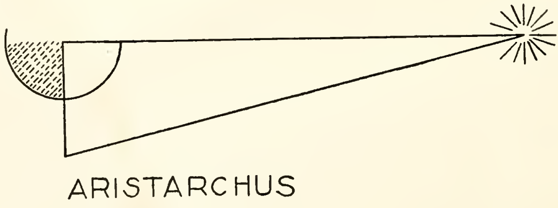
# THE TRIANGLE AS AN AID TO DISCOVERY

BY JAMES CARLILE

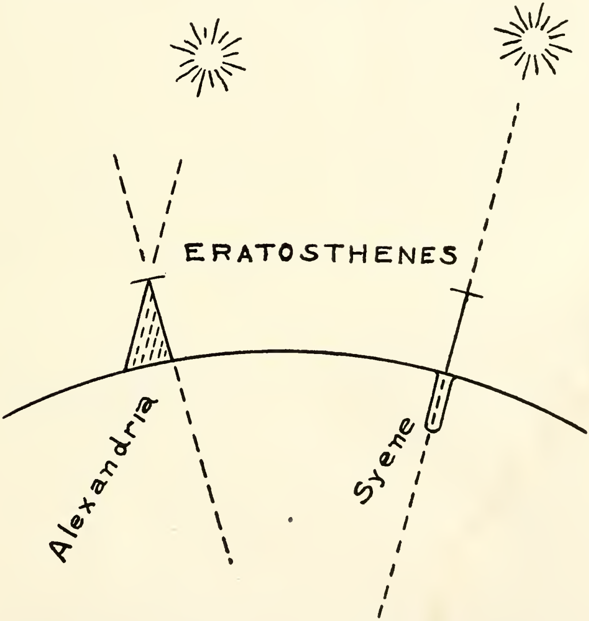
RATIONAL ASTRONOMY begins in the third century B. C. with the use of the right angled triangle by Aristarchus. He conceived the idea that when the moon is at her first and her third quarters the three bodies, sun, earth and moon, are situated at the points of a triangle, the moon being at the right angle. If then he could draw a line from his point of observation at Alexandria in the direction of the sun and another line from the same point in the direction of the moon, and could measure the angle between the two lines, he would have the means of ascertaining the ratio of the sides enclosing the right angle and of calculating the relative distances of the moon from the sun and from the earth. The means of measurement at his disposal were not such as to enable him to approach accuracy. All he could conclude was that the moon's distance from the sun was more than twenty times her distance from the earth. But his method was the important thing; it enabled him to take a new view of the universe and convinced him that the earth was not the center of the heavenly bodies but was only a member of a Solar



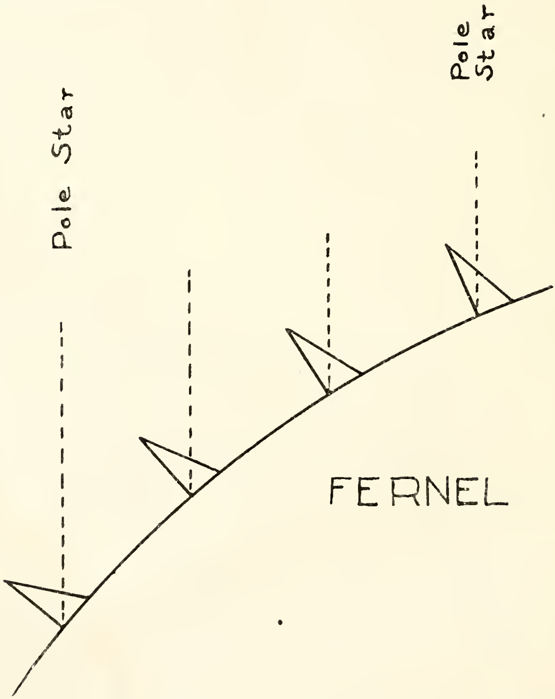
ARISTARCHUS

System. He anticipated Copernican doctrine by seventeen hundred years, and at the same time gave the first example of trigonometrical reasoning long before trigonometry was practised or even thought of.

It was in Alexandria in the third century B. C. that Aristarchus thus exhibited the true method of research as applied to the heavenly bodies: it was in the same city and in the same century that Eratosthenes applied the right angled triangle to determine the figure of the Earth. There was a deep well at Syene in which, at the summer solstice, the image of the sun could be seen in the water at the bottom of the well. Therefore, argued Eratosthenes, the sun is directly overhead, and a line from the sun to the well makes a right angle with the surface of the Earth. Suppose then a line to be drawn on the ground due northward to Alexandria, and at Alexandria a rod be erected and its shadow observed. Assuming the Earth to be a globe, then the length of the shadow would be a measure of the curvature; the rod (or gnomon), its shadow, and a line drawn from




the top of the rod to the end of the shadow would form a small triangle similar to the large triangle made by lines drawn from the center of the Earth to Alexandria and Syene respectively, and the North to South line joining those two places. Eratosthenes measured the angle made by the rod and the oblique line joining the top of the rod with the end of its shadow; this he found to be one-fiftieth part of our right angles. In other words, if a circle were described with the top of the rod for a center then the length of the shadow would be one-fiftieth part of the circumference. Therefore, he concluded, the distance from Syene to Alexandria is one-fiftieth part of the Earth's circumference; and as Eratosthenes knew that distance to be 5,000 stadia he inferred that the Earth's circumference was 250,000 stadia.

