THE STATICS OF LEONARDO DA VINCI

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PART II

MISCELLANEOUS PRACTICAL APPLICATIONS

E MAY now turn to a number of practical applications of the lever principle. We find references to the Chinese windlass in the *Codex Atlanticus*. Against one sketch (Fig. 1), Leonardo writes, "If one pulls at the arm of the wind-

lass *as* with the cord *bc*, it is just the same as turning a balance ab."¹ On the same page is to be found a similar mechanism for winding heavy loads, represented as a lever with unequal arms.

The lever arm on which the force acts has a length equal to nineteen times the radius of the winding cyl-

FIGURE TWO



inder, thus providing a leverage of one in twenty. In Manuscript

"A" ² is to be found an ingenious device for weighing bodies in which the principle of the lever is directly applied. It consists of an equilateral triangular frame (Fig. 2) of wooden mouldings suspended from one corner. The base is graduated in such a way as to indicate (by means of a plumb line suspended from the top angle) the weight of an object n suspended at one extremity in terms of weight m at the other extremity. A variation of this is also to be found in a sketch³ unaccompanied by any note, in which the triangle is replaced by a semi-

circle, the right half of which is presumably weighted to counter act ¹ Codex Atlanticus, fol. 25, r.b.

² Ms. A., fol. 52 r.

³ Ms. 2038, Bibliothèque Nationale, Paris.

the effect of a scale-pan suspended from the left extremity of the diameter. The left quadrant is again graduated to read the load in the scale-pan directly.

Leonardo also applied the principle of the balance to the design of hygrometers. Hygrometry was a subject which attracted some attention in the fifteenth century, and both Nicholas of Cusa in Germany, and Leon Battista Alberti in Italy (the latter of whom was known to Leonardo) had devised various forms of hygrometers. Leonardo was in this respect little more than a follower, but his designs were both inegnious and interesting. In one form⁴ we see a balance whose arm is triangular, in the left scale-pan of which is a



bullet of cotton, and in the right a bullet of wax. These are arranged to balance in dry weather. The absorption of moisture in humid atmospheres by the cotton bullet is then clearly indicated by the depression of the left scale-pan. A second form of hygrometer is found later in the same manuscript (Fig. 3⁵). A balanced rod carries the absorbent bullet on the left extremity, and the dry bullet on the right, and the absorption of moisture now draws the left arm down over a graduated quadrant of a circle. The graduations indicate the relative humidity of the atmosphere.

THE SIMPLE PULLEY

In view of Leonardo's activities in the role of engineer, it is not surprising that his theoretical investigations in those branches of mechanics which are capable of easy practical application were

⁴ Codex Atlanticus, fol. 18 v.b.

⁵ Codex Atlanticus, fol. 249 v.a.

peculiarly successful. This especially applies to his studies on pulley systems. Here, however, we must make due allowance for the fact that he had at hand a rich store of achievement handed down from the mechanics and engineering of antiquity. Aristotle, Archimedes, Vitruvius, Heron, Ctesibus, Pappus and others had freely contributed to the subject, and Leonardo was acquainted with their work. Accordingly we find in the summation of his notes a very thorough discussion of the whole subject, and a clear grasp of the practical



possibilities which arise from it. The theoretical side of Leonardo's work on pulley systems has b e e n f ully summarized by Schuster.⁶ It is evident that the treatment is based on the important conceptions of (1) the lever laws, and (2) the principle of virtual velocities. The initial

position is simply stated by da Vinci in the Codex Atlanticus in a note (accompanied by two sketches (Fig. 4) which reads, "The line of movement is *ab*, the line of the force is *ad*. The line of the movement in the balance is the distance of the middle points of the pulleys

from their circumferences, especially from the direction of the force which acts on the circles as a tangent, i. e., *ab.* At the point of contact of the rope from which the weights acts with the circle right angles continually arise between this rope and the radius of contact."⁷ The simple pulley is therefore



a simple balance, as in (A), or a virtual balance with potential arms as in (B).

We have next to ask ourselves whether Leonardo appreciated the simple function of a pulley as a means of changing the direction of a force. Here unfortunately we find him distinctly at fault. Referring to Figure 5, he asks. "Which of the ropes *of*, *on* and *om* have more stretch, and further, how much, and why?"⁸ Leonardo must clearly have been confusing this with the problem of the inclined plane; because, he must have argued, with the changing

⁷ Codex Atlanticus, fol. 149 r.a.

⁶ Schuster, Zur Mechanik, etc., pp. 114 to 132.

⁸ Codex Atlanticus, fol. 346 v.a.

inclination of the plane, the pull on the rope supporting the body on the plane steadily changes, so will the pull on the rope passing over a single pulley change with its angle of inclination to the vertical. The error in reasoning is still more strikingly shown in a sketch³

of a system of two four-pound weights suspended from the ends of a rope that passes over two faced pulleys, i. e., there is a horizontal portion of rope joining the two pulleys, and two vertical portions. The tension in the horizontal portion of the rope is given as eight pounds, whilst those in the vertical portions are given as four pounds. We are unable therefore to say that da Vinci properly understood the simple function of change of direction of a force which all simple pulleys possess.

8 FIGURE SIN

Let us next consider Leonardo's interpretation of the purposes of the *moveable* pulley.



THE MOVABLE PULLEY

Here, happily, we are on more fortunate ground. Attached to a sketch in the *Codex Atlanticus* (Fig. 6) is the note "Pay heed that g is the half of the weight h, and that the path of g is twice as great as that of h."¹⁰ Nothing could be more explicit, both from the point of view of mechanical advantage and of velocity ratio.

As a direct application of this property of the moveable pulley, we may quote again from the *Codex Atlanticus*, from which Figure 7 is taken. Here we

have a system of two fixed and two moveable pulleys. Leonardo speaks of the end of the rope carrying the power as the "arganica," and of the end of the rope carrying the cord as the "retinente." Referring to the sketch, he writes, "If one divides the burden lifted through the pulley tackle by the number of pulleys, one obtains a weight that, fastened at the 'arganica' makes equilibrium."¹¹ Here then is the statement regarding the mechanical advantage obtained

^o Codex Atlanticus, fol. 104 v.b. ¹¹ Codex Atlanticus, fol. 321 r.a. ¹⁰ Codex Atlanticus, fol. 321 r.a.

by this system. Continuing, however, he says, "The path of the 'arganica' which lifts the burden is so many times as long as the path of the burden which becomes lifted through the pulley tackle as the number of pulleys," which completes the statement regarding the velocity ratio.

PULLEY SYSTEMS

It is interesting to note, in turning to the various pulley systems with which Leonardo was familiar, that practically all forms of pulley combinations such as are met in the modern text-book are to to be found in one place or another among the various manuscripts. Figure 8¹² shows a typical example, with four fixed and four movable pulleys, and Leonardo reduces the discussion to the study of the tension in the rope. "The weight divides itself into eight pieces of rope. The ninth opposed to this simply holds the equipcise of the eight." A curious case is presented by a note in Manuscript A



accompanying a sketch of a system of two fixed and two movable pulleys (Fig. 9). The load is twenty pounds, and the power should clearly be five pounds. Yet Leonardo makes it six pounds. His note reads, "If the burden borne be twenty pounds, then I say let ten pounds act on the pulley l and ten pounds on the pulley k, these being the points of suspension of the twenty pounds load. That is to say, that o takes off from l five pounds, and p also takes five pounds from l, and five pounds from k. Finally k transmits five pounds to q. If one wishes to overcome the five pounds, one must apply at x an opposing weight of six pounds. So long as one applies six pounds at the extreme point against the five pounds at x, and so long as each of the four pieces of string which bear the twenty pounds themselves only experience the five pounds pull, then because the active extra weight on the rope q.x finds nothing to equal it in the opposing effective pieces of rope, the tension will be over-

¹² Codex Atlanticus, fol. 104 v.b.

come, and movement will result.¹¹³ Clearly the point here is that whilst the theoretical value for the power is recognized by Leonardo as five pounds, nevertheless frictional and other resistances require an addition to this value in order that motion may ensue. At the same time it is just a little difficult to appreciate exactly what was in his mind regarding this extra pound pull. On the previous page



in the same note-book,¹⁴ for example, Leonardo deals with the same system of pulleys, except that he now has three movable pulleys (Fig. 10). The discussion proceeds as follows: "On movement and force. Among one and the same causes of moved bodies which receive greater velocity, more loss is caused to the moving body. For example, in the case of the blocks ag and bh, let the lower one be



raised to the line mn. I maintain that necessarily in this case the piece of rope gh so falls that the point g comes into the position of h." He then goes on to point out that for the balk to move to mn, the pieces of rope mbo, pdq, etc., must all pass round and below q, so that the power applied at h moves through a big distance. He refers to the movement involved by any point as the "fatigue," and he points out that the fatigue at h is greater than that at f. that at f greater than that at d, and so on. We are unable to say that the term "fatigue" was employed here as a technical term to express linear displacement; indeed Schuster expresses his view that

Leonardo was here introducing physiological considerations which ought to have been foreign to the abstract theory of the pulley system under consideration.

