"If a man die, shall he live again?   Job, xiv, 14.

"Then shall the dust return to the earth as it was, and the spirit shall return unto God who gave it."   Ecclesiastes, xii, 7.

"The Lord himself shall descend from heaven with a shout, with the voice of the archangel, and with the trump of God: and the dead in Christ shall rise first."   I. Thessalonians, iv, 16.

*"If there be no resurrection of the dead,   *   *   *   then is our preaching vain, and your faith also is vain   *   *   *   The trumpet shall sound, and the dead shall be raised incorruptible   *   *   For this corruptible must put on incorruption, and this mortal must put on immortality."*   I. Corinthians, xv, 13, 14, and 52, 53.

"When the Son of man shall come in his glory, and all the angels with him, then shall he sit upon the throne of his glory: And before him shall be gathered all the nations   *   *   And these" [the wicked] "shall go away into eternal punishment, but the righteous into eternal life."   Matthew, xxv, 31, 32, and 46.

"The hour cometh in which all that are in the tombs shall hear his voice, and shall come forth."   John v, 28 and 29.

"I believe in   *   *   the resurrection of the body, and the life everlasting.   Amen."   Apostles' Creed.

The author of the matter on the following pages has listened to a great many funeral discourses, in churches of different denominations and elsewhere, but does not remember to have heard the doctrine of a final, simultaneous, resurrection of the dead insisted upon in a single one of the number.   In numerous instances the Pauline sentences, quoted above, were *read* in course as part of the regular service, but "the resurrection of the body" was not specifically alluded to otherwise by Protestant clergymen.   On the other hand, the immortality of the soul was alluded to with more or less fullness in the great majority of their addresses, and generally was spoken of as if belief in it as a verity were taken for granted by the members of the audience as well as by the speaker.   Of course it is known that Roman Catholics feel bound to believe that there is a purgatory for the soul while the body is in the grave.   But, as a rule, the remarks made by Protestant clergymen were such as to justify the inference that they believe the spirit, immediately after the death of the body, returns "unto God who gave it," and has its status fixed without waiting for a judgment that is to follow a general resurrection.   The author supposes that the preceding statement holds true, approximately, if not accurately, in the case of many millions of funeral dis-

courses that have been delivered within the last half-century. He, however, does not wish to be understood as claiming that many or any of the clergy deliberately abstain from preaching the doctrine of a final resurrection of the dead, which, for many thousands of millions who have died already, and probably for untold millions who will die in the future, would occur long after the complete decomposition of the body (except here and there a few bones), and the recombination of the decomposed material into other organic forms. Yet it can not be denied truly that in recent years there has been a growing and widening inclination among regular attendants on church services, as well as outside their ranks, to doubt the truth of that particular doctrine; and it were no wonder if the leaven of doubt has entered the minds of some of those who have pledged themselves in ordination vows to preach "the faith which was once for all delivered unto the saints."

Also, it hardly will be denied that if any such doubt be entertained it is based in large part upon considerations named in the few lines last preceding this, which constitute real or supposed scientific grounds for scepticism, or that the doctrine of the immortality of the soul would be objected to less if it were not associated with that of a resurrection of the body. It, therefore, should be recognized as a matter of some interest to ascertain if scientific reasoning can be brought to bear on the subject, either directly or indirectly. The author has ventured to discuss the problem along mathematical lines, having in mind a remark made by Professor Huxley, that "the facts of consciousness are, practically, interpretable only by the methods and the formulæ of physics."

The first section of the following essay should not be difficult reading to any one who has mastered the notation of elementary algebra, because the author has explained with sufficient fullness the representation of lineal magnitudes by algebraic symbols, so far as they are required to be used in the work. He can promise that such a reader will be able to understand this first section, provided he is willing to go through it carefully. The second part is a little more abstruse, but correspondingly more comprehensive. For both sections, it is hoped that the new line of thought they are intended to open up will be found interesting enough to repay the labor of a patient perusal.

No. 2 Groveland Park, Chicago.   C.

      Easter, 1900.

CURVE OF IMMORTALITY.

MATHEMATICAL ANALOGY TO DEATH AND THE RESURRECTION.

