ON THE DETERMINATION OF HIGH TEMPERATURES.[1]

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REFERENCE must here be made in connection with our discussions of the concept of temperature to the so-called *pyrometric* methods, or expedients for determining high temperatures. Newton[2] is the first to have devised a method of this kind, and we shall simply state his idea, without at present making any critical comment.

Newton observed, by the aid of a linseed-oil thermometer, that the loss of temperature of a hot body exposed to a uniformly cold current of air was for the same interval of time proportional to the temperature-difference of the body and the air, and *assumed* that this relation held universally for all temperatures, however high. Imagine two bodies, A and A', alike in all respects, save that the difference between the temperature of the air and that of A' is twice the corresponding difference for the air and A. Allowing these bodies to cool during the same element of time t, A' will lose twice as much as A, and the excess of its temperature above that of the air will at the end of time t be again twice that of A. The same reasoning holds true for the succeeding element t_2, and so for the rest. Hence, in the process of cooling during any interval of time t, A' will lose twice as much as A. The generalisation is obvious.

Now let a body A at a very high temperature cool, and call the *equal* intervals into which the total time of cooling is divided, t_1, t_2 ....t_{n-1}, t_n. Suppose the excess of temperature of the body at the beginning of the last interval t_n is $2u$, but at the end of it it is u, then on the preceding assumption it follows that at the beginning

[1] Translated from Mach's *Principien der Wärmelehre* by Thomas J. McCormack.

[2] *Newtoni Opuscula*, Lausanne and Geneva, 1744, Vol. II., p. 419. *Scala graduum caloris et frigoris, Phil. Trans.*, 1701, XXII., p. 824.

of the equal intervals t_{n-1}, t_{n-2}, t_{n-3}, ...it would show respectively the excesses of temperature, $4u = 2^2u$, $8u = 2^3u$, $16u = 2^4u$. Newton ascertained the time t_n and the value of u by means of a linseed-oil thermometer, and was thus able to assign the temperature at every other prior period of the cooling.

The body A was a red-hot mass of iron exposed to a current of air. On it particles of different metals and their alloys were placed and the *time* noted at which they congealed, the idea being to determine the *temperatures of congelation.* From the melting-point of tin downwards the process of cooling could be followed with a linseed-oil thermometer. Newton made the temperature-numbers of this thermometer *proportional* to the voluminal incre-ment of the linseed-oil above the melting-point of ice.

According to Newton, the temperature of boiling water is not quite three times (2.83) that of the human blood (37° C.), whence 104° C. would follow for the temperature of boiling. For the melt-ing-point of tin (5.83 × 37) he obtained 215° C. (new researches give 230°); for the temperature of lead (8 × 37) he got 296° (new determinations give 326°), and for the temperature of red-heat (16.25 × 37) 600° C.

At the conclusion of his paper Newton remarks that, owing to the uniformity of the air-current, the same number of air-particles is heated in equal intervals of time, by an amount proportional to the heat of the iron, and that therefore the losses of heat suffered by the iron must be proportional to its heat. But since these losses are in point of fact also proportional to the indications of the lin-seed-oil thermometer, therefore we are justified in assuming that the heat of a body is proportional to the increase of volume of the linseed-oil thermometer.[1] From this reasoning, in which by the way no distinction is yet made between the concepts "tempera-ture" and "quantity of heat," it would appear that Newton, here as elsewhere, is guided in his enunciations partly by instinct and partly by observation, making the suggestions of the one correct those of the other. It appeared to him *antecedently* obvious that the "losses of the heat" should be proportional to the "heat," and likewise that the "expansion" should be proportional to the "heat." Observation tallied with these views, and so the concep-tions were retained.

