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The Effect of Instruction in Alternative Solutions on American Ninth-Grade Algebra I Students' Problem Solving Performance

Erin Elizabeth Sagaskie
Southern Illinois University Carbondale, esagaskie@gmail.com

THE EFFECT OF INSTRUCTION IN ALTERNATIVE SOLUTIONS ON AMERICAN NINTH-GRADE ALGEBRA I STUDENTS' PROBLEM SOLVING PERFORMANCE

by

Erin E. Sagaskie

B.S., Southern Illinois University, 2001
M.S., Southern Illinois University, 2004

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Approved by:

Dr. Cheng-Yao Lin, Chair

Dr. Jerry P. Becker

Dr. D. John McIntyre

Dr. Todd C. Headrick

Dr. Mary H. Wright

Graduate School
Southern Illinois University Carbondale
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AN ABSTRACT OF THE DISSERTATION OF

ERIN E. SAGASKIE, for the Doctor of Philosophy degree in CURRICULUM AND INSTRUCTION, presented on October 10, 2014, at Southern Illinois University Carbondale.

TITLE: THE EFFECT OF INSTRUCTION IN ALTERNATIVE SOLUTIONS ON AMERICAN NINTH-GRADE STUDENTS' ALGEBRA I PROBLEM SOLVING PERFORMANCE

MAJOR PROFESSOR: Dr. Cheng-Yao Lin

The purpose of this study was to investigate the effect of the use of an Alternative-Solution Worksheet (ASW) on American ninth-grade students' problem solving performance, and to determine the extent to which instruction in alternative solutions promotes "look back" strategies. "Look back" strategies are based on Polya's (1973) problem solving steps, and they are an examination of what was done or learned previously. The ASW was designed to encourage students to utilize "look back" strategies by generating alternative solutions to the problems.

This mixed-methods study was conducted with two existing groups of ninth-grade Algebra I students. An experimental group of 18 students received instruction in utilizing the ASW for two 55-minute class periods a week for a period of four weeks. A comparison group of 14 students did not receive any instruction. Data for this study were collected by pre- and post-testing, ASWs, focus groups, and one student's "think aloud" process.

For the quantitative analysis, a one-way ANCOVA was conducted to determine if there was a significant difference in the mean post-test scores between the experimental group and the comparison group. The students' pre-test score was the covariate. The findings indicated that the experimental group scored slightly better on the post-test, and $R^2 = .345$, a medium effect size. There were no significant correlations between the ASW scores and the pre- and post-test scores,

but the ASW scores were significantly correlated with the students' EXPLORE9 math and reading percentiles.

The qualitative findings indicated that "look back" occurred at all six levels of Bloom's Revised Taxonomy, but it is the "look back" that occurs at the upper three levels, in the context of higher order thinking skills, that results in better mathematical problem solving abilities. In addition, positive affective changes were evident despite little improvement in students' mathematical problem solving abilities. The results of this study indicated that higher order thinking skills need to be practiced regularly so students can use them effectively.

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Finally, yet importantly, I am thankful for husband, Spencer, my son Eli, and my mom and stepfather; Jo Carol and R.B. for the countless hours of babysitting, sacrificed family time, and for listening to me as I navigated this process.

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CHAPTER 1

INTRODUCTION

The 2010 adoption of the Common Core State Standards (CCSS) by most of the United States and the anticipated upcoming assessments bring a sense of urgency for teachers to be prepared to teach to those standards. The Common Core State Standards for Mathematics (CCSSM) require students to use critical thinking skills to solve problems in a way that has not been assessed on past high stakes assessments. The NCTM Research Committee (2013) suggested that, "Current studies should continue to seek to identify specific pedagogies that promote student engagement, performance, and learning of mathematics content" (p. 347). The use of Alternative Solutions Worksheets (ASW) to promote "look back" strategies coupled with the Open Approach to teaching mathematics is one such method (Lee, 2009). The Open Approach to teaching mathematics is a strategy for teaching mathematical problem solving and mathematical thinking. Students are presented with an open-ended problem and asked to solve it in as many ways as possible; combining individual and group work and class discussion as a part of the process (Becker & Shimada, 1997). Perhaps the most well known mathematical problem solving approach is that of George Polya (1973), who proposed a four-phase approach. The four phases are understanding the problem, making a plan to solve it, carrying out the plan, and looking back at the completed solution to review and discuss it. The final step of looking back is the most neglected of the four steps (Cai & Brook, 2006; Jacobbe, 2007; Polya, 1973; Taback, 1988). In neglecting this step, students miss alternative solutions/methods and, more importantly, they lose the opportunity to validate their solution, or make sure the solution is reasonable and appropriate. Requiring students to use an ASW encourages them to complete the looking back phase of Polya's four-phase approach.

Purpose of the Study

The purpose of this study was to investigate the effect of the use of an Alternative-Solution Worksheet (ASW, see appendix A) with American ninth-grade students' problem solving performance and to determine the extent to which instruction in the formulation of alternative solutions promoted "look back" strategies in the study. The classroom teacher used an ASW to encourage students to "look back" by generating alternative solutions to mathematical problems. A triangulation mixed methods study was designed to investigate the following research questions.

1. What relationship is there between problem solving scores on pre, posttests, and students' performance on ASW activities?
2. What relationship is there between students' EXPLORE9 (see ACT, Inc., 2013) scores and scores on the pre, posttests, and ASW activities?
3. To what extent do students report the use of "look back" strategies when completing ASW activities?
4. What is the relationship between the subjects' reported use of "look back" strategies and their performance on ASW activities?

The problem solving intervention questions as well as the assessments were largely centered on an integrated approach to factoring, allowing students to make a connection between area and factoring. The participants were ninth-grade students enrolled in an Algebra I class. The CCSSM suggests that a typical ninth-grade Algebra I course covers various aspects of linear, quadratic, and exponential functions. In a publication regarding Algebra I topics from the Charles A. Dana Center at the University of Texas, Austin and Agile Mind Inc. (2012), the authors claim, "The critical areas (of the CCSSM), called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions” (p. 2). Currently, linear relationships are included in a typical ninth-grade Algebra I curriculum. Quadratic expressions and equations are covered under the umbrella of factoring, but with little depth. It is important to follow the suggested curriculum of the CCSSM for a ninth-grade Algebra I course, thus including the study of quadratic functions at a deeper level. Many of the intervention and pre/post test questions for this study involved quadratic functions. The questions were similar in difficulty and level of abstractness to some examples seen on the Partnership for Assessment of Readiness for College and Careers (PARCC) assessments for the CCSSM, thus giving the reader a small glimpse into the challenges of implementing such a change in mathematics education.

Importance of the Study

According the Schoenfeld (1983), “The primary responsibility of mathematics faculty is to teach their students to think: to question and to probe, to get to the mathematical heart of the matter, and to be able to employ ideas rather than simply to regurgitate them” (p. 2). Teaching students to think continues to be an elusive goal for many math teachers. In an attempt to cover all of the required curriculum and include some problem solving, it has been my practice to assign one open-ended type problem as a part of a larger homework assignment with the hope that the students will work on it, and then discuss it as a class the following day. Rarely do the majority of the students even try the problem, and those who do are often not interested in discussing it. The teacher ends up doing most of the talking, and the students are not much better off for it. To deal with this issue, it is important and necessary to present the students with a situation where they must practice mathematical problem solving because understanding how the students think about mathematical problem solving can only be done by studying students' thinking in the process. The ultimate goal is to improve the teaching of mathematical problem solving and thinking thus improving students' performance and perceptions towards problem solving (Hino, 2007). This study is an attempt to address those issues.

Literature Review

Rationale and Theoretical Background

Standards-based reform is not new to the world of mathematics education. When the National Council of Teachers of Mathematics (NCTM) released its *Curriculum and Evaluation Standards for School Mathematics* in 1989, there were skeptics. Some remembered the failed implementation of the "new math" of the 1960's (Herrera & Owens, 2001). Others simply believed the future of mathematics education could only go in one of two ways; radical reform or back to basics (Schoen, Fey, Hirsch, & Coxford, 1999). Critics did not believe there could be a middle ground where both conceptual and procedural fluency could be attained while mastering reasoning and sense making skills at the same time. After the *Curriculum and Evaluation Standards* were criticized for lacking an emphasis on proof and basic skills, the NCTM revised them in their 2000 publication *Principles and Standards for School Mathematics*, in which five process standards and five content standards were included to address those criticisms.

The critics of the reform movement in mathematics have two arguments. The first is regarding the curriculum associated with standards-based materials. Standards-based materials are comprehensive, coherent, and promote sense making by engaging students in problems and tasks (Trafton, Reys, & Wasman, 2001). They are generally an integrated, one-size-fits-all sequence of courses meant for a heterogeneous group of students. Those opposing this type of curriculum question its rigor and its appropriateness for low, average, and gifted students in the same classroom.

Second, critics note the misalignment of high stakes test questions with the goals of standards-based materials. High stakes tests are those for which results are used to make decisions that have serious consequences such as grade-level promotion, course placement, high school graduation, or college entrance (Wilson, 2007). Test items are usually procedural tasks that do not make reasoning and sense making a priority, thus resulting in teaching to the test and the narrowing of curriculum and instruction (Neill, 2006). In some states, such as Illinois, the high stakes tests were also used to evaluate the effectiveness of schools per the No Child Left Behind Act (NCLB). Currently, as the implementation of the CCSSM rolls out, new tests will be used to evaluate students, schools, and individual teachers. When so much is at stake for both the schools and the students, the curriculum will be aligned to the test. Since reasoning and sense making were not tested much in the past, they were not a priority in the schools. Krupa (2011) argued that current state assessments emphasize items with low cognitive demand and new assessments should be tests worth teaching to, including formative assessments to guide instruction and flesh out student misconceptions. It seems that the new assessments that align with the CCSSM include such problems.

The NCTM (2009) agreed with this criticism and called for a redesign of high stakes tests to better assess students' reasoning and sense making. An attempt has been made to answer that call with the 2010 adoption of the CCSS. The CCSS have currently been adopted by 45 states, the District of Columbia, four territories, and the Department of Defense Education Activity. The target date for the new assessment system varies by state and grade level, but will be in place over the next five years. Once the new system is in place, for the first time in our educational history, there will be near uniformity in the learning goals and the assessments used to measure student achievement (NCTM Research Committee, 2013). The new tests will measure skills such as critical thinking and the application of skills to solve complex problems as well as growth and proficiency (Center for K-12 Assessment and Performance Management, 2012).

The goal of the CCSS is to better prepare students for success in college and the workforce in a competitive global economy (Illinois State Board of Education, 2013). That goal is shared by the NCTM and laid out in publications such as *Principles and Standards for School Mathematics*, *Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics*, and *Focus in High School Mathematics: Reasoning and Sense Making*. According to the NCTM (2000; 2006; 2009), everyone needs to be able to use mathematics in his or her personal life, in the workplace, and in further study. Students need to learn a new set of mathematics basics that enable them to compute fluently and to solve problems creatively and resourcefully. This type of mathematical thinking requires the reasoning and sense making skills the NCTM documents mention.

This type of problem solving has not been a pedagogical focus in school mathematics. The weight of high stakes tests such as the ACT has driven curriculum for many years. The implementation of the CCSSM including new and different high stakes tests lead many to believe that “the common standards demand significant changes in pedagogy, and, in some cases, teachers’ content knowledge” (Gewertz, 2013). It seems that the ongoing reform efforts in mathematics education will eventually be implemented on a large scale with the CCSSM. Teachers must learn how to facilitate students’ construction of their own mathematical understanding (Heibert et al., 1997; Heibert & Carpenter, 1992; NCTM, 2000). If teachers are not properly trained, the implementation of the CCSSM will fail, as did other past reform efforts such as the “New Math” (Wooten, 1965). One such way to help students construct their own understanding is through open-ended problems.

Instruction in Alternative Solutions

Research has shown that instruction in alternative solutions is a promising reform-based pedagogical practice (Heibert et al., 1997; Lampert, 1990, 2001). An Alternative-Solutions Worksheet (ASW) is a tool that encourages students to come up with alternate methods for solving a problem. George Polya’s (1973) four-phase problem solving model requires students to understand the problem, devise a plan, carry out the plan, and look back at the completed solution, by reconsidering and reexamining the result and the path that led to it. The final step of looking back is the most neglected of the four steps, as mentioned earlier (Cai & Brook, 2006; Jacobbe, 2007; Polya, 1973; Taback, 1988). The use of an alternative solutions worksheet encourages students to “look back” (Cai & Brook, 2006; Kersh & McDonald, 1991; Krulik & Rudnick, 1994; Lee, 2009).

Students need more guidance to effectively “look back” than just having the ASW, however they need to be taught to problem solve reflectively and to share their thinking and to listen to the thinking of others (NCTM, 2009). One way teachers can facilitate this type of interaction is to establish a community of learning in which a multitude of approaches and solutions are respected and encouraged like in The Open-Ended Approach to mathematical problem solving (Becker & Shimada, 1997; Kwon, Park, & Park, 2006).

The Open-Ended Approach

The problems that are suggested for the CCSSM are similar to the types of math problems that work well with the Open-Ended Approach. Much like problems presented to medical and law students, the CCSSM problems will present students with a complex scenario that requires them to apply higher-order thinking skills to develop a solution strategy (NCTM Research Committee, 2013). When implementing the Open-Ended Approach, teachers give students a problem with either no single solution or more than one method to solve it. Students can then mathematize the situation, find rules or relations, solve, and check (Becker & Shimada, 1997). Students can also see others' results, compare and examine methods and solutions, modify their solutions and further develop them, a "looking back" strategy. The Open-Ended Approach offers an opportunity for this type of discussion after the students complete their individual work and the classroom teacher has a chance to compile the students' strategies for discussion.

Existing Research

There are a limited number of existing empirical studies on alternative solutions and problem solving performance (Groβe & Renkl, 2006; Silver et al., 2005). Kantowski (1977) investigated ninth-grade Algebra students' development of process involved in solving complex, non-routine Geometry problems as they received instruction utilizing heuristic instructional techniques over four months. Her research was based on the use of alternative solutions and on Polya's four-step process, but her findings indicated that the use of "looking back" strategies did not seem to be related to problem solving success.

Fouche (1993) investigated the multiple solution methods employed by Pre-Algebra and Algebra eighth-grade students. Students who received instruction in multiple solution methods found more solution methods and seemed to understand and be able to generalize their solution methods better than those who did not.

Hwang, Chen, Dung, and Yang (2007) investigated Taiwanese sixth-grade students' use of multiple representations while solving mathematical problems using a developed online multimedia whiteboard system. They found that students who used multiple representations performed better in the problem solving than students who used fewer representations.

Herman (2007) explored how teaching multiple representations in a ten-week Algebra course affected college students' problem solving ability. She found that students preferred symbolic manipulation as a primary solution strategy even after being exposed to the other representations. They agreed that using multiple representations improved their understanding, but primarily used those other methods only for checking their work. Herman also found that students used more solution methods on the posttest, and were more likely to get correct answers when they used more representations.

Huntley and Davis (2008) interviewed 44 pairs of high achieving third year high school students to investigate their approaches to solving Algebra problems. They found that the majority of the students used symbol manipulation initially and as their primary method of solving. They also found it common for students to use other representations such as graphical and tabular when they were checking their answers or looking for alternative solution methods. It was suggested that students who are adept at using multiple representation strategies are less reliant on their teachers for detecting and correcting errors (Huntley & Davis, 2008). Last, they found that graphing calculators were rarely used even though the students used them frequently in their current math classes. All of the problems in the study were presented in the symbolic form, which likely influenced the students to use symbol manipulation as a primary method.

Pugalee (2004) investigated the impact of writing during mathematical problem solving. He provided ninth-grade Algebra I students with open-ended problems, alternating verbal and written solutions. Students who wrote descriptions of their thinking were more successful than those who only verbalized their thought process. Additionally, the number of varying solution methods on a single problem increased as the scores decreased, suggesting that students try more methods on difficult problems, but the use of more methods does not necessarily result in correct solutions. Finally, it was noted that although students do engage in “look back” behaviors at various times during the problem solving process, the majority do not investigate the validity of their final answer.

This study seeks to replicate and extend an important study by Lee (2009). She investigated “the effect of the use of Alternative Solutions Worksheets on Taiwanese eighth-grade students’ problem solving performance and determined the extent to which instruction in the formulation of alternative solutions promotes “look back” strategies” (p.75). Lee found that using instruction based on the Open Approach was important in the implementation of the ASW techniques. In addition, students’ improvement in problem solving was positively correlated with their performance on the ASW.

Differences from Lee’s Study

In this research, I decided to report as much information about the participants as I could. In a small sample sized, mixed-methods study, applicability of the results can only be determined if the participants and the context are described thoroughly. According to Guba and Lincoln (1982) some degree of transferability is possible if enough “thick description” is available about both how the study was conducted and how the results were analyzed.

In this study, there were 32 participants. Each student’s math and reading percentiles on the standardized test, EXPLORE9, were included in the data analysis because they are often a good predictor of success in high school math courses (ACT, Inc., 2013). The students’ semester grades in Algebra I were also analyzed. Demographic information about each participant was used to establish context. Information about the high school and the community is included to paint a more complete picture.

For the sake of triangulation of the data, an observation component was included in this study. I sat in the classroom once a week during the discussion day of the intervention. In addition, two focus groups were conducted in which the participants provided more insight into their process and thoughts. This data, along with the interview and focus group data and the ASWs helped answer the third research question about the extent students report the use of “look back” activities on the ASWs.

Lee (2009) reported a coding scheme that was developed to determine the extent to which the student “looked back” during the interview. Beyond this, open coding was used initially in this study. As categories emerged, constant comparison was utilized until no new properties emerged. Lee reported coding problems as a limitation of her study. In her study, there were problems when two or more “look back” indicators were present in one sentence, or when there were no “look back” indicators at a time when “look back” was occurring. For this study, “look back” was analyzed in terms of the context in which it was happening, and then categorized as strong or weak “look back”, with the help of Bloom’s Revised Taxonomy.

For this mixed-methods study, details such as how the students were trained to “think aloud”, how the graders were trained, and the teacher/researcher bias were explained. These types of details are necessary for an author to gain trust from the audience according to Guba and Lincoln (1982).

Extending Lee’s Study

Lee (2009), implemented instruction in alternative solutions over a period of four weeks with one class of thirty-four Taiwanese eighth-grade students. This study was also conducted over a period of four weeks, but with one class of American ninth-grade Algebra I students. In addition, another Algebra I class with a similar demographic make-up served as a comparison group. The inclusion of a comparison group was one way that Lee's study was extended. Lee (2009) indicated, "Students who did not improve their problem solving performance on the posttest were not well equipped with either mathematical content knowledge or procedural facility or both" but it was unclear how this was known (p. 72). It can be assumed that since she was also the classroom teacher, she had a good understanding of each student's prior abilities. For this study, existing data such as the students' standardized test scores were used to determine the students' abilities. Lee also noted that the coding scheme should be refined to better distinguish between strong and weak "look back" indicators, which would also affect how points were awarded for "look back". Thus, following Lee, a more flexible coding scheme was developed for this study. Participants in Lee's study were eighth-grade Taiwanese students studying Algebra I topics. The subjects for this study are average to below average Algebra I students. It will be interesting to see how Lee's results compare with the results of this group. Some methodological issues noted in the qualitative portion of Lee's study will be addressed to enhance the credibility, transferability, dependability, and confirmability, four characteristics of good qualitative research according to Guba and Lincoln (1982).