It often has been asserted that physical science furnishes no support to the claim that there is a spirit entity in man the individuality of which persists after the death of the body. It is true that Paul sought to establish a parallel between the resurrection of a "spiritual body" and the growth from a vegetable seed which can not be "quickened except it die"; and that lesser lights have adduced insect metamorphosis as another parallel to a claimed persistence from a material existence into one that is spiritual. But both of these are poor as illustrations, and utterly valueless as alleged proof. Paul virtually admits that the vegetable entity which springs from the seed is no higher or lower in the scale of existence than was its immediate predecessor, being identical with it in "kind." In the case of the insect phenomenon cited, the alleged illustration is a much poorer one, for the organism does not rot while in the pupa state, and in very many of the Class the moth is exceedingly short lived, dying immediately after having provided for the perpetuation of its species. There is nothing in either of these assumed parallels, or illustrations, that points to a higher or better state of existence, still less to one of persistence of the individual entity through a long course of ages. The writer, some years since, called attention to the persistence of memory while the material constituents of the body are giving place to new ones, as the strongest scientific indication we possess toward a continuance of individual existence after the death of the body; but even that is no proof. And we can not believe in the eternity of matter while refusing to admit that force is eternally existent, without being severely illogical; but we have no more scientific warrant for believing in a perpetuated individuality for any named collection of physical force than we have for that of any known aggregation of material units.

5

An argument against the value of belief in a future state of existence has been drawn from the mathematics, and many have regarded it as a strong one. It may be of interest to exhibit this, in brief, and all the more so as we can derive in the opposite direction an argument that is equally strong, if not stronger. In order to do this, we employ the method of rectangular coördinates.

Through any conveniently situated point in a vertical plane, we may draw a horizontal straight line and a vertical straight line, and take these lines as the axes of any desired number of rectangular coördinates. They intersect at right angles, and may be supposed to extend to any required distance in each direction in the plane in which they are drawn.

Any distance measured	*Is designated by*
To the right from the vertical axis,	x
" " left " " " "	$-x$
Upward from the horizontal axis,	y
Downward " " " "	$-y$

each distance being measured on a line which is parallel to an axis.

The radius of the circle is denoted by a. The equation for the circle as referred to an origin at the circumference, is $y^2 = 2ax - x^2$; and for every possible numerical value of x, when substituted in this equation, we obtain two equal values of y, one above the horizontal axis and the other below it, because $+y^2 = (-y)^2$ and $= (+y)^2$. Furthermore, any value of x that is negative, or that is greater than $2a$, will give a minus value for y^2, showing that there is no point in the curve to the left from D or to the right from A, in our first diagram.

Now, if we introduce another value, p, and write

$$y^2 = 2px - (p / a) . x^2$$

we shall have an equation which holds good for all the conic sections, p being a perpendicular from the focus in each case, and the focus becoming the center in the case of the circle. Also, if we take q to designate the distance from the origin to the focus, then in the circle, $p = q$; in the ellipse, p is greater than q, but less than $2q$; in the parabola, $p = 2q$; and in the hyperbola p, is greater than $2q$, the value of a being minus in the last-named curve, so that

— $(p / a) . x^2$ is a positive quantity to be added to $2 p x$ in order to obtain the value of y^2 in the hyperbola. In the parabola a is infinite, whence $y^2 = 2 p x$.

We present here a figure of the ellipse, because it is that one of the conic sections with which the progress of human life most frequently has been supposedly compared.
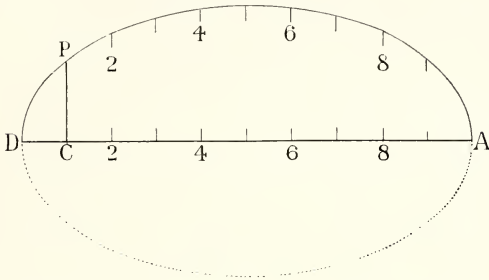
In this diagram $D A = 2 a$; $C P = p$; and $D C = q$. If we take q as the lineal unit, and measure off two, four, etc., units from D, on the horizontal axis, the perpendiculars 2, 2; 4, 4: etc., will be the corresponding values of $+ y$. Then, if we suppose the increase of x from zero to the length $D C$ to correspond to the first seven years of life, that each of the succeeding increases of x by a unit corresponds to seven years, and that the resulting values of y be the measures of vital power at the different times, we shall have a pretty close parallel, not to every human life, but to the average of all the lives which are not terminated by accident or by disease other than those which are incident to old age. The measure of power increases from nothing at or just before the beginning of life, to its maximum between the ages of thirty and forty, declining thence to the death zero at about the point assigned by the Psalmist as "three score and ten." We may extend x as far as we please in the minus direction (to the left) from D, or as far as we please in the plus direction to the right beyond A, without obtaining for y any value that is other than imaginary. Hence, if this curve really be entitled to be called the equation of human life, then it holds out no hope of life beyond the grave.