[1] The original of the passage in question reads: " Locavi autem ferrum, non in aere tran-quillo, sed in vento uniformiter spirante, ut aer a ferro calefactus semper abriperetur a vento, et aer frigidus in locum ejus uniformi cum motu succederet. Sic enim aeris partes aequalibus temporibus calefactae sunt, et calorem conceperunt calori proportionalem. Calores autem sic inventi eandem habuerunt rationem inter se, cum caloribus per thermometrum inventis; et propterea rarefactiones olei ipsius caloribus proportionales esse recte assumpsimus."

Critically viewed, matters stand as follows. The temperature-numbers repose on an *arbitrary* convention. They may be taken proportional to the voluminal increments or they may not. But after a decision regarding them has been reached, *observation* alone can decide whether the losses are proportional to the temperatures. On the other hand, the temperature-numbers could be so *chosen* that the losses would be proportional to the temperatures even on the assumption of some different law of cooling from that actually obtaining.

There is thus no necessary connection between Newton's propositions. Nothing whatever follows from his observations regarding the correctness or incorrectness of his scale of temperature. Dulong and Petit have in fact shown, as we shall see later, that the harmony between Newton's assertions is immediately ruptured if the observations on cooling are made with a thermometer within wide limits of temperature and with greater care than Newton bestowed upon them. Newton's two assumptions contain, so to speak, two different scales of temperature.

But nothing would prevent our employing Newton's pyrometric principle as a *definition of a scale of temperature,* by considering on some principle of co-ordination the times counted backwards as *inventorial numbers* of the corresponding thermal states of the cooling body. Whether this definition is or is not independent of the nature of the bodies and what is the relation of this scale to any other now in common use, could be ascertained only by special experiments and only to the extent to which the two scales under comparison were actually and simultaneously accessible (without extrapolation) to experiment.

Another pyrometric method, early devised by Amontons[1] in imperfect form, was employed by Biot. Biot[2] showed by experiment and from considerations of theory that in a metal bar one end of which has been exposed sufficiently long to a constant source of heat, the excesses of the temperature of the bar over that of the air decrease in geometrical progression as we move away in arithmetical progression from the heated end,—as far at least as the process can be followed with a thermometer. Ascertaining the ratio of the progression at the colder end and assuming that the law holds *without limit* for all temperatures, however high, we can infer the temperatures of the places which by reason of their great heat are inaccessible to direct thermometric examination. Amontons had assumed that the temperatures increased from the cold to the

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[1] *Histoire de l'Académie*, 1703, p. 6.   [2] *Journal de Mines*, 1804, Vol. XVII., p. 203.

hot end by the law of a straight line. But since the ratio of the above-mentioned progression depends on the dimensions and the material of the bar, it will be seen that the temperature-numbers obtained by Amonton's principle would depart very considerably from those obtained by Biot's. Examining Biot's case within wide ranges of temperature and with greater exactness, as Forbes has recently done, it appears that even within the limits accessible to a thermometer the ratio of the geometrical progression depends on the temperature. Thus Biot's pyrometric principle also, if it is to be consistently maintained, involves a new definition of temperature, and what was said regarding Newton's principle holds true substantially regarding Biot's. As for the rest, the relation between the two methods is simple. In Newton's method the temperatures to be determined *succeed one another*, in Biot's they occur *side by side*. The temperature-numbers employed as inventorial numbers are obtained in the first instance as *measures of time* and in the second as *measures of length*. Newton's idea may have suggested Biot's. Even Lambert[1] sought to correct Amonton's principle after the manner of Biot.[2]

Black also devised a pyrometric method, based on his researches in calorimetry. If a body of mass m be cooled in a quantity of water M from the temperature u_1 to the temperature u, then, as thermometric observation shows, the water M will be heated by an amount proportional to the product $ms(u_1-u)$, where s is a constant peculiar to the cooling body (viz., its specific heat). If M be the mass of the water and u_2 its initial temperature, the equation will obtain

$$m s(u_1 - u) = M(u - u_2),$$

from which follows for the initial temperature u_1 of the cooled body

$$u_1 = u + \frac{M(u - u_2)}{m s}$$

If m and s be small and M large, u and u_2 will remain within reach of the ordinary thermometric scale, even when the body to be cooled has been heated to a degree far beyond it. Assuming with Black the unlimited validity of the principle, the initial temperature u_1 can still be ascertained from the above equation. For example, we can cool in a large mass of water a piece of iron of known weight and specific heat which has been taken from a fur-

1 *Pyrometrie*, pp. 184-187.