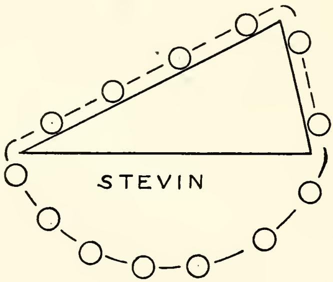



A gap of 1750 years separates the work of the Alexandrian astronomers from the beginnings of modern science. During that time the Greek Geometry was forgotten and dreams usurped the place of thoughts. It was in the year 1500, seven years after the first voyage of Columbus, that Jean Fernel undertook a journey on foot from

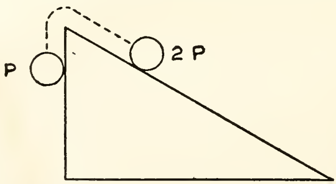
Paris due north to Amiens, equipped with a coach wheel and a large right angled triangle constructed of wood. His problem was to ascertain how many times his coach wheel would revolve before the pole had risen by one degree of circular measure. His line of thought seems to have been this: the Earth is circular in section and the zenith of the heavens is a prolongation of the radius; on the other hand, the true north maintains its position wherever seen from the Earth's surface. Therefore, the perpendicular to the Earth's surface and the skypole line make an angle which in the tropics is a right angle and diminishes as we move northward. If he could measure the rate of decrease of this angle he would not only demonstrate the true figure of the Earth, but also determine its exact circumference.

The coachwheel trundled before him measured the length of each day's pilgrimage. The triangle with its base on the ground enabled him to mark each night the altitude of the pole star, and by the end of his journey he was able to satisfy himself that the pole star rose regularly one degree in altitude for every sixty-nine miles of his northward progress. Beyond adding a few decimal places, modern methods and appliances have but confirmed Fernel's result. Short as his journey had been it had been more conclusive than the voyage of Columbus, which had likewise been undertaken with the object of demonstrating the true figure of the Earth. Had Bacon but known of Fernel's investigation he might have taken it as the type of the *experimentum crucis*.

The use of the triangle made by Stevin of Bruges, about fifty years after Fernel, may be taken as the last word of the old classical science in which the element of time is wholly ignored. Remarkable as were the achievements of the Alexandrian Geometers, of Archimedes, of Fernel and of Stevin himself, they admitted of no further expansion until it was recognized that motion was all important and that statical conceptions were merely instantaneous glimpses into a continuum. What Stevin did was to take a right angled triangle with the hypothenuse parallel with the ground and suspend over it a string on which fourteen balls were strung at equal distances. The sides of the triangle were as two to one; on the longer side four balls rested, and on the shorter side two balls, while the remaining eight hung in a catenary curve below the hypothenuse. In that position the balls remained immovable, and Stevin found that in whatever position the chain of balls is placed it remains at rest. He then cut away the festoon of eight balls and found the two balls on the shorter side supported by the four balls on the longer. From this experiment he proceeds immediately to the principle which contains in essence the whole of Statics, namely that any stresses operating in the direction of three sides of a triangle will balance and produce equilibrium if they are proportional in magnitude to the lengths of the three sides. It is true that he could not give adequate expression to the principles of his new Science. His language is chaotic, sometimes unintelligible; how he satisfied himself of the truth of his principle does not appear. He is still under the influence of certain medieval conceptions, and he did not understand what is implied in the word "weight." But he saw clearly that which he could not quite justify in words; he had recalled mankind to the right path of scientific enquiry and he deserves the statue which Bruges has erected in his honor.




The different way in which Galileo handles the problem of Stevinus marks the beginning of the new Science in which rest is regarded as virtual velocity. When a weight hanging perpendicularly supports a double weight resting on an incline which is double the length of the perpendicular, Galileo sees in the phenomenon one weight tending to descend to the Earth at a certain rate and a double

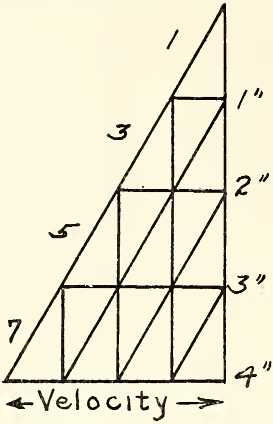



weight tending to descend at half that rate. He has much mental struggle before he liberates himself from the bonds of a Geometry which refuses to recognize the idea of movement.

