The Open Court

A MONTHLY MAGAZINE

Devoted to the Science of Religion, the Religion of Science, and the Extension of the Religious Parliament Idea

Editor: DR. PAUL CARUS. Associates: { E. C. HRGELER. MARY CARUS. VOL. XIV. (NO. 7) JULY, 1900. NO. 530 CONTENTS: Frontispiece. COPERNICUS.

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CHICAGO

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JUST PUBLISHED

A BRIEF

HISTORY OF MATHEMATICS

AN AUTHORISED TRANSLATION OF

DR. KARL FINK'S GESCHICHTE DER ELEMENTAR-MATHEMATIK

ВΥ

WOOSTER WOODRUFF BEMAN

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AND

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NICOLAUS COPERNICUS.

(1473-1543.)

From a picture in the possession of the Royal Society, presented by Dr. Wolf of Dantzic, June 6, 1776. Engraved by E. Scriven.

Frontispiece to The Open Court.

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COPERNICUS, TYCHO BRAHE, AND KEPLER.

BY CARUS STERNE.

F we review the long line of fighters who freed the human mind from the oppressive bonds of its early subjection, two investigators, Copernicus and Kepler, always stand out prominently as leaders. These men, however, did not stand in the fore-front of battle; they worked in comparative obscurity; but they rank as real leaders through the weight of their investigations, and through the irresistible force of the proofs obtained by their patient observation of nature.

Although originally destined for the priestly office, each rose above the narrow principles of Church doctrine as received in theological lecture-rooms. Nor must it be forgotten that the Protestant camp from which Kepler came was at that time quite as intolerant as the Catholic, as witness the case of Michael Servetus, discoverer of the circulation of the blood. Indeed at the time of the rise of Copernicus, the Church of Rome felt itself still so firm and unshaken in its sense of power, that it believed it might grant considerable liberty in the observation and explanation of nature.

It must not be forgotten that a long time before Copernicus the old belief in the geocentric system was shaken, but the Church then acted as though this was a matter of no concern to her. Nicholas Krebs of Cusa (1401-1464) had made no secret of his conviction that the earth moves, and yet several popes had advanced him to the highest offices in the Church. Leonardo da Vinci speaks repeatedly, in his written notes, of the movement of

¹Translated from the German by Dr. David Eugene Smith, of the State Normal School Brockport, N. Y. The publishers are also indebted to Dr. Smith for having courteously placed at their disposal the originals of the portraits which adorn the present article. For the remaining illustrations they are under obligations to W. Engelmann, of Leipsic.

the earth as a matter of course. But all these views, differing from the prevailing teaching, had been only philosophical speculations



FROM AN OLD PRINT. A SCIENTIFIC INTERPRETATION.

which in part were awakened by the study of classical authors, in part were clarified by independent reflexion, yet did not rest upon the foundation of thorough observations. To have made such supporting and confirming observations, with the simplest instruments and with untiring patience, remains the undying merit of Copernicus (1473-1543).

Through his uncle Lucas von Watzelrode, Bishop of Ermland, Copernicus was led to the priestly office, although in Cracow, besides his theological work, he was interested in mathematical and astronomical studies, in which Albert Bruzewsky was his teacher. The youth of twenty-three continued these many-sided occupations in Bologna, then the indispensable source of scientific knowledge. Insatiable in his thirst for learning, he then went to Padua where he added medical studies to the theological, mathematical, and astronomical which he had already pursued. From Rome, where he received a professorship at the university in 1500, our scholar, who apparently cared little for splendor and fame, returned to his bleak northern home. Here he obtained, through the mediation of his uncle, a position as canon in Frauenburg (1510), which allowed him to prosecute his astronomical researches in all tranquillity.

Since the year 1507 the thought had come to him and had become more and more fixed, that the old geocentric idea was false. Finally, through unremitting observations, he became convinced of the movement of the earth and the planets round the sun, not publishing his views, however, save to friendly astronomers or to amateurs, many of whom flocked to him for instruction and for the removal of their doubts. Copernicus possessed a universal mind similar to that of Leonardo da Vinci, a mind which seemed to succeed in all it undertook. He therefore was able to carry on the government of the Chapter after the death of his uncle, to appear as its representative at the Prussian diet, to undertake the regulation of the Prussian system of minting and coinage, to carry on the work of a popular physician in his district, and to advance to successful conclusion a difficult construction of some flood gates. Just as thoroughly and systematically did he proceed in his observations of the heavens, so that his undying services to astronomy are in nowise prejudiced by such predecessors as Nicholas von Cusa. The views of the latter were still so confused that he seems never to have drawn even the most important of the consequences of the movement of the earth, namely, the immobilising of the sphere of the fixed stars.

In the year 1530 the great work of Copernicus, On the Revolution of the Heavenly Bodies (De revolutionibus orbium coelestium, libre VI.), was finished in outline, —a work which prepared the way for a great revolution in thought. Of this work the Archbishop of Capua, Nicholas von Schönberg, who was of German extraction, and one of the sincerest admirers of Copernicus, is said to have received a manuscript copy in 1536. This good patron also encouraged Copernicus to publish the work, its contents being already so much talked of in learned circles. He is also said to have borne



DIAGRAM OF THE COPERNICAN SYSTEM. From Copernicus's work, *De revolutionibus* (1530).¹

Copernicus says: "The first and highest of all the spheres is that of the fixed stars, enclosing itself and the others and therefore immovable, being the place of the universe to which the mo tion and position of all the other stars are referred. Then follows the outermost planet, Saturn which completes its revolution round the sun in thirty years [the planets, Uranus and Neptune had not yet been discovered]; then Jupiter, which has a period of twolve years; then Mars, with a period of two years. The fourth sphere in order is that of the yearly revolution, and in it is contained the earth, having the orbit of the moon as an epicycle; in the fifth place, Venus revolves in nine months; the sixth place is occupied by Mercury, which performs its revolution in a period of eighty days. In the middle of all stands the sun : for who could think of another or better place in this most beautiful temple for so brilliant a luminary? The sun, thus, seated on is kingly throne, guides the movements of the stars that circle round it."

the cost of printing, and to have recommended the dedication of the work to Pope Paul III., one of the most ardent admirers of

Reproduced from a cut in Friedrich Dannemann's Grundriss einer Geschichte der Naturwissenschaften, 2 vols. Leipsic: W. Engelmann. astronomy. It was therefore probably scientific caution, rather than apprehension as to its reception by the Church authorities, that led Copernicus to defer so long the publication of this work. This is the more probable because, only a short time before (1533), the German astronomer Widmansstedt, who held similar views, had met with a kind reception from Pope Clement VII.

That he would find manifold and lively opposition among



DIAGRAM OF THE PTOLEMAIC SYSTEM OF THE UNIVERSE (160 A. D.). From Guericke's *De vacuo spatio*. (After Dannemann).

Reproduced for comparison with the system of Copernicus. The first sphere contains the moon, which has a period of revolution of one twelfth of a year; the second contains Mercury having a period of our to far year; the third Venus, having a period of two thirds of a year the fourth the sun, having a period of one year; the fifth Mars, having a period of two years; the sixth Jupiter, having a period of twelve years; and the seventh Saturn, having a period of thirty years.

Catholic and Protestant scholars and laymen, Copernicus must have understood from the very beginning. It is, however, remarkable that the first attacks of the Protestant spokesmen were almost more violent than those of the Catholic, and this may have been brought about by the well-received dedication to the Pope. It is known that Luther was one of the most determined opponents of the theory, and in the *Table-Talk* he says of the Canon of Frauenburg with little consideration : "Mention was made of a contemporary astrologer who tried to prove that the Earth moved and turned round, but not the Heavens, nor the Firmament, nor the



FROM AN ORIGINAL CLAIMED TO BE AUTHENTIC. AN ECCLESIASTICAL INTERPRETATION.

Sun, nor the Moon; just as when a person is seated in a wagon or on a boat and is in motion, and fancies he is sitting still and at rest while the earth and the trees are moving. But this is the way of the world now: when a person is bent on being thought clever, he must perforce make up something of his own, which has to be the best that is, just as he makes it. This fool will upset the whole Science of Astronomy. But the Holy Scriptures tell us, Joshua bade the *Sun* stand still and not the Earth."

One may well think from this that Luther followed the views of his friend Melanchthon, who was an ardent adherent of astrology, a science which remained, as we shall presently see, closely bound up with the geocentric theory. Luther, however, believed less firmly in astrology, and several times even declaimed vehemently against it, so that the accusations brought against him, that he made use of the astrological superstition of the time for his own ends, is probably not tenable.