2 Black, *Lectures on Chemistry*, German translation by Crell, 1804, Vol. 1., pp. 108, 277.

nace, and ascertain in this way the temperature of the furnace. Inasmuch as the careful inquiries of Dulong and Petit have demonstrated that s depends on the temperature even within the limits of the ordinary scale, and since any investigation of s outside the limits of this scale is impossible, it will be seen that Black's pyrometric principle also involves a new definition of temperature. Substantially the same remarks may be made with respect to this method as were advanced regarding the methods discussed above.

A pyrometric method can be constructed on the basis of any physical property which varies with the thermal state. Pyrometers have been devised that rest on variations of volume or pressure, and others have been conceived which indicate the thermal state by melting, boiling, dissociation, and alterations of tenacity. The spectral photometer, the polaristrobometer, have also been put to pyrometric use. Acoustic pyrometers are based on the changes in the pitch and the wave-length of a note with the temperature. Finally, change of magnetic moment has been thought of in connection with temperature, and attempts have been made to put to pyrometric use the dependence of electric resistance on the temperature, as well as the alteration of thermoelectromotive force with the temperature. The writings of Weinhold,[1] Bolz,[2] Holborn and Wien,[3] as well as the more recent work of Barus,[4] contain explicit information on all these points, including a rich bibliography.[5]

After the foregoing there will be no doubt that each individual pyrometric method simply furnishes an index of a thermal state by means of which that state can again be recognised and reproduced. For many practical purposes this is in itself very valuable and is often quite sufficient. The number which is the result of any pyrometric observation has therefore no other significance than that of an *inventorial number*. If from three observations we obtain three numbers, $a < b < c$, all the information that these numbers furnish is that the thermal state to which b belongs lies between the two states to which a and c belong. It is antecedently unreasonable to expect any agreement between the numbers obtained by the *different* pyrometric methods, for the reason that in general every pyro-

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[1] A. Weinhold, *Pyrometrische Versuche, Poggendorff's Annalen*, Vol. 149, 1873, p. 186.

[2] C. H. Bolz, *Die Pyrometer*, Berlin, Springer, 1888.

[3] L. Holborn and W. Wien, *Ueber die Messung hoher Temperaturen, Wiedemann's Annalen*, Vol. 47, 1892, p. 107.

[4] C. Barus, *Die physikalische Behandlung und die Messung hoher Temperaturen*, Leipzig, J. A. Barth, 1892.

[5] For the bibliography of thermometry see also H. Griesbach, *Physikalisch-chemische Propædeutik*, Leipzig, Engelmann, 1900, Chapter 27, pp. 1–88.—*Tr.*

metric method involves a *special* definition of temperature. The reduction of pyrometric numbers to the Celsius scale can only be performed to the extent within which this method can be employed simultaneously with the air-thermometer. Reductions of this kind have been attempted by Weinhold, Holborn and Wien, to mention only the most important.[1] Sir William Siemens[2] speaks of the calculations of the temperature of the sun which were made by Secchi, Zöllner, and others, and which amounted respectively to 10,000,000° C. and 27,100° C. Apart from the objections which may be raised against the premises of this calculation and the methods of computation, it is to be remarked that indications in *degrees Celsius* far outside the possible limits of employing the air-thermometer have absolutely no meaning whatever.

[1] See the works cited above.

[2] *On the Conservation of Solar Energy*, German translation, Berlin, Springer, 1885, p. 144.