13 Ms. A., fol. 62 r.

¹⁴ Ms. A., fol. 61 v.

The use of sheaves of pulleys is frequently met with in da Vinci's manuscripts,¹⁵ whilst many other pulley systems are illustrated freely in the *Codex Atlanticus*. One such (Fig. 11) is of especial interest on account of the clear enunciation of the relationship between mechanical advantage and velocity ratio given in the text beside the drawing. It will be noticed that the figure, in so far as the scheme of tensions in the ropes and the relationship between power and load are concerned, is accurate and complete. The note reads, "Just as one can here find a rule of diminishing force for the mover, so can one also lay down a rule for the increase in the velocity of the movement. The path of *m* stands in proportion to that of *n* as the weight *n* is to the weight m." ¹⁶

As further illustrative of the extent and variety of pulley systems dealt with by Leonardo, we may refer to yet another page in the *Codex Atlanticus*.¹⁷ Here are shown a number of diagrams, selfexplanatory (which may possibly account for the fact that they are unaccompanied by notes in the text) giving pulley systems of various complications, in each of which the mechanical advantage is accurately worked out in terms of the tensions in the strings.

THE MOTION OF SYSTEMS OF CONNECTED WEIGHTS

One of the most interesting aspects of Leonardo's work on pulleys is that in which he discusses the nature of the motion that must ensue when two different weights are connected by a string passing over a pulley. A number of sketches deal with this class of problem. Naturally, a correct solution, being dependent upon a knowledge of the value of the acceleration due to gravity, such as was unknown in his days, was beyond him. Nevertheless that he even recognized the existence of the problem and attempted a solution is all to his credit. Certain broad and admittedly ill-defined facts regarding such systems did emerge from his experiments. Thus he realized that whilst the pull on the pulley was equal to the sum of the weights in the case when no motion ensues, in the case of unequal weights setting up a resulting motion the pull on the pulley was *less* than the sum of the two weights.¹⁸

¹⁵ E. g., Codex Atlanticus, fol. 141 v.a.

- ¹⁶ Codex Atlanticus, fol. 120 v.c.
- ¹⁷ Codex Atlanticus, fol. 153 r.a.
- 18 Codex Atlanticus, fol. 249 r.b.; 323 r.a.; Ms. G., fol. 95 v., etc.

THE TRANSMISSION OF POWER

Turning next to the transmission of power, we come again to a field which, by the fifteenth century, was already full of historical associations.¹⁰ Indeed, from the time when Aristotle discussed the gearing of three wheels in rough contact onwards.²⁰ the theory of



FIGURE TWELVE

power transmission, mainly through toothed gearing, progressed rapidly, to the accompaniment of an equally rapid development in practice. Hence, not only do we find innummerable examples of machines, structures, cranes, and all kinds of practical mechanism freely drawn and described throughout Leonardo's note-books, some of them indeed extremely elaborate and complicated in construction, but we also meet with a number of theoretical discussions. The ordinary cases of toothed wheels in gear with each other are naturally very frequent, but in addition, Leonardo had

a partiality for the method of transmission illustrated in Figure 12,²¹ in which a wheel Λ is made to rotate by causing a number of pieces projecting from it at equal intervals to engage in the spaces between the cylindrical uprights of a rotating spindle B. By this means power is transmitted through a right angle.

The general scheme of Leonardo's theoretical treatment can perhaps be illustrated by a typical example (Fig. 13) taken from the *Codc.r. Atlanticus.*²² The scheme is as follows: There are three large wheels, A, C, and E, O² of which C and E are toothed. A has



no teeth, but carries the power load at the extremity of a cord. The axles B, D and E of each of A, C and E are also toothed, and the radii of the axle wheels B and D are one-tenth of those of the larger

¹⁰ A very comprehensive summary of this history is to be seen in F. M. Feldhaus, *Die Geseechtleshe Entwicklunp des Zahnrades in Theorie and Praxis*, Berlin, 1911.