Furthermore: if we choose to adopt into this discussion the very extensively entertained idea that the principle of negation is the principle of evil (as indicated all the way down from the "ye shall *not* surely die," said to have been spoken by the serpent in the garden of Eden, to the "acarm" of the Sanscrit, and thence along the stream of time to the latest claim that "unbelief is sin"), we may take the plus values of y, those which are situated above the horizontal axis, to represent the activities of the good and useful life, and the negative values of y, those which are situated below the horizontal axis, to represent the "pernicious activities" of the sinful life, these also being extinguished by death, and not followed by punishment beyond the grave.

It is true that in the parabola and hyperbola the value of y continually increases with every augment in the value of x, (and that we can construct a companion hyperbola to the left from the vertical axis), the one coördinate becoming infinite when the other is so; but in each of these cases, the curve is continuous, and presents nothing that could be adduced as a parallel to a real or apparent discontinuity between life in the body and life after death of the body. So with the cycloid, which is described by a point in the circumference of a circle while the circle is rolled along a straight line, as a carriage wheel rolls along a road. By keeping the circle rolling far enough we may produce any desired number of cycloidal curves, but all of them are equal in dimension to each other, comparable in this respect with the succession of individuals in a genealogical line of plant or animal existences; and there is no blank between them, each member of the succession of cycloids beginning at the same point as the one in which its immediate predecessor terminates. So, while the cycloid may be taken as typifying the progress of human life, it is not susceptible of application or similitude to a higher and more prolonged existence after death.

There is, however, a curve which contains the remarkable properties here suggested. It is represented by the equation

$$m^2 y^2 = x \cdot (x - b) \cdot (x + c).$$

where m, b, and c retain the same numerical values while we assign to x any desired number of different numerical values, in succession, and thence determine from the equation the corresponding values of y. For instance, if we take $b = 1$, and $c = 3$, we may construct the following table of approximate values for m y;

| x	$x-1$	$x+3$	$m^2 y^2$	$m\,y$
$-3\cdot$	$-4\cdot$	$0\cdot$	$0\cdot$	$0\cdot$
$2\cdot7$	$3\cdot7$	$+0\cdot3$	$3\cdot0$	$1\cdot73$
$2\cdot4$	$3\cdot4$	$0\cdot6$	$4\cdot9$	$2\cdot21$
$2\cdot1$	$3\cdot1$	$0\cdot9$	$5\cdot86$	$2\cdot42$
$1\cdot8$	$2\cdot8$	$1\cdot2$	$6\cdot05$	$2\cdot46$
$1\cdot5$	$2\cdot5$	$1\cdot5$	$5\cdot6$	$2\cdot37$
$1\cdot2$	$2\cdot2$	$1\cdot8$	$4\cdot75$	$2\cdot18$
$0\cdot9$	$1\cdot9$	$2\cdot1$	$3\cdot6$	$1\cdot9$
$0\cdot6$	$1\cdot6$	$2\cdot4$	$2\cdot3$	$1\cdot52$
$-0\cdot3$	$1\cdot3$	$2\cdot7$	$1\cdot05$	$1\cdot03$
$0\cdot$	$-1\cdot0$	$3\cdot0$	$0\cdot$	$0\cdot$
$+1\cdot$	$0\cdot$	$4\cdot0$	$0\cdot$	$0\cdot$
$1\cdot25$	$+0\cdot25$	$4\cdot25$	$1\cdot3$	$1\cdot15$
$1\cdot5$	$0\cdot5$	$4\cdot5$	$3\cdot4$	$1\cdot75$
$2\cdot0$	$1\cdot0$	$5\cdot0$	$10\cdot0$	$3\cdot16$
$3\cdot$	$2\cdot$	$6\cdot$	$36\cdot$	$6\cdot$
$4\cdot$	$3\cdot$	$7\cdot$	$84\cdot$	$9\cdot2$
$5\cdot$	$4\cdot$	$8\cdot$	$160\cdot$	$12\cdot7$
$+6\cdot$	$+5\cdot$	$+9\cdot$	$270\cdot$	$16\cdot4$

Multiplying together the numbers in the first three columns we obtain the results in the fourth column, the product being zero in each case where one of the factors is zero. Then, assigning to m any desired value, we divide by it the numbers in the fifth column, and may construct a diagram such as the accompanying one, in which $m =$ about $3\cdot9$. It will be noted that the whole of the closed curve is on the minus side of the vertical axis, and that this part of the curve is not elliptical but oval shaped; and that any value of x which is less than -3, also any value of x between the limits of zero and $+b$, makes y^2 a minus quantity, for which y has no real value.

