CRITIQUE OF THE CONCEPT OF TEMPERATURE.[1]

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IT appears from what has preceded that the *volume* of a body may be employed as a *mark* or *index* of its *thermal state*, and that consequently change of volume may be looked upon as indicating a change of thermal state. It stands to reason that the changes of volume here involved are not such as are determined by alterations of pressure or electric force, or by any other circumstances inducing change of volume though known from experience to be independent of the thermal state. Concomitantly with the thermal sensation which a body provokes in us, other properties of the body also undergo alteration,—as, for example, its electric resistance, its dielectric constant, its thermoelectric motive force, its index of refraction, etc. And not only might these properties be employed as indices of the thermal state, but they actually have found such employment. In the preferment of volume, therefore, as a test of states of heat, there is involved, despite the manifest practical advantages of the choice, a certain *caprice*; and in the general adoption of this choice, a *convention*.

A body employed as a thermoscope initially indicates only its *own* state of heat. But observation informs us that two bodies, *A* and *B*, which at the start provoke in us unlike sensations of heat, after prolonged contact excite in us precisely the *same* sensations, that is, equalise the difference of their thermal states. Transferring this empirical discovery by analogy to volumes as indices of thermal states, we assume that a thermoscopic body indicates not only its own state but also that of any other body with which it has been sufficiently long in contact. But in so summarily proceeding we are acting without warrant. For sensation of heat and volume are two entirely disparate elements of observation. The

fact of their connection has been determined by experience; the manner and extent of their connection it also remains for experience to determine.

We may convince ourselves easily that volume and sensation of heat are indices of widely different *sensitiveness*, and generally of different *character*. By means of volume we can perceive changes of state that utterly escape our sensations of heat. And owing to the dissimilar properties of the thermoscope and the sensory organ of heat, these instruments may give not only different, but even diametrically opposed, indications. The instances adduced on page 643 of the November *Open Court* amply illustrate this fact. But the indications may also be different with respect to equalised thermal states. Two pieces of iron after sufficient contact give the *same* sensations of heat. A piece of wood and a piece of iron after contact also show on the thermoscope the same indications. But if both *feel warm*, the iron will feel the *warmer* of the two, no matter how long they have been in contact; and if both feel *cold*, it will feel the *colder*. This, as is well known, is due to the greater conductivity of the iron, which imparts its thermal state to the hand more rapidly than the wood.

Volume being a more sensitive index of the thermal state than sensations of heat, it is more advantageous and rational for us to resort for our empirical results to observations on volume, as it is also to base upon these our definitions. Observations based on sensations of heat may serve us for guidance, but to employ them outright and uncritically is, as we now know, inadmissible. We assume with this perception an entirely new point of view, and one which is essentially different from that occupied by the original founders of thermometry. The defective separation of these two points of view, which owing to the gradual transition of the one into the other was unavoidable, became, as we shall subsequently see, the occasion of many obscure speculations.

The fact that a thermoscope shows an increase of volume when in contact with a body that is perceptibly warmer, and a diminution of volume when in contact with one that is perceptibly colder, is indisputable. But it is without the power of our *sensations of heat* to inform us whether this continues so until the thermal states are completely equalised. On the other hand, we can, consonantly with our new point of view, arbitrarily lay down the following definition: *Those thermal states are to be regarded as the same in which bodies produce in one another no alterations of volume* (mechanical pressures, electric forces, etc., excluded). This definition may be

applied immediately to the thermoscope, which indicates the thermal state of the body it touches the moment mutual alteration of volume by contact ceases.

If two bodies A and B are, as the common phraseology goes, both as *warm* as, or, both provoke the same sensations of heat as, a third body C, then is A, in the same sense, *just as warm as* the body B. This is a logical necessity, and we are incapable of thinking it otherwise. The contrary would involve our holding two *sensations* to be at the same time *alike* and *different*. But we are not permitted by our definition to assume outright that if A and B both do not produce alterations of volume in C, A likewise will produce none in B. For this last result is an *experience*, the outcome of which we have to await, and which is not co-determined by the two first-mentioned experiences. This is a simple consequence of the position above assumed.