Naturally there was no lack of satirical inuendoes against the new theory of Copernicus, which had become known long before the appearance of his work. To such attacks on the part of meddling critics Copernicus addresses himself in the dedicatory letter to Pope Paul III., published before the work appeared. Here he speaks, in a tone of perfect confidence, of those vain babblers who, without possessing mathematical knowledge of their own, would condemn his work because it was at variance with a few purposely distorted passages in the Bible. Thus did the holy Lactantius, in his ignorance, once childishly scoff at the spherical form of the earth; but the learned must overlock with contempt such objections of non-mathematicians. Just as boldly did he oppose prevailing prejudices by his eulogy of the new system, delivered with noble pride and self-confidence: "Through no other arrangement," he says, "have I been able to find such wonderful symmetry of the universe and such harmonious connexion of the orbits, as when I place the sun, the light of the world, as ruler of the whole family of circling stars in the midst of the high temple of nature, as though upon a kingly throne. Who indeed could find in all glorious nature a better place for the sun than that from which it can give light to the whole?"

It will be seen that in his works, which were finally given to the press, appearing however only after his death (which occurred on the following May 24), Copernicus spoke out with manly firmness for the truth of the results of his investigations. It is evident, too, that the supposition that he was spared by the ecclesiastical censor only because one of the editors (Andreas Osiander) had sent out in advance an anonymous preface, could be true only on the hypothesis that the censors had read neither the dedication to the pope nor the work itself. This preface of Osiander designated



Tycho Brahe. (1546-1601.) (From a very rare print.)

the new doctrine as a mere hypothesis which "need be neither true nor probable," as it was to serve only to calculate more easily the phenomena of the heavens, and it is not probable that it was



ALTAZIMUTH OF TYCHO BRAHE, THE ARCHETYPE OF THE MODERN THEODOLITE (From Tycho Brahe's *De mundi actherei*, Prague, 1603. After Dannemann.)

This beautiful instrument was constructed of brass and served for determining both azimuths and alitudes. The azimuth-circle, NP, rested on four pillars; the altitude-circle had a radius of almost two yards, and was provided with a scale of minutes, EC, and an alidade, DE.

added with the consent of Copernicus. He could not, however, protest against what had been done, as he received the first copy only on his death-bed.

The great successor of Copernicus in the field of observation was the Danish nobleman Tycho Brahe. In the estimation of the world he generally passes for the outspoken opponent of Copernicus, and as the inventor of a new cosmology which left the earth in the center of the universe, and made the sun and moon revolve around it, but the planets around the sun,—a system which has



ARMILLARY SPHERE OF TYCHO BRAHE.¹

This instrument, like all the others of the great Danish astronomer, was manufactured in Tycho Brahe's own workshop. The elegance and exactitude with which they were executed in every detail, are beautifully shown by this illustration.

sion to the prevailing belief. But the traditional estimate of this admirable investigator, who pursued his studies in Leipsic, Wittenberg, and Augsburg, is a very unjust one. In point of fact, he was the most ardent admirer of Copernicus that could be imagined. In the Sternenburg (Uranienburg) which his royal patron Frederick II. of Denmark had constructed for Tycho Brahe on the island Hyeen, the picture of the Canon of Frauenburg, adorned with palms and laurels, occupied the place of honor in the room of state. When the heirs and successors of Copernicus heard of this worship, they sent as a gift the simple wooden instrument with which the latter had made his observations. Tycho celebrated the happy day of its reception (July 13, 1584) by

been explained as a conces-

a Latin poem, in which it is said of Copernicus:

"He succeeded in snatching the sun from the heavens, And placing it firmly. Around it then he guided the earth, As around the earth the moon."

And of the instrument, which had no lenses :

¹From Gerland and Traumüller, Geschichte der physikalischen Experimentirkunst, Leipsic W. Engelmann. "... O monument of the great And immortal man! You are perishable wood, But shining gold will look on you with envy."

But Tycho had so improved this instrument, although he likewise had to do without lenses, and he was besides so sharp an observer, that he could not but perceive the defects still adhering to the Copernican system as well as the discrepancies between the facts and the calculations based upon it. Untiring observations of the orbit of Mars showed clearly that the *circles* assumed as the



TYCHO BRAHE'S SYSTEM OF THE UNIVERSE. (1587.)