CHAPTER 2

METHODOLOGY

The data for this study were collected over the last four weeks of the 2013-2014 school year. The pre-test, post-test, and intervention questions were designed by Lee (2009). Since the intervention involved knowledge of factoring, it was conducted after the students completed the chapter on factoring in their Algebra I classes.

Both qualitative and quantitative data were collected for this study. Quantitative measures consisted of students' scores on the pretest, posttest, and Alternative-Solutions Worksheets (ASWs). Qualitative measures were comprised of two focus groups, classroom observations, and "think aloud" sessions with one student. Existing data such as student demographic information, Algebra I grades, and standardized test scores were also used for analysis and description. To describe the methods and procedures that were used in collection of these data, this chapter includes the following sections: Subjects and Setting, Materials and Instruments, Scoring, Data Collection, and Instructional Procedures, and Position of the Researcher.

Subjects and Setting

The subjects for this study were students enrolled in two intact ninth-grade Algebra I support classes at a Midwestern high school. One of the classes served as a comparison group. The comparison group was composed of 14 students. Nine were African American, three were white, one was Hispanic, and one Asian. Twelve of the 14 participants in the comparison group came from low-income households. The experimental group was composed of 18 students. Eight were African American, eight were white, one was Hispanic, and one was Asian. Thirteen of the 18 students in the experimental group came from low-income households. The students were not randomly assigned to groups, but the two groups have similar mean standardized test scores and Algebra averages (See Table 1).

Table 1

Descriptive Statistics of the Participants' EXPLORE9 Math and Reading Percentiles and Second Semester Algebra I Averages

	Experimental Group			Comparison Group		
Score	*n*	*M (SD)*	95% CI	*n*	*M (SD)*	95% CI
EXPLORE Math	18	43.94 (23.06)	[32.48, 55.41]	14	54.078 (23.83)	[40.31, 67.83]
EXPLORE Reading	18	55.17 (25.88)	[36.83, 60.17]	14	48.50 (20.21)	[36.83, 60.17]
Algebra Average	18	79.17 (13.76)	[72.33, 86.01]	14	74.64 (8.76)	[69.59, 79.70]

Students enrolled in the Algebra I support class typically have standardized test scores near the bottom cut-off percentile to take Algebra I (around the 50th percentile). It was determined that these students could be successful in Algebra I if they concurrently took the support class. A result of tracking, the students in Algebra I are typically average students because the high achieving students and the low achieving students are tracked into other classes. Even with the tracking, the Algebra I classes are a heterogeneous group of students with a wide range of mathematical abilities. Math teachers teach the two support classes. The participants had one of three teachers for Algebra I. I was not a teacher of any of the participants.

The high school had approximately 1130 students, with 44% minority and 49% considered economically disadvantaged. Under the No Child Left Behind Mandate, the school had undergone restructuring in recent years because it remained on the Academic Watch List of schools who were not meeting the needs of subgroups, and had been audited by the state because of the gap in performance between students who were served by special education and those who were not served by the special education program. One of the restructuring efforts that the school had undertaken included adding a math support class for freshmen in Algebra I whose standardized test scores indicated that they struggled with math and/or reading. The students in the two sections of the Algebra I support class were the participants for this study. These students were chosen because the classroom teachers for the support classes were willing to allow the study to take place during class time. One of the support class sections happened to be during my planning period, so I chose that group to be the experimental group so I could do classroom observations and facilitate focus groups. Prior to the start of the intervention, I spoke to both groups to explain the purpose of the study and exactly what was going to take place during the intervention. The classroom teachers assigned participation points for each day the students completed a task for this study. I was not the classroom teacher of any of the participants.

Materials and Instruments

Alternative-Solutions Worksheets (ASWs)

Lee (2009) developed the Alternative-Solutions Worksheets (ASWs) to “encourage students to use “look back” strategies while finding alternative solutions to mathematical problems” (p. 33). The ASWs have two sections, initial and alternative solutions. For the purpose of this study, the “solutions” include both answers and methods. Some problems have only one correct answer but several methods to obtain it (see Appendix E). The ASWs were graded with a rubric by another math teacher, a math aide, and me. The graders were not involved in the intervention. To familiarize the graders with the rubric and how to grade open-ended questions, a handout was provided. (See Appendix G.) The three of us graded each problem concurrently, and if two of three agreed on a score, it was accepted, therefore creating intergrader reliability.

The Pretest and the Posttest Instruments

All students took the pre-test before the intervention began. All students took the post-test at the end of the fourth week of the intervention. The intervention took place during the last four weeks of the 2013-2014 school year. Both the pretest and the posttest consisted of four open response questions that required the students to provide alternate solutions to each problem. The questions on the pre and posttests were similar to those used during the intervention. The posttest was comparable to the pretest in content and level of difficulty. Students were instructed to show all work and not to erase or black out anything they write. To assess the students' performance on the pre and posttests, the same rubric as the intervention was used. The graders and the procedures were the same as for the ASWs.

Scoring

Each attempted problem solving approach on the ASW, and the pre- and post-tests was scored using a rubric (see appendix D). Each unique attempt to solve a problem was assigned a score of zero to four per the rubric. The students were instructed to solve each problem in as many ways as they could, so theoretically there was no maximum score. For example, if a student made two unique attempts to solve a problem, then his or her total score would be between zero and eight.

EXPLORE9 Standardized Test Scores

In October 2013, all ninth-grade students took the EXPLORE9 standardized test. EXPLORE9 is a nationally normed, scaled test. It is scored on the same scale as the ACT. According to ACT, Inc. (2013), "EXPLORE9 is a point of entry into the secondary-school level of ACT's college and career readiness system and it is designed to measure student development from Grade 8 through Grade 12" (p.1). EXPLORE9 contains four multiple-choice tests: English, Mathematics, Reading, and Science. I chose to analyze the participants' math and reading percentiles because those two measures are considered when placing students in support classes such as the Algebra I support classes from which my participants were selected.

The mathematics test. The EXPLORE9 mathematics test is composed of 30 items. The test has a time limit of 30 minutes. The students are allowed to use a calculator on this portion of the test. The emphasis is on solving practical, quantitative problems the students may have encountered in middle school mathematics as well as during the current school year (ACT, Inc., 2013). See Figure F1 for additional information about specific content on the EXPLORE9 mathematics test.

The reading test. The EXPLORE9 reading test is composed of 30 items. The test has a time limit of 30 minutes. "The test items require students to derive meaning from several texts by referring to what is explicitly stated, and reasoning to determine implicit meanings" (ACT, Inc., 2013, p.6). See Figure F1 for additional information about specific content on the EXPLORE9 reading test.

Reliability and validity of EXPLORE9. Reliability coefficients and average standard error of measurements for all of the subtests as well as the composite are given in Figure F2. Kuder-Richardson 20 internal consistency reliability coefficients of raw scores are listed first. Scale sore reliability coefficients and standard errors of measurement are listed next. Validity of the EXPLORE9 test to measure students' educational achievement in particular subject areas and that students' preparedness for future education and careers has been explored by ACT in a number of ways. First, the test items were analyzed to make sure they were representative of current middle school curricula. The EXPLORE9 was also found to be related to course grades, plans to take college-prep course work, growth in educational achievement, and college and career readiness benchmarks. See Figures F3 and F4 for additional validity information.

Data Collection

Each student in both groups took a pretest prior to the intervention. Over a period of four weeks, the students in the experimental group were given two questions to solve each week. Each student filled out an ASW for each question and that was scored. After four weeks, each student in both groups took a posttest. During weeks 1, 2, and 3 of the intervention, one student was selected to be recorded doing a “think aloud” while he worked. This particular student was selected because he was willing to do it, and because I thought he would do it well. He seemed very outgoing in class, and his teacher told me that he was very good to work with one-on-one. Time constraints prevented me from recording other students thinking aloud. Each week I sat in the classroom and took field notes on the second day of the intervention while the students were discussing their solutions. Lastly, two focus groups were conducted during the intervention. One took place at the end of the first week of the intervention, and the other took place after the post-test. (See Figure F5.) Field notes, transcriptions of the “think aloud” process and the focus groups, as well as student work were coded using open coding, then analytical coding. Additionally, the participants’ Algebra I second semester class averages, and their math and reading percentiles on the EXPLORE9 standardized tests were used for analysis.

Coding Phase 1

Open coding techniques were used to generate descriptive codes in which I categorized the field notes, think-aloud transcripts, and documents into broad topic areas (Gibbs, 2007). For example, I identified recurring patterns in the students’ work like switching between adding, subtracting, multiplying, and dividing to generate more solutions. After identifying common practices, I looked more specifically at what those practices were and analyzed them further.

Coding Phase 2

Corbin and Strauss (2008) explained, "As analysts work with data, their minds automatically make connections because, after all, the connections come from the data" (p. 198). While analyzing the data, I made connections to Bloom's Revised Taxonomy (2001). My lens as a high school math teacher filtered the ways I looked at the students' work, and I attempted to figure out why they were struggling so much with the problems. Bloom's Revised Taxonomy helped me to understand why they struggled and how they used "look back" techniques.

Validity

Yin (2009) explains that utilizing multiple data sources can minimize threats to construct validity (pp. 115-118). With this in mind, I did classroom observations, analyzed student papers, conducted focus groups, and had one student "think aloud". These multiple data sources have allowed me to triangulate the data.

Instructional Procedures

For this study, the classroom teacher instructed the students in the treatment group on the "look back" technique by use of the alternate-solutions worksheet using the Open-Ended Approach two class periods per week for a four-week period. One class period was 55 minutes long. The classroom teacher was a certified math teacher.

Each week, the students were given two problems to solve. On the first day, they worked independently, filling out an ASW for each of the two questions. To allow each student to be individually graded on the ASW, the classroom teacher collected the ASWs when all the students were finished. They were graded by three graders. Positive feedback was written on the students' papers. Another day the same week, the graded ASWs were returned to the students. Some students were asked to share their solutions with the class. Different approaches were strategically picked so that varying viewpoints were shared. The teacher guided the class to analyze and compare the solutions, and wrapped up with a discussion summarizing what was learned. Following the guidelines in *The Open-Ended Approach* (Becker & Shimada, 1997) the classroom teacher posed the problem, allowed time for individual work and the sharing of responses, and then concluded with a class discussion. The classroom teacher was trained on the instructional procedures by the investigator prior to the start of the intervention. The teacher was instructed on the amount of time to spend on each section. The weekly intervention was put on a power point presentation with appropriate hints. If ever an additional hint was needed, the teacher was instructed to give it to the entire class rather than an individual student. The students were required to provide an alternate solution in addition to their initial solution on all ASWs and the pre and posttests. To encourage students' engagement in generating different solutions on the ASWs, the classroom teacher explained how their responses on the ASWs would be evaluated. The rubric gave credit based on the number of solutions attempted and whether each solution was correct. This type of scoring is a simplified way to evaluate fluency, flexibility, and originality as per *The Open-Ended Approach* (Becker & Shimada, 1997). For additional encouragement, I provided a bag of assorted candies. The students got one piece of candy for every point they received on the weekly ASWs. In addition, the students' names were entered in a weekly drawing for a free lunch card.

The Problems for the Intervention and Their Context

All of the following problems were given to ninth-grade students who were currently taking an Algebra I course. Since many of the problems required skills in operations with polynomials such as factoring and combining like terms, the intervention took place after the students completed the chapters on polynomials and factoring. This was necessary because "when students solve an open-ended problem, they need to use previously learned mathematical knowledge and skills" (Becker & Shimada, 1997, p. 31). The following paragraphs include detailed lesson plans for each of the four weeks of the intervention. The format of the lesson plans follow the structure used in *The Open-Ended Approach*. The three sections included in the lesson plans for each question are: The Problem and Its Context, Expected Responses, and Record of the Classroom Teaching (Becker & Shimada, 1997). The problem and its context lists the problem and explains how it connects to the Algebra I curriculum. The expected responses are a teacher-generated list of likely responses and approaches to the problems. They are classified by viewpoints. The record of classroom teaching details what actually took place in the classroom during the intervention.

Week One Problem 1

The Problem and Its Context

The problem. Find both the total length and the total width of the following rectangle in terms of a and b. The expression inside each individual rectangle is the area of that rectangle.

ab	ab	b^2
a^2	a^2	ab

1. You will be evaluated on this task based on the number of approaches you use to solve this problem. Each approach will be given credit based on accuracy and completeness. It

is important that you show all of your work, and do not erase or black out anything you write.

Pedagogical context. The students are familiar with the concept of length and width of a rectangle. They have also learned about adding polynomials. It is unlikely that the students have had experience putting the two topics together in this way.

Expected Responses

The students should understand they need to find an expression for the length and width of the rectangle. Most students will try to add the expressions using either the correct mathematical rules or incorrect ones. Some students might try to multiply the expressions instead. Expected responses follow.

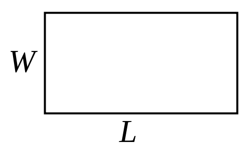
1. $2a + b$ and $a + b$ This is the correct response.
2. a^2b and ab In this response, the students multiplied the expressions instead of adding them.
3. $a^2 + b$ and $a + b$ In this response, the students added the exponents rather than the coefficients when combining like terms.
4. $3ab$ and $2ab$ In this response, the students added the coefficients after they combined like terms.
5. $3b$ and $2a$ In this response, the students transposed the length and the width of the rectangles with area equal to ab.
6. b^3 and a^2 In this response, the students transposed the length and the width of the rectangles with area equal to ab and then multiplied the expressions rather than adding them.

The expected responses numbered 1-4 are the result of correctly labeling the dimensions of the individual rectangles. The responses numbered 5-6 result from incorrectly labeling one of the individual rectangles.

Record of Classroom Teaching

Teaching the lesson. The purpose of this lesson is to introduce the students to the alternative solutions worksheet and to introduce the concept of rectangles that have algebraic expressions for dimensions. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and an alternative solutions worksheet. After every student had each of the handouts, the teacher projected the problem on the board, and read the problem aloud as the students followed along. What was projected was identical to the students' handouts.
2. Since the concept of area of a rectangle was required for this problem, the teacher reviewed the formula with the students and wrote it on the board as shown below.



$$A = LW$$

3. The students were instructed to solve the problem individually and to record their solutions on the alternative solutions worksheet. Since the students were solving two problems during the class period, they were instructed to split their time between the problems as needed. The teacher collected their solutions at the end of the class period.
4. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agree on a score, that score was

assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.

5. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher wrapped up the lesson with a discussion of what was learned.

Week One Problem 2

The Problem and Its Context

The problem. Which of the following two jobs gets better pay?

(Job 1) At Bob's Burgers, you will be paid $9.00 per hour and will be expected to work 20 hours per week. You are required to buy a uniform for $30.

(Job 2) At Terry's Tacos, you will be paid $8.50 per hour and will be expected to work 20 hours per week. There is no required special attire.

This problem was adapted from *Looking Back in Problem Solving* (Cai & Brook, 2006).

Pedagogical context. This problem is not related to the problems on the pre- and post-tests, rather it was designed to keep the interest of the students, and for them to practice solving open-ended problems using the alternative solutions worksheet. The students should have the prerequisite knowledge to come up with solutions to this problem.

Expected Responses

The students could approach this problem in many different ways. The way the students answer the question depends on how they view the cost for the uniform and if they consider how long one intends to work at the job.

1. Some students might make a chart to see how much they would earn from working a 20 hour week at each job. They might just figure out the earnings for one week, or they may look at earnings over multiple weeks. If they look at multiple weeks, they might figure out that the $30 uniform is paid for in three weeks with the extra $.50 per hour.
2. Some students might try writing and graphing the function for each of the two jobs.
3. Some students might just try to write an equation and substitute in values.

Record of Classroom Teaching

Teaching the lesson. The purpose of this lesson was to introduce the students to an open-ended problem and the alternative solutions worksheet. The students already studied linear functions and graphing, so they have all the necessary prerequisite skills to find a solution. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and the alternative solutions worksheet. She also projected the problem onto the board. She read the question aloud and answered questions that pertained to understanding the problem.
2. The students were instructed to solve the problem individually. The teacher explained that they were to come up with at least two solutions. Since they worked on two problems during the class period, they were instructed to split their time between the problems as necessary to finish in the allotted time (one class period). The teacher collected the solutions at the end of the class period.

3. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.
4. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher will wrapped up the lesson with a discussion of what was learned.

Week Two Problem 1

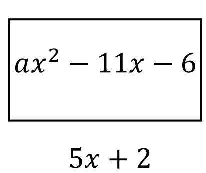
The Problem and Its Context

The problem. Suppose the area of a rectangular billboard is $ax^2 - 11x - 6$ and the length of one of its sides is $5x + 2$. Find the value of a.

Pedagogical context. The students are familiar with the concept of length and width of a rectangle. They have learned how to factor quadratic expressions as well. They have probably seen a similar problem in which the area of a rectangle was given as a quadratic expression and one of the dimensions was given. They had to find the other dimension.

Expected Responses

Since this problem is similar to some found in the textbook, some students may try factoring as a solution method. The unknown quantity *a* makes this problem more challenging than the problems in the book.



1. Students might draw a picture like the one shown here. One way to solve the problem is to factor. Those the recognize that $5x + 2$ is a factor might write the following:

$$(bx - 3)(5x + 2) = ax^2 - 11x - 6$$

2. Those who understand the concept of polynomial multiplication will know that $5b = a$ and $2b - 15 = -11$. Following from the latter two equations, $b = 2$ and $a = 10$. This is the correct answer.
3. Many incorrect solutions can be obtained from this process if students do not fully understand the concepts involved. For example, students might believe that $\frac{a}{2}$ has to be the coefficient of both of the factors, thus writing the equation $\frac{a}{2} = 5$ and coincidentally obtaining the correct solution $a = 10$. Students might believe that $a = 5$ because the other coefficient is 5.
4. Some students might try to divide the polynomials like this $\frac{ax^2-11x-6}{5x+2}$. They do not know how to do synthetic division and it is unlikely they will attempt polynomial long division, so the most likely result would be incorrectly "canceling" the linear and constant terms to obtain $ax^2 - \frac{11}{5} - 3$. Although this is not an equation, students at this level like to "solve" rather than simplify, so they might write $ax^2 = \frac{26}{5}$ and extract some sort of a value for a.

Record of Classroom Teaching

Teaching the lesson. This lesson continued the intervention by expanding on question one from the first week. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and an alternative solutions worksheet. After every student had each of the handouts, the teacher projected the problem on the

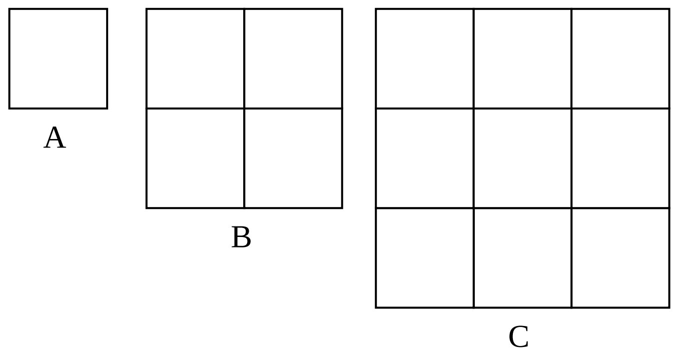
board, and read the problem aloud as the students followed along. What was projected was identical to the students' handouts.