²⁰ Aristotle, Mechanica, Chap. I.

²¹ Ms. H., fol. 86 v.; Ms. G., fol. 26 v.

22 Codev Atlanticus, fol. 153 v.d.

wheels A and C, whilst that of F is *one-fifth* of E. F carries an endless chain along which the lifted load is distributed. Leonardo's treatment is very simple, and is based entirely on the principle of the lever. He reduces the system to what he calls an "interrupted balance," which is really a sequence of equivalent simple levers working from left to right as shown in Figure 14. In this we see that with a power of two pounds the system AB gives an equivalent load of twenty pounds, the

system CD an equivalent load of 200 pounds, and EF a final load of 1,000 pounds. So far the reasoning is per-

FIGURE FOURTEEN

fect. Where, however, Leonardo now goes astray is in introducing the further (and incorrect) notion of an equivalent *uninterrupted* balance (Fig. 15), the left arm of which is equal to the sum of the



"interrupted" left arms of Figure 14 (i. e., 10+10+10=30).
and the right arm of which is equal to the sum of the right arms of the "interrupted" scheme (i. e., 1+1+2=4). This

for a power of two pounds as before, gives a load $\frac{2 \times 30}{4} = 15$ pounds. Reverting to the original value of 1,000 pounds. Leonardo now points out that fifteen pounds is contained in the 1,000 pounds $66\frac{2}{3}$.

times, and thus concludes that the movement of the heavy weight is $66\frac{2}{3}$ times as slow as that of the power. In view of the fact that Leonardo had shown himself to be quite clear as to the relationship between velocity ratio and mechanical advantage, it is difficult to understand why he should have brought in this incorrect and unnecess



sary notion of the "uninterrupted balance." We offer one more illustrations of Leonardo's applications of the principle of the lever to the transmission of power in Figure 10.²³

Here we have a combination of two wheels and axles, each of which have radii ratios of three to one. The power being ten pounds, Leonardo readily deduces the load to be ninety pounds.

²³ Ms. J., fol. 132 r.

REACTIONS ON SUPPORTS OF LOADED BODIES

We turn next to another branch of inquiry in Statics-that which belongs today to the strength of structures. Leonardo was again naturally interested as an engineer and architect, but it was not alone in such roles as these that he approached the subject. His study of bird flight also brought him into contact with this class of problem, and in his manuscript On the Flight of Birds, he asks, "In which part of the under surface of the width of the bird does the wing press the air more than in any other part of the length of the wings?"²⁴ So he begins with the consideration of a rigid structure supported at equal intervals. "All bodies which do not bend," he says, "will exert equal pressures on all the supports that are equally distant from the center of gravity, the center being the middle of the substance of such a body. One proves how the above mentioned weight exerts equal pressure on its supports; let us assume that it is four pounds, and that it is sustained by the supports *ab*. I say that the body being unhindered in its fall except by the two supports ab, that these supports will sustain equal parts of this weight, that is to say, two and two."²⁵ He then points out that this would not apply to the structure if it were not uniform. A similar example is to be seen in the Codex Atlanticus.²⁶ A sketch shows a uniform beam of six pounds weight, and the reactions at the ends are given as three pounds each. Adjoining this,²⁷ he next considers what happens when one support is moved inward two divisions (the beam has six equal divisions of length), and he correctly shows the reactions to be four and one-half and one and one-half pounds, respectively; and the problem is later expressed in more general terms²⁸ by the assertion that if one support is kept unaltered at one end, and the other is moved towards it, the pull on the moved support steadily increases, whilst that on the fixed one equally diminishes. He gives a number of calculations in illustration of this, many of which are incorrect. Some, however, are quite correct.²⁹ so that the errors are

- ²⁵ Sul l'olo degli Uccelli, fol. 4 v.
- ²⁶ Codex Atlanticus, fol. 101 r.a.
- 27 See also Codex Atlanticus, fol. 185 r.a.
- 28 Codex Atlanticus, fol. 141 v.a.
- ²⁹ E. g., Codex Atlanticus, fol. 152 v.b.

²⁴ Sul Volo degli Uccelli, fol. 4 v.

not due to faulty principles, but, as happens so frequently with da Vinci, to faulty mathematics.