But experience shows that if there be a series of bodies A, B, C, D....each of which has been sufficiently long in contact with that which follows, the thermoscope will give the same indication for the one as for the other. And, furthermore, we should be led into singular contradictions with our daily thermal experience, were we to assume that the equality of the physical condition of A and B, and B and C, conformably to the above definition, did not likewise determine the equality of the physical condition of A and C. Inverting the order of the bodies, which now do not induce alterations of volume in one another, would result in new alterations. But as far as our thermoscopic experience extends, this nowhere occurs.

To my knowledge, Maxwell is the first who drew attention to this point, and it may not be amiss to mention that Maxwell's remarks are quite similar to those which I advanced respecting the concept of mass.[1] It is extremely important to note that whenever we foist a definition upon Nature, it is imperative to wait and observe whether it accords perfectly with her constitution. We may indeed frame our concepts as our caprice dictates, but with the exception of pure mathematics, we are bound, even in geometry, and far more so in physics, to investigate minutely the extent to which reality conforms to our concepts.

Any conception, therefore, of the *experiences* familiar to us, if

[1] Maxwell, *Theory of Heat*, 9th edition, London, 1888. I surmise that the remarks cited were contained in the first edition of 1871; but I am unable to verify my conjecture, as I have had access only to Auerbach's translation of the fourth edition (1877). My considerations on the concept of mass were published in 1868 in the fourth volume of *Carl's Repertorium*, again in 1872 in my tract *Erhaltung der Arbeit*, and finally in 1883 in my *Mechanics* (Eng. trans., Chicago, 2nd edition, 1902).

it is to be free from contradiction, demands the assumption that two bodies A and B which are in the same thermal state as regards a third body C are in the same thermal state as regards each other.

The stronger the thermal sensation, the greater the volume of the thermoscopic substance. Hence again, by analogy, the following arbitrary definition may be set up : *Those thermal states are to be regarded as the more intense in which bodies produce in the thermoscope greater augmentations of volume.* After the analogy of the thermal processes observable by sensation, we should then expect that of two bodies A and B that which produced in the thermoscope the greater augmentation of volume would on contact also induce in the other an augmentation of volume, but in itself a diminution. But while the analogy holds generally true, it may fail utterly in special cases. Water furnishes an example where the analogy is misguiding. Two masses of water at $+3°$ C. and $+5°$ C. both show a diminution of volume on contact. Two masses of water at $10°$ C. and $15°$ C. present the normal case. Two masses at $1°$ C. and $3°$ C. present a case diametrically opposed to the analogy.

It will be seen from the foregoing that water as a thermoscope could, under certain circumstances, give the same indication for two thermal states for which other thermoscopes would give different indications. The use of water as a thermoscope, at least in the thermal field under consideration, is accordingly to be avoided.

Our *sensations of heat*, like the thermoscopic volumes, form a simple series, *a simple continuous manifold*; but it does not follow from this that *states of heat* form also such a manifold. The properties of the system of symbols we employ are not decisive of the properties of the states symbolised. If we were to take, for example, as our criterion of the state of a body K the pull exerted by K on an iron ball suspended from a balance, these pulls, the aggregate of which as symbols likewise constitute a *simple* manifold, could be determined indifferently by the electric, magnetic, and gravitational properties of K, and would be the symbolic corre-

Fig. 28.

spondent consequently of a *threefold* manifold. Inquiry must determine in each case whether the symbolic system chosen is the appropriate one.

Let A, B, C, D, E be a series of bodies, of which each exhibits a more intense thermal state than that which follows. (Fig. 28.) As far as our experience goes, a body can be transported from the state of A to that of E only by way of the states B, C, D and

the states intermediate to them. There is nothing in the domain of experience to suggest that this could also be effected through a succession of conditions *MN* situate outside of the series *B, C, D.* The assumption of a *simple continuous manifold of thermal states* is sufficient.