2. The students reviewed the formula for the area of a rectangle the previous week. The classroom teacher asked the class to recall the formula and wrote it on the board.
3. The students were instructed to solve the problem individually and to record their solutions on the alternative solutions worksheet. Since the students were solving two problems during the class period, they were instructed to split their time between the problems as needed. The teacher collected their solutions at the end of the class period.
4. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.
5. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher wrapped up the lesson with a discussion of what was learned.

Week Two Problem 2

The Problem and Its Context

The problem. Below, in example A, 4 sticks make a 1 by 1 square. In B, 12 sticks make a 2 by 2 square. In C, 24 sticks make a 3 by 3 square. How many sticks does it take to make a 10 by 10 square?



Pedagogical context. This problem is not related to the problems on the pre- and post-tests, rather it was designed to keep the interest of the students and for them to practice problem solving open-ended problems using the alternative solutions worksheet. The students have the prerequisite knowledge to come up with solutions to this problem.

Expected Responses

Students will either try to draw the ten by ten figure or look for a pattern to avoid drawing it. In looking for a pattern, students might double count sticks.

1. If looking for a pattern, students might make chart like follows.

Dimension	1	2	3
Number of Sticks	4	12	24

From here, any number of patterns and subsequent steps might follow. Students might see that the dimension is multiplied by 4, then 6, then 8. From there they might expand the chart to go up to 10, thus getting 220 sticks in the ten by ten.

2. Alternately, students might look at the number of sticks that are added each time the dimensions are increased.

Dimension	1	2	3
Sticks added	4	4+8	4+8+12

From here students might notice the solution is just 4 times the sum of the dimensions, i. e. 4(1+2+3+4+5+6+7+8+9+10) = 220 sticks.

3. If students tried to look for a pattern in the picture, they might have noticed there were 24 sticks in the three by three, and incorrectly deduced that a nine by nine is made up of 9 of those so 24 times 9 is 216 sticks, and then try to count the remaining sticks that would make the ten by ten. The problem with that approach or other similar ones is that sticks on the perimeter are counted twice. Similarly, students might think a ten by ten is made up of 100 one by ones with 4 sticks each, so 400 total sticks.

Record of Classroom Teaching

Teaching the lesson. This is an open-ended problem in the sense that there are multiple approaches to find the solution. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and the alternative solutions worksheet. She also projected the problem on the board. She read the questions aloud and answered any questions that pertained to understanding the problem.
2. The students were instructed to solve the problem individually. The teacher explained that they were to come up with at least two solutions. Since they were working on two problems during the class period, they were instructed to split their time between the problems as necessary to finish in the allotted time (1 class period). The teacher collected the solutions at the end of the class period.

3. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.
4. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher wrapped up the lesson with a discussion of what was learned.

Week Three Problem 1

The Problem and Its Context

The problem. Factor $(x-2)^2 + a(x-2) + 3(x-2) + 3a$ completely.

Pedagogical context. The students learned how to factor by grouping and about common factors in the unit on factoring. Problems like this one where a quantity has to be considered as a factor have not been practiced regularly. Although the students have the knowledge to solve this problem, it will be a challenge.

Expected Responses

As students try to solve this problem, some will want to multiply first. Those who understand quantities as factors might try the following.

1. $(x-2)^2 + a(x-2) + 3(x-2) + 3a = (x-2)(x-2+3) + a(x-2+3) =$

   $(x-2+3)(x-2+a) = (x+1)(x-2+a)$ This is the correct answer.

2. Similarly, students might see the common $(x-2)$ and choose to substitute first, say $y = x - 2$, to simplify the expression. Factoring by grouping will yield the same result as above unless the student forgets to substitute back at the end leaving $y(y+3) + a(y+3) = (y+3)(y+a)$.

Record of Classroom Teaching

Teaching the lesson. This lesson continued the intervention by expanding on the topic of factoring. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and an alternative solutions worksheet. After every student had each of the handouts, the teacher projected the problem on the board, and read the problem aloud as the students followed along. What was projected was identical to the students' handouts.
2. The students were instructed to solve the problem individually and to record their solutions on the alternative solutions worksheet. Since the students were solving two problems during the class period, they were instructed to split their time between the problems as needed. The teacher collected their solutions at the end of the class period.
3. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.
4. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss

them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher wrapped up the lesson with a discussion of what was learned.

Week Three Problem 2

The Problem and Its Context

The problem. John solved the equation, $2x^2 - bx + a = 0$, and he got the correct answer: $x = \frac{3}{2} \pm \frac{\sqrt{15}}{2}$. Find the value of a.

Pedagogical context. This problem is related to the other problems in that it is a quadratic equation. The students must be familiar with the symbol ± to solve this problem. The students should also be familiar with solving systems of equations.

Expected Responses

Students might have trouble solving this problem because of the format of the values of x because radical expressions are more difficult to work with than integers. After substituting in the two values of x, the students have to choose how and when to simplify, and if they should round or keep the exact values throughout. Students may make errors in arithmetic and algebra throughout the lengthy process of solving this problem. A possible solution path follows.

1. Substitute each value of x into the equation. $2(\frac{3}{2} + \frac{\sqrt{15}}{2})^2 - b\left(\frac{3}{2} + \frac{\sqrt{15}}{2}\right) + a = 0$ and $2(\frac{3}{2} - \frac{\sqrt{15}}{2})^2 - b\left(\frac{3}{2} - \frac{\sqrt{15}}{2}\right) + a = 0$. If students were able to get this far, then they might notice they have a system of equations to solve. This system lends itself to the elimination/linear combination method. If the second equation is multiplied by -1 and the equations are added together, then the a's will be eliminated. After a lot of simplification, the remaining equation is $6\sqrt{15} - b\sqrt{15} = 0$, thus $b = 6$. Substituting the value of b

back into one of the equations gives $a = -3$. I believe all of the students will approach this from an algebraic viewpoint, but few will get the correct answer.

Record of Classroom Teaching

Teaching the lesson. This is an open-ended problem in the sense that there are multiple approaches to find the solution. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and the alternative solutions worksheet. She also projected the problem on the board. She read the questions aloud and answered any questions that pertained to understanding the problem.
2. The students were instructed to solve the problem individually. The teacher explained that they are to come up with at least two approaches to solving the problem. Since they were working on two problems during the class period, they were instructed to split their time between the problems as necessary to finish in the allotted time (1 class period). The teacher collected the solutions at the end of the class period.
3. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.
4. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare

solutions according to the viewpoint involved. The teacher will wrapped up the lesson with a discussion of what was learned.

Week Four Problem 1

The Problem and Its Context

The problem. Solve the equation: $4(2x-3)^2-36=0$.

Pedagogical context. Since the students have been working on quadratic expressions and equations for the last three weeks, they should feel comfortable approaching this question from multiple viewpoints.

Expected Responses

Even though this problem can be approached from multiple viewpoints, I believe most students will try the algebraic approach first. As a second solution method, some may try a graphic solution.

1. The following is an example of an algebraic approach.

$$4(2x-3)^2-36=0$$

$$4(2x-3)^2=36$$

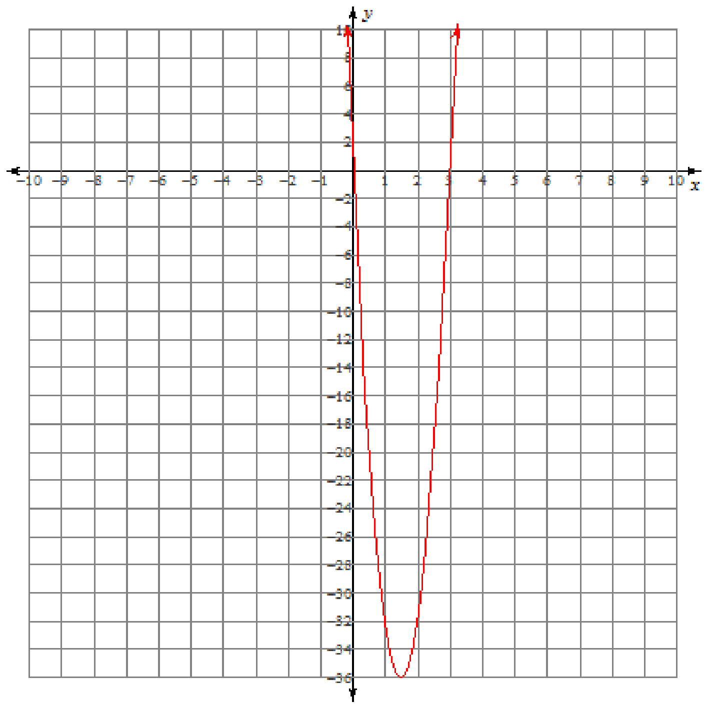
$$(2x-3)^2=9$$

$$2x-3=\pm 3$$

$$x=\frac{3\pm 3}{2}$$

$$x=3 \text{ or } 0$$

2. Graphically, students might do the following. If they understand the concept of zeros, they will see the solutions.



Record of Classroom Teaching

Teaching the lesson. This lesson continued the intervention by expanding on the topic of factoring. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and an alternative solutions worksheet. After every student had each of the handouts, the teacher projected the problem on the board, and read the problem aloud as the students followed along. What was projected was identical to the students' handouts.
2. The students were instructed to solve the problem individually and to record their solutions on the alternative solutions worksheet. Since the students were solving two problems during the class period, they were instructed to split their time between the problems as needed. The teacher collected their solutions at the end of the class period.
3. To evaluate each student's performance on the alternative solutions worksheet, the scoring rubric was used. Three graders independently scored each student's paper and the

results were compared. If two out of three agreed on a score, that score was assigned. In the case that all three disagreed, the score was discussed until at least two agreed. Positive comments were written on each student's paper if possible to encourage future participation.

4. During a second class period the same week, the graded worksheets were given back to the students. The teacher had some of the students present their solutions and discuss them with the whole class. The students were encouraged to analyze and compare solutions according to the viewpoint involved. The teacher wrapped up the lesson with a discussion of what was learned.

Week Four Problem 2

The Problem and Its Context

The problem. Suppose $x^2 - 6x + b = 0$ and $y^2 - 6y + b = 2$ are equivalent to $(x - a)^2 = 7$ and $(y - a)^2 = k$, respectively. Find the value of k.

Pedagogical context. Solving this problem requires the same tools the students have been using throughout the intervention.

Expected Responses

This is an especially challenging problem. The students will likely use an algebraic approach. What follows is one approach.

1. Completing the square on the first equation yields $(x - 3)^2 = 9 - b$. If this equation is equivalent to $(x - a)^2 = 7$, then if $a = 3$ then $b = 2$. Completing the square on the second equation yields $(y - 3)^2 = 11 - b$. Since this equation is equivalent to $(y - a)^2 = k$, we can see that $a = 3$ so it follows that $b = 2$ and $k = 9$.

Record of Classroom Teaching

Teaching the lesson. This is an open-ended problem in the sense that there are multiple approaches to find the solution. The lesson proceeded as follows.

1. The classroom teacher handed out the problem and the alternative solutions worksheet. She also projected the problem on the board. She read the questions aloud and answered any questions that pertained to understanding the problem.
2. The students were instructed to solve the problem individually. The teacher explained that they were come up with at least two solutions. Since they were working on two problems during the class period, they were instructed to split their time between the problems as necessary to finish in the allotted time (1 class period). The teacher collected the solutions at the end of the class period.
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Position of the Researcher

I am a high school mathematics teacher. I have been teaching high school for 11 years. The majority of the classes I have been teaching for the last 6 years are low track freshman Algebra/Geometry. Most of my students are considered at-risk due to low standardized test scores in reading and mathematics, and/or a history of failure prior to high school. I have co-taught these math courses with a special education teacher for the last 6 years, although not all of my students have IEP's. The close interactions I have had with my co-teacher have helped shape my views towards struggling students. Before I worked with an expert in special education, I did not believe the impact a student's reading level could have on his/her math performance. I knew it affected performance on word problems, but I never realized the full extent of the issue as it applies to note taking, instructions, and disposition. As such, I am an advocate for students who are academically at-risk. I spend my time outside of the classroom searching for ways to improve mathematics instruction for those students.

I am representative demographically of the most common teacher in the current workforce because I am white and middle class (Terrill & Mark, 2001). As explained by Hammersly and Atkinson (1995), access is not limited to the ability to be physically at the research setting, but includes gaining a social and psychological entrance into that which is being studied. It may seem that my appearance and culture could set me at a disadvantage when studying at-risk students, but the relationships I developed with current and former students during my tenure at the high school earned me a favorable reputation with the at-risk group, thus allowing me to gain access to the participants of this study, even though they were not my students.

CHAPTER 3

RESULTS

This study investigated the effectiveness of an intervention where students in the experimental group were instructed in the use of Alternative-Solution Worksheets (ASWs) for a period of four weeks. A comparison group did not receive the intervention. This study also explored the relationships between problem solving scores on pre-tests, post-tests, and ASW activities, as well as the students' EXPLORE9 scores. Lastly, this study investigated the extent that students report the use of "look back" strategies when completing ASW activities and the relationship between their reported use of "look back" strategies and their performance on ASW activities.

The quantitative findings of this study indicated that students in the experimental group had better post-test scores than students in the comparison group after controlling for the students' pre-test scores. Additionally, the students' EXPLORE9 math and reading percentiles were significantly positively correlated with their average ASW scores. The qualitative findings of this study indicated that although "look back" can occur at many levels, it is the "look back" that occurs in the context of higher order thinking skills that may result in increased mathematical problem solving abilities. Additionally, positive affective changes were evident despite little improvement in the students' mathematical problem solving abilities.

Quantitative Findings

A one-way analysis of covariance (ANCOVA) was conducted to determine if there was a significant difference in the mean post-test scores of the two groups. The independent variable, treatment group, had two categories: comparison group and experimental group. The dependent variable was the students' post-test score and the covariate was the students' pre-test score. A preliminary test evaluating the homogeneity-of-regression (slopes) assumption indicated that the relationship between the covariate and the dependent variable did not differ significantly as a function of the independent variable, $F(1, 28) = .08$, $p = .78$, thus the assumption is met. To meet the normality assumption, a logarithmic transformation was done on the post-test scores. After the transformation, 95% Bootstrap CIs were computed for each group (14, 18) using 10,000 resamples for kurtosis and skew. The 95% CIs for kurtosis and skew for the experimental group are [-0.9502, 5.9251], and [-1.1519, 2.1442], respectively. The 95% CIs for kurtosis and skew for the comparison group are [-1.9510, 0.5376], and [-0.9597, 0.7237], respectively. Since zero is contained in all of the confidence intervals, the data meets the normality assumption because the population parameters for normal skew and kurtosis are both zero. Levene's Test for homogeneity-of-variances indicated that the variances of the post-test scores are approximately equal between the two groups, $F(1, 29) = .003$, $p = .960$. The ANCOVA was computed with four covariates: pre-test scores, EXPLORE9 math percentile, EXPLORE9 reading percentile, and the students' second semester Algebra I final average. The results were significant, but only the pre-test covariate significantly contributed to the variance in post-test scores, so another ANCOVA was run using only the pre-test as a covariate. This ANCOVA was significant, $F(1, 29) = 4.84$, $p = .036$, and $R^2 = .345$, a medium effect size according to Cohen (1992). The adjusted means for the experimental group and the comparison group were 1.07 and .65 respectively, while the actual means for the experimental group and the comparison group were 1.01 and .73 respectively, thus justifying the use of ANCOVA with nonequivalent groups.

To more completely answer the first part of the first research question, "what relationship is there between problem solving scores on pre-, post-tests, and students' performance on ASW activities," descriptive statistics and a t-test analysis were used to examine score differences between scores on the pre- and post-tests as well as the difference in the number of responses given on the pre- and post-tests (see Table 2 and Table 3). Both the mean score and the number of responses decreased between the pre- and post-tests for both groups. The decreases were significantly different from the pre-tests in all cases except for the number of responses the experimental group gave, which means that despite getting lower scores on the post-test, the experimental group still made an equivalent number of attempts to solve the problems on the post-test as they did on the pre-test, whereas the comparison group's number of attempts decreased along with their scores.

Table 2

Descriptive Statistics of Problem Solving Scores and the Number of Responses on the Pre- and Post-Tests

	Experimental Group			Comparison Group		
Measure	*n*	*M (SD)*	95% CI	*n*	*M (SD)*	95% CI
Pre-Test Score	18	4.11 (2.59)	[2.82, 5.40]	14	5.14 (2.83)	[3.51, 6.77]
Post-Test Score	18	2.28 (2.05)	[1.26, 3.30]	14	1.43 (1.34)	[.65, 2.20]
Pre-Test Responses	18	5.50 (2.33)	[4.34, 6.66]	14	7.07 (1.90)	[5.97, 8.17]
Post-Test Responses	18	4.56 (1.25)	[3.94, 5.18]	14	5.21 (1.76)	[4.20, 6.23]

Table 3

Paired Samples T-Test for the Difference in Problem Solving Scores and the Number of Responses on the Pre- and Post-Tests

	Paired Differences							
	M	SD	SEM	95% CI		*t*	*df*	Sig. (2-tailed)
				Lower	Upper			
Pre EG vs. Post EG	1.83	2.20	.519	.74	2.93	3.53	17	.003
Pre CG vs. Post CG	3.71	2.16	.58	2.47	4.96	6.42	13	.000
Res.Pre EG vs. Res.Post EG	.94	2.51	.591	-.30	2.19	1.60	17	.129
Res.Pre CG vs. Res.Post CG	1.86	1.61	.43	.93	2.79	4.32	13	.001

Note. EG = Experimental Group; CG = Comparison Group; Pre = pre-test score; Post = post-test score; Res = number of responses

To answer the second part of the first research question about the students' performance on the ASW activities, a correlation analysis was performed to understand the relationship between the weekly intervention scores and the scores on the pre- and post-tests (see Table 4). There were no significant correlations between the students' average ASW scores and the pre- and post-test scores.

To answer the second research question, "what relationship is there between the students' EXPLORE9 scores and scores on the pre- and post-tests, and ASW activities, a correlation analysis was done. As seen in Table 4, the student's average ASW scores were significantly correlated with both their EXPLORE9 math and reading percentiles.

Table 4

Pearson Correlations between Average ASW Performance, Pre-Test, Post-Test, the Difference of the Pre- and Post-Test Scores, and EXPLORE9 Math and Reading Percentiles

Measure	1	2	3	4	5	6
1. Pre-Test	-	.57*	.45	-.64**	.32	.36
2. Post-Test	.57*	-	.17	.26	.25	.25
3. Average ASW	.45	.17	-	-.37	.63**	.50*
4. Difference in Pre- and Post-Test	-.64**	.26	-.37	-	-.15	-.19
5. EXPLORE9 Math Percentile	.32	.25	.63**	-.15	-	.51*
6. EXPLORE9 Reading Percentile	.36	.25	.50*	-.19	.51*	-

*p < .05, two-tailed. **p < .01, two-tailed.

Qualitative Findings

This study sought to investigate the extent that students report “look back” strategies when completing Alternative Solution Worksheet (ASW) activities, and the relationship between the subjects’ reported use of “look back” strategies and their performance on ASW activities. Although this study initially sought to investigate the extent to which students report the use of “look back” strategies, I observed that the participants of this study encountered a tremendous obstacle to using “look back” strategies to get correct solutions in the context of the intervention. Perhaps the questions were too difficult, or maybe the format of open-ended questions were so different from the students’ classroom experiences that they were unable to use “look back” effectively most of the time. Once I realized how much difficulty the students had using “look back” successfully, I began to investigate why they had trouble and how they were using “look back". Despite the idea that the participants were unable to utilize "look back" strategies effectively, several patterns emerged within the transcripts of "think alouds", focus groups, observation notes, and the students' writings. These patterns include examples of student "look back" within specified levels of Bloom's Revised Taxonomy (BRT), student reactions to the process, affective changes, and strong and weak uses of "look back."