The unit of measurement chosen for x in our diagram may be supposed to correspond to about twenty-five years, or a little less, in the duration of human life. Then, for that part of the curve which is above the horizontal axis we may trace the parallel somewhat as follows: Beginning at the extreme left we have a rising from nothingness to a maximum of vigor that is attained between the ages of twenty-five and thirty, which certainly for the physical (example in

the case of athletes), and with very many for the mental power, is a
closer parallel than is offered by the ellipse as previously considered.
Then comes a decline, slow at first, and proceeding more rapidly with
a lessening of the distance from the vertical axis which is the nor-
mal terminus for bodily life. Following this (still proceeding toward
the right), is a period of apparent nothingness, which may be com-
pared with that of "rest" in the grave. Then the curve suddenly
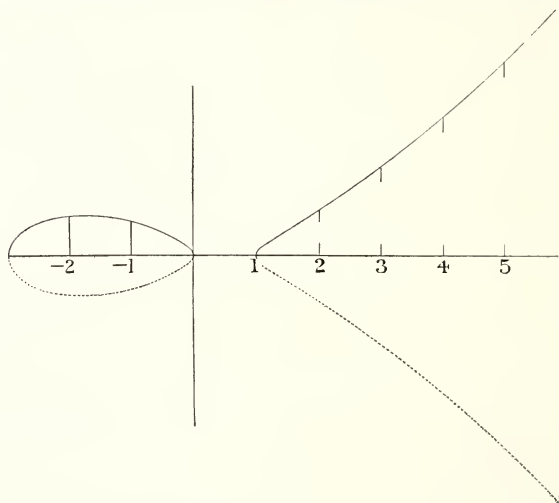
starts upward, and its distance above the horizontal axis continu-
ally increases with every increase in the value of x, the one becom-
ing infinite if the other be increased to infinity. It is not difficult
to regard this right-hand portion of the curve as illustrating the
resurrection of what St. Paul expressly stated will be "a spiritual
body," in contradistinction to the "natural body" which was buried
in the grave, and a continuous ascent thereafter toward the infinity
of perfection and power to enjoy through a blissful eternity. Fur-
thermore, if we choose to consider that portion of the curve which
is below the horizontal axis as representing the career of the wicked
(page 8), the idea will agree with what has been claimed by the
orthodox churches through a long course of ages, to be reserved for
sinners.

The assigning of minus values of x for that portion of the curve which is situated to the left from the vertical axis, harmonizes with the idea expressed by Benjamin Franklin in his "last words" which were: "A man is never perfectly born till after his death."

It should be remembered that this curve, with its apparently complete discontinuity between branches that may seem to be totally different in character, the one being a closed curve and the other an open one, however far extended, is obtained by the successive substitution of different numerical values of a single variable quantity(x), in a single equation. It, therefore, logically may be compared with the theological formula according to which a single vital force is sufficient to produce all the observed phenomena of human life on this earth, and after the death of the bodily organism, to revive, and then persist as an individual entity through all eternity.

The writer ventures to suggest still another comparison, though aware that some may deem it fanciful: The constant quantity c measures that change in the value of x which corresponds to the duration of life in the body, while the constant b measures the change in x that corresponds to the time elapsing between "death" and the resurrection; and the variable x is the quantity the unlimited extension of which gives the infinite branch of the curve, this being the one that corresponds to the immortal part of our existence. So, extracting the square roots, we have for the value of y in every position the product of the three factors, $\sqrt{x} / m$, $\sqrt{x - b}$, and $\sqrt{x + c}$; and these may be considered as corresponding to the "spirit, and soul, and body" which St. Paul seems to have believed constitute man as a whole. Also, it is obvious that while the relative strength of these components varies as between individuals, we may choose other numerical values than those previously assigned to the constant quantities in the mathematical equation; the result being an alteration in the relative magnitudes of the different parts of the diagram, but no change from the essential characteristics of the curve as already described.