It was remarked above that there was an *arbitrary convention* involved in the choice of *volume* as a thermoscopic index. There is a further arbitrary choice involved in the adoption of a thermoscopic *substance.* Yet if the substance selected were universally adopted, the resulting thermoscope would substantially accomplish everything that could be demanded of it. The thermoscope would be exposed to the greatest possible number of thermal states, established as invariable by cessation of change on the part of the thermoscope, and these points of cessation would be distinguished by *marks* and *names;* such as the freezing-point of mercury, the melting-point of ice, the congealing-point of linseed-oil and aniseed-oil, the melting-point of butter, blood-heat, the boiling-point of water, the boiling-point of mercury, etc. *These marks would then enable us not only to recognise a recurring state of heat, but also to reproduce a state already known to us.* But in accomplishing this, the essential function of the thermoscope is achieved.

The inconveniences of such a system, which as a matter of fact long prevailed, would soon be manifest. The more delicate the inquiry, the more fixed points of this sort would be necessary; and ultimately they would not be attainable. Furthermore, the number of the names to be remembered would be annoyingly augmented, and

Fig. 29.

it would be impossible to discover from the character of these names the *order* in which the thermal states under consideration succeeded one another. This order would needs be specially noted in each individual case.

But there exists a system of names which is *at the same time* a system of *ordinal symbols,* permitting of indefinite extension and refinement, viz., numbers. Substituting numbers for names as our designations of thermoscopic marks, the inconveniences in question are eliminated. Numbers may be continued into infinity without effort; between two numbers any number of other numbers may be mechanically interpolated; it is apparent immediately from the very nature of a number between what other numbers it lies. This could not have escaped the notice of the inventors of the early

thermoscopes; and the idea was actually applied, though to vary-
ing extent and with varying appropriateness.

For the introduction of this more appropriate system, a new
convention was necessary,—a convention respecting the *manner* in
which the numbers should be *coördinated* with the thermoscopic
marks. And here new difficulties arose.

One of the methods proposed consisted in scratching on the
capillary tube of the thermoscopic envelope *two* fixed points (the
melting-point of ice and the boiling-point of water). The *apparent*
voluminal increment of the thermometric substance (neglecting the
dilatation of the vessel) was next divided into 100 parts (degrees),
and *this* division was then *continued* beyond the boiling and melting-
points. By means of these fixed points and the principle of co-
ordination referred to, every number *appeared* to be *univocally* con-
nected with a physically determined thermal state.

But this connection is immediately broken when some other
thermoscopic substance or some other enveloping material is
chosen. Laying off the volumes of any given substance as abscis-

Fig 30.

sas and erecting those of any other in
the same thermal states as ordinates,
we obtain, according to Dulong and
Petit, by joining the extremities of the
ordinates, not a straight line, but a
curve, similar to that pictured in Fig-
ure 30, and differing for every two dif-
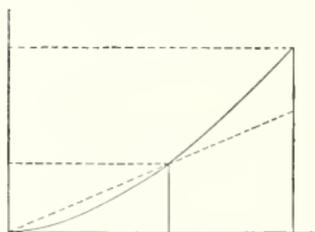ferent substances. In point of fact,
substances do not expand proportion-
ally to one another when subjected to the same thermal changes,
as we have already learned. Hence, on the *same* principle of
coördination, sensibly different numbers are assigned to the same
thermal states for each and every thermoscopic substance.

Even adopting exclusively mercury as our thermal substance,
the expansion of the glass of the containing vessel, which is not a
vanishing quantity comparatively, exercises an appreciable influ-
ence upon the march of the apparent expansion, and this influence
is peculiar to every different kind of glass. Therefore, even though
the same principle of coördination be employed, strictly speaking
the connection between numbers and thermal states is again pecu-
liar to each thermoscope.

When attention was directed to the like behavior of gases un-
der the same thermal conditions, the choice of a *gas* as a standard
thermoscopic substance was, by reason of this property, regarded

as *less conventional* and as having deeper roots in *Nature*. But while it will appear that this opinion is erroneous, yet there are other reasons which make for this choice, which was a felicitous one, though at the time it was made no one could have been aware of the fact.

