CRITIQUE OF THE CONCEPT OF TEMPERATURE.[1]

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[CONCLUDED.]

IT is remarkable how long a period elapsed before it definitively dawned upon inquirers that the designation of *thermal states* by *numbers* reposed on a *convention*. Thermal states exist in nature, but the concept of temperature exists only by virtue of our arbitrary *definition*, which could very well have taken another form. Yet until very recently inquirers in this field appear more or less unconsciously to have sought after a *natural* measure of temperature, a real temperature, a sort of Platonic Idea of temperature, of which the temperatures read from the thermometric scales were only the imperfect and inexact expression.

The concepts temperature and quantity of heat were never kept clearly apart by Black and Lambert, and for both these ideas, between which we now distinguish, Richmann uses the same word, *calor*. At this stage, therefore, we are unwarranted in expecting clearness. But the obscurity extends farther than we should have thought. Let us look at the facts.

Lambert[2] well characterises the state of opinion of his time when he says: "Inquirers doubted whether the *actual* degrees of heat were in reality proportional to the degrees of the expansion. And even granting that this were so, the further question arose, at what degree the counting should begin." He then discusses Renaldini's proposition to graduate thermometers by means of water-mixtures, and he appears to have regarded this last scale as a natural one.

Dalton has the following passage:[3] "Liquids have been tried, and found to expand unequally, all of them expanding more in the

[1] Translated from Mach's *Principien der Wärmelehre* by Thomas J. McCormack.

[2] Lambert, *Pyrometrie*, p. 52.

[3] *Loc. cit.*, p. 9.

higher temperatures than in the lower, but no two exactly alike. Mercury has appeared to have the least variation, or approach nearest to uniform expansion."

Gay-Lussac says: "The thermometer, as it exists to-day, cannot indicate the exact relationships of *heat*, for we are not yet cognisant of the connection obtaining between the degrees of the thermometer and the *quantities* of heat which these degrees possibly indicate. It is generally believed, indeed, that the equal divisions of this scale represent equal *tensions* [expansive forces] of the caloric; but this opinion is based on no very positive fact."[1] Manifestly Gay-Lussac was in a fair way to overcome the obscurity of his contemporaries on this point, but he was nevertheless unsuccessful.

It is very singular that inquirers of the exactness of Dulong and Petit, who were the first to introduce clearness into this field, continually lapse, in their expressions at least, to the old points of view. We read in one place:[2] "It will be seen, from the deviation that occurs at so low a temperature as 300°, how greatly glass departs from *uniformity*." We ask in astonishment: "By what criterion is the 'uniformity' or 'lack of uniformity' of glass to be estimated and measured?" The following passage is also characteristic:[3] "We are constrained to say, nevertheless, that the well-known uniformity in the principal physical properties of all gases, and especially the identity of their laws of dilatation, render it very *probable* that in this class of bodies the *disturbing* causes do not produce the same effects as in solids and liquids; and that consequently the changes of volume produced by the action of the heat are in the present instance *more immediately dependent on the force that produces them*."[4]

This vacillation between a physical and a metaphysical point of view has not been entirely overcome, even to-day. In an excellent modern text-book by a distinguished inquirer in this field, we

[1] *Ann. de chim.*, XLIII., 1802, p. 139: "Le thermomètre, tel qu'il est aujourd'hui ne peut servir à indiquer des rapports exacts de la *chaleur*, parce que l'on ne sait pas encore quel rapport il y a entre les degrés du thermomètre et les quantités de chaleur qu'ils peuvent indiquer. On croit, il est vrai, généralement, que des divisions égales de son échelle représentent des tensions égales de calorique; mais cette opinion n'est fondée sur aucun fait bien positif."

[2] *Ann. de chim.*, VII., 1817, p. 139.

[3] *Ann: de chim.*, VII., 1817, p. 153.

[4] "Nous devons dire cependant que l'uniformité bien connue dans les principales proprietés physiques de tous les gaz, et sourtout l'identité parfaite de leurs lois de dilatation, rendent très-vraisemblable que, dans cette classe de corps, les causes perturbatrices n'ont plus la même influence que dans les solides et liquides; et que par conséquent les changements de volume produits par l'action de la chaleur y sont dans une dépendance plus immédiate de la force qui les produit."

read: "The indications of the air-thermometer are comparable. But it by no means follows from this that the air-thermometer actually measures that which we *conceive as temperature*; it has, in fact, never been proved that the increase of the pressure of gases is *proportional to the increase of the temperature*, for hitherto we have only *assumed* this."