It would be absurd to claim that this equation is *proof* of the doctrine of a sentient individual existence for man after death; the term "man" being intended to include "woman," Mohammedan doctrine to the contrary notwithstanding. But the writer does submit it as important, in this respect that it offers a close mathematical analogy, which he is not aware has been claimed previously to

exist (though the equation itself is not new); and that it is of far
greater value as such analogy than are the growth of a plant from
seed or the metamorphosis of insects, whatever *their* value may be.
Inasmuch, however, as those puerile analogies have been insisted
upon, the first by the "Apostle to the Gentiles," and the second by
numerous theologians of later ages, the one still being cited at a
large percentage of the funeral services held in civilized countries
today, and the other semi-occasionally mentioned in the pulpits of
christians of all denominations, it does seem to the writer that the
equation offered and described in preceding pages is worthy of wide-
spread attention. It offers the only close analogy yet discovered in
the field of pure scientific reasoning to the doctrine of a resurrection
of the body; and fairly may be commended to theologians of all
sects as a good illustration of what they believe to be an important
truth:—if "this, and nothing more."

SECOND SECTION.

The matter on the seven and a half pages next preceding this par-
agraph was printed separately from that which follows, and a copy
of the former sent to each of thirty-three clergymen in this city,
accompanied by the statement that "the author would be pleased to
know that those to whom it is submitted deem it worthy of a wider
circulation." The indicated division was made for the reason that
what follows involves a little higher order of mathematical thought
than was employed in the earlier presentation of the subject, though
in Hutton's "Course of Mathematics" the imaginary unit and its
powers are treated of on the very next page after the one which con-
tains the statement that "no quantity can be found which, when
raised to an even power, can give a negative result."

The author aimed at a mathematical treatment of the subject
which would not be above comprehension by an average graduate
from a Chicago High School. He did this because he hoped that
each of the gentlemen would understand it as a result of careful
reading, but in this respect he was mistaken. Some of them expressed
regret that they were not familiar with the line of reasoning; and not
one of the number offered any real criticism of the effort to point
out a "Mathematical Analogy to Death and the Resurrection." So,
the essay is presented in more complete form, without any such com-
mendation as was hoped for, but also without fault finding. It would
not be fair in this case to claim that "silence gives consent."

The author, however, does feel it to be his unpleasant duty to enter a mild protest against the claim by any one to be called "an educated man" if he does not know enough to enable him easily to follow out the line of thought presented on preceding pages. Mathematics constitute such an important part of the science of logic that one well may doubt ability to reason correctly on any abstruse subject by one who has not climbed up the ladder as far as the extraction of the square root in Algebra, and the comparing of right lines drawn in a circle for the study of Plane Trigonometry.

If we let c denote the diameter, then the equation for the circle (page 6) may be written:

$$y^2 = c\,x - x^2; \quad = x\,(c - x).$$

Making $c = 10$, assigning to x successive values from zero to 10, and performing the muliplication, we shall have the corresponding values of y^2, as in the third column of the following table:

x	$c - x$	Products.	y Values.	
0	+ 10	0	0	0
+ 1	9	+ 9	3	1·8
2	8	16	4	2·4
3	7	21	4·58	2·75
4	6	24	4·9	2·94
5	5	25	5·0	3
6	4	24	4·9	2·94
7	3	21	4·58	2·75
8	2	16	4	2·4
9	+ 1	+ 9	3	1·8
10	0	0	0	0
11	− 1	− 11	3·32 i	1·99 i
12	2	24	4·9 i	2·94 i
13	3	39	6·24 i	3·75 i
14	4	56	7·48 i	4·49 i
15	5	75	8·66 i	5·20 i
16	6	96	9·8 i	5·88 i
+ 17	− 7	− 119	10·91 i	6·54 i
etc.	etc.	etc.	etc.	etc.

Extracting the square root of each of these products we obtain the values of y for the circle, as in the fourth column, these being the

distances from the horizontal axis to several points in the curve, above and below that axis. Now, if each of these quantities thus denoted by y be multiplied into 0·6, the results will be those given in the fifth column, which are the values of y in the ellipse the principal axes of which are in the ratio of 6 to 10, as represented in the diagram on page 7.

If we take m to denote the ratio of the principal axes in the ellipse, we shall have, for that curve:

$$y^2 = m^2 . x . (c - x); \quad \text{or} \quad \pm y = m . \sqrt{x . (c - x)}.$$

which becomes the equation for the circle when m equals unity.