This system occupies an intermediary place between the geocentric system of Ptolemy and the heliocentric system of Copernicus. In Tycho's system the earth is at the center; the sun Mars, Jupiter, and Saturn revolve about the earth, while Mercury and Venus perform secondary epicyclical revolutions about the sun. (From Guericke's *De vacue spatio*. After Dannemann.)

planetary orbits by Copernicus did not exist. Furthermore, Tycho found good reason to deny the third movement of the earth (around the pole of the ecliptic) presupposed by Copernicus. Hence he is not to be blamed if he held provisionally to his own system, which had in common with the ecclesiastical conception the geocentric idea only; for naturally he had never doubted the revolution of the earth on its axis. Neither did Tycho Brahe ever publicly set up his system in opposition to that of Copernicus. His theory was not published until three years after his death, in a book appearing in Frankfort in 1604; and the essay contained in it "On the System of the Universe" is, probably with good reason, attributed to his pupil B. Ursus. The fruits of Tycho's labors did not ripen until later, after Kepler was able to build further on the foundation of these observations, the most exact that any astronomer had made before the discovery of the telescope.

Johannes Kepler (born December 27, 1571) was on account of his weak frame, like Copernicus, originally destined for the ministry; or rather we may say he had grown up into it, for his parents, having ended after various vicissitudes in life in keeping a tavern, had placed the boy in the school attached to the monastery of Hirsau. From there he went to the school of the former Cistercian monastery Maulbronn, where in 1516 Dr. Faust is said to have taught Abbot Entenfuss alchemy and to have passed the last years of his life. We might believe that something of the Faust-spirit there descended upon the young man, who later removed to the seminary of Tübingen (1589) in order to study Protestant theology.

Kepler fortunately found at Maulbronn a fatherly friend and adviser in Michael Mästlin, a theologian versed in astronomy and an adherent of Copernicus. It is he whom Galileo also honored as a teacher, and who inspired Kepler with all the more interest for astronomy as his warm attachment to theology was repulsed by the extreme views of most of the other teachers there. In particular Kepler would not profess Luther's dogma of the omnipresence of the body of Christ, and as he was already suspected on account of his fondness for the stars, he seems to have been in a difficult position. Probably also he did not understand keeping his conviction of the truth of the Copernican system as secret as was necessary, as his cautious and timid teacher had done for years and also recommended to him. Accordingly serious conflicts soon arose, and Mästlin as well as several other teachers advised Kepler to give up his theological studies entirely and accept a position as teacher of mathematics at the Gymnasium at Graz, which was to be filled in the spring of 1594. Kepler, who was a zealous Protestant, even though not according to the strict Lutheran fashion, went unwillingly to the Catholic country, but he accommodated himself to circumstances and supplied the Styrian provincial almanac with all the astrological lumber which was at that time deemed to be the main requisite of a calendar.

It is remarkable and at the same time instructive to observe how Kepler, with his strong inclination to fantastic dreaming and



JOHANN KEPLER. (1571–1630). Probably from contemporary sources.

to poetical ideas of things in general and their relations, wrested himself almost entirely free from the seductive allurements of the astrological craze of that time. It was apparently, next to his mathematical vein, his religious conviction of the perfection of the structure of the universe that kept him from this aberration. The words of the Bible, that the universe is duly disposed according to number, measure, and weight,¹ which more than two hundred years later led the chemist of the Berlin porcelain factory, J. B. Richter, to the discovery of the stoicheiometric relationships between the chemical elements, impelled him also to seek the mathematical law of the structure of the universe.

Led astray on this quest by his classical education, he first took up with the speculations of the Pythagoreans, who had alternately compared the five regular solids to the five worlds and to



Fig. 1.

KEPLER'S CONSTRUCTION OF THE PLANETARY SPHERES.

Exhibiting the dimensions and distances of the planetary orbits by means of the five Platonic solids. (From Kepler's Mysterium Cosmographicum, Tübingen 1506. After Dannemann.)

Kepler says: "The orbit of the earth gives the circle which constitutes the measure of all the others. About this circle (η in the figure) describe a dodecahedron; in the sphere which encloses this solid lies the orbit of Mars (e in the present figure). About the Martian sphere describe a tetrahedron; the spherical surface described about this solid would contain the orbit of Jupiter (see γ in Fig. 2). Describe about the latter a cube; the sphere enclosing the cube (a, Fig 2, contains the orbit of Saturn. Further, construct within the terrestrial sphere an icosahedron; the spherical surface inscribed within the same contains the orbit of Venus (in the present figure). Describe within this last sphere an octahedron, and this body will enclose the sphere Mercury."

the five senses of man, and conjectured that by them possibly the five spaces between the six planetary orbits might be typified. He accordingly imagined the octahedron, icosahedron, dodecahedron, tetrahedron, and cube, placed successively one within another, with the sun at the center; and describing spheres between each

1 Wisdom, 11, 22.

two successive solids to touch the outer angles of the smaller and the inner surfaces of the larger, he conceived the great circles of these spheres to represent the orbits of the planets, and the spaces between them the distances of the orbits.

