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Globally Optimal Computable Distributed Decision Fusion

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GLOBALLY OPTIMAL COMPUTABLE DISTRIBUTED DECISION FUSION

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ABSTRACT

The problem of distributed decision fusion in parallel sensor configuration is considered. It is shown that the optimal combining scheme is the Neyman-Pearson test at the sensors and the decision fusion. Computationally efficient algorithms that allow the determination of near-to-optimal solutions are developed. The algorithms are shown to perform very close to the optimal solution in all the examined cases.

INTRODUCTION

The problem of distributed decision fusion where a number of sensors transmit a compact form of information about a common observation space has attracted considerable attention recently, [1] through [7]. In this paper we consider the optimal decision scheme for the parallel sensor configuration, Fig. 1. According to this scheme, a number of sensors monitoring the same geographical volume, transmit their decisions in regards with the nature of the true (binary) hypothesis to the fusion center which is responsible for combining the sensor decisions into a final one. We assume that the sensor decisions are independent. Under these conditions, it was shown by Thomopoulos and al. [4], [5] that, if a sensor employs a Neyman-Pearson test, the same test can improve the performance of the fusion center beyond that of the best sensor provided that there are more than two sensors.

MAIN RESULTS

In [7] it was shown that the optimal combining rule in a parallel sensor configuration is the N-P test at the fusion and the sensors. The proof in [7] is general and does not depend on the Lagrange multipliers method [6] which fails to maintain optimality of the solution when the solution lies on the boundaries of the optimization space [7].

Due to the limitations in space in this paper we focus mainly on two numericallly efficient algorithms for the solution of the optimal decision scheme. The algorithms are based on the sequential optimization of the Lagