NAMES AND NUMBERS.[1]

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A NAME is an *acoustic attribute*, which I *add* to the other sensory attributes of a thing or complexus of things, and which I *engrave* in my memory. Even in themselves alone, names are important. Of all the attributes of a complexus of things, they are the most invariable. They constitute thus the most convenient representative of that complexus as an entirety, and around them the remaining and more variable attributes cluster in memory as around a nucleus.

But the facility with which these attributes called names permit of being spread and communicated is more important still. Each observer is likely to discover different attributes in a thing; one person will notice this, another will notice that; with the result that they will not necessarily come to an understanding regarding the thing, or for that matter even be able to come to an understanding. But the name, which always remains the same, is imprinted as a *common* attribute in the memories of *all* persons. It is like a label that has been attached to a thing and is known to all persons. But it is not only attached to things, it is also preserved in the memories of men, and leaps forth at the sight of these things, of its own accord.

The importance of names in technical fields has never been a subject of doubt. The possibility of procuring things which are not within our immediate reach, the producing of effects at a distance through a chain of human beings, are attributable to names. The ethical achievements of names are perhaps even more important still. Names particularise individuals; they create personalities. Without names there is neither glory nor disgrace, neither defensible personal rights, nor prosecutable crime. And by the

[1] Translated from the German by T. J. McCormack.

use of written names these marvellous performances have been enhanced to a stupendous degree.

When two persons part company, each soon shrinks for the other to a mere perspective point. Without names it would be almost impossible for the one to find the other. The fact that we know more of some men than of others, that some men mean more to us than others, is owing to names. Without names we should be utter strangers to one another, as are the animals.

Fancy for a moment how I should be obliged to mimic, caricature, and portray a person that I was seeking, in order that some small group of people, who were perfectly familiar with my methods, could assist me in my search. But if I know that the name of the person I am seeking is Jacques Montmartre, that he lives in France, and in addition in Paris, at No. 45 Rue St.-Pierre Fourier, then I am always in a position to find him by means of these names, —names which countless numbers of different individuals associate with the *same* objects, although they may know these objects under entirely different aspects and in greatly varying degree, sometimes themselves *by name only*. I can thoroughly appreciate the marvellous achievement involved in these performances by imagining myself making such a search without a knowledge of names. I should then have to travel from country to country and from city to city, like the people in *The Arabian Nights*, until I found by accident the person whom I was seeking,—which happens only in fairy tales. I should be in the situation of the lost child who could tell no more than that she belonged to the "mother" who "lived in the house."

A name is the product of a convention, reached unconsciously under the favoring influence of accident, by a limited circle of people having common interests, and gradually communicated by that circle to wider groups.

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What are numbers? Numbers are also names. Numbers would never have originated had we possessed the capability of picturing with absolute distinctness to ourselves the members of a set of like objects as *different*. We count where we desire to make a distinction between like things; in doing so, we assign to each of the like things a name, a distinguishing sign. If the distinction to be made between the things is not effected, we have "miscounted." To accomplish our purpose, the signs employed must be better known and must admit more readily of distinction than the things to be designated. Counting, accordingly, begins with the use of the familiar objects known as fingers, the names of which have in this manner gradually come to be the names of numbers.[1] The association of the fingers with the things is accomplished, without effort or design, in a definite *order*. In this manner, numbers are quite unconsciously transformed into *ordinal* symbols.[2] As a consequence of this invariable order, and as a consequence of it alone, the last sign associated with the things comes to represent all the previous associations; this last sign is the *number* of the things counted.

If there are not enough fingers to associate with the things, the original series of associations is simply repeated, and the *several* series of associations so obtained are then themselves supplied with ordinal symbols, as before. Our system of numbers becomes in this manner a system of purely *ordinal signs*, which can be extended at pleasure. If the objects counted be made up of like parts, and in each of these parts there be discovered parts which again are alike, and so on, the same principle may be employed for the enumeration of these parts of parts. Our system of ordinal signs, accordingly, admits of indefinite refinement. Numbers are an orderly system of names which admit directly and readily of indefinite extension and refinement.

Where a few objects only are to be designated, and these are readily distinguished from one another by salient attributes, proper names as a rule are preferred; countries, cities, friends, are not numbered. But objects that are numerous and which constitute in any way a system in which the properties of the individual things forming the system constitute a gradation, are always numbered. Thus, numbers and not names are given to the houses of a street, and in regularly laid out cities, also to the streets themselves. Degrees on a thermometer are numbered, and proper names are given to the freezing and boiling points only. The advantage here, in addition to the mnemotechnic feature of the plan, consists in the fact that one can easily discover by the sign of the thing the position which it occupies in the system,—an advantage not appreciated by the inhabitants of small towns, where the houses are unnumbered and where there are consequently no municipal co-ordinates to assist a stranger in finding his way.

The operation of counting may again be applied to the numbers themselves; in this manner, not only is the development of the number-system carried to a point considerably beyond that of

1 Cantor, *Mathem. Beiträge zum Culturleben der Völker*. Cantor, *Geschichte der Mathematik*. Tylor, *Primitive Culture*. Tylor, *Early History of Mankind*.