No less a man than Clausius has similarly expressed himself: "We may infer from certain properties of gases that the mutual attraction of their molecules is very weak at their average distances and hence offers a very slight resistance to the expansion of the gases, so that it is the walls of the containing vessel that have to offset by their resistance nearly the entire effect of the action of the heat. The outward, sensible pressure of the gas, accordingly, forms an *approximate* measure of the repellent force of the heat contained in the gas, and, therefore, conformably to the preceding law, this pressure must be *approximately* proportional to the absolute temperature. The correctness of this inference has, indeed, so much intrinsic probability that many physicists since Gay-Lussac and Dalton have assumed it outright, and based upon it their calculations (!) of the absolute temperature."[1]

In a valuable treatise on pyrometry we find the following:[2] "In view of Gay-Lussac's discovery, made as early as 1802, that all gases suffer, under the action of heat, like expansions for like increases of temperature, the *hypothesis* is doubtless justified that the expansion in question is *uniform* for all *degrees of temperature*, inasmuch as it is *more probable* that the expansion should be uniform than that all gases should exhibit *the same variations*."

On the other hand, it is to be particularly noted, that W. Thomson, as early as 1848, in propounding his absolute thermodynamic scale of temperature, was very clear on this matter and went critically to the bottom of it, as we shall see in a later chapter in detail.

After what has just been adduced, the preceding exposition, however obvious it may appear to individual physicists, will not, I trust, be regarded as altogether redundant. We repeat, the question is always one of a scale of temperature that shall be *universally comparable and that can be constructed* with accuracy and certainty, and never one of a "real" or "natural" scale.

It could be easily shown, by analogous examples from other departments of physics, that men generally are inclined to hypostatise their abstract ideas, and to ascribe to them a reality outside

1 *Mechanische Wärmetheorie*, 1864, I., p. 248. 2 Bolz, *Die Pyrometer*, Berlin, 1888, p. 38.

of consciousness. Plato, in his doctrine of Ideas, merely exploited this tendency. Even inquirers of the rank of Newton, despite their precepts, were not always discreet enough in this respect; it will therefore repay the trouble to inquire in what the difficulty in the present case consists. We start in our investigations from the *sensation of heat*, and find ourselves later obliged to substitute for this original criterion of the behavior of bodies *other criteria*. But between these criteria, which may be quite distinct, *no exact parallelism* obtains. For this reason, latently and unconsciously, the original sensation of heat, which was replaced by these non-conforming criteria, remains the *nucleus* about which our ideas cluster. Then, on our theoretically discovering that this sensation of heat is in its turn nothing but a symbol for the collective behavior of the body, which we already know and shall later know better,[1] our thinking compels us to group these varying phases of collective behavior under some *single* head and to designate them by a *single* symbol called *state of heat*. Scrutinising our procedure closely, we again discover this same *sensation of heat*, which is the initial and the most natural *representative* of the group in its entirety, as the indistinct nucleus of the symbol last reached. And to this symbol, which is after all not entirely our arbitrary creation, we appear to be forced to attribute reality. Thus, the impression arises of a "real temperature," of which that read from the thermoscope is only a more or less inexact expression.

Newton's conceptions of "absolute time, "absolute space," etc., which I have discussed in another place,[2] originated in a quite similar manner. In our conceptions of time the *sensation of duration* plays the same part with regard to the various measures of time as the sensation of heat played in the instance just adduced.[3] The situation is similar with respect to our conceptions of space.

Once we have clearly comprehended that by the adoption of a new, arbitrarily fixed, more sensitive and more delicate criterion of the thermal state an entirely *new* point of view has been assumed, and that henceforward the new criterion alone is the basis of our investigations, the entire illusion will be dispelled. This new criterion, or *indicium*, of the thermal state is the *temperature-number*, or more briefly, the *temperature*, which reposes on an arbitrary convention in three respects,—first with regard to the selection of vol-

[1] Compare Mach, *Analysis of the Sensations*, Eng. trans., Chicago, 1897, pp. 18 et seq. Also Popper, *Elektrische Kraftübertragung*, Vienna, 1884, p. 16.