Blooms' Revised Taxonomy

The six levels of BRT (see Figure F6) provided a lens that I was able to use to analyze students' "look back" techniques as related to their struggles. BRT helped me understand how the students were using "look back" and why they were largely ineffective. One particular example, from a student's "think aloud" illustrates the taxonomy at work.

Remembering. The lowest level of BRT is simply recalling information. I observed the students doing this frequently throughout the process. During the "think aloud" process, one student repeated "area equals length times width" after he read the question as well as at other times. On students' ASW worksheets, many students wrote down the area formula before proceeding with the problem. In Figure F7, the student "remembered" that $(x-1)^3 = (x-1)(x-1)(x-1)$, but failed to "understand" how to find the product when she incorrectly simplified it as $x^3 - 3$ in both of her solutions. In Figure F8, this student "remembered" proper exponent rules but did not "understand" the context enough to "apply" them and execute a solution.

Understanding. The next level of BRT is explaining or interpreting ideas or concepts. For example, after the "think aloud" in which one student recalled the formula for area of a rectangle in the understanding phase, he noted that "Area of the rectangle equals $ax^2 - 11x - 6$, and one of the sides is $5x + 2$, so I'm going to try $5x + 2$ in parenthesis multiplied by

$ax^2 - 11x - 6$, no we're dividing." He even drew a figure that accurately represented the information (see Figure F9). This student understands how each piece of the given information fits in the context of the problem. Figures F10 and F11 show two different examples of understanding from the same question. In Figure F10, the student understands the relationship between area and length to realize she should divide. In Figure F11, this student also recognizes that relationship when she notates $(5x + 2)(?) = ax^2 - 11x - 6$. In all three of the above instances, the students understand the relationship between the expressions given, but they cannot move on to the next level of BRT to get a solution.

Applying. The third level of BRT is using the given information to execute a solution. At this level, I observed the students trying to do some mathematical operations to solve the problem. Many students were stuck at this point. During the "think aloud", one student wavered between the applying level and the next level, analyzing. In the previous example, he realized he should be dividing, rather than multiplying. His process follows.

> So $(5x + 2) \div (ax^2 - 11x - 6)$ that makes …no… I will multiply it. [He goes back to multiplying even though he knows he should be dividing.] So $5x + 2$ multiplied by that, is $5ax^3 - 55x^2 - 30x$, then 2 times ax^2 which would be $2ax^2$, then 2 times $-11x$ is $-22x$ and then 2 times -6 is -12. Like terms so it would be $5ax^3 - 107x + 2ax^2 - 12$, then that wouldn't work because it's supposed to be length times width equals area and the area is $ax^2 - 11x - 6$, so we will do $5x + 2$…I don't know if I add, divide, multiply, or subtract, yeah I can try to subtract, that's what I will do. [Again, he realizes that he should not have multiplied. He thinks he must add, subtract, multiply, or divide, so he settles for subtraction next even though he already stated he has to divide.] So I can take $ax^2 - 11x - 6$ and subtract $5x + 2$ from that and that gives me $ax^2 - 16x - 4$. No, I gotta find the area I can't factor it so I'm going to come back to this one later. [Once again, he knows what he did was wrong, but he cannot figure out how to make it right. He is using look back here, but at the levels of understanding and applying.] Question 1 again. Area equals length times width. Let's see, we can't factor the $ax^2 - 11x - 6$. We can't factor it, and we can't subtract it so how are we supposed to …So we may have to divide, no we can't divide. So $ax^2 - 11x - 6 = 5x + 2$ so we can subtract the $11x$ from both sides,

> then we can…this looks right. So $ax^2 = 16x + 8$, then divide the x^2 so it will be $a = \frac{16x+8}{x^2}$ and that's what the area is…no that's not what the area is…no, it's supposed to be area equals length times width and the $5x + 2$ has to be the length or the width and the $16x + 8$ has to be something. So my final answer will be $5x + 2$ equals the length and then $16x + 8$ equals the width and the area is $ax^2 - 11x - 6$.

At this point in the "think aloud", the student is trying to analyze his solution (the next level of BRT), but he stopped short of actually doing it. Figure F9 is the student's written work for this problem.

Analyzing. The fourth level of BRT is breaking information into parts to explore understandings and relationships among the data. In mathematical problem solving, this level differs from the previous level, applying, in that students who are analyzing are pointedly pursuing a solution based on their understandings of the relationships in the problem rather than just randomly doing mathematical operations for the sake of doing something. In the example above, the student never really got to this level. He knew he was supposed to divide, as he stated multiple times, but since he did not figure out how to do the division, he settled for multiplying and subtracting.

Evaluating. The fifth level of BRT is justifying a decision or course of action. In mathematical problem solving, evaluating can be accomplished by explaining why one chose a certain solution method and then checking the solution obtained to validate it. In the prior example, the student gave up and ended the "think aloud". He settled for subtracting, even though he stated earlier that subtracting was not a correct method. He also knew that length times width should equal area, but he did not actually multiply his solutions for length and width to try it. When "look back" is most important, to assess the validity of a solution, he failed to do it.

Creating. The sixth and highest level of BRT is generating new ways of viewing things, or generating alternative solution methods, as was requested of the students during this study. The students were largely unsuccessful at this level.

Students' Reactions to the Intervention

When comparing student responses from transcriptions of my observations and focus groups, several patterns became evident. The following section on student reactions includes tell me what to do, if I got an answer, why should I do it again, if I'm writing, I'm trying, and I was thinking about it, but I didn't write it down.

Tell me what to do. I observed that the students seemed to be uncomfortable with the open-ended format of the questions. The first focus group was conducted on a Friday; the day after the experimental group had their first discussion day. When asked to describe their feelings about the intervention questions, one student said, "It was kind of sort of hard because you weren't very specific and not everyone's assumptions are the same." Another student added, "like we were supposed to know what we were supposed to do, but it didn't say." When I asked how the questions could be improved, one student said, "be more specific" and added "technically my answer was both of them since you weren't specific about now or weeks passing" as an excuse for not having a clear solution to the Bob's Burgers and Terry's Tacos question.

If I got an answer, why should I do it again? I observed that students did not want to find alternate solutions if they believed they already had a correct answer. During the second focus group, one student remarked "it was kind of difficult because I pictured how I was going to do it in my mind, but to picture it to do it in another was kind of tricky for me." Another example of BRT at work, finding an alternate solution requires the students to evaluate their current solution and then create by finding an alternate solution.

If I'm writing, I'm trying. I observed that when students were not sure how to proceed with the problem, they just wrote down more and different math. It was as if they believed that they must do different math to find an alternate solution (or any solution). During the second focus group, one student said, "people think they should just write down stuff, you're at least trying when you are writing." Another student said, "it (hearing a correct solution) kind of made me feel like I wasn't thinking about it as much as I should have, I kind of just wrote something down and didn't get it right." After addressing the issue of doing different math just for the sake of writing something down, one student responded, "that's like saying I wrote an essay, it just wasn't on the topic we were supposed to write about." The other students agreed and admitted they had done that before in math class.

During the discussion day of week two, three viewpoints were written on the board for question 2a. Two of the three were incorrect methods. The three viewpoints were:

1. $(5x + 2)(ax^2 - 11x - 6)$
2. $(ax^2 - 11x - 6) - (5x + 2)$
3. $(5x + 2)(w) = ax^2 - 11x - 6$

When the teacher asked the students to comment on the three methods, they quickly offered appropriate comments. One said, "#1 can't work because you can't take side times area". Another student said, "#2 doesn't work because you can't subtract the length from the area". Several students then admitted that they tried those two methods despite knowing better.

During the fourth week, one student wrote on the worksheet, "I know my answers are wrong. What I did was I plugged the $(x - a)^2 = 7$ in for *x* and $(y - a)^2 = k$ in for y. It was the only thing I could think". Her equation had multiple equals signs because she was substituting in an equation for a variable in another equation.

There were numerous instances when students verbalized their frustration about not getting points when they wrote so much. One student said, "I thought I would get 3 not 1 because I explained so much" and another student said "I really tried hard, I tried so hard I got a zero."

This theme is an example of student stopping at the applying level of BRT. When they are doing random mathematical operations, they are not stopping to analyze if what they are doing is correct for the problem. The students did not analyze until they were questioned by the teacher on the second day of the intervention. At that point, they were verbally able to analyze the process and evaluate the solutions. The discrepancy between the students verbal and written work was another theme.

I was thinking about it, but I didn't write it down. On multiple occasions, I observed students' thought process' did not match what was written on their papers. Two of the instances involved the second question from week 1 about Bob's Burgers and Terry's Tacos. On the students paper, all she wrote was, "Job 2 because you won't have to pay for the uniform fee. You will keep more money." She did not show any calculations. When asked about that question during the focus group, she said, "I wrote down Terry's Tacos because I was just thinking because since at Bob's Burgers you have to pay the $30 for the uniform and every so often you have to buy a new uniform because if it ripped, so that's just what I was thinking." This student would have gotten more points had she written down that explanation.

Another student made many calculations on her paper. For the initial solution, she calculated the weekly pay for both Bob's and Terry's. She got $150 and $170, respectively. To answer the question, she wrote, "Bob's Burgers pays you $9.00 an hour + the cost of uniforms.

Terry’s Tacos pays you $8.50 an hour, no uniform. Terry’s Tacos pays more.” When asked about that question during the focus group, she replied, “most work places require you to buy at least five uniforms in the beginning so you have some for the week and you are not coming to work smelly, like if it’s a required uniform, they usually have you buy at least five at a time, not just one.” If she had added that explanation on the paper she turned in, she would have gotten more points per the rubric.

A third student also had a discrepancy between what he was thinking and what he actually wrote down. On the first question for week 2, when asked how many sticks make up a 10 by 10 square, on his paper he drew a 5 by 5 square and a 10 by 10 square. He had two answers written down. His first solution was, “10X10 it would be 98 sticks”. He also had “10X10=220” written further down on his paper under alternative solutions. Lastly, he had “I got that answer by drawing the 10X10” written on his paper but it was unclear which answer it went with. This particular student was recorded thinking aloud during this question. The transcript of his “think aloud” tells a much different story about how he got the solution, as follows:

> So I’m going to draw it and see how many sticks make a 10 by 10.
> 1 box is 4 sticks so 2 boxes is 8 sticks so 3 boxes has to be 12 sticks, no 12 sticks make a 2 by 2 square, wait 12 sticks in a 2 by 2…I’m going to draw the 24 sticks which makes a 3 by 3 and keep adding to make a 4 by 4. Oh snap, that’s what it means, so a 10 by 10 would be 10 across and 10 down, I will continue off the 3 by 3, ok I’m going to make 4 by 4 now and add up the sticks, so 40 for a 4 by 4, continue to make a 5 by 5, I am counting the sticks down and then across and then adding the together, so it would be 49 sticks for a 5 by 5.
> I’m going to draw another 5 by 5 so my answer will be for the 10 by 10, it will be 98 sticks (49 times 2). I got the 5 by 5, which has 49 sticks and then another 5 by 5 which is 49, so 49+49=98, wait, oh no it’s a 4 by 5, if forgot to add another one to that side. Now I have to count all over again. Oh so a 5 by 5 equals 60 not 49, my mistake and if I add 5 by 5 again it’s another 60 so that’s 120, but if I made a mistake, I’m going to check it, I’m going to do 10 squares across and 10 squares down so this is going to be a lot of writing and a lot of squares, whoa that’s a lot of sticks, it looks like its 120 maybe possibly more, hope its 120 though.

> Alright I drew the 10 by 10 now I just got to count the sticks [counting out loud, gets near 120] ok let me check [counting some more]. 11 times 10 is 110, oh yeah, so 110 sticks going up and down, now counting horizontal oh so it's 11 again times 10 so 110 +110=220. I got that answer by drawing the 10 by 10 and counting the sticks up and down in each row so 11 times 10 is 110 and its 11 sticks across times 10 so 110 +110 =220.

That thought process was not visible in his written work at all (see Figure F12). Verbally, during the think-aloud, the student was able to navigate all the levels of BRT, leading him to a correct solution. Despite this, he was not able to communicate his thought process on paper.

Affective Changes

Initially I was interested in the students' dispositions towards the intervention. I observed in the beginning of the intervention that the students were discouraged. However, a clear pattern of change in student disposition became evident over the course of the four-week intervention.

Week 1. During the first focus group, when asked about her feelings towards the pre-test, one student said, "that was like really hard, I felt like I would never learn it, like know how to do it." On the discussion day of the first week, as she walked into the classroom, she said, "that stuff is impossible, like Albert Einstein stuff, like college." Also during the first focus group, some other students commented, "it made me feel stupid, I thought it was stupid as well" and "it made me feel like I was just dumb, like I didn't know nothing." As the teacher was leading the discussion, I observed that many students were not engaged, perhaps because they thought the problems were too hard. In particular, there were four girls sitting close to me who were talking to each other rather than listening to the teacher.

Week 2. On the discussion day the second week, however, the overall atmosphere in the classroom seemed more positive. Only three of the students scored higher than the previous week, but perhaps the questions were more accessible. Of the four girls who had not paid attention last week, three were engaged. Near the beginning of the discussion, one of the girls turned around and told her friend to “shut up and listen”. She was one of the three students who scored higher than the first week.

Week 3. On the discussion day of the third week, I observed that every student was engaged. Seven students scored higher than they did the second week, but the scores were not great overall. The average ASW scores for weeks 1 through 4 were 2.5, 2, 2, and 1.5 respectively. Even with the low scores, the students were interested. One student called the teacher over to his desk to find out how he could have done better.

Week 4. The trend continued during the discussion day the fourth week. Students wanted to come up to the board to share their answers. One was even eager to share her mistake. She placed a parenthesis in the wrong place. At one point, a student was talking instead of listening and another student shushed him. My observation notes reflected that the students were listening, engaged, and excited even though they did not score very many points on their ASWs.

During the last focus group, I observed an improvement in the students’ attitudes towards the intervention. The focus group took place on the day before the post-test. In the conversation, one student randomly offered her thoughts on the upcoming post-test. She said, “I think I can do pretty good on the post-test”. Others chimed in, “yeah, yeah, we feel confident”. Even though their scores had not improved, they still thought they could do well.

Strong and Weak “Look Back”

Utilizing both BRT and Lee’s (2009) findings of strong “look back” and weak “look back” strategies, I compared data to look for patterns that could be categorized accordingly. As I analyzed student iterations, I noted weak “look back” represented student work at lower levels of BRT, whereas strong “look back” represented student work at higher levels of BRT. Weak “look back" generally did not result in the student finding a correct solution to the problem, and strong "look back" generally did result in a correct solution. The following sections provide examples of this analysis in terms of weak "look back" and strong "look back".

Weak "look back". I observed many instances of weak "look back" during the study. At first, I did not even recognize those instances as "look back". It was not until after I started coding the data through the lens of BRT, that I could see the students' attempts. For example, on the first question, the student says "Area of the rectangle equals $ax^2 - 11x - 6$, and one of the sides is $5x + 2$, so I'm going to try $5x + 2$ in parenthesis multiplied by $ax^2 - 11x - 6$, no we're dividing, so $(5x + 2) \div (ax^2 - 11x - 6)$ that makes …no… I will multiply it". When he decided that he should divide, he was using "look back", however he did not actually end up dividing, so I would consider that weak "look back" because he did not follow through. Instead, he took another route. He goes back to multiplying even though he knows he should be dividing. This student continued to utilize weak "look back" in the following passage.

> So $5x + 2$ multiplied by that, is $5ax^3 - 55x^2 - 30x$, then 2 times ax^2 which would be $2ax^2$, then 2 times $-11x$ is $-22x$ and then 2 times -6 is -12. Like terms so it would be $5ax^3 - 107x + 2ax^2 - 12$, then that wouldn't work because it's supposed to be length times width equals area and the area is $ax^2 - 11x - 6$, so we will do $5x + 2$…I don't know if I add, divide, multiply, or subtract, yeah I can try to subtract, that's what I will do.

After he multiplied, he again used "look back" when he stated that it is supposed to be length times width equals area. This time, the "look back" had the potential to be strong "look back", but fell short when the student decided to subtract after he already indicated that he knew he should be dividing and that length times width equals area. I stated earlier that during this episode, the student was wavering between the applying and the analyzing levels of BRT, which also explains why these incidences of "look back" had the potential to become strong "look back". He just could not make the connection between factoring and division to get past the idea that he must add, subtract, multiply or divide. Attempting to divide, he said, "So I can take $ax^2 - 11x - 6$ and subtract $5x + 2$ from that and that gives me $ax^2 - 16x - 4$. No, I gotta find the area I can't factor it so I'm going to come back to this one later". The weak "look back" here did not allow him to get a solution. When he came back to the problem later, more weak "look back" occurred. He said, "Question 1 again. Area equals length times width. Let's see, we can't factor the $ax^2 - 11x - 6$. We can't factor it, and we can't subtract it so how are we supposed to …So we may have to divide, no we can't divide." He kept going back to division, more examples of weak "look back", but ended up doing something else each time. Next, he set the area equal to the side and worked on solving the resulting equation.

> So $ax^2 - 11x - 6 = 5x + 2$ so we can subtract the $11x$ from both sides, then we can…this looks right. So $ax^2 = 16x + 8$, then divide the x^2 so it will be $a = \frac{16x+8}{x^2}$ and that's what the area is…no that's not what the area is…no, it's supposed to be area equals length times width and the $5x + 2$ has to be the length or the width and the $16x + 8$ has to be something. So my final answer will be $5x + 2$ equals the length and then $16x + 8$ equals the width and the area is $ax^2 - 11x - 6$.

In this final passage, he utilized "look back" when he said "no, it's supposed to be area equals length times width and $5x + 2$ has to be the length or the width", but that was back to the remembering and understanding levels of BRT, thus all examples of weak "look back". He did not get a correct solution. See Figure F9 for the student's written work for this question.

Strong "look back". When working on the second question for week two, the same student utilized strong "look back". While in the process of drawing a 10 by 10 square to count the sticks, the student first thought he could just draw a 5 by 5 and double the number of sticks for a 10 by 10. During his "think aloud" he said:

> I am counting the sticks down and then across and then adding them together so it's 49 sticks for the 5 by 5. I'm going to draw another 5 by 5 so my answer will be for the 10 by

> 10 it will be 98 sticks, 49 times 2, wait, oh no it's a 4 by 5, I forgot to add another one to the side." Realizing he made a mistake, he used strong "look back" to evaluate what we was doing, and then correct his mistake. Next, he got the correct number of sticks for a 5 by 5 (60 sticks) and theorizes that a 10 by 10 has 120 sticks (60 times 2).