2 Mach, *Mechanics*, page 486.

its original simplicity, as by the formation of the decimal system of writing and of performing operations with numbers, but the entire science of arithmetic, the entire science of mathematics, takes its being from this application. The perception, for example, that $4+3=7$, arises from the application of the ordinal signs or numbers of the upper horizontal row of the following diagram, to the numbers of the row which is beneath:

1	2	3	4	5	6	7
1	2	3	4	1	2	3

I conceive the truths of arithmetic to be propositions that have been reached by experience, understanding by experience here inner experience; and I long ago characterised mathematics as a system of economically ordered experiences of counting, made ready for immediate use, and designed to replace direct counting, which is frequently impossible, by operations previously performed, and hence accomplishing a great *saving* of time and trouble.[1] I am here substantially in accord with the views which Helmholtz expressed in 1887.[2] This is of course not as yet a theory of mathematics, but merely a programme of such a theory. The interesting psychological questions presented here may be seen from the work of E. Schröder[3] who was the first to inquire why the *number* of the objects is independent of the order in which they are counted. As Helmholtz remarks,[4] in any succession of objects that have been counted in a definite order any two adjacent objects may be interchanged, whereby ultimately any order of succession whatever of the objects may be produced without changing the succession of the numbers, or causing either objects or numbers to be dropped. The non-dependence of the sum on the order of the things added follows from this consideration. But this inquiry cannot be pursued farther here.

Although in the first instance counting supplies the necessary means of distinguishing objects which are in themselves difficult to

[1] Comp. "Ueber die ökonomische Natur der physikalischen Forschung," *Almanach der Wiener Akademie*, 1882, p. 167. (Engl. trans. in *Popular Scientific Lectures*. Chicago, 1898, p. 186.) Also, *Mechanik* (1883), p. 458. (Eng. trans., Chicago, 1893, page 486.) Also, *Analyse der Empfindungen*, 1886, p. 165. (Eng. trans., Chicago, 1897, page 178.)

[2] Helmholtz, "Zählen und Messen," in *Philosophische Aufsätze, Eduard Zeller gewidmet* Compare especially pp. 17 and 20.

[3] *Lehrbuch der Arithmetik und Algebra*. Leipsic, 1873, p. 14. I became acquainted with Schröder's book, which is based upon Grassmann's work, through a quotation in the aforementioned paper of Helmholtz.

[4] *Loc. cit.*, pp. 30 et seq. Conf. also Kronecker, *loc. cit.*, p. 268.

distinguish, it is nevertheless afterwards applied to objects which, while clearly distinguishable, are yet in some certain respect regarded by us as the same, and so are interchangeable in this respect. The properties with respect to which objects may be considered the same differ greatly and vary almost from mere existence at a given point of space or moment of time to absolute undistinguishability. We count *different* objects as the same *only in so far* as they are the same; dimes, dollars, shillings, sovereigns, francs, marks, and gulden are counted, not as dimes, dollars, shillings, etc., but as coins. Thermometers and induction coils are counted as physical apparatus, or as items of an inventory, but not as thermometers and induction coils.

Objects counted, which are alike in some particular respect, and which may replace one another in this respect, are called units. What is it that is counted, for example, by the number representing a temperature? In the first place it is the divisions of the scale, the real or apparent increments of volume or of pressure of the thermometric substance. *Geometrically* or *dynamically* regarded, the objects here counted may be substituted for one another, indifferently; but with reference to the thermal state these objects are signs or indices merely of that state, and not equivalent, enumerable parts of a *universal* property of the thermal state *itself*.

This may be made very clear by the consideration that the number measuring a potential for example does quantitatively determine a universal property of the potential. If I cause the electric potential of a charged body to sink from 51 to 50 or from 31 to 30, I am able by so doing to raise the charge of any other body having the same capacity one degree, indifferently whether it be from 10 to 11 or from 24 to 25. Different single degrees of potential may be substituted for one another.

A relation of like simplicity does not exist for scales of temperature. A thermometer is raised *approximately* one degree of temperature when some other thermometer of the same capacity is lowered one degree of temperature in some other part of the scale. But this relation is not exact; the deviations vary with the thermometric substance selected for either one or both thermometers, and with the position of the degrees in the scale; the deviations are furthermore individual in character, according to the substance and to the position in the thermometric scale; they are vanishingly small only in the gas scale. We may say that by cooling off a gas thermometer one degree in any part of the scale, any other body may be made to receive always the same alteration of thermal state. This property might have served as a definition of equal degrees of temperature. Yet it is worthy of remark that this property is not shared by all bodies whatsoever that pass through the course of temperature-changes indicated by the gas thermometer, for the reason that their specific heat is in general dependent upon the temperature. It is no less deserving of mention that this principle was not intentionally embodied in the construction of the temperature-scale, but subsequently proved itself by accident to be substantially fulfilled. The conscious and rational introduction of a scale of temperature having universal validity analogous to the potential scale was first made by Sir William Thomson. The temperature-numbers of the common scale are virtually inventorial numbers of the thermal states.