and more diminished, just as when the speed of a boat is increased a similar phenomenon is noticed in connexion with the bow-wave. In fact, we can from an instantaneous photograph so taken approximately estimate the speed with which the projectile is travelling.

The explanation of the bow-wave of a ship and that of the head-wave of a body travelling in atmospheric space both repose upon the same principle, long ago employed by Huygens. Conceive a number of pebbles to be cast into a pond of water at regular intervals in such wise that all the spots struck are situate in the same straight line, and that every spot subsequently struck lies a short space farther to the right. The spots first struck will furnish then the wave-circles which are widest, and all of them together will, at the points where they are thickest, form a sort of cornucopia closely resembling the bow-wave. The resemblance is greater the smaller the pebbles are, and the more quickly they succeed each other. If a rod be dipped into the water and quickly carried along its surface, the falling of the pebbles will then take place, so to speak, uninterruptedly, and we shall have a real bow-wave. If we put the compressed air-wave in the place of the surface-waves of the water, we shall have the head-wave of the projectile.

You may be disposed to say now, it is all very pretty and interesting to observe a projectile in its flight, but of what practical use is it?

It is true, I reply, one cannot *wage war* with photographed projectiles. And I have likewise often had to say to medical students attending my lectures on physics, when they inquired for the practical value of some physical observation, "You cannot, gentlemen, cure diseases with it." I had also once to give my opinion regarding how much physics should be taught at a school for millers, supposing the instruction there to be confined *exactly* to what was necessary for a miller. I was obliged to reply: "A miller always *needs* exactly as much physics as he *knows*." Knowledge which one does not possess one cannot use.

Let us forego entirely the consideration that as a general thing every scientific advance, every new problem elucidated, every extension or enrichment of our knowledge of facts, affords a better foundation for practical pursuits. Let us rather put the special question, Is it not possible to derive some really practical knowledge from our theoretical acquaintance with the phenomena which take place in the space surrounding a projectile?

No physicist who has ever studied waves of sound or photographed them will have the least doubt regarding the sound-wave character of the atmospheric condensation encompassing the head of a flying projectile. We have therefore, without ado, called this condensation the head-wave.

Knowing this, it follows that the view of Melsens according to which the projectile carries along with it masses of air which it forces into the bodies struck, is untenable. A forward-moving sound-wave is not a forward-moving mass of matter but a forward-moving form of motion, just as a water-wave or the waves of a field of wheat are only forward-moving forms of motion and not movements of masses of water or masses of wheat.

By interference-experiments, on which I cannot touch here but which will be found roughly represented in Figure 7, it was found that the bell-shaped head-wave in question is an extremely thin shell and that the condensations of the same are quite moderate, scarcely exceeding two-tenths of an atmosphere. There can be no question, therefore, of explosive effects in the body struck by the projectile through so slight a degree of atmospheric compression. The phenomena attending wounds from rifle balls, for example, are not to be explained as Melsens and Busch explain them, but are due, as Kocher and Reger maintain, to the effects of the impact of the projectile itself.

A simple experiment will show how insignificant is the part played by the friction of the air, or the supposed conveyance of the air along with the moving projectile. If the photograph of the projectile be taken while passing through a flame, i. e., a visible gas, the flame will be seen to be, not torn and deformed, but smoothly and cleanly perforated, like any solid body. Within and around the flame the contours of the head-wave will be seen. The flickering, the extinction of the flame, etc., take place only after the projectile has travelled on a considerable distance in its path, and is then affected by the powder gases which hurry after the bullet or by the air preceding the powder-gases.

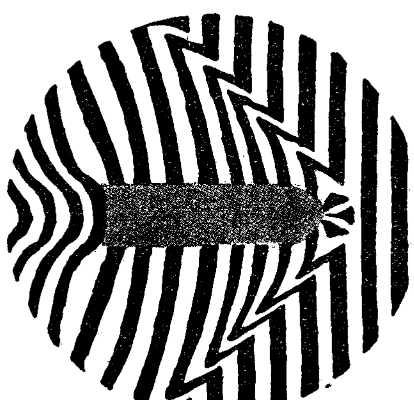

Fig. 7.