[2] *Science of Mechanics*, Eng. trans., 2nd ed., pp. 222–238 and 541.

[3] *Analysis of the Sensations*, Eng. trans., pp. 109 et seq.

ume as the index, secondly with regard to the thermoscopic substance employed, and thirdly with regard to the principle by which the numbers are coördinated with the volume.

An illusion of another sort is involved in a peculiar, almost universally accepted, process of reasoning which we shall now discuss. Taking the numbers indicative of the temperatures as proportional to the pressures exerted by a mass of gas at constant volume, it will be seen that while the pressures and the temperatures may increase without limit, they can never fall below zero. The equation

$$p = p_0(1 + \alpha t)$$

asserts that for every degree increase of temperature the pressure increases by $\frac{1}{273}$ of its amount at the point of melting ice; or rather, contrariwise, that when the pressure increases $\frac{1}{273}$, *we reckon* the temperature one degree higher. For temperatures below the point of melting ice we should have

$$p = p_0(1 - \alpha t),$$

from which it will be apparent that if $\frac{1}{273}$ of the pressure p_0 be deducted 273 times, and the temperature $-273°$ C. attained, the pressure will be zero. The favorite mode of conception now is, that when a gas has been cooled off to this point it no longer contains any "heat"; that consequently any further cooling below this temperature is impossible; that, in other words, the thermal states have apparently *no upper* limit, but possess a *lower* limit at $-273°$ C.

The principle of coördination employed by Dalton[1] did not remain in use, but not the slightest objection can be made to its admissibility. On this principle, when the pressure of the gas increases by 1.0179, the temperature increases ten Daltonian degrees. When the pressure diminishes by 1.0179, the temperature sinks ten degrees. We can repeat this last operation as often as we wish without ever reaching a pressure zero. If Dalton's scale were used, the idea need never have occurred to us that a thermal state could exist having the gaseous pressure zero,—that the series of thermal states had a lower limit. The possibility of a gaseous pressure zero would not, indeed, have been affected by this fact, because Dalton does not reach the lower limit for the reason that he moves toward it, like Achilles toward his tortoise in the famous paradox, with steps of diminishing magnitude. The essential point to be empha-

[1] See *The Open Court* for February, p. 102.

sised here is the precariousness of regarding outright the properties of a *system of symbols* as the properties of the *things symbolised* by them.

Amontons, in propounding his scale of temperature, starts from the idea that the pressure of a gas is produced by "heat." But this absolute zero-point is not the only one that has been proposed, nor is it the only one that could be proposed on the ground of equally sound ideas. Taking the coefficient of expansion of mercury, and pursuing the same train of reasoning as with air, we should obtain —5000° C. as our absolute zero. As with air and with every other body, so likewise here with mercury, the coefficient of *expansive force* might be employed instead of the coefficient of expansion, in order to eliminate the distressing idea of a body losing its volume when it loses its heat.

Dalton's[1] conception is that a body contains a certain quantity of caloric. Increasing the caloric raises the temperature; withdrawing it altogether reduces the body to the absolute zero-point. This idea of heat as a substance (caloric) was derived from Black, although the latter inquirer was no friend of speculations of the stripe we are now discussing. If ice at 0° C. is converted into water at 0° C., and for every kilogramme in this process eighty kilogramme-calories are absorbed, Gadolin[2] and Dalton contend that owing to the doubling of the capacity for heat by the liquefaction of the water, the entire loss of caloric from the absolute zero-point to 0° C. is compensated for by the eighty thermal units in question. Whence it follows that the absolute zero-point lies at $2 \times 80 = 160°$ C. below the melting-point of ice. The same zero-point is on the same premises obtained for many other bodies. But for mercury, which has a low fusing-point and which exhibits a very slight difference of specific heat in its solid and liquid conditions, 2021° C. below the melting-point of ice is obtained as the absolute zero. If two bodies, A and B, of like temperature, be mixed together, and the mixture $A + B$ shows an alteration of temperature, we can in an analogous manner, after determining the specific heats of A and B and $A + B$, deduce the absolute zero-point from the change in the temperature. By mixing water and sulphuric acid Gadolin found the absolute zero-point to lie between —830° C. and —1720° C. Other mixtures, and also chemical combinations, have been similarly treated, and have again yielded different results.