It is evident that if we assign to x any values greater than c, which in the present instance is assumed equal to 10, the resulting values of y^2 will be minus quantities, which can not have real roots. But, we may consider any particular value of $- y^2$ as equal to plus y squared, multiplied into minus unity, that is $- y^2 = y^2(-1)$; and then also we may write its square root as equal to $\pm y . \sqrt{-1}$, the expression, $\sqrt{-1}$, denoting what is called "the imaginary unit," and often is represented by i. With this notation the latter part of the table gives, in the fourth column, the results for the equilateral hyperbola, and in the fifth column those results multiplied into 0·6, which we have assumed as the value of m for the ellipse in the cut on page 7.

These products of an actual quantity into an imaginary one might be rationalized by assuming the difference between c and x to be a positive quantity, whichever of those two be the greater, or by regarding m^2 as negative in the hyperbola while it is positive in the ellipse. The truth is that in analytical researches it now is preferred by mathematicians to regard the square of the ratio of the minor axis by the major axis, which we denote here by m^2, as negative for the ellipse and positive for the hyperbola. To adopt this would be simply to change our equation to:

$$y^2 = m^2 . x . (x - c); \quad \text{or} \quad \pm y = m\sqrt{x . (x - c)}.$$

It is not difficult to state a satisfactory reason for this convention, which might be thought arbitrary unless explained: The quantity m^2 also is equal to $e^2 - 1$; and e, the eccentricity, is greater than unity for the hyperbola, while it is less than unity for the ellipse, and reduces to zero for the circle.

The statements made in the last fifteen lines are added only for the purpose of precluding any possible objection that otherwise might be raised to the effect that the author is not well up in "Conic Sections" because he does not treat them according to the formulæ laid down in the books. The line of thought to which this additional discussion is intended to lead up is one which recognizes the supposition that the imaginary unit may be the factor the equivalent of which, in the equation of human life, distinguishes the purely spiritual from the corporeal part of our existence.

First of all, let us look at the features of correspondence with the line of thought as developed in the eight pages which were first sent out; this on the supposition that some satisfactory relation afterward may be discovered between the technically named "imaginary unit" and an actual spirit existence apart from the body. We have the ellipse, as described on page 7, corresponding approximately to the progress of human life from the cradle to the grave, and then an immediate starting up in the value of y, to give the hyperbolic curve, which (page 8) continually widens out with every increase in the value of x, the one coördinate becoming infinite when the other is so. The analogy here is to an eternal existence of the spirit after it leaves the body, but without any suspension of spirit life, or of manifestation of power, between death and a resurrection of either natural or spiritual body. The "grave" component, $\sqrt{x - b}$, (page 11, line 24) drops out from the equation given on page 8, and we have $c - x$ instead of $c + x$, with m becoming a multiplier instead of a divisor on the right hand side of the same equation. The last-named consideration simply is one of terms, because m may represent either a stated number or its reciprocal, so that the only essential change from the equation of page 8 to the one of page 14 consists in the making of b equal to zero while we retain its accompanying x; and this means but that the after-death portion of the curve widens out a little less rapidly in the first case than in the second.

Next, we may consider the fact that the table on page 13 can be extended upward as well as downward. If, in measuring to the left from D, in the diagram on page 7, we take x equal to -1, -2, -3, etc., in succession, the corresponding values of $c - x$ will be 11, 12, 13, etc., and the products are -11, -24, -39, etc., exactly the same as those in the lower part of the third column in the table.

Hence, the values of y are precisely the same for equal distances to the left from D, and to the right from A, in the diagram; and the ellipse may be regarded as situated between two equal and opposite hyperbolas, each of which has D A (equal to c), for its major axis, one stretching out toward infinity to the left, and the other stretching out toward infinity to the right. Therefore, if the quantities in the extended table are analogous to different phases in the existence of man, they point to an existence previous to his birth as well as to one subsequent to his decease, indicating that the spirit is not called into being with the first animation of the body any more than it is extinguished at the death of the body.