Instead of ending the problem at that point, when he had an answer, he decided to check his answer, illustrating the evaluating level of BRT. At first he was going to draw the 10 by 10 and count all of the sticks, but during that process he recognized a pattern, and went with it, which was utilizing an alternative solution technique, which is at the creating level of BRT. In his final solution, he stated: "Alright I drew the 10 by 10 now I just got to count the sticks, ok let me check, 11 times 10 is 110, oh yeah, so 110 sticks going up and down and vertical its 11 again times 10 so 110 plus 110 equals 220". As he recognized the vertical and horizontal pattern, he said "ok let me check". That phrase is where the strong "look back" is taking place. It could only be categorized as strong "look back" from the context of the "think aloud".

Conclusion

This study sought to investigate the effectiveness of a four-week intervention during which students were instructed in the use of ASWs. Additional comparisons explored the relationships between problem solving scores on the pre-, post-tests, ASW activities, and the students' EXPLORE9 scores. This study also investigated the extent to which students utilized "look back" strategies as well as the relationship between their reported use of "look back" and their performance on the ASWs.

The quantitative findings indicated that students in the experimental group did perform better on the post-test than students in the comparison group, after controlling for their pre-test scores. In addition, there was a positive correlation between students' EXPLORE9 math and reading percentiles and their performance on the ASWs.

The qualitative findings indicated that “look back” occurred at all six levels of BRT, but it was only the strong “look back”, which occurred at the upper three levels that resulted in correct solutions, thus student performance on ASW activities depended on the type of “look back” being utilized.

CHAPTER 4

DISCUSSION

The goal of this study was to see if requiring students to provide alternate solutions to open-ended math problems would increase the instances of “look back”, thus improving overall problem solving skills. Understanding how and why students do or do not utilize “look back” strategies effectively became a secondary goal once I realized there was little improvement happening with the problem solving scores. Using BRT provided a cognitive lens to the students’ thinking process and levels of development that went above and beyond just studying the students’ procedures during the problem solving process (Hashim, 2014). I entered this study with the belief that the participants would improve their mathematical problem solving skills despite their “at-risk” labels. I thought students would build upon the recurring concepts in the intervention, and acquire “look back” strategies on the discussion day to use the following weeks. Although the scores on the pre- and post-tests and the ASWs do not indicate an improvement, some affective changes and other findings emerged during the qualitative analysis.

Research Question 1

The first research question sought to investigate the relationship between problem solving scores on the pre-test, post-test, and ASW activities. Some unexpected results of this study were the decreases in problem solving scores from the pre-test to the post-test as well as on the weekly intervention problems (see Table 2). There were a few possible reasons for these results:

First, the post-test was given on the last full day of school. High school students’ motivation often wanes by the last day of school. In addition, that last week was only a four day week because of the Memorial Day holiday, and all four of those days had an intervention activity in the experimental group. The students may have been tired of doing the problem solving by Friday.

Second, the students in the two groups were classified as at-risk by their standardized test scores in math and reading. Many of them were struggling with basic Algebra even with the support class. The intervention was only four weeks long, and only one day each week was dedicated to discussion of the problems with the teacher. Perhaps the time frame was not long enough for the students to become comfortable enough with the open-ended problems to make significant improvement. This supports the idea that understanding and ability to use problem solving processes develop slowly over time (Lesh, 1985). Pugalee (2004), however, conducted an intervention over a six-day period, during which the students were able to be successful. In contrast to this study, his data collection was preceded by two weeks of intensive journaling during which the students were trained to answer open-ended math problems. Teacher feedback was given so the students could correct and extend their solutions in both quantity and quality. This suggests that students need to be properly trained to communicate mathematically, and that process should include ongoing feedback so the students can rethink and revise, and opportunities to engage with other students about the process (Green & Emerson, 2010; Webb et al., 2014).

The number of responses given also decreased for both groups from pre-test to post-test, but a paired samples t-test indicated that that decrease was not significant for the experimental group (see Table 3). The students in the experimental group made approximately as many attempts to solve the problems on the post-test as they did on the pre-test. They did not score as well, but at least they were trying. Perhaps the confidence they built over the course of the intervention is the reason why made more attempts than the comparison group.

Research Question 2

The second research question sought to investigate the relationship between the students' EXPLORE9 standardized test scores and their scores on the pre-test, post-test, and ASW activities. Table 4 summarizes the Pearson correlation coefficients for all of those measures. The only significant correlations were between the students' average ASW scores and their EXPLORE9 math and reading percentiles. This makes sense because the EXPLORE9 test measures students' reasoning skills. In particular, the mathematics test measures the students' abilities to, "solve practical quantitative problems at four cognitive levels: knowledge and skills, direct application, understanding concepts, and integrating conceptual understanding" (ACT, Inc., 2013, p.5). The reading test measures "students' level of reading comprehension as a product of skill in referring and reasoning" (ACT, Inc., 2013, p.6). Thus students who did not score well on the EXPLORE also did poorly on the ASW activities. This result is similar to Lee's (2009) finding that the students in the low ability group also performed poorly on the ASW activities. In addition, the correlation between EXPLORE9 scores and the ASW activities supports the claim that students who used only used weak "look back" did not do well on the ASW activities because weak "look back" occurs at the lower levels of BRT where higher order thinking skills do not take place. Students who are lacking the abilities necessary to use higher order thinking skills can neither score high on a standards based, standardized test, nor can they make the transition to excel on open-ended mathematics problems when they have not been exposed to them on a regular basis, thus supporting the need for more mathematical writing tasks in the curriculum.

Vygotsky (1987) suggested that writing requires structuring of a web of meaning, and helps the writer make connections between prior, current, and new knowledge. To articulate mathematical ideas in writing requires students to select appropriate information and strategies and revise that plan throughout the process, thus utilizing "look back" at various stages. The research shows that writing supports metacognition (Pugalee, 2001; Powell, 1997; Artz & Armour-Thomas, 1992; Carr & Biddlecomb, 1998). This also supports the idea mentioned previously that students need the opportunity to write, get teacher feedback, have discussion with their peers, and then revise to refine their mathematical problem solving skills. The Open-Approach to teaching mathematics (Becker & Shimada, 1997) is a way this can be integrated.

Research Question 3

The third research question sought to investigate the extent to which students reported "look back" strategies when completing the ASW's. I used the three "think aloud" transcripts and the focus group transcripts to answer this question because the students' written work alone was not sufficient to determine when "look back" was occurring. When looking at the students' written work on the pre-test and the first week's intervention questions, it did not appear they were using "look back" strategies at all. It was not until the first focus group that I began to understand why the students were having difficulties. Question 2 from the first week was a real eye-opener. On paper, most of the students answered that Job 2, Terry's Tacos, paid better than Job 1 based on just one week of pay. I wondered why they would answer that an $8.50 per hour job paid better than a $9.00 per hour job. It seemed that they did not understand how hourly pay worked. In the focus group, however, the students made it clear that they did understand that the $9.00 per hour job would eventually pay more. Instead, it was the open-ended format of the question that confused them. Instead of thoroughly answering the question in terms of time spent working at the two restaurants, many students believed it was a poorly worded question, so it was not their faults if they did not answer completely. According to Newell and Simon (1972) and Schoenfeld and Hermann (1982), correct problem identification initiates a straightforward strategy, while incorrect problem identification results in unproductive strategies. Thus students need to be taught how to approach open-ended questions. If open-ended questions were used more frequently in the curriculum, then students would be more apt to offer thoughtful solutions from varying perspectives.

A second phenomenon I encountered when analyzing the students' use of "look back" was in the utilization of the alternative solution strategy. First, students typically wanted to use the method of symbolic manipulation for any solution attempts. Similarly, Huntley and Davis (2008) also found that the majority of students in their study used symbolic manipulation as a primary method of Algebra problem solving, but a contrasting finding was that their participants tended to use other representations such as tables or graphs when they were checking their answers or as an alternative solution method. In this study, the participants tended to use symbolic manipulation for an initial solution, and then attempted to manipulate the expressions or equations in a different way for an alternative solution. Figure F13 is a good example of that. The student made three different attempts to solve the problem. She explained her thought process for each one. On her first attempt, the solution was in the correct format for a completely factored expression, but she made a mistake factoring out $(x-2)$ as a common factor when it was not. Her second attempt made the least mathematical sense. She separated all of the $(x-2)$s from the other terms, then "cancelled" them out. In her third attempt, she correctly distributed and combined like terms, but then she did not attempt to factor. She spent considerable time carefully explaining her thought process for the three attempts, but did not analyze three different solutions she got. Based on what she wrote, it does not appear that she did any error analysis. She was stuck at the “applying” level of BRT because she was carrying out three solution methods without “analyzing” what she was doing or “evaluating” the solutions.

Research Question 4

The fourth research question sought to investigate the relationship between the subjects’ reported use of “look back” strategies and their performance on ASW activities. The most common types of “look back” utilized were at the lowest three levels of BRT; remembering, understanding, and applying, and perhaps that is why their performance on the ASW activities was poor. In addition, I believe that “look back” at any of the lower three levels should be considered weak “look back”, and “look back” at any of the upper three levels should be considered strong “look back’. Thus, strong “look back” occurs in the context of higher-order thinking. Additionally, it’s the strong “look back” that results in successful problem solving attempts. The students were able to do some look back strategies, but many of them were at the lower levels of Bloom’s Revised Taxonomy, so they were not helpful in getting a correct solution. According to Pohl (2000), Bloom’s Revised Taxonomy provides a way to organize thinking skills into six levels, from the most basic to the most complex levels of thinking (see Figure F4). The upper three levels of BRT, analyzing, evaluating, and creating are where the higher-order thinking skills take place and where strong “look back” occurs. The lower three levels of BRT, remembering, understanding, and applying are where weak “look back” occurs. The participants for this study often got “stuck” at the applying level of BRT. They would choose to do a mathematical operation seemingly randomly without rationale as to why that would work, and often it was not an appropriate solution method. This finding supports the idea that using multiple solution methods leads to better performance, greater understanding, and improved error analysis (Fouch, 1993; Hwang, Chen, Dung, & Yang, 2007; Herman, 2007;

Huntley & Davis, 2008). In addition, even when students were using “look back” effectively, they were not able to communicate it in writing on their papers, resulting in a low score.

Additional Findings

Affective Changes

Going into this study, I anticipated the students would have a negative attitude towards the problem solving based on my experiences teaching high school math. Comments made by the students during the first week of the intervention affirmed my suspicion. Despite the problem solving scores decreasing each week, the students’ attitudes toward participating in the intervention seemed to improve. Even though the problems were hard, the students were more engaged in the class discussions over time, and they tried harder on the ASWs by writing more down. According to the National Research Council (2001), “one important factor is attaining a productive disposition toward mathematics and maintaining the motivation required to learn it is the extent to which children perceive achievement as the product of effort as opposed to fixed ability” (p. 171). The participants of this study were in a math support class because they were at-risk of failure due to academic deficiencies in math and/or reading so it is possible that they were aware of their difficulties in mathematics, but they believed that their efforts would facilitate achievement otherwise they would not have tried so hard.

Findings that Differ from Lee’s

Lee (2009) found that “students who looked back more, with respect to either degree or frequency during ASW activities, tended to improve more from pretest to posttest” (p.79). The findings of this study indicate that only the degree of “look back” increased ASW performance if it was at the upper three levels of BRT. Students who “looked back” frequently, but only at the lower three levels of BRT did not perform well on the ASW activities.

Conclusion

Although the participants in this study were not able to effectively utilize “look back” strategies or alternative solution methods, some important lessons were learned from the results. In this section, I will summarize the findings and how they tie in to the literature, discuss their implications, and apply them more generally to mathematics instruction.

Writing in Mathematics

Students need to be taught how to communicate mathematically both verbally and in writing. I came into this study with the assumption that the students would attempt to find alternative solution methods because I directed them to with the ASW. I anticipated the students would initially have some difficulty, but I thought they would improve through the intervention process. Perhaps because they had little to no prior experience with open-ended problems or alternative solutions, in their attempt to follow the directions, they took the “if I’m writing, then I’m trying” approach that was disclosed in results section. The result was multiple attempts to use symbolic manipulation to find a solution with little to no regard to the question that was posed or the rules of Algebra. The choice to use symbolic manipulation was also noted in Huntley and Davis’ (2008) study on high school students’ approaches to solving Algebra problems. If the students are going to choose symbolic manipulation as a preferred method of problem solving, then they need to be taught to be more critical of the process.

One way to teach students to be more critical is to require them to write, then revise based on teacher feedback, similar to constructing multiple drafts on an English essay. To have the opportunity to write, the students must be presented with an open-ended question to work with. Providing the students with constructive feedback and then allowing them to refine their solutions is a way to celebrate the process rather than the solution, thus placing value on doing mathematics. When students are required to explain their ideas, they perform better (Webb et al., 2014).

Discussion in Mathematics

Another way to encourage students to be critical of the process they are using, is to provide an opportunity for engagement with others. According to Webb et al. (2014), the level of engagement with other's ideas predicted achievement over and above providing explanations. In this study, the students were supposed to engage with other students on the discussion day each week. What actually happened was the classroom teacher had a few students present their solutions, and then she discussed some of the incorrect attempts as well. Perhaps because the individual work had been done earlier in the week, and there were no grades attached to additional participation, the students were largely unengaged. I believe the group work would have been more productive if they had the opportunity to work together before the classroom teacher got involved in the conversation, and before they knew the correct answer. Mercer et al. (1999) gave an example of how one teacher set ground rules for group work that were meant to generate exploratory talk (which included sharing information, providing reasons, accepting challenges, discussing alternatives, and reaching agreement). I believe the participants of this study might have improved more had they engaged in more meaningful discourse on the discussion day each week. Multiple previous studies support this hypothesis that active student participation benefits student learning (Brown & Palincsar, 1989; Chinn, O'Donnell, & Jinks, 2000; Fuchs et al., 1997; Gillies & Ashman, 1998; Howe & Tolmie, 2003; Howe et al., 2007; King, 1992; Mercer, Dawes, Wegerif, & Sams, 2004; Nattiv, 1994; Saxe, Gearhart, Note, & Paduano, 1993; Slavin, 1987; Veenman, Denessen, van den Akker, & van der Rijt, 2005; Webb & Palincsar, 1996; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

CHAPTER 5

SUMMARY, CONCLUSIONS, RECOMMENDATIONS

A summary of the study and its findings, followed by limitations of the study, implications for teaching, and recommendations for future research are included in this chapter.

Summary of the Study and Its Findings

The purpose of this study was to investigate the effect of the use of the Alternative-Solution Worksheet (ASW, see appendix A) on American ninth-grade students' problem solving performance and to determine the extent to which instruction in the formulation of alternative solutions promoted "look back" strategies. The specific questions to be answered by this study included:

1. What relationship is there between problem solving scores on pre, posttests, and students' performance on ASW activities?
2. What relationship is there between students' EXPLORE9 scores and scores on the pre, posttests, and ASW activities?
3. To what extent do students report the use of "look back" strategies when completing ASW activities?
4. What is the relationship between the subjects' reported use of "look back" strategies and their performance on ASW activities?

This mixed-methods study was conducted over of a period of four weeks with two intact classes of ninth-grade, American, Algebra I students. The experimental group had eighteen students and the comparison group had fourteen students. The experimental group participated in an intervention where they practiced using alternative solutions to solve open-ended problems two days per week for four weeks. The comparison group did not receive this intervention.

Data for this study were collected by pre- and post-testing, Alternative Solution Worksheet use, “think alouds”, focus groups, document analysis (ASWs) and classroom observations. Each student’s pre-test, post-test, standardized test scores, and ASW performance scores were used to answer research questions 1 and 2. Qualitative data from the “think alouds”, focus groups, classroom observations, and document analysis were used to answer research question 3. Finally, a combination of the qualitative data from question 3 and the quantitative data from question 1 was used to answer research question 4. Answers to the research questions follow.

Research Questions 1 and 2

An ANCOVA was conducted to determine if there was a difference in the mean post-test scores of the two groups. After controlling for the pre-test scores, there was a significant difference in favor of the experimental group on the post-test. A correlation analysis showed that the students’ average ASW scores were significantly correlated with their EXPLORE9 math and reading percentiles.

Research Question 3

I observed students reporting “look back” activities in the context of mathematical operations such as combing like terms, or adding subtracting, multiplying or dividing to solve a problem. Most of the instances of observed “look back” were considered weak “look back”, and did not result in a correct solution. The students were not equipped with the higher-order thinking skills necessary to effectively use strong “look back” and get correct solutions.

Research Question 4

Since most of the “look back” that occurred was weak “look back”, it did not result in increased problem solving performance.

Limitations of the Study

This study followed a quasi-experimental, nonequivalent control group design, and thus was subject to certain threats to external and internal validity. Cook and Campbell (1979) claim, however, it is possible to draw strong conclusions if all of the threats to validity are considered and accounted for. Threats to internal validity that are of concern in this study include selection-maturation interaction and instrumentation.

Selection-maturation interaction is a threat because the participants in this study were selected from two intact classes rather than by random selection. The existing differences between the nonequivalent groups could have made an impact on the findings. This threat to internal validity is minimized in a few ways. First, the students were randomly assigned to the two sections of the math support class by the school's computer system. Second, analysis of the mean EXPLORE9 math and reading percentiles as well as the mean Algebra I semester averages indicate the two groups are equivalent by those measures. Last, the use of ANCOVA eliminated any differences in the pre-test scores of the two groups.

A second threat to internal validity is that of instrumentation. More specifically, the grading of the pre- and post-tests and the ASWs may not have been consistent even though there were three graders as outlined in the methodology. The grading of the papers always took place after school, and sometimes took several hours to complete. Fatigue could have been a factor in the decisions that were made regarding assignment of points. Even though a rubric was used for the grading, the process of assigning points was still subjective in that it was based on our interpretation of the students' work and how it fit in the rubric.

Additional limitations include the overall poor performance of all of the participants, and the inclusion of only one student "thinking aloud". The level of difficulty of the intervention questions as well as the pre- and post-test questions is a limitation of this study because the students were not able to improve over the four-week period. The participants of this study did not understand what was expected of them to adequately answer the open-ended questions on the ASW. The students needed to have an opportunity to practice writing in mathematics and using alternative solutions before the study began. I was only able to record one student “thinking aloud” during this study. In future studies, it would be beneficial to record more students “thinking aloud”.

Lastly, the results of this study are not generalizable because the participants were not randomly selected, but they could be applicable under similar circumstances. Most importantly, the participants were considered at-risk of not succeeding in the college-prep track by their standardized test scores or by teacher recommendation. Regardless whether tracking exists in a high school or not, it is likely that an Algebra I class will be composed of a heterogeneous group of students, some of which are struggling. Although the participants of this study were from the at-risk group, they could be applied to learners of mathematics at other academic levels as well.

Implications for Teaching

Perhaps the most interesting finding was the discrepancy between the students’ written work and their verbal process. Verbally, students could explain their thought processes and give a nice thoughtful solution. What ended up on their papers, at times, did not match what they said. According to Pintrich (2002), “Regardless of their theoretical perspective, researchers agree that with development, students become more aware of their own thinking as well as more knowledgeable about cognition in general” (p. 219). For teachers, that means students need to practice honing their mathematical thought process and effectively communicating it on paper. One way this can be accomplished is by having the students work in pairs to solve problems.

One student verbally solves the problem while the other student writes. The students can switch roles to solve another similar problem, then analyze both of their solutions for accuracy.

Teachers need to examine the types of problems they are practicing in their classrooms. If they all fall within the lower three levels of Bloom's Revised Taxonomy, then that is all the students will be able to master. Students need to practice open-ended questions to use higher order thinking skills, and they need to do it often so it becomes a habit.