We have thus a multitude of different absolute zeros. To-day

[1] *Loc. cit.* [2] Cited by Dalton in another work.

only one of these is in use, that of Amontons, which, conformably to the dynamic theory of gases, has been connected with the destruction of the velocity of the moving gaseous molecules. But all these deductions alike rest on hypotheses regarding the processes by which we conceive the phenomena of heat to be produced. Whatever value we may attribute to these hypothetical constructions, we must yet admit that they are unproved and unprovable, and cannot antecedently determine facts which may at some time be rendered amenable to observation.

We now revert to the point which we were discussing. The pressures of gases are *indices* or *symbols* of the thermal states. When the pressures vanish, the symbols likewise vanish; our gas is rendered unserviceable as a thermoscope; we must seek another. That the thing symbolised also disappears, does not at all follow. For example, if a thermoelectromotive force on approaching a certain high temperature should diminish, or become zero, it would doubtless be thought extremely rash were this temperature to be regarded as indicating an *upper* limit to the states of heat.

The temperature-numbers again are symbols of the symbols. From the fact that our fortuitously chosen system of symbols has a limit, nothing whatever follows as to the limits of the thing symbolised. I may represent *sensations of tone* by *rates of vibration.* These latter, as positive numbers, have a lower limit at zero, but no upper limit. I may also represent sensations of tone by the *logarithms* of the rates of vibration, and obtain a much better view of the musical intervals. In which case, my system of symbols (running, as they do, from $-\infty$ to $+\infty$) has neither a lower *nor* an upper limit. But the system of tone-sensations is not a whit disturbed by this; it has *both* an upper and a lower limit. I may *define* an infinitely high or an infinitely low tone by my system of symbols, but it in no wise follows from this that such a tone *exists.*

The entire train of reasoning reminds one vividly of the so-called ontological proof of the existence of God; it is scholastic to a degree. The concept is defined, and existence is predicated of its *attributes*; whence follows forthwith the existence of what has been defined. It will scarcely be gainsaid that a similar logical looseness is unpermissible in modern physics.

We may accordingly assert, that even granting it were possible by cooling a gas to reduce its pressure to zero, this result would simply prove the unfitness of gases as thermoscopic substances from this point downward. But that the thermal states have or have not a lower limit, would in no wise follow from it.

And, similarly, nothing follows as to an *upper* limit for thermal states from the fact that the pressure of a gas may be *conceived* to increase without limit, or from the fact that the numbers expressing the temperatures have no upper limit. A body melts and boils at certain temperatures. And the question naturally arises whether a gas can attain indefinitely high temperatures without suffering important alterations of character.

Experience alone can determine whether the series of thermal states has a lower or an upper limit. Given a body of definite thermal conditions and supposing no other can be produced that is hotter or colder than it, then and then only can such a limit be established.

The view here taken does not exclude our conceding to Amontons's zero the rôle of a *fiction*, or our investing the Law of Boyle and Gay-Lussac with the simple form before referred to,[1] whereby many discussions to be later developed are very materially simplified.

From the foregoing it will be readily seen that *temperature* is nothing but the *specification* or *designation* of a thermal state by a *number*. This temperature-number has exclusively the properties of an *inventorial number*, by means of which the same thermal state can again be recognised, and, if necessary, sought for and reproduced. This number likewise informs us in what *order* the designated thermal states succeed one another and *between* what other states a given state is situated. In the investigations to follow it will appear that the temperature-numbers fulfil still other, and indeed extremely comprehensive, functions. But this was not due to the acumen of the physicists that propounded the system of temperature-numbers, but was the outcome of several fortunate circumstances, which no one could foresee and no one control.

The *concept of temperature* is a *concept of level*, like the height of a heavy body, the velocity of a moving mass, electric and magnetic potential, and chemical difference. Thermal action takes place between bodies of different temperature, as electric action does between bodies of different potential. But whilst the concept of potential was deliberately framed in perfect consciousness of its advantages, in the case of the concept of temperature these advantages were a matter of good luck and accident.

In most departments of physics the *differences* alone of the *level values* play a determinative part. But temperature appears to share in common with chemical level the property that its level values are *per se* determinative. The fixed fusing-points, melting-points, boiling-points, critical temperatures, temperatures of combustion and dissociation, are obvious instances.

[1] See *The Open Court* for December, p. 738.