The unit for the orbit-distances was given by the orbit of the earth, which was assumed to be on the sphere between the icosahedron and the dodecahedron.

This device, when closely examined, will be found not unworthy of a poetising mathematician. As the mean distances of the orbits of the planets, not then known with the strictest accu-



Fig. 2. Kepler's Construction of the Planetary Spheres.

(For description see Fig. 1.)

racy, corresponded fairly well with those reckoned in this way, he was convinced that he had discerned the skilful plan of the architect of the universe, and accordingly he made known this cosmical secret in his maiden work, the *Mysterium cosmographicum* (1596).

Genuine enthusiasm for the perfection of creation as revealed by Copernicus had furnished the original incentive for all Kepler's

calculations and investigations, and so he begins his work with the following words :

"Inspired, full of holy joy, David cries aloud, calling upon the world itself: 'Praise ye him, sun and moon: praise him all ye stars of light.' But what voice has been bestowed upon the heavens, and what upon the stars, fit to praise God like that of man? Because they give reasons for praising God, we may say, they praise God himself. Since then we are endeavoring to make this voice of the heavens and of all nature more perceptible and clear, let no one say we are pursuing vain studies or exerting ourselves for naught."

After interpreting, as far as in him lies, this marvellous construction, and emphasising the lack of a planet between the orbits of Mars and Jupiter, his enthusiasm breaks forth once more in a lofty hymn, a few lines of which may here be quoted :

"Great Artist of the universe, with admiration I look upon the works Of thy hands, which constructed them according to an ingenious plan. In their midst the sun, dispenser of light and life, Which curbs the earth according to sacred law and guides her In her changing course. I see the toil of the moon, And stars scattered on the infinite meadow.... Sovereign of the world! Thou eternal power! Thine infinite glory Soars on the wings of light through all the worlds!"

The impression of this work, which to-day possesses value only as a poem and picture of the fancy, was a very mixed one. Kepler's Tübingen teachers were not in accord with it. "God forbid," Professor Hafenraffer wrote (1597) with discernment but with kindness, "that you should ever try publicly to bring your hypothesis into agreement with Holy Scriptures; act, I beg of you, entirely as a mathematician and do not disturb the repose of the Church."

But Galileo wrote an enthusiastic letter dated the fourth of August, 1597:

"I consider myself happy to know of so great an ally in the search for truth and consequently such a friend of truth itself. It is really pitiful that there are so few who strive for the truth, and care to depart from perverted methods of philosophising. But this is not the place to lament the wretchedness of our time; rather should I wish you good luck in those splendid investigations, by which you strengthen the truth... I should risk publishing my own speculations, if there were more like you. But since this is not the case, I postpone it, for fear of sharing the fate of our master Copernicus, who although he has won undying fame with a few, has nevertheless with very many—so great is the number of fools !—become an object of ridicule and contempt."¹

The work was of great advantage to Kepler in that it brought the young astronomer to the notice of Tycho Brahe of Prague, and

1K. von Gebler, Galileo Galelei und die römische Kurie, Stuttgart, 1876.

caused him to invite Kepler to come to that place as his assistant. This was so much the more important for Kepler, as his position in Graz had in the meantime become untenable. In the year 1598 Archduke Ferdinand had, by an edict, banished all Protestant



JOHANN KEPLER.

From a picture in the collection of Godefroy Kraenner, merchant at Ratisbon. Engraved by F. Mackenzie.

teachers and priests from Styria, and Kepler alone was allowed to remain, it was said through the intercession of the Jesuits, who needed his astronomical calculations for their missions in China. But in the year 1600 there was a repetition of the storm, and Kepler might count himself happy in having found employment in the well-appointed observatory of the Hradschin at Prague.