One of the greatest advantages that gases offer is their remarkable expansibility and the consequent enhanced sensitiveness of the thermoscopes. Furthermore, the disturbing effect of the variable envelopes is very considerably reduced by this great expansibility. The expansion of mercury is only about seven times as great as that of glass. The expansion of the glass and the variation of this material find, therefore, very perceptible expression in the apparent expansion of the mercury. But the expansion of a gas is 146 times as great as that of glass.[1] The expansion of the glass, therefore, has only a very slight effect upon the apparent expansion of the gas, and a vanishing effect upon the variations in the different kinds of glass. In the case of gas-thermometers, therefore, when the fixed points and the principle of coördination have been determined upon, the connection between the numbers and the thermal states is far exacter than with any other thermoscope. The envelope selected, or more briefly, the individuality of the thermoscope, can have only a very inconsiderable influence upon this relationship; the thermoscopes are rendered in high degree *comparable*,—a point which confirms the critique of Dulong and Petit. We shall in the considerations to follow make tacit reference to an air-thermoscope.

That number which, conformably to any chosen principle of coördination, is *uniquely coördinated with a voluminal indication of the thermoscope, and consequently with a state of heat, is called the temperature of that state.* It will be generally denoted in the following by *t*. The temperature-numbers are dependent on the principle of coördination, $t = f(v)$, where *v* is the thermoscopic volume, and, consequently, for the same state of heat they will vary greatly according to the principle adopted.

It is instructive to note that different principles of coördination actually have been propounded, although only one has proved of actual practical scientific value and hence remained in use. One of these principles may be termed the Galilean. It makes the temperature-numbers proportional to the real or apparent voluminal increments from a definite initial volume v_0, corrresponding to a definite thermal state.

[1] Cf. Pfaundler, *Lehrbuch der Physik*, II., 2. See also *Open Court* for November, 1902, p. 651.

To the volume: v_0, $v_0(1 + a)$, $v_0(1 + 2a)$, ... $v_0(1 + ta)$,
corresponds

the temperature: 0,   1,   2,   ... t,

For a here we take the hundredth part of the coefficient of the voluminal increment from the melting-point of ice to the boiling-point of water (viz., $\frac{1}{273}$), the temperature-number 100 falling to the last-named point. The same principle admits of extension beyond the boiling and melting points, the temperature-numbers in the latter case being reckoned negatively.

An entirely different principle of coördination is that of Dalton. It is as follows:

To the volume: ... $\dfrac{v_0}{(1.0179)^2}$, $\dfrac{v_0}{1.0179}$, v_0, $v_0 \times 1.0179$, $v_0 \times (1.0179)^2$,...
corresponds

the temperature... —20,   —10,   0,   +10,   +20   ...

If we take with Amontons and Lambert the expansive force of a mass of gas of constant volume as our thermoscopic index, and make the numbers indicative of the temperatures *proportional* to the expansive force of the gas, we shall again have, strictly speaking, a different principle. But owing to the validity of the Law of Boyle and Gay-Lussac within wide limits, and the slight deviation of the coefficient of expansive force from the coefficient of expansion,—facts which at the time this scale was proposed were only imperfectly known,—it happens that the properties of Amontons's scale are not sensibly different from those of Galileo's.

Calling p the pressure of a mass of gas of constant volume, p_0 the pressure at the melting-point of ice, and k a constant, Amontons's principle of coördination is expressed by the equation $t = \dfrac{kp}{p_0}$. A second fundamental point is unnecessary on this scale.[1] Since p and p_0 depend in the same manner on the thermal states that v and v_0 do, the new scale has precisely the same properties as the old. For $p = 0$, $t = 0$. Putting $k = 273$, the degrees assume their customary magnitude: for the melting-point $t = 273$, for the boiling-point $t = 373$. The new scale coincides absolutely with the old scale, if the zero-point be placed on the melting-point, and the temperature-numbers downward be reckoned negatively.

The employment of the air-thermometer involves, whether volumes or pressures be taken as the thermoscopic indices, a *defini-*

[1] See *The Open Court* for November, 1902, p. 647.

tion of temperature.   Starting from the equations $p = p_0 (1 + at)$, or $v = v_0 (1 + at)$, we *arbitrarily* posit, *that the temperature t shall be* given by the equation.

$$t = \frac{p - p_0}{a\,p_0} \text{ or } t = \frac{v - v_0}{a\,v_0}.$$

Amontons's temperature, which is designated by way of distinction the *absolute temperature,* and denoted by T, is defined by the equation

$$T = \frac{273\,p}{p_0};$$

its relation with that first defined is indicated above.

[TO BE CONCLUDED.]