Recommendations for Future Research

There were a few discrepancies between the findings of this study and similar studies. First, in Huntley and Davis' (2008) study, participants used multiple representations for alternative solutions and for checking, even though they primarily used symbolic manipulation as a primary method. The participants in this study only tried to use symbolic manipulation, which tended to result in major conceptual errors. A future study could investigate how and why students make the decision to choose one representation over another to solve a problem, and/or what are the defining characteristics of students who choose symbolic manipulation over others.

Second, Pugalee (2004) found that students' written work resulted in fewer procedural errors than "think aloud". That finding is the opposite of what I observed in this study with the one student who was recorded. His verbal explanations were much more accurate and meaningful than his written work. Pugalee (2004) did not compare written work and verbal processes on single problems. A future study could compare students' written work to their verbal process. Students of varying mathematical abilities could be studied.

Final Thoughts

Teachers need to analyze the types of problems they are using in their classrooms as well as the opportunities they are providing for their students to communicate mathematically. The ability to accurately navigate through mathematical procedures is meaningless in the real world unless they can be applied to other situations. Students will generally not naturally make the connection between classroom mathematics and real world mathematics if it is not presented in a way that allows connection. The classroom teacher must provide the opportunity for students to communicate mathematics both verbally and in writing as well as provide rich learning experiences during which the students can make inferences and draw conclusions. A very talented mathematician, Carl Friedrich Gauss, once said, “Mathematics is the queen of sciences.” According to the Oxford English Dictionary, “science” refers to a way of pursuing knowledge that explains the phenomena of the universe, so if classroom teachers can find a way to introduce the science of mathematics through meaningful discourse, I believe a renewed appreciation for mathematics will follow, along with increased skills.

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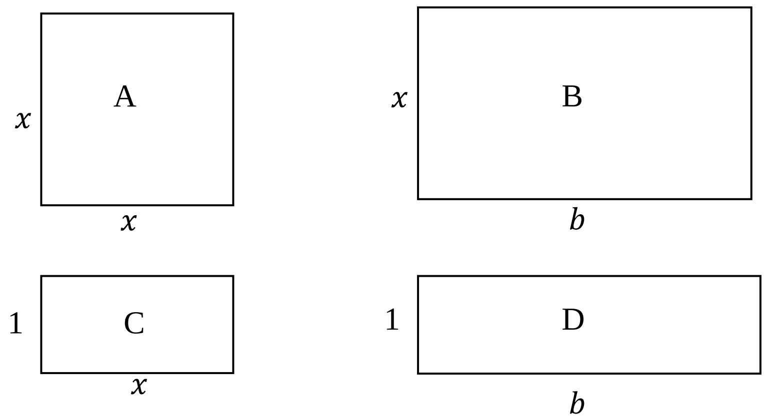
APPENDICES

APPENDIX A: SAMPLE OF THE ALTERNATIVE-SOLUTION WORKSHEET

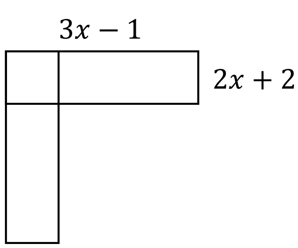
Problem:
Initial Solution:
Alternative Solutions:

APPENDIX B: PRETEST

1. Below are four different rectangles A, B, C and D, each of which has length and width of x, x and b, x and 1, b and 1, respectively. Suppose there is a big rectangle constructed of 3 As, 1 B, 9 Cs and 3 Ds. Find the length and width of this big rectangle. The solution will be in terms of x and b.

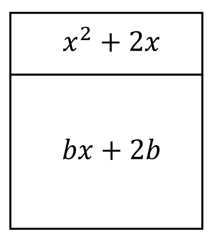


2. Two identical rectangular strips, each of which has a length of $3x - 1$ and a width of $2x + 2$, are overlapped on a desk. Suppose the area of another rectangular strip equals the covered desk area. Find the length and width of this rectangular strip in terms of x.



3. Factor $x^3 + 2x^2 + x$ completely.

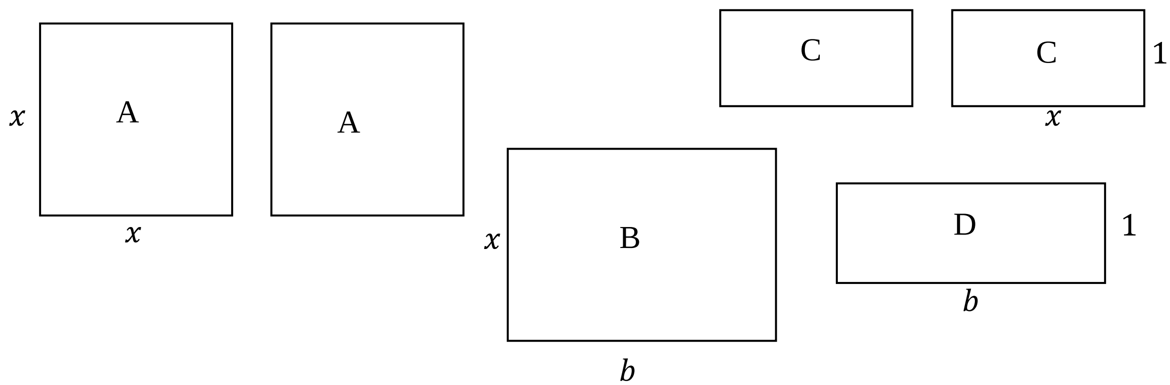
4. Suppose the area of the following rectangle is $x^2 + 2x + bx + 2b$. Find the length and width of this rectangle in terms of x and b.



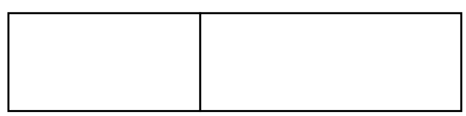
APPENDIX C: POSTTEST

1. Factor $(x-1)^3 + 4(x-1)^2 + 3(x-1)$ completely.

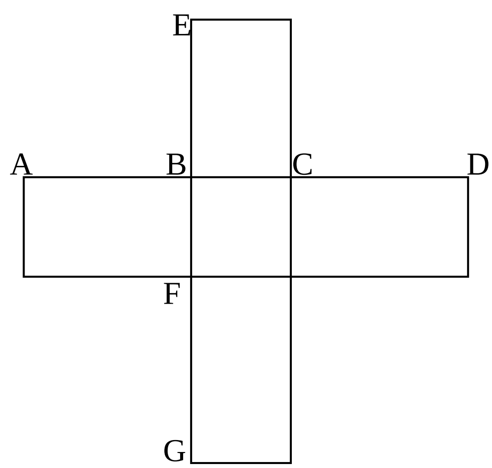
2. There are six rectangles below. As shown below, there are 2 As, 1 B, 2 Cs, and 1 D, where A, B, C, and D have length and widths of x, x and b, x and 1, and 1 and b, respectively. Now a new rectangle is made up of these six rectangles. Find the length and width of this new rectangle in terms of x and b.



3. A rectangular flier is designed as follows. Suppose the area of each component rectangle is $2a - a^2x$ and $4x - 2ax^2$. Find the length and width of this flier in terms of x and a.



4. Below a crossroad consists of two same rectangular roads, with $\mathrm{AB} = \mathrm{BC} = \mathrm{CD} = \mathrm{EB} = \mathrm{BF} = \mathrm{FG} = x + 1$. If the area of a square garden equals the area of the crossroad, how long is one of the sides of this square in terms of x?



APPENDIX D: ASW SCORING RUBRIC

Each attempted problem solving approach to the problem presented on an ASW, the pre and post tests will be scored as follows. A total score will be computed by adding up points of each attempted problem solving approach.

4 points

Student utilizes a correct problem solving approach, and has a correct solution.

3 points

Student utilizes a correct problem solving approach, but a little incompleteness or a few errors.

2 points

Student utilizes a correct problem solving approach but solves the problem with some incompleteness or some errors.

1 point

Student minimally understands the problem. It seems the students is aware of a correct problem solving approach, but a correct approach is not pursued at all.

0 points

Student utilizes a wrong problem solving approaches or incorrectly indentifies the problem to be solved. This student does not understand the problem.

APPENDIX E: PROBLEMS IN ALTERNATIVE-SOLUTIONS WORKSHEETS

Week 1:

1a. Find both the total length and the total width of the following rectangle in terms of a and b.

ab	ab	b^2
a^2	a^2	ab

1b. Which of the following two jobs gets better pay?

(Job 1) At Bob's Burgers, you will be paid $9.00 per hour and will be expected to work 20 hours per week. You are required to buy a uniform for $30.

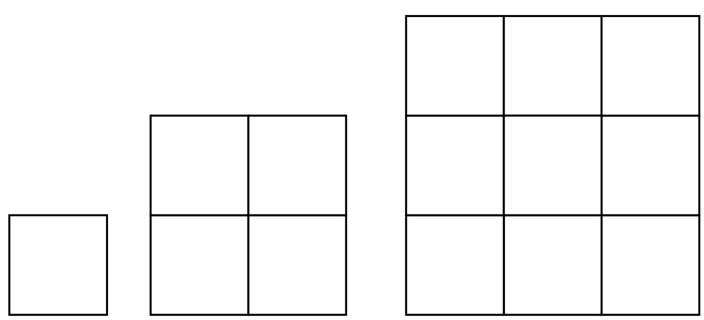
(Job 2) At Terry's Tacos, you will be paid $8.50 per hour and will be expected to work 20 hours per week. There is no required special attire.

This problem was adapted from Looking Back in Problem Solving (Cai & Brook, 2006).

Week 2:

2a. Suppose the area of a rectangular billboard is $ax^2 - 11x - 6$ and the length of one of its sides is $5x + 2$. Find a.

2b. Below, in example A, 4 sticks make a square with 1 stick on each side. In B, 12 sticks make a square with 2 sticks on each side. In C, 24 sticks make a square with 3 sticks on each side. How many sticks does it take to make a square with 10 sticks on each side?



A B C

Week 3:

3a. Factor $(x-2)^2 + a(x-2) + 3(x-2) + 3a$ completely.

3b. John solved the equation, $2x^2 - bx + a = 0$, and he got the correct answer: $x = \frac{3}{2} \pm \frac{\sqrt{15}}{2}$. Find a.

Week 4:

4a. Solve the equation: $4(2x-3)^2 - 36 = 0$.

4b. Suppose $x^2 - 6x + b = 0$ and $y^2 - 6y + b = 2$ are equivalent to $(x-a)^2 = 7$ and $(y-a)^2 = k$, respectively. Find k.

APPENDIX F: HANDOUT FOR THE GRADERS

Explanation of the Process

The alternative solution worksheet (ASW) could address “look back” in problem solving in several ways. It could encourage students to review a completed solution to find more efficient solutions. To complete an ASW, they should reconsider and reexamine their previous problem solving paths and improve their understanding of the problem. Moreover, the ASW could encourage students to compare different solutions they may have generated.

To assess students’ performance, each problem solving approach attempted will be graded based on the ASW scoring rubric (APPENDIX D). Based on the rubric, each problem solving approach will be graded based on accuracy and completeness, so students must write down all work and not erase or black out anything they write down.

(Lee, 2010)

APPENDIX G: FIGURES

Figure F1: Content Distribution of the EXPLORE Mathematics and Reading Tests

Table 2.2

Content Specifications for the ACT Explore Mathematics Test

The items in the Mathematics Test are classified according to four content categories. These categories and the approximate proportion of the test devoted to each are given below.

Mathematics content area	Proportion of test	Number of items
Pre-Algebra	.33	10
Elementary Algebra	.30	9
Geometry	.23	7
Statistics/Probability	.14	4
Total	**1.00**	**30**

a. *Pre-Algebra.* Items in this category are based on operations with whole numbers, integers, decimals, and fractions. The topics covered include place value, square roots, scientific notation, factors, ratio, and proportion and percent. Formal variables are not used.

b. *Elementary Algebra.* The items in this category are based on operations with algebraic expressions. The operations include evaluation of algebraic expressions by substitution; use of variables to express functional relationships, solution of linear equations in one variable, use of real number lines to represent numbers, and graphing of points in the standard coordinate plane.

c. *Geometry.* Items in this category cover such topics as the use of scales and measurement systems, plane and solid geometric figures and associated relationships and concepts, the concept of angles and their measures, parallelism, relationships of triangles, properties of a circle, and the Pythagorean theorem. All of these topics are addressed at a level preceding formal geometry.

d. *Statistics/Probability.* Items in this category cover such topics as elementary counting and rudimentary probability; data collection, representation, and interpretation; and reading and relating graphs, charts, and other representations of data. These topics are addressed at a level preceding formal statistics.

Table 2.3

Content Specifications for the ACT Explore Reading Test

The items in the Reading Test are based on the prose passages that are representative of the kinds of writing commonly encountered in middle-school and junior-high school curricula, including the social sciences, prose fiction, and the humanities. The three content areas and the approximate proportion of the test devoted to each are given below.

Reading passage content	Proportion of test	Number of items
Prose Fiction	.33	10
Social Sciences	.33	10
Humanities	.33	10
Total	**1.00**	**30**

a. *Prose Fiction.* The items in this category are based on short stories or excerpts from short stories or novels.

b. *Humanities.* The items in this category are based on passages from memoirs and personal essays, and in the content areas of architecture, art, dance, ethics, film, language, literary criticism, music, philosophy, radio, television, or theater.

c. *Social Sciences.* The items in this category are based on passages in anthropology, archaeology, biography, business, economics, education, geography, history, political science, psychology, or sociology.

(ACT, Inc., 2013, p.8)

Figure F2: Reliabilities and Standard Errors for the EXPLORE Test.

Table 4.24
Estimated Reliabilities and Standard Errors of Measurement for ACT Explore Tests and Subscores

Statistic	English	Usage/ Mechanics	Rhetorical Skills	Mathematics	Reading	Science	Composite
			Form A Grade 8				
Raw Scores							
Reliability	0.87	0.78	0.75	0.83	0.90	0.86	—
Scale Scores							
Reliability	0.84	0.76	0.74	0.76	0.86	0.79	0.94
SEM	1.66	1.16	1.11	1.71	1.44	1.53	0.79
			Form A Grade 9				
Raw Scores							
Reliability	0.88	0.80	0.78	0.85	0.91	0.88	—
Scale Scores							
Reliability	0.86	0.79	0.76	0.80	0.86	0.82	0.95
SEM	1.69	1.11	1.10	1.69	1.60	1.52	0.81
			Form B Grade 8				
Raw Scores							
Reliability	0.88	0.82	0.75	0.80	0.90	0.82	—
Scale Scores							
Reliability	0.85	0.81	0.73	0.72	0.82	0.76	0.93
SEM	1.55	1.01	1.20	1.74	1.63	1.58	0.81
			Form C Grade 8				
Raw Scores							
Reliability	0.87	0.80	0.73	0.82	0.88	0.84	—
Scale Scores							
Reliability	0.84	0.79	0.72	0.74	0.84	0.77	0.94
SEM	1.69	1.06	1.19	1.70	1.52	1.55	0.81

(ACT, Inc., 2013, p.43)

Figure F3: Validity Information for the EXPLORE Test

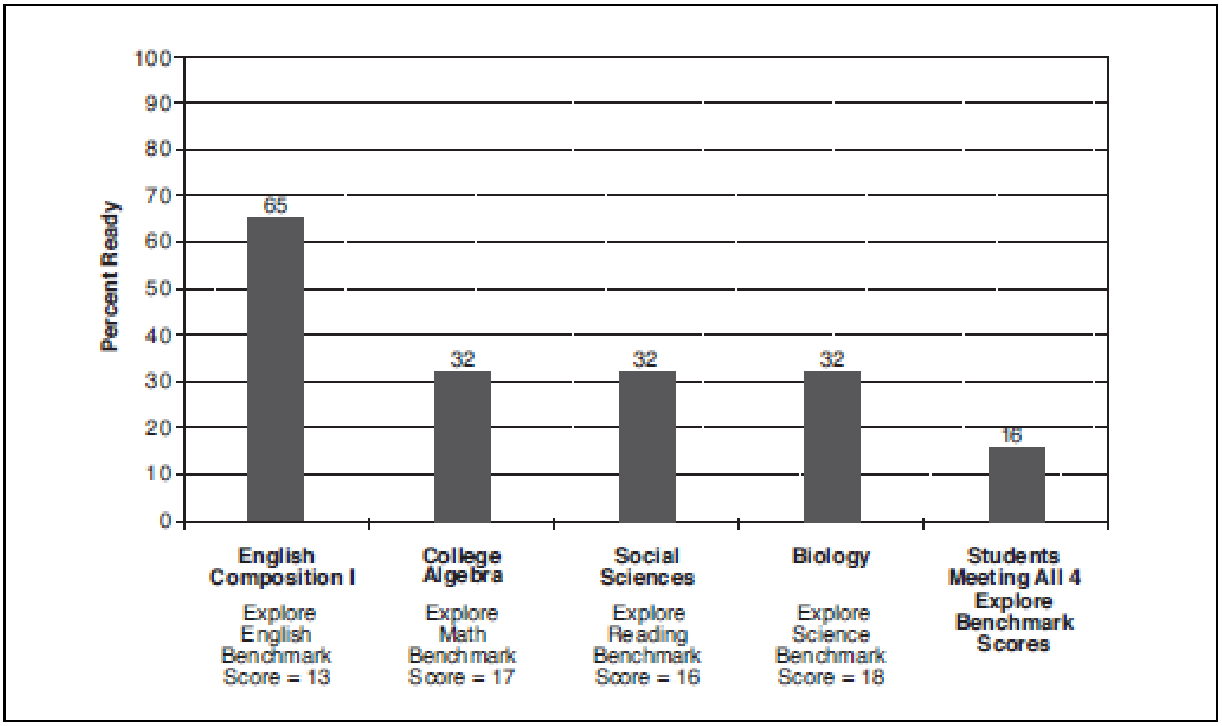


Figure 4.8. 2011–2012 national ACT Explore-tested 8th-grade students likely to be ready for college-level work (in percent).