But his position there with the haughty Danish astronomer. busy with the improvement of the Copernican system, seems not to have been the most agreeable. Indeed it would probably have become unendurable, owing to the great difference of temperament of the two men, had not Tycho Brahe's unexpected and early death (on the 23d of October 1601) put an end to this unsatisfactory alliance. The result was that Kepler was raised to the position of the imperial astronomer and mathematician. Not without manifold difficulties, however, did he come into possession of the priceless results of his predecessor's observations, which were to serve in the calculation of the Rudolphine Tables (of the movements of the planets). This material was so much the more indispensable for Kepler's labors, because on account of the weakness of his eyes he would never have been able to acquire it for himself. And even now it would have gone badly with his mission, had not industry been aided by imagination, that freest daughter of the mind, which raised him above the preconceived but respectable errors of his master, driving him incessantly into new combinations and conjectures. Thus we see that not the gift of observation and the art of calculating alone suffice for the making of great discoveries, but that science often has still more to gain from the consistent working out of hypotheses. On the other hand, Kepler was always irresistibly impelled to check the creations of his imagination by calculation. But he was successful in doing this only in three discoveries, namely those relating to the movements of the planets, which were alone sufficient to have made his name immortal. He dreamed, however, of many another, which it remained for Newton, and even for Laplace, to furnish a firm support.

It was above all the irregularities which Tycho Brahe had established in the revolution of Mars that attracted the attention of Kepler, and he gave voice to the conviction that "through the planet Mars we must reach the secrets of astronomy, or remain forever ignorant in this science."

In the preface to the New Astronomy or the Commentary on the Planet Mars (1609) he gives an account to the Emperor Rudolph of the result of the "struggle with the heathen god of war, in which General Tycho Brahe won the highest fame, inasmuch as he discovered in the night-watches of twenty years all the habits, positions, and stratagems of his enemy. O that he [Kepler] might now bring this most noble lord a prisoner before the Emperor!" Copernicus had, as was before mentioned, believed that the planets move in circles round the sun, and from this assumption had arisen the appearance of irregularities in these orbits. Kepler now perceived that Mars and the other planets moved not in circular but in elliptic orbits round the sun, which therefore is not in the center of the orbit, but in one of the foci (*Kepler's first law*). At the same time he perceived that the planet hurries forward faster in its orbit in perihelion than in aphelion, but that the radius of rotation describes equal areas in the same time in all portions of the orbit. (*Kepler's second law*.)

He did not hesitate, in spite of the opposition made hitherto by the Church, to announce openly these great discoveries, which, as he rightly assumed, removed the last difficulties from the Co-

pernican system of the universe. Accepting the Joshua miracle, he says that Joshua merely expressed wrongly his prayer commanding the sun to stand still, just as we still every day express ourselves wrongly when we say we wish the sun would soon rise above the horizon. He adds:

"In theology the weight of authority may decide, in philosophy we must have reasons. Holy is Lactantius, who doubted the spherical form of the earth; holy is Augustine, who conceded this but denied the existence of antipodes;... but holier to me is truth, when I, with all respect for the Church, prove by science that the earth is round, is inhabited by antipodes, is a little dot in the universe, and wanders among the stars!"



DIAGRAM ILLUSTRATING KEPLER'S SECOND LAW.

If the distances tt' and TT' are traversed in equal times, then the segments tt'S and TT'S are equal in area.

Having now discovered in the law of areas a new confirmation of the structure of the universe according to number, he bent all his energies to find why the planets did not hasten round the sun with uniform swiftness in the more perfect circular orbit, but revolved, as he had found, with changing swiftness in elliptic orbits. Again it was a dream of the ancients that captivated his poet-soul, the Pythagorean dream of the harmony of the spheres, of the music of the universe, audible only to spirits specially blessed. By this means he hoped to reduce apparent anomalies to a mutual balance, to harmonise the courses of the planets, so that every dissonance which a single planet might produce when regarded outside of its connexion with the system, would be by such law removed from the celestial concert. "Straying in this labyrinth of delusion," as a stern critic of these ideas would say, "at last, at last," he discovered on the 15th of May, 1618, at Linz, where he meanwhile had found a position as gymnasium professor, his third law. This is the law according to which the squares of the times of revolution of the planets are proportional to the cubes of the mean distances. This discovery followed upon a failure that calls to mind the discovery of universal gravity by Newton; for suddenly, as Kepler expresses himself, the perception of the truth triumphed "over the darkness of my mind with such conformity to my seventeen years' work on the observations of Tycho, that I at first thought I was dreaming and that I had taken for granted that which I was seeking."

Certain of his critics have utterly failed to comprehend this combination of dreaming and mathematical genius, and in regard to the discovery of the third law, have cried out: "Whence suddenly so much light after such deep obscurity?" (Bertrand.) They have also spoken of his gambler's luck; but Whewell has pointed out, in his *History of the Inductive Sciences*, that this combination of imagination and penetration is the peculiar characteristic of most great discoverers. He further notes that Kepler is distinguished from most of the others only in that he describes at length his mistakes and aberrations in his search for the truth, and admits that truth would now hide herself from his gaze, and again incite him to pursuit, like Virgil's Galatea:

> "Galatea throws apples after me, the roguish maiden, Then back she flies to the pastures, yet wishes first to be seen."