The physicist who examines the head-wave and recognises its sound-wave character also sees that the wave in question is of the same kind with the short sharp waves produced by electric sparks, that it is a *noise*-wave. Hence, whenever any portion of the head-wave strikes the ear it will be heard as a report. Appearances point to the conclusion that the projectile carries this report along with it. In addition to this report, which advances with the velocity of the projectile and so usually travels at a speed greater than the velocity of sound, there is also to be heard the report of the exploding powder which travels forward with the ordinary velocity of sound. Hence two explosions will be heard, each distinct in time. The circumstance that this fact was long misconstrued by practical observers but when actually noticed frequently received grotesque explanations and that ultimately my view was accepted as the correct one, appears to me in itself a sufficient justification that researches such as we are here speaking of are not utterly superfluous even in practical directions. That the flashes and sounds of discharging artillery are used for estimating the distances of batteries is well known, and it stands to reason that any unclear theoretical conception of the facts here involved will seriously affect the correctness of practical calculations.

It may appear astonishing to a person hearing it for the first time, that a single shot has a double report due to two different velocities of propagation. But the reflexion that projectiles whose velocity is less than the velocity of sound produce no head-waves (because every impulse imparted to the air travels forward, that is, ahead, with exactly the velocity of sound), throws full light when logically developed upon the peculiar circumstance above mentioned. If the projectile moves faster than sound, the air ahead of it cannot recede from it quickly enough. The air is condensed and warmed, and thereupon, as all know, the velocity of sound is augmented until the head-wave travels forward as rapidly as the projectile itself, so that there is no need whatever of any additional augmentation of the velocity of propagation. If such a wave were left entirely to itself, it would increase in length and soon pass into an ordinary sound-wave, travelling with less velocity. But the projectile is always behind it and so maintains it at its proper density and velocity. Even if the projectile penetrates a piece of cardboard or a board of wood, which catches and obstructs the head-wave, there will, as Figure 8 shows, immediately appear at the emerging apex a newly formed, not to say newly born, head-wave. We may observe on the cardboard the reflexion and diffraction of the head-wave, and by means of a flame its refraction, so that no doubt as to its nature can remain.

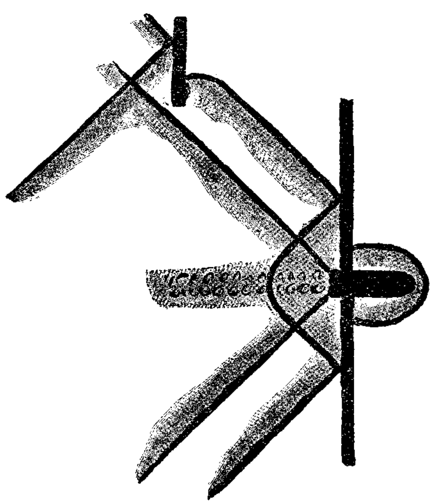

Fig. 8.

Permit me, now, to illustrate the most essential of the points that I have just adduced, by means of a few rough drawings taken from older and less perfect photographs.

In the sketch of Figure 9 you see the projectile, which has just left the barrel of the rifle, touch a wire and disengage the illuminating spark. At the apex of the projectile you already see the beginnings of a powerful head-wave, and in front of the wave a transparent fungiform cluster. This latter is the air which has been forced out of the barrel by the projectile. Circular sound-waves, noise-waves, which are soon overtaken by the projectile, also issue from the barrel. But behind the projectile opaque puffs of powder-gas rush forth. It is scarcely necessary to add that many other questions in ballistics may be studied by this method, as, for example, the movement of the gun-carriage.

A distinguished French artillerist, M. Gossot, has applied the views of the head-wave here given in quite a different manner. The practice in measuring the velocity of projectiles is to cause the projectile to pass through wire screens placed at different points in its path, and by the tearing of these screens to give rise to electromagnetic time-signals on falling slabs or rotating drums. Gossot caused these signals to be made directly by the impact of the head-wave, did away thus with the wire screens, and carried the method so far as to be able to measure the velocities of projectiles travelling in high altitudes, where the use of wire screens was quite out of the question.

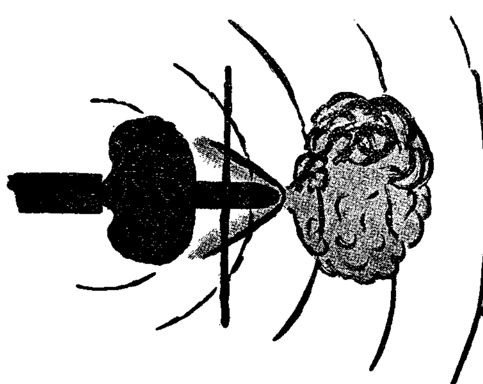

Fig. 9.