(ACT, Inc., 2013, p.54)

Figure F4: Validity Information for the EXPLORE Test

Table 4.31
Correlations Between ACT Explore Scores and Course Grades

Grade/grade average	No. of students	Mean	Correlation for Explore score: English	Mathematics	Reading	Science	Composite
English							
English 9	175464	3.00	.41				.44
English 10	80801	3.06	.38				.41
English GPA	178539	2.99	.42				.46
Mathematics							
Algebra 1	158716	2.97		.42			.43
Geometry	80776	3.13		.40			.42
Algebra 2	34801	3.21		.39			.40
Mathematics GPA	166355	2.92		.42			.44
Social Studies							
U.S. History	61780	3.14			.38		.42
World History	70044	3.16			.37		.42
Government/Civics	42408	3.17			.39		.44
World Cultures/ Global Studies	13539	3.21			.39		.44
Geography	74309	3.31			.28		.32
Economics	11367	3.16			.38		.43
Social Studies GPA	172668	3.18			.37		.42
Science							
Physical/Earth/ General Science	123078	3.03				.38	.43
Biology 1	95581	3.06				.39	.44
Chemistry 1	18383	3.24				.34	.40
Science GPA	168395	3.02				.40	.45
Overall							
Overall GPA	148640	3.08	.49	.48	.47	.48	.55

(ACT, Inc., 2013, p.49)

Figure F5: Schedule of Events and Data Collection

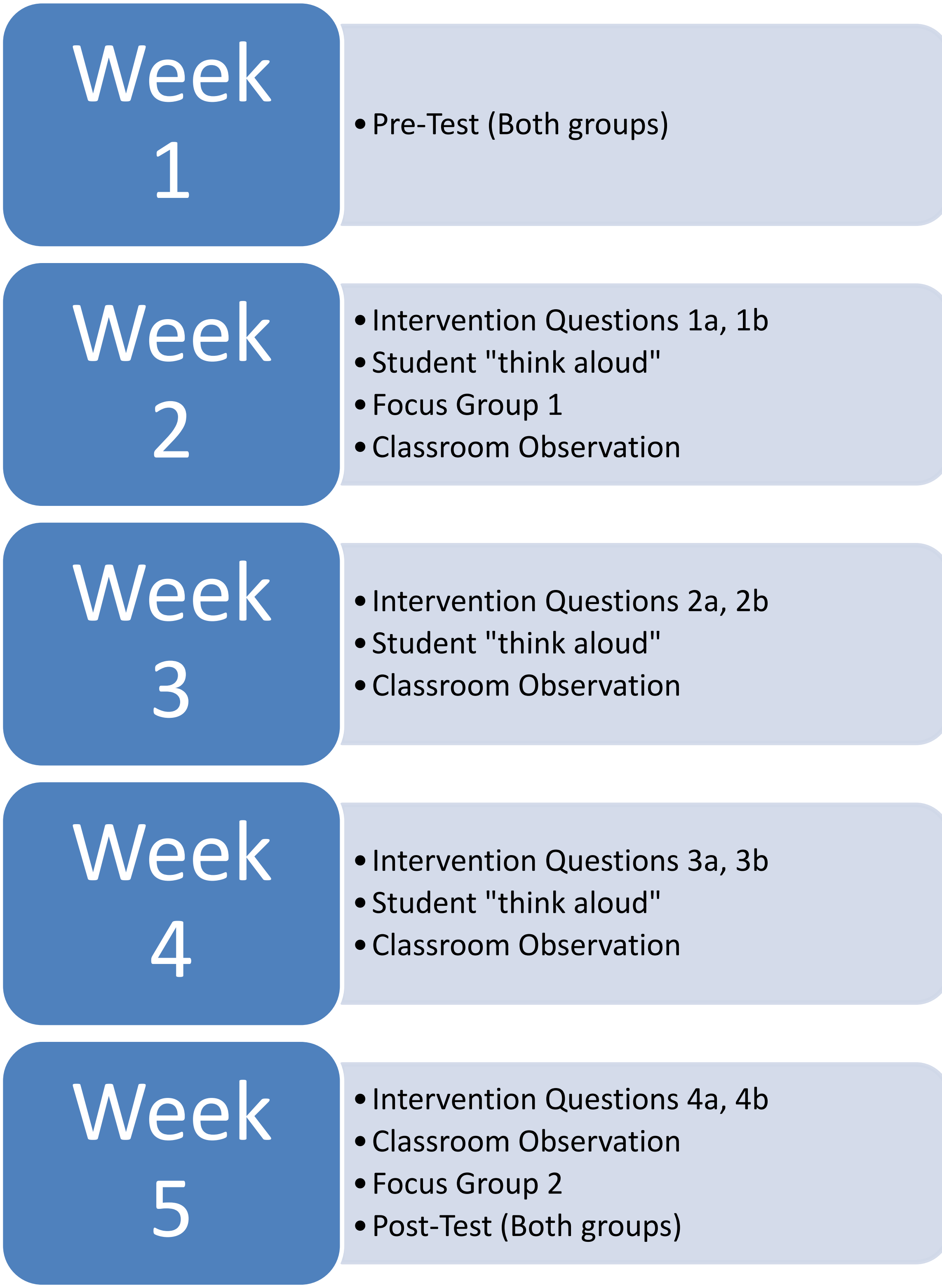
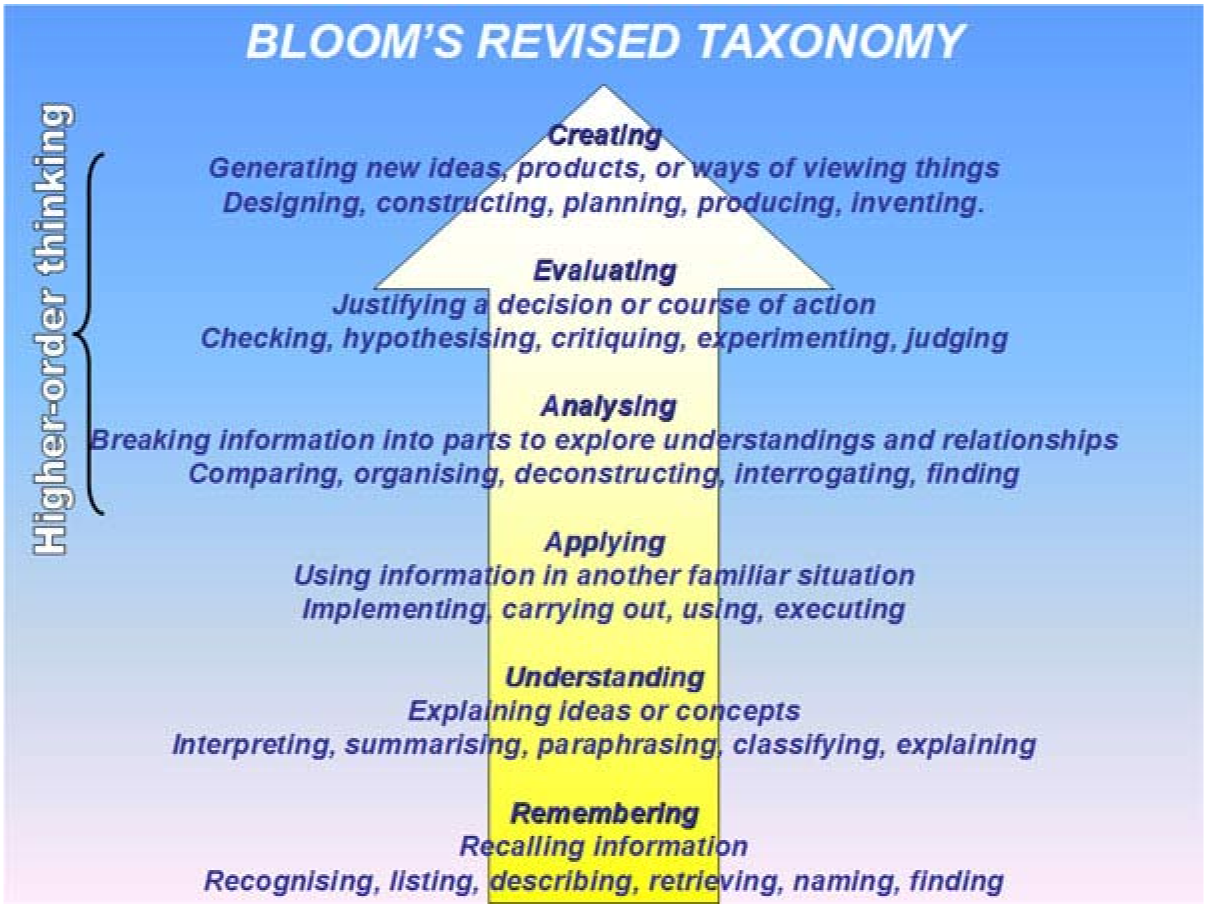
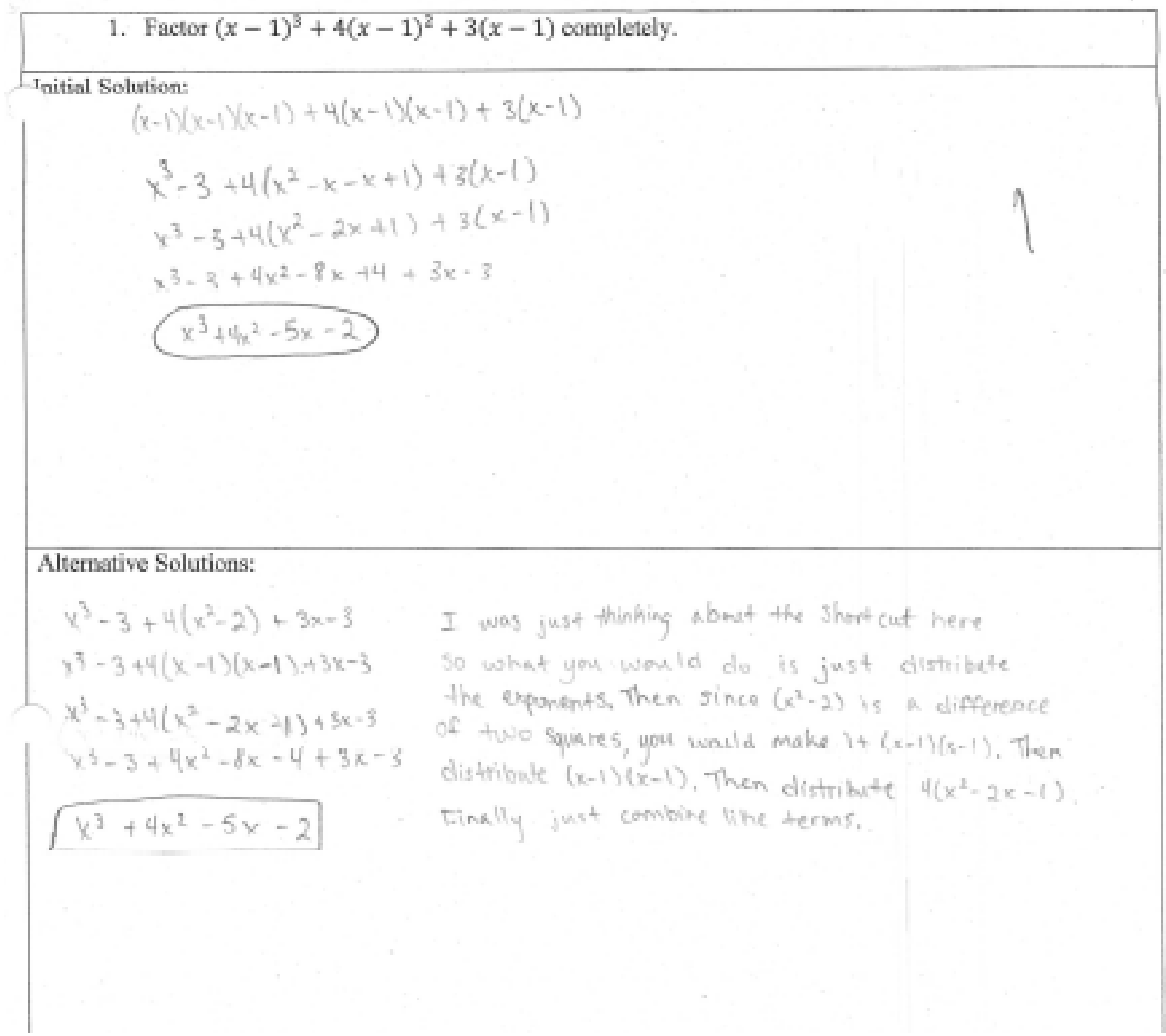


Figure F6. Bloom's Revised Taxonomy



Retrieved from: http://saraeffron.files.wordpress.com/2012/10/blooms-revised-taxonomy.jpg

Figure F7: Student's Written Work for Question 1 on the Post-Test



1. Factor $(x-1)^3 + 4(x-1)^2 + 3(x-1)$ completely.

Initial Solution:

$(x-1)(x-1)(x-1) + 4(x-1)(x-1) + 3(x-1)$

$x^3 - 3 + 4(x^2 - x - x + 1) + 3(x-1)$

$x^3 - 3 + 4(x^2 - 2x + 1) + 3(x-1)$

$x^3 - 3 + 4x^2 - 8x + 4 + 3x - 3$

$x^3 + 4x^2 - 5x - 2$

1

Alternative Solutions:

$x^3 - 3 + 4(x^2 - 2) + 3x - 3$

$x^3 - 3 + 4(x-1)(x-1) + 3x - 3$

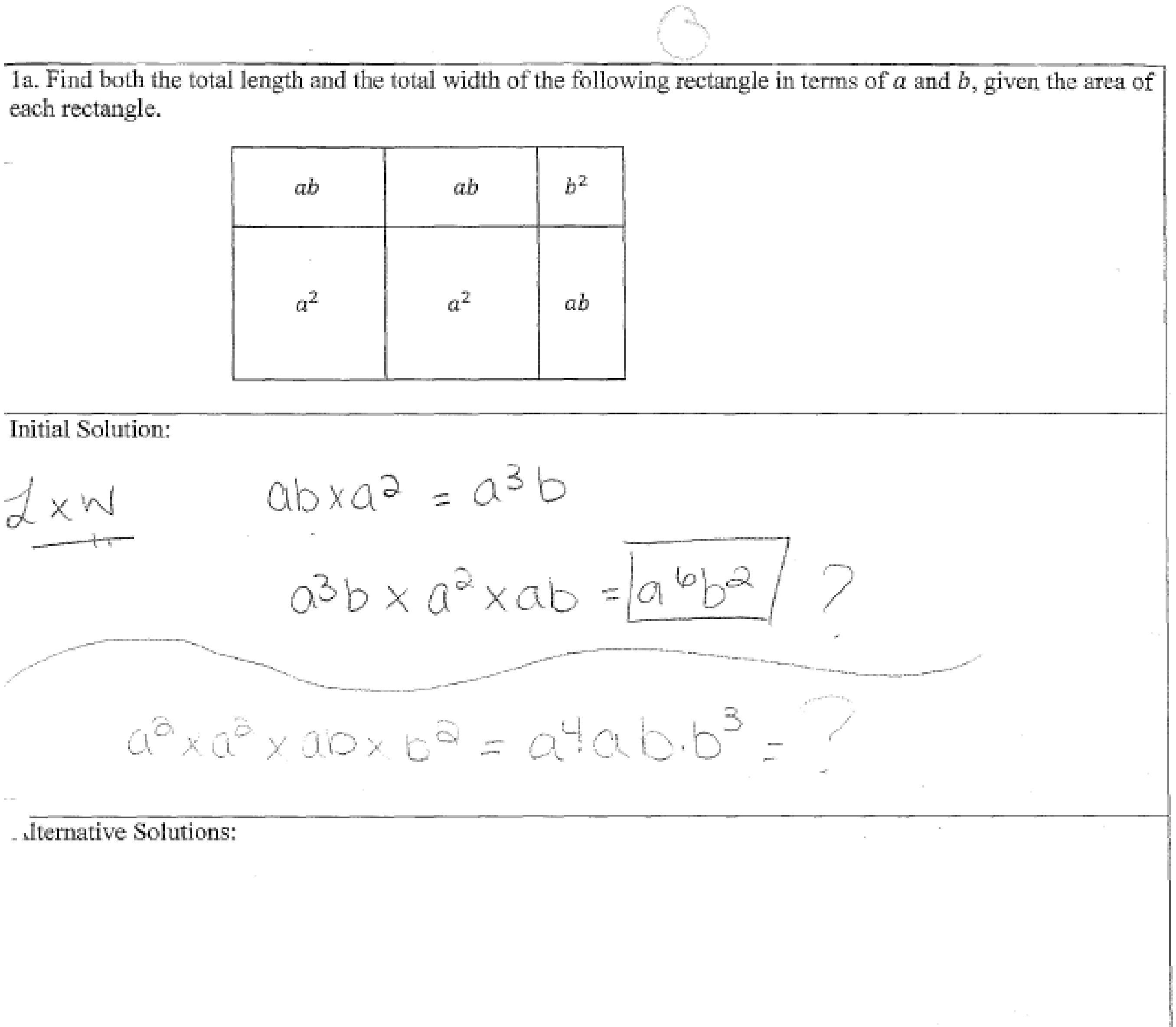
$x^3 - 3 + 4(x^2 - 2x + 1) + 3x - 3$

$x^3 - 3 + 4x^2 - 8x - 4 + 3x - 3$

$x^3 + 4x^2 - 5x - 2$

I was just thinking about the shortcut here so what you would do is just distribute the exponents. Then since (x^2-2) is a difference of two squares, you would make it $(x-1)(x-1)$. Then distribute $(x-1)(x-1)$. Then distribute $4(x^2-2x-1)$. Finally just combine like terms.

Figure F8: Student's Written Work on Question 1a.



1a. Find both the total length and the total width of the following rectangle in terms of a and b, given the area of each rectangle.

ab	ab	b^2
a^2	a^2	ab

Initial Solution:

$l \times w$

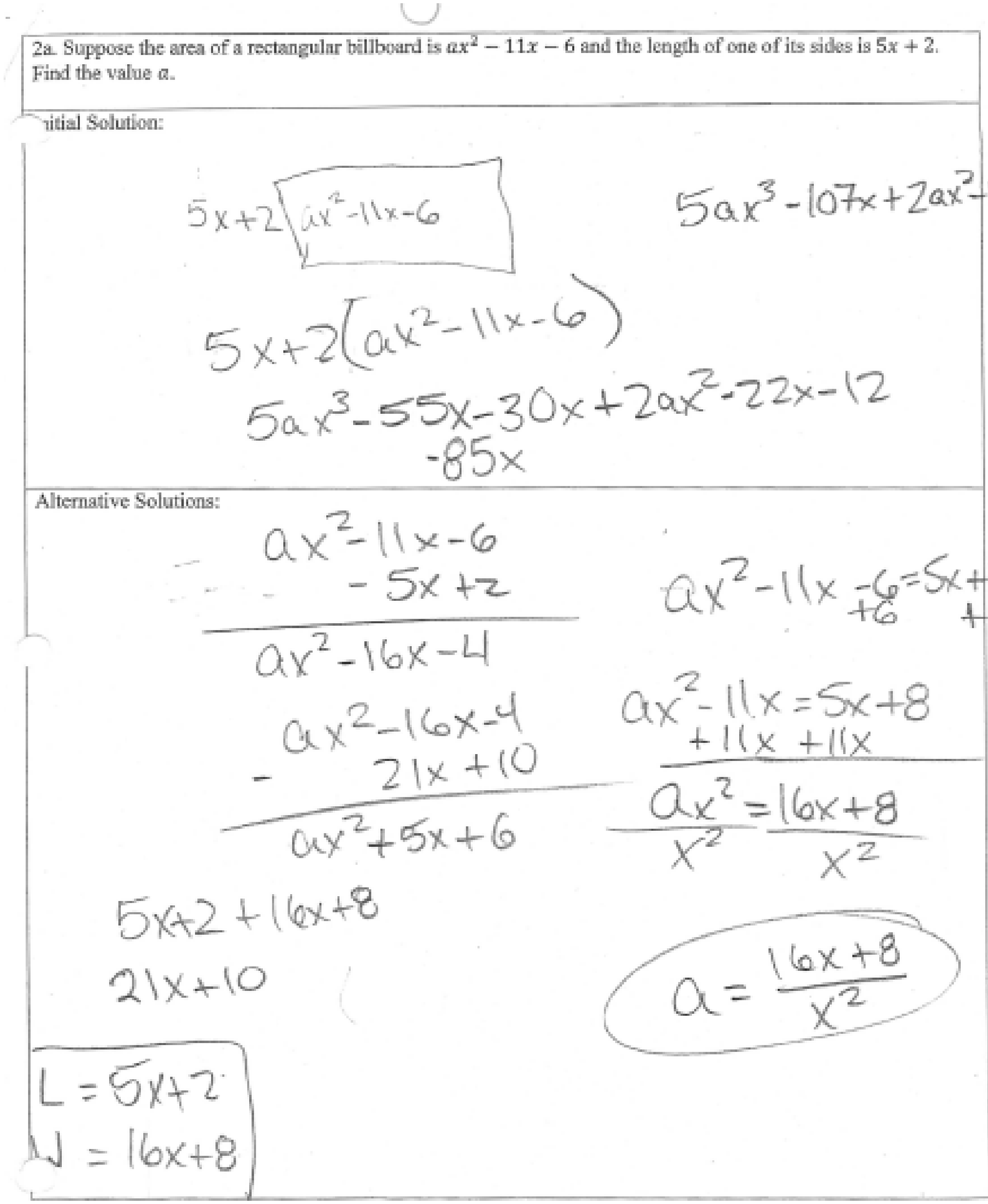
$ab \times a^2 = a^3b$

$a^3b \times a^2 \times ab = a^6b^2$?