This view is in harmony with the idea, which is probable in a scientific sense though far from being proven, that the sum total of force in the universe is constant, if that of matter be so. Also it harmonizes with that small part of "theosophic" doctrine which declares that the vital principle, the essential "Atman" which is held to be loaded with the "Karma," exists before the body is formed as well as persists after its dissolution. But the notion is at radical disagreement with the theosophic conception of a long series of sentient existences for the one mentality, and even more inconsistent with what is declared by Mrs. Besant to be an integral part of the theosophic doctrine; namely, that these sequent phases of existence are separated by long intervals of "sleep." The analogy rather would be with a gradual ingathering of vital force from the fund, which is infinite in extent while the process is infinite in duration, and with the theory that after a temporary concentration of that gathered force into bodily form it gradually dissipates into the vast fund, though the identity is not absolutely lost in any assignable period of time, because any such period falls short of infinity. Hence, the absolute Nirvana of Hindoo philosophy could not be attained by any one of those who aspire after it, so long as the Universe endures, unless the term of its existence be eternal.

According to the theory of Helmholtz, which many scientific investigators now are inclined to consider the most probable basic theory of the constitution of matter, the material atom simply is a whorl in an ether which pervades all known space, and must be supposed to be a perfectly elastic entity, which roughly may be likened to a jelly, though having no weight, and being in reality the only imponderable substance in Nature—if we are justified in speaking of it at all as

a substance. This ether is the medium that transmits, or through which are transmitted, vibrations from the constituent particles of one mass to those of another. And if we accept the whorl theory it follows that we must regard the material atom as nothing more or less than a differentiated ether, just as we must consider the protoplasm which is the fundamental material of organic life to be a differentiation from the inorganic atom. We then may fall back on the language of the calculus, in the absence of anything better, and state the inorganic and protoplasmic forms as the first and second differentials of the vast etheric integral—the "Constant" of which will be the "Great First Cause," though we can not hope that scientific investigation ever will enable us to ascertain whether that Great Cause be personal or impersonal. Now, if we suppose that the employment of the imaginary unit corresponds wholly or in part to the process of differentiation, and that the ether itself is essentially positive, the inorganic form of matter must be deemed imaginary positive, and the protoplasmic form negative (because $i^2 = -1$). The protoplasmic then will be rational, and will need a negative multiplier to bring it into the state of the integral. It is well known that in celestial mechanics we have:

$$\text{Force} = \text{Mass} / r^2; \quad = -d^2 s / d t^2.$$

If we dispense with the simile of differentiation, as being above the comprehension of those who have not had the advantage of a course of mathematical study which appears to be exceptional, we may conceive the following parallel, which can be drawn without resort to the language of the calculus: Starting with the previously stated conception that the state of the ether is essentially positive, and multiplying by the imaginary unit four times in succession, we shall reach the positive again, and may conceive the several results to correspond to conditions in the grand scheme of Nature, thus:

$+ 1$	corresponds to Ether.	
$+ \sqrt{-1}$	"	" Inorganic matter.
$- 1$	"	" Organic matter,
$- \sqrt{-1}$	"	" Unknown. (?)
$+ 1$	"	" Spirit entity.

The last four expressions in the first column represent the "four roots of unity," being the analytical values of radius unity at angular distances of $90,°$ $180,°$ $270,°$ and $360°$ from the origin.

In regard to this comparison we remark that one well may hesitate at assigning a parallel to the fourth mathematical term. If the square root of minus unity be imaginary, its negative hardly is conceivable, and we may suppose it corresponds to some to us totally unknown mode of existence. Also, the third and fifth terms suggest the taking of the negative and positive values of y as collating with the inferior and superior parts of man's nature, the animal and the spiritual, represented by positions below and above the horizontal axis. To accept the latter idea would be to discard the suggestion (pages 8 and 10), about goodness and wickedness, and consider the whole of the varying breadths of the figure, measured in a perpendicular direction for successive values of x, as representing the measures of total force at different epochs in the term of our existence. Then follows the thought that the fifth term in the series is not necessarily identical with the first. They numerically are equal, but the last is the fourth power of the second, and a potential conversion is involved in a return from it to the original quantity if not to the original form. These analogies, and deductions from them, fitly may be described as "reasoning in a circle," but the process is not necessarily vicious on that account, though it can not be denied that, at least in one sense of the word, it is largely "imaginary."