The laws of the resistance of fluids and of air to bodies travelling in them form an extremely complicated problem, which can be reasoned out very simply and prettily as a matter of pure philosophy but in practice offers not a few difficulties. The same body having the velocity 2, 3, 4 . . . . displaces in the same interval 2, 3, 4 . . . times the same mass of air, or the same mass of fluid, and imparts to it *in addition* 2, 3, 4, ... times the same velocity. But for this, plainly, 4, 9, 16 . . . . times the original force is required. Hence, the resistance, it is said, increases with the square of the velocity. This is all very pretty and simple and obvious. But practice and theory are at daggers' points here. Practice tells us that when we increase the velocity the law of the resistance changes. For every portion of the velocity the law is different.

The studies of the talented English naval architect, Froude, have thrown light upon this question. Froude has shown that the resistance is conditioned by a combination of the most multifarious phenomena. A ship in motion is subjected to the friction of the water. It causes eddies and it generates in addition waves which radiate outward from it. Every one of these phenomena are dependent upon the velocity in some different manner, and it is consequently not astonishing that the law of the resistance should be a complicated one.

The preceding observations suggest quite analogous reflexions for projectiles. Here also we have friction, the formation of eddies, and the generation of waves. Here, also, therefore, we should not be surprised at finding the law of the resistance of the air a complicated one, nor puzzled at learning that in actuality the law of resistance changes as soon as the speed of the projectile exceeds the velocity of sound, for this is the precise point at which one important element of the resistance, namely, the formation of waves, first comes into play.

No one doubts that a pointed bullet pierces the air with less resistance than a blunt bullet. The photographs themselves show that the head-wave is weaker for a pointed projectile. It is not impossible, similarly, that forms of bullets will be invented which generate fewer eddies, etc., and that we shall study these phenomena also by photography. I am of opinion from the few experiments which I have made in this direction that not much more can be done by changing the form of the projectile when the velocity is very great, but I have not gone into the question thoroughly. Researches of the kind we are considering can certainly not be detrimental to practical artillery, no less than experiments by artillerists on a large scale will be of undoubted benefit to physics.

No one who has had the opportunity of studying modern guns and projectiles in their marvellous perfection, their power and precision, can help confessing that a high technical and scientific achievement has found its incarnation in these objects. We may surrender ourselves so completely to this impression as to forget for a moment the terrible purposes they serve.

Permit me, therefore, before we separate, to say a few words on this glaring contrast. The greatest man of war and of silence which the present age has produced once asserted that perpetual peace is a dream, and not a beautiful dream at that. We may accord to this profound student of mankind a judgment in these matters and can also appreciate the soldier's horror of stagnation from all too lengthy peace. But it requires a strong belief in the insuperableness of mediæval barbarism to hope for and to expect no great improvement in international relations. Think of our forefathers and of the times when club law ruled supreme, when within the same country and the same state brutal assaults and equally brutal self-defence were universal and self-evident. This state of affairs grew so oppressive that finally a thousand and one circumstances compelled people to put an end to it, and the cannon had most to say in accomplishing the work. Yet the rule of club law was not abolished so quickly after all. It had simply passed to other clubs. We must not abandon ourselves to dreams of the Rousseau type. Questions of law will forever remain in a certain sense questions of might. Even in the United States where every one is as a matter of principle entitled to the same privileges, the ballot according to Stallo's pertinent remark is but a milder substitute for the club. I need not tell you that many of our own fellow-citizens even are still enamored of the old original methods. Very, very gradually, however, as civilisation progresses, the intercourse of men takes on gentler forms and no one who really knows the good old times will ever honestly wish them back again, however beautifully they may be painted and rhymed about.

In the intercourse of the nations, however, the old club law still reigns supreme. But since its rule is taxing the intellectual, the moral, and the material resources of the nations to the utmost and constitutes scarcely less a burden in peace than in war, scarcely less a yoke for the victor than for the vanquished, it must necessarily grow more and more unendurable. Reason, fortunately, is no longer the exclusive possession of those who modestly call themselves the upper ten thousand. Here, as everywhere, the evil itself will awaken the intellectual and ethical forces which are destined to mitigate it. Let the hate of races and of nationalities run riot as it may, the intercourse of nations will still increase and grow more intimate. By the side of the problems which separate nations, the great and common ideals which claim the exclusive powers of the men of the future appear one after another in greater distinctness and in greater might.