$a^2 \times a^2 \times ab \times b^2 = a^4ab \cdot b^3 =$?

Alternative Solutions:

Figure F9: Student's Written Work for Question 2a



2a. Suppose the area of a rectangular billboard is $ax^2 - 11x - 6$ and the length of one of its sides is $5x + 2$. Find the value a.

Initial Solution:

$5x+2 \overline{) ax^2-11x-6}$

$5ax^3 - 107x + 2ax^2 - 12$

$5x+2(ax^2-11x-6)$

$5ax^3 - 55x - 30x + 2ax^2 - 22x - 12$

$-85x$

Alternative Solutions:

$ax^2-11x-6$
$-\ 5x+2$
$ax^2-16x-4$

$ax^2-16x-4$
$-\ 21x+10$
ax^2+5x+6

$5x+2+16x+8$
$21x+10$

$L = 5x+2$
$W = 16x+8$

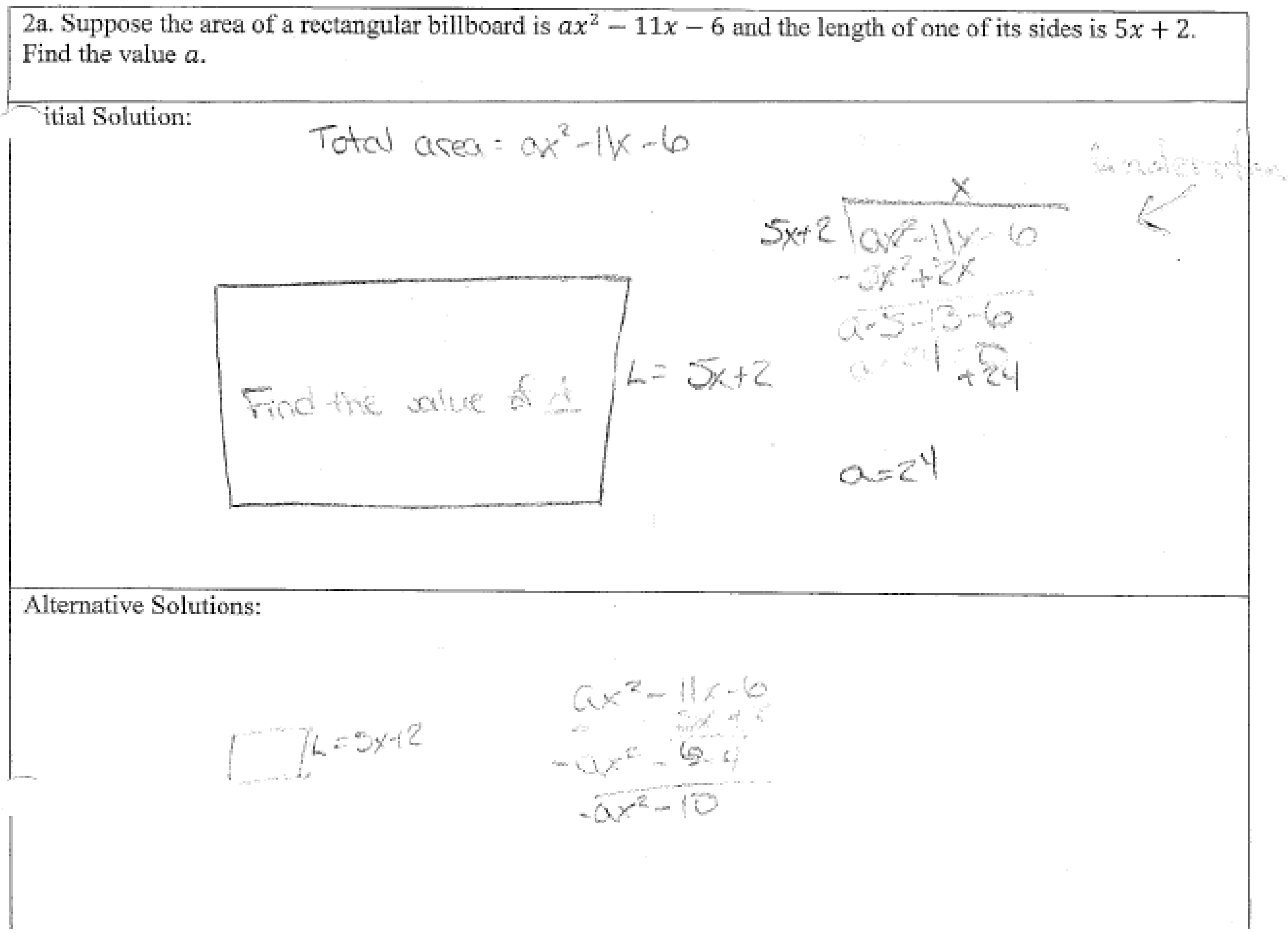
$ax^2-11x-6 = 5x+2$
$+6 \quad +6$

$ax^2-11x = 5x+8$
$+11x \quad +11x$

$\frac{ax^2}{x^2} = \frac{16x+8}{x^2}$

$a = \frac{16x+8}{x^2}$

Figure F10: Student's Written Work for Question 2a



2a. Suppose the area of a rectangular billboard is $ax^2 - 11x - 6$ and the length of one of its sides is $5x + 2$. Find the value a.

itial Solution:

Total area = $ax^2 - 11x - 6$

Find the value of a

L = 5x+2

a=24

Alternative Solutions:

L = 5x+2

Figure F11: Student's Written Work for Question 2a.

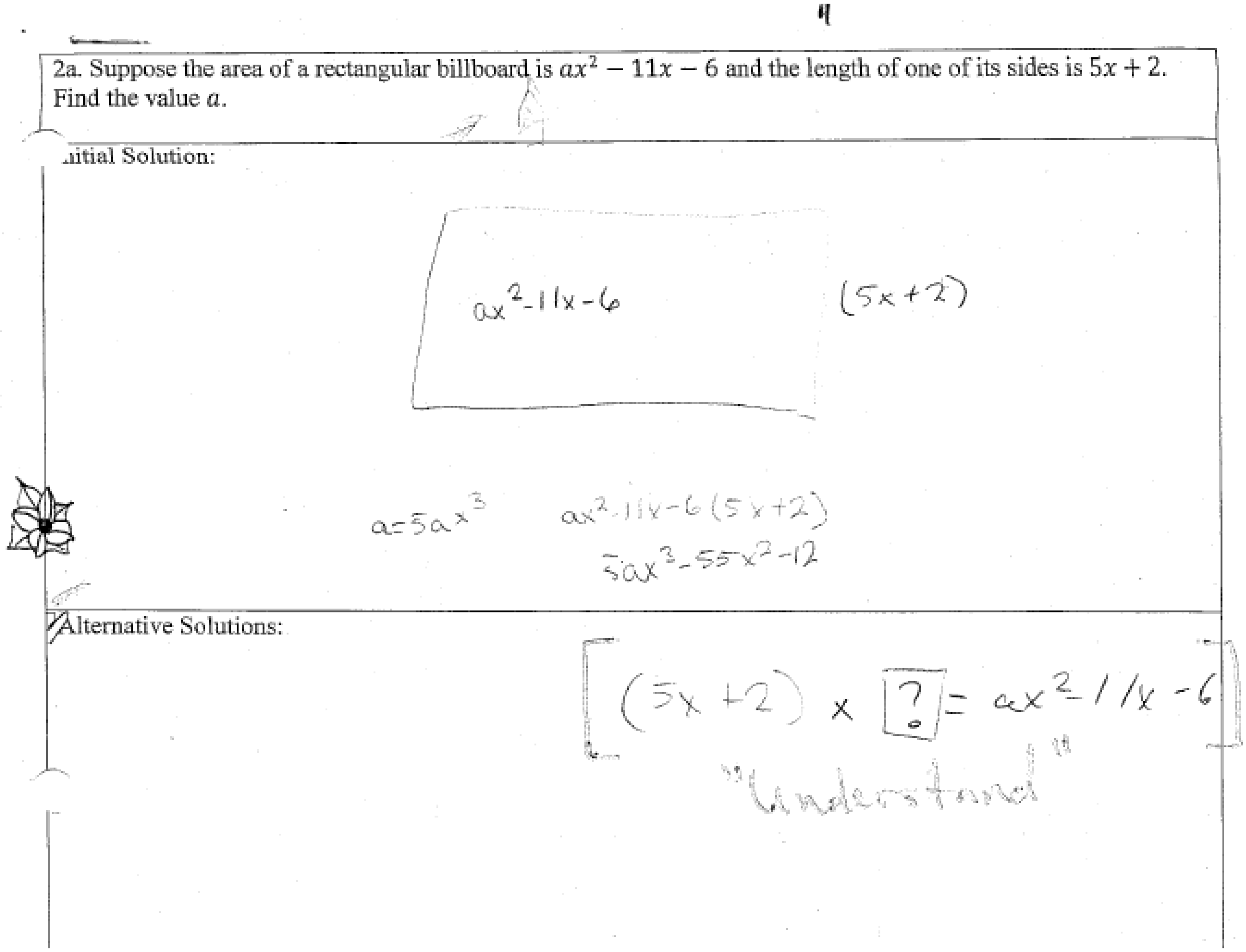


Figure F12: Student's Written Work for Question 2b.

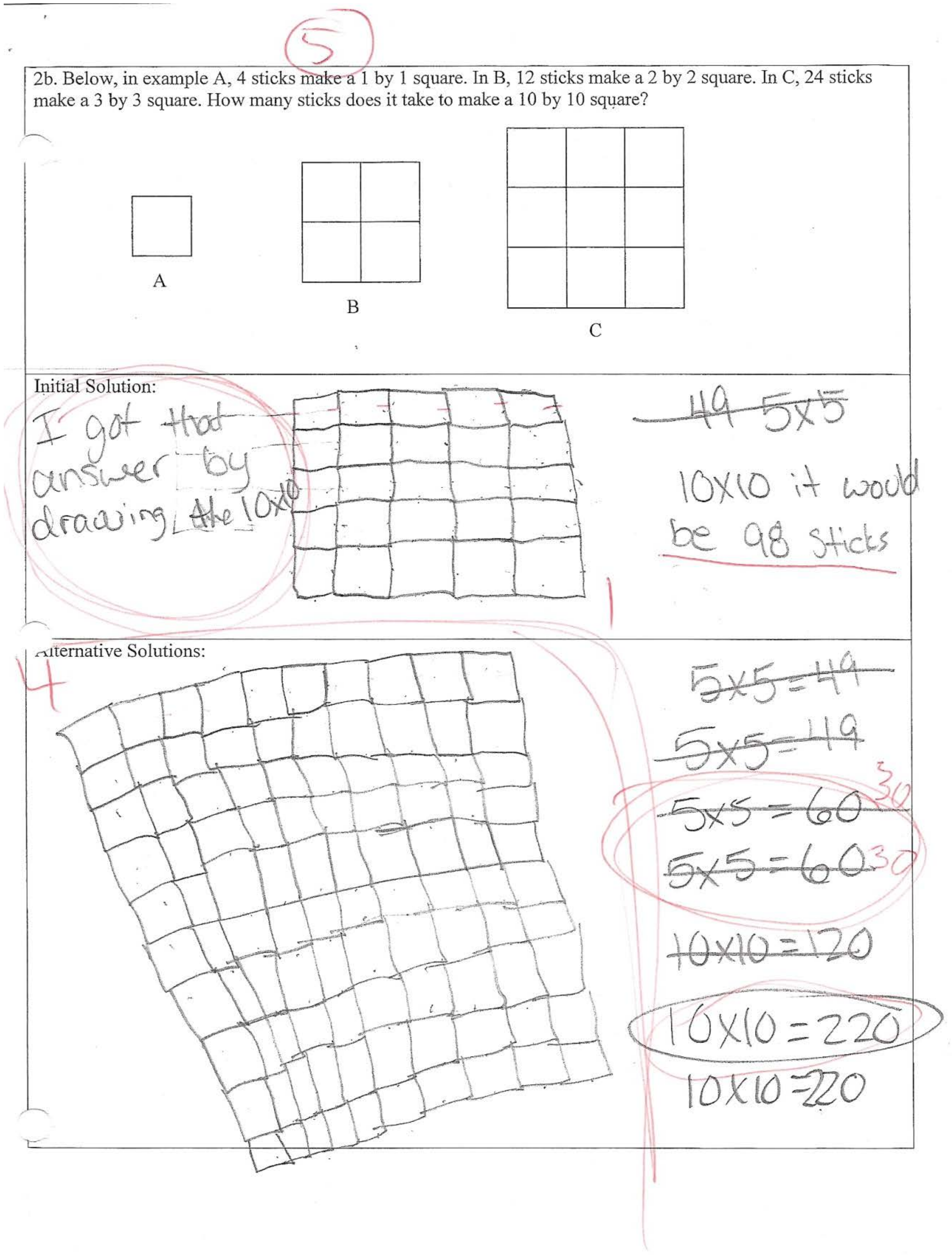
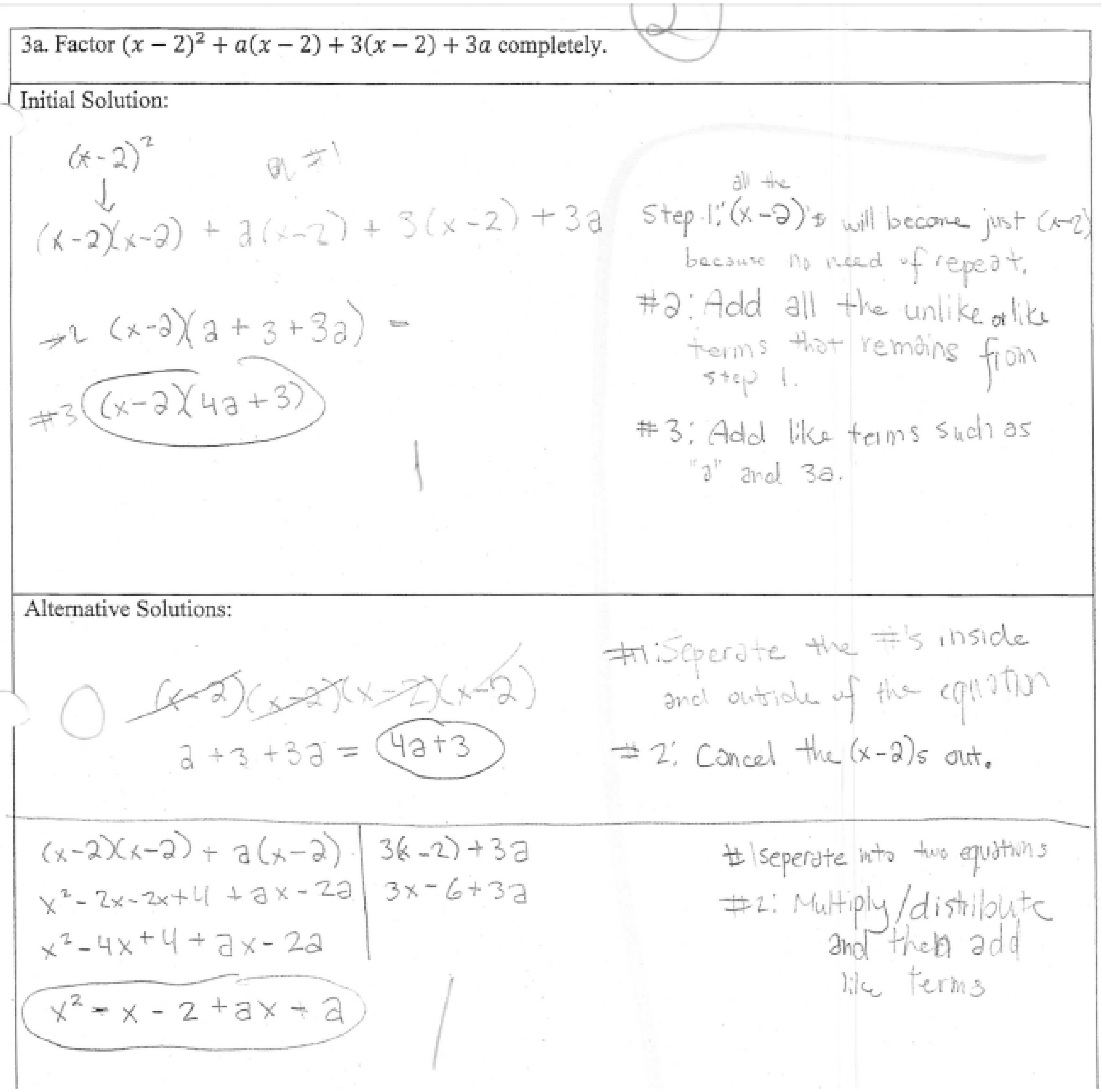


Figure F13: Student’s Written Work on Problem 3a



3a. Factor $(x-2)^2 + a(x-2) + 3(x-2) + 3a$ completely.

Initial Solution:

$(x-2)^2$ ↓

$(x-2)(x-2) + a(x-2) + 3(x-2) + 3a$

#2 $(x-2)(a+3+3a) =$

#3 $(x-2)(4a+3)$

Step 1: all the $(x-2)$'s will become just $(x-2)$ because no need of repeat.

#2: Add all the unlike or like terms that remains from step 1.

#3: Add like terms such as "a" and 3a.

Alternative Solutions:

~~$(x-2)(x-2)(x-2)(x-2)$~~

$a + 3 + 3a = 4a+3$

#1: Seperate the #'s inside and outside of the equation

#2: Cancel the $(x-2)$s out.

$(x-2)(x-2) + a(x-2)$

$x^2 - 2x - 2x + 4 + ax - 2a$

$x^2 - 4x + 4 + ax - 2a$

$x^2 - x - 2 + ax + a$

$3(x-2) + 3a$

$3x - 6 + 3a$

#1 seperate into two equations

#2: Multiply/distribute and then add like terms

VITA

Graduate School
Southern Illinois University

Erin E. Sagaskie

esagaskie@gmail.com

Southern Illinois University Carbondale
Bachelor of Science, Mathematics, May 2001

Southern Illinois University Carbondale
Master of Science in Mathematics, College of Science, May 2004

Special Honors and Awards:
John M. H. Olmstead Outstanding Master's Teaching Assistant, May 2004

Dissertation:
The Effect of Instruction in Alternative Solutions on American Ninth-Grade Algebra I Students' Problem Solving Performance

Major Professor: Cheng-Yao Lin