STRENGTH OF LOADED STRUTS

Standing in a class by themselves are Leonardo's studies of the load which vertical struts, pillars, etc., are capable of sustaining. Most of his notes on this subject are to be found in Manuscript A of the collection at the Institute of France, though others also occur in the Codex Atlanticus, and they constitute the first scientific attempt of their kind. All previous efforts were frankly empirical. Of da Vinci's we can at least say that it attempted scientifically to combine theory with practice. He begins early in the former manuscript by pointing out that a number of pillars or supports held together are stronger collectively than a single "equivalent" pillar. "Many little supports held together," i. e., in a bundle, he writes. "are capable of bearing a greater load than if they are separated from each other. Of 1,000 such rushes of the same thickness and length which are separated from one another, each one will bend if you stick it upright and load it with a common weight. And if you bind them together with cords so that they touch each other. they will be able to carry a weight such that each single rush is in the position of supporting twelve times more weight than formerly.³⁰ Later in the same manuscript³¹ he insists that this increase in the carrying capacity is entirely dependent upon the firmness with which the bundle is bound together, so that if the connection is loose, the total load possible becomes merely equal to the sum of the loads of which each rush is separately capable, instead of a multiple of that sum.

It is interesting to note the nature of the experiments carried out by Leonardo in verification of the above. He applies a vertical load on a tightened iron wire strip, and increases the load until a plumb line shows the wire to begin to bend. He then repeats the experiment with two wires bound together, and then with three, and so on. We quote his own words, "Make the following experiment: Take two pieces of iron wire which have been stretched in a fourcornered wire drawer, and fasten one of them from below with two supports, and load it above with a given weight. Notice exactly

³⁰ Ms. A., fol. 3 v. ³¹ Ms. A., fol. 40 v. when it begins to incline, and further investigate with a thread which carries a plumb at which weight this inclination occurs. Next, double the iron wire, bind the two with a fine silk cord, and see whether this investigation agrees with what I have maintained. And similarly repeat the experiment with a fourfold increase, and so often as desired, continuing to bind the new ones with silk."³²

Turning next to a comparison of the loading capacities of two struts of equal height, but of different cross-section (Fig. 17), Leonardo writes. "A support with twice the diameter (width and depth?) will carry eight times as much weight as another, both having the same height."⁵⁰ His proof of this is not very convincing, but he concludes in effect that the ability to carry a load is directly proportional to the area of the carrying surface, and inversely proportional to the relation of the height to the diameter. Schuster³⁴ expresses Leonardo's results generally by the formula

Possible load $= \frac{\text{Carrying surface}}{\text{height}}$

Applying this to Figure 17, we see that there is a factor of four in



FIGURE SEVENTEEN

respect of surface areas, and an inverse factor of one-half in respect of the denominator, giving one load, eight times the other. Leonardo's application of this rule is again frequently faulty in his arithmetic. Thus he quotes one example in which the comparison of loads shows according to the rule 320 : 80, whereas, he makes it 320 : 60, a note in the text giving the product of 16 and 5 as 60.

Leonardo now passes on to a comparison of the carrying capacities of struts of equal cross-section, but of different heights, and concludes that these are inversely as their heights. "So often as the short staff is contained in the longer," he writes, "so much greater is the load which it can carry than the long one."³⁵

32	Ms.	А.,	fol.	47	r.	³⁴ Schuster, Zur Mechanik, etc., p.	136.
33	Ms.	Α.,	fol.	47	r.	³⁵ Ms. A., fol. 49; see also fol. 48	v.

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LOADED BEAMS

Finally, we may refer briefly to Leonardo's work on loaded beams, the chief notes on which are to be found in the *Codex Atlanticus*, on one page of which we find a sketch (Fig. 18) with the forlowing note: "You will find the same power of support by binding



FIGURE EIGHTEEN

together nine balks as by the ninth part of one of these parts: ab, which consists of nine balks, carries twenty-seven units of weight. Further, cd, which is the ninth part of the same in cross-section, carries three; thereby can ef, which is the ninth part of the length of cd, carry twenty-seven units of weight, since it is nine times as short as the former." ³⁶ Whatever else we may say of this result, it does embody certain generally correct features regarding the in-



FIGURE NINETEEN

fluence of length, breadth and depth. Leonardo, however, proceeds later to an actual discussion of the amount of deflection produced in the beams which is also interesting (Fig. 19). He compares the deflection of three beams of which cd is twice as long as ef, and ab twice as long as cd. His note proceeds as follows: "If ab bends through an eighth of its length when loaded with eight units, then cd, if it is, as I believe twice as strong as ab, will not bend itself through an eighth of its length with a smaller weight than sixteen units; since it is half as long as ab. Similarly will ef, since it is half as

long as cd, be twice as strong and will bend one eighth of its length with a load of thirty-two units. One must also note that the beam cd which is twice as strong as ab on a load double that of ab will not bend one-eighth of its length, but, speaking much more exactly, will bend one-sixteenth of its length."³⁷ The formula for the deflection x of a beam loaded in the middle and supported at the ends is given by $x = \frac{Wl^3}{4Ebd^3}$, where W is the load, l the length, b the breadth, and d the depth of the beam. An application of this to the case cited by Leonardo in Figure 19 will show that the correction he offers in the last paragraph of the note is right, the former result being wrong. Again, however, we must stress the point that although he accomplished little in the way of accurate results, he was the first man to attempt the problems scientifically, and to that extent at least he deserved considerable credit.

FRICTION

We conclude our study of Leonardo's researches in statics with a note as to his experiments on friction. Here, undoubtedly, the experimental scientist in da Vinci showed itself with distinction. It is regrettable that his work was lost to posterity. It was incomplete, and in some respects incorrect. But Amontous in 1699 and Coulomb after him had to cover much the same ground through ignorance of his work, where otherwise they might have gone much farther than they did. Leonardo's writings on friction are almost exclusively to be found in the *Coder Atlanticus.*³⁸ His experiments were similar in scope to those which obtain today—the sliding of a given surface over both horizontal and inclined planes by means of a weight passing over a pulley; and he carefully distinguished between sliding friction and rolling friction ("which, moving forward by infinitely small steps, touches rather than rubs").³⁹

Leonardo's experiments led him to conclude that the amount of friction was independent of the areas of the surfaces in contact; that artificial smoothing or lubricating reduces friction; that for bodies polished or smoothed to an equal degree, the friction is proportional to the pressure between them, and that on a horizontal polished surface, the resistance to friction by all bodies is with a force equal to one-quarter of the weight. This last result is most

³⁷ Codex Atlanticus, fol. 332 r.b.

³⁸ Codex Atlanticus, fols. 16, 47, 64, etc.

³⁹ Codex Atlanticus, fol. 195 r.

interesting. It recognizes a definite coefficient of friction, but with a value common to all bodies and equal to twenty-five. Nevertheless, in the general importance of the conclusions as a whole—since they afford us the first presentation in scientific history of any laws of friction whatever—we may well excuse Leonardo this incorrect result.

Conclusion

In the foregoing we have endeavored to summarize as completely as possible the main achievements of Leonardo da Vinci in the science of statics. In so doing we have endeavored to confine ourselves to a survey of broad scientific principles rather than to a detailed and exhaustive scheduling of notes and experiments.

Such notes and experiments as we have quoted have been introduced rather for the purpose of illustrating the particular principle or theme under discussion. Indeed, so prolific was our philosopher in the writing of his notes, that to attempt anything more than this would be an almost overwhelming task. At the same time it must be pointed out that we have purposely omitted a treatment of Leonardo's work in hydrostatics and hydraulics on the one hand, and in mechanical technology on the other. Each of these offers a very wide and very important field in itself, and it is hoped to reserve their survey for a special and a later study elsewhere.

Of the two branches of dynamics and statics into which Leonardo's work in mechanics may be divided, the former, although perhaps smaller in extent than the statics, offered more that was purely new and original. In statics, Leonardo's role was rather that of "carrying on." Nevertheless, we may regard him as having been conspicuously successful in both roles. He "carried on" where work had already been done, he created where there had been nothing upon which to build. In so doing, we may claim for him that he was a genius, and inasmuch as his work breathed the whole spirm and atmosphere of true scientific method, he was a scientific genius. Nor was his genius entirely lost to posterity. The devoted labors, spread over many years, of Pierre Duhem in the works which we have already so frequently quoted, have shown only too conclusively⁴⁰ that Leonardo's influence in the world of mechanical science was, indirectly perhaps, but decisively and effectively nevertheless, handed down through Jerome Cardan to Bernardino Baldi, thence to Roberval (and through him to Descartes) and to Galileo.

⁴⁰ P. Duhem, Les Etudes de Leonardo da Vinci, Vol. I, p. 127, et seq.