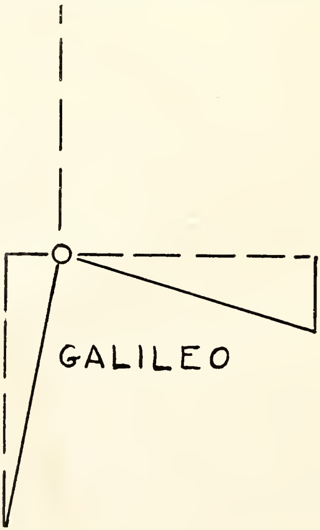
His dilemma about the rate of fall has been much misunderstood. It seems to have presented itself to him in the following way. A body falls from a certain height and afterwards from twice the height. On the second occasion it is moving much faster; but how much faster? Let us say twice as fast, or in other words that velocity varies directly as height of fall. That phrase has a different meaning to us from that which it presented to Galileo at the outset of his enquiries. "Velocity" meant to him "intensity of purpose." The body falling from a double height has a twofold intensity of purpose; but the upper half of its course is of the same length as the whole course of its first fall. Does the double purpose exhaust itself in this upper half of the fall leaving the rest to be accomplished instantaneously? At this point he realized that speed could not be discussed without taking duration into consideration. He recurs to the triangle and supposes the vertical to represent time elapsed and the base to represent speed at that instant; if the vertical line is produced to double its original length then the new base will be double, and the triangle becomes the graphic expression of a law that speed varies directly as the time. His experiments on the pendulum and the inclined plane enable him not only to recognize this explanation as satisfactory but to generalize and assert that undisturbed bodies continue in uniform motion and that the fall of a body is a state of continually disturbed or accelerated movement, and not what had always been supposed the general rule of nature in bodies affected with heaviness.

To expound to himself the exact rate at which bodies approach the Earth, Galileo again had recourse to the right angled triangle. Suppose the vertical line prolonged and at equal intervals lines to be drawn parallel to the base. These successive bases cut off areas which are continually increasing. The second base cuts off an area equal to three of the original triangles; the third base cuts off an area equal to five of the original triangles; the fourth an area equal to seven, and so on. The vertical side measures intervals of time, say seconds; the area between two horizontal bases represents the space covered in that particular second; the two bases represent the velocity at the beginning and end of the second. The total area enclosed by the vertical and the hypothenuse prolonged, and the last constructed base, represents the total space covered in the fall from rest. To construct such a triangle is to understand the law of falling bodies far more clearly than it could be apprehended by the aid of any algebraic formula.

It is characteristic of Galileo's method of thought that in dealing with the problems of the lever he employs the conceptions of pure




geometry. He takes a lever or unequal armed balance, the longer arm being four times the length of the short arm. A weight depends



from the long arm and four times the weight depends from the short arm. The position is one of equilibrium and the equilibrium is illustrated by the equality of two triangles two sides of which measure the masses and the spaces which they cover in equal times.

The triangle has thus since the days of Galileo acquired an importance in Mechanics such as it formerly possessed in Geometry. Every one of its fundamental properties has a dynamic significance. The statement that two sides of a triangle are together greater than the third translates into the proposition that any force exerted diagonally diminishes in effectiveness as the angle at which it is applied increases. Let a cord pass over two pulleys on a horizontal beam; attach different weights to the two ends, and suspend a third weight from the middle between the two pulleys; then the rope and the beam form a triangle. Suppose the two weights at the end of the cord are five pounds and twelve pounds, then a weight of thirteen pounds in the middle will draw the rope into a position of equilibrium in which it forms a right angled triangle with its sides

PASCAL

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| 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 |

in the proportion of 5, 12, 13. Any weight in the center, however small, will establish a triangle of equilibrium, and the triangle will never disappear so long as there is any weight in the middle.

Pascal's Arithmetical Triangle is often regarded as a toy or mathematical recreation. But it was not so regarded by him. He took it up at a late period of his life when he was engaged in founding the doctrine of chances. *"C'est un chose étrange,"* he said, *"combien il est fertile en propriétés."* John Bernoulli was enthusiastic about it and hailed it as the foundation of most of the important discoveries that had then been made in mathematics. It certainly furnished Newton with a hint for the construction of the binomical theorem in its general form. But as it in no way uses the geometrical properties of the triangle it may be regarded as merely an orderly arrangement of numbers which might with even more propriety have been arranged as the successive ordinates of a curve the celebrated bell-curve of the Theory of Probabilities.