It, however, is possible to conceive of the spirit essence returning to the ether condition, if not directly then, by another conversion not indicated in the preceding table of supposed changes. Then, in sequence to the line of thought sketched briefly in this section, we even may indulge a flight of fancy to the extent of supposing that man is a microcosm of the macrocosmic universe in a far wider sense than hitherto has been understood in the use of that phrase. One talks of the "eternity of matter," but that simply is because within the limits of human experience the elasticity of the atom is permanent, and rates of vibratory movement of atom or molecule vary only with temperature. It is legitimate to imagine that the duration of existence of an atom is "eternal" only in the sense that it lasts longer than the planet, as that of the earth compared with the life of an organism on its surface, and as the life of the organism with that of one of the countless millions of cells the aggregation of which makes up the form of animal or vegetable. Hence, one may venture to think it not impossible that the pulsations of the ether vortices which we call material atoms may decrease to the zero of pulsative activity in some portions of space, while more active in other regions,

in obedience to what we feel forced to believe is a law—that the sum total of energy in the whole of Nature's vast domain is a constant quantity. In this view of the case we may speculate on the possibility of a single solar system, or millions of such systems, springing into existence out of nothing (in the sense in which the ether may be spoken of as "nothing), and after the lapse of millions of ages dying again into the same kind of nothingness, while the operation of creative force on the very same kind of nothingness is in progress in other portions of infinite space. This is a more comprehensive sweep, if not a more daring one, than even is the expansion of the nebular theory by its author into the thought that the universe as known to us may be only one out of a vast series, each of which is evolved from the material ruins of an immediately preëxisting one.

According to the enlarged view here suggested, though not insisted on, the life of a vegetable or animal cell, that of the aggregation of cells which constitutes a unit of animated existence as seen by the unaided eye, the existence of the planet whose surface is the theater of such vitality, that of the system to which the planet belongs, that of ths clustering molecule and even of "the ultimate atom": each and all may be regarded as only temporary phenomena, the time occupied by the exhibition of which is no greater in comparison with an eternity of duration than is the massing of vapor drops in the atmosphere and the collection of electric force due to the gathering of those vapor specks into raindrops that form a passing shower.

We can reason strictly and surely in regard to the protoplasm which is the physical basis of all life, animal as well as vegetable, and of human life equally with that of the lowest forms of animated existence, that it always is dying, and could not live unless it died: such death involving a resolution into its mineral and lifeless constituents, which subsequently are so scattered and diffused that the recombination of the same material into a consequent individual entity identical with the previous one, is highly improbable if not absolutely impossible. And so we may reason toward the theory that those constituents themselves ultimately resolve into the universal ether. To think along similar lines in regard to the vital force is not more illogical than are speculations about the supposed possibility of a fourth dimension in space, and hardly can it be more conclusive. But it may be worth while to note that the ability to indulge in such speculative imaginings furnishes a by no means weak argument in favor of the belief that the human spirit is immortal.

It is fair to suppose that: (1.) The Christians of the Apostolic age fully expected that many of their own number would live till the "Second Coming," and would welcome their Lord in company with those who had "fallen asleep." (2.) That when several years had elapsed without bringing the desired consummation some began to doubt, and not far from a quarter of a century after the Crucifixion Paul wrote to the Corinthians with an express purpose of answering some among them who said "there is no resurrection of the dead": these objectors reasoning to the effect that the bodies of many of the witnesses must have become completely disintegrated, rendering impossible such a resurrection as they had been led to expect, and Paul undertaking to show, for the first time, that such objection was not a valid one, the body that is buried being "natural," while the one to be resurrected will be "a spiritual body." (3.) That notwithstanding this explanation the general belief in the churches during many centuries seems to have been in a reanimation of the body that was buried, and this appears to have been the principal reason why the churches have systematically frowned on cremation. Within the last quarter of this century a distinctly forward step has been taken by a few clergymen in admitting that the statement of the case made by St. Paul in his first Epistle to the Corinthians is not inconsistent with belief in the possibility of resurrection from the ashes of a body that has been cremated.

The reanimation of many millions of human bodies, each of which when "raised up" shall be composed of the very same material atoms and molecules that made up its substance in a former life, would be a miracle indeed: involving what well may be designated as a physical impossibility. The doctrine of "the resurrection of the body," on the plan stated by St. Paul, is not open to the same sweeping kind of objection; but it does not appear to the author to be in line with the average trend of modern scientific thought. Whether or not the Pauline resurrection of a spiritual body, if it occur at all, will be simultaneous, and at the sound of an archangel's trumpet or otherwise, scientific research is powerless to find an answer. But the truly scientific man hardly will dare to assume the responsibility of asserting that such resurrection is impossible—knowing, as he does, how easy it is to find instances of seeming folly in one age becoming recognized as wisdom in a succeeding one: and vice versa. It was remarked, a long time ago, that the student of science ought not to feel sure of anything in regard to which there is a reasonable doubt.

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