"We may be surprised," says Reuschle,¹ at Kepler's wonder ful luck in disentangling truth from the wildest, most preposterous fancies; yet we *know* that with our hero the one is productive of the other, that both are strangely interwoven into a very singular whole." We must also agree with what Reuschle has said about his poetical bent and the enthusiasm that always reanimated his courage when extended calculations threatened to wear out his spirit, or his dire poverty seemed about to prostrate him. All his life long he was obliged to beg of Tycho Brahe, as well as of the emperor and the empire, for the salary rightfully belonging to him and for the money to print his books. Indeed, he met his death while on a begging expedition, made on foot, in early winter, to the imperial diet at Regensburg, November 15, 1630. He had to struggle not only with the opponents of Copernicus and Tycho,

1C. G. Reuschle, Kepler und die Astronomie, Frankfort, 1871.

men like Chiaromonte, Riccioli, etc., but also with the most absurd prejudices of the people and with a fanatic priesthood. To add to all his misery, through the efforts of his own degenerate brother Christopher, his old mother was accused of witchcraft, so that she was saved from the stake only by the greatest exertions and after six years of legal contest.

Kepler was also forced incidentally to pay court to that "coquettish daughter of astronomy," astrology, both in Gratz and in

Prague, in order to keep his court position and earn his living. This was done, however, with very little regard for her, and with bitter complaint that he must so play the charlatan. It all had its humorous side, however; he writes in one place: "This astrology is indeed a foolish little daughter, but—*lieber Gott!*—where would her mother, the highly rational astronomy, be, if she did not have this foolish offspring? The world is even more foolish, so foolish in fact, that this sensible old mother must for her own benefit cajole and deceive it, through her daughter's foolish, idle talk."

And although on the whole he was very fortunate in his prophecies, yet he says frankly: "Since the guessing is after all only a matter of Yes or No, we are sure to hit the mark half the time, and miss it only the other half. The successful guesses are remembered after the manner of women, but the failures are forgotten, because they are nothing peculiar, and so the astrologer is still held in honor." Even Wallenstein thought once of making Kepler his astrologer, and met him in Sagan, after he had lost his professorship in Linz through the



KEPLER OBSERVES A SUN-SPOT WHICH HE ERRONE-OUSLY TAKES FOR MER-CURY.¹

expulsion of the protestants under Ferdinand II. But Wallenstein saw very well that Kepler did not believe in his own prophecies

¹⁰pera Omnia, II., 793. After Dannemann. This observation was made in 1607, before the invention of the telescope. Tradition, dating from the days of Charlemagne, asserted Mercury to be visible, when in conjunction, as a minute dark spot on the surface of the sun. Allowing the rays of the sun to pass through a narrow orifice in a dark chamber, Kepler saw in the image of the sun, caught on a paper screen, a minute flocculent speck, which he took for Mercury. It was a sun-spot.

himself, and so he gave him a professorship in Rostock instead, a position where he ran against the same old difficulty of work without salary.

In spite of all these distressing circumstances, Kepler was at least a fortunate man through his discoveries, and but few investigators can have tasted such hours of rapture as he. It was after the discovery of his third law, that he could write:

"But now nothing more holds me back; a year and a half ago the first dawn, a few months ago the full day, a few days ago the pure sun of the most wonderful contemplation, have come upon me. Now I will revel in holy ecstacy; now will I scoff at the children of men, with the simple avowal that I am stealing the golden vessels of the Egyptians, in order to build a tabernacle for my God, far distant from the land of Egypt. If they forgive, I shall be glad; if they are angry, I shall bear it; here I cast the die, and write a book to be read by my contemporaries or by posterity, it matters not; it may wait for its reader thousands of years, since God himself waited six thousand years for him who should behold his work."

After this preface he unrolls in his favorite work, the *Harmonice mundi* (1619), a picture of the universe which would not cast discredit upon the greatest of poets. In the eyes of many the scheme, however, is discrediting to an astronomer, for it contains, besides many glorious thoughts and discoveries, fanciful speculations in regard to the earth-beast, its sleeping, waking, breathing, etc., as well as in regard to the spiritual relations of the heavenly bodies to one another. We may be allowed to quote a few more words from the epilogue, in order to show the beauty of the language.

"From the music of heaven to its hearers," he cries, "from the muses to Apollo, the great chorister, from the six circling planets, that discourse the music to the sun which in the midst of their orbits revolves about itself alone, without change of place!"

From the complete harmony obtaining between the smallest and greatest movements of the planets, from their strong tendency toward the center, Kepler inferred not only that the sun influenced the planets, but that the latter also reacted upon the sun, which he mystically designates as the contemplation and interchange of their homage.

"What the nature of that seeing or that perception, in short, the nature of that soul in the sun, may be, it is difficult to gness; it is, however, true that the assumption of the six principal orbits around the sun, which do homage to the latter with constant revolutions, and above all the further existence of harmony, the trace of the highest wisdom doing homage in the solar system, compels me to the assertion, that not only *from* the sun does light go ont into all parts of the world, life and warmth as from the heart, motion as from the seat of power and might, but that also, vice versa, these tributes of the most delightful harmony gather from all provinces of the universe. In short, in the sun are found counsel and favor for the whole kingdom of nature."

This continual outbreaking of poetical language gives to Kepler's works (of which we have received in modern times a model complete edition by Frisch¹) a living charm, and often lends to them a transporting power. He took occasion to speak in verse only in inspired

moments, as in the lines given above from the Mysterium cosmographicum, and he then reaches such heights that he must be counted among the great "German classical authors," discovered by Daniel Strauss. "who wrote in Latin." Not seldom he was impelled to set forth his astronomical discoveries in the form of popular tales, of which we find several examples. among his works, as for instance his story of Jupiter's satellites, of the star which had recently appeared in the Swan, and his "Dream of the World." Exceedingly vivid episodes are also frequently found in his other writings, as for instance in his work on the new star, which had appeared with great radiance in the foot of Serpentarius, in 1604. It was this appearance which raised again the question, whether the heavens could really be called unchangeable, according to Aristotle, when new stars actually appeared in it. Did this star originate recently from the light-exhalations of the universe, and if so, were perhaps all stars of the universe such incidental productions of fate? Against such assumptions, Kepler's deeply religious nature struggled, and the idea of Cicero came into his mind, that just as well might Homer's *Iliad* have been thrown together (as one would throw dice) from the twenty-four letters of the alphabet, as the harmony of the universe from a lot of whirling atoms.

"Yesterday, in the midst of my meditations," he writes, "I was called to the table, as my young wife was serving a salad. 'Do you think,' I asked, 'that if pewter dishes, lettuce-leaves, grains of



salt, drops of oil and vinegar, with hard-boiled eggs, had been flying about in this room higgledy-piggledy since creation, chance would ever have been able to gather them together to-day into a salad?' 'Certainly not into so well and skilfully mixed a one as this,' answered my beautiful wife."

²Gonstruction given in the *Dioptrics* (1610). After Daunemann. For the convex and concave lenses of the Dutch and Galilean telescopes Kepler substituted two suitably disposed convex lenses, giving an inverted but clearer image.

¹ In eight volumes, Frankfort, 1858-1872.

When to the newly discovered telescope, which in the beginning was very imperfect and was constructed on the principle of the opera-glass, he had given the arrangement of the astronomical telescope still in use, he addressed the new instrument in the preface to his *Dioptrics* as follows: "O knowledge-fraught perspicil, more precious than any sceptre! Does not he who holds thee in his right hand, stand like a king and a master of the works of God?"

As often as I try to search out in the history of German investigation the prototype of the practical ideal of German philosophy, the ideal of Faust, who, wandering, but not confused, struggles forward to the solution of the great world-problems, I always come back again to Kepler, who, by his profound meditation, embodied as has none other, the specifically German bent of mind.

How proudly he asserts in his Harmony of the World, that he wrote this book as a German, according to the German manner and habit of philosophising, freely and without constraint. All his works and all his actions are in the most beautiful accord with this same reflective German spirit, which descends to the profoundest depths of speculation, yet ever remains self-conscious. He declined the call to Bologna because he was a German, and did not wish to renounce German liberty of speech and investigation; and although in constant distress in Prague, because of failure to receive his salary, he answered the invitation of the king of England, that only ingratitude could make him think of leaving Austria, his second fatherland. Without envy he recognised foreign merit, rejoiced over Galileo's discoveries, and admired in Copernicus still more than his learning, his "free spirit." Yet, if all who pass judgment on this German would do him like justice, they would have to say with Galileo: "While I hold Kepler in exceedingly high esteem on account of his fine unprejudiced mind, yet his manner of philosophising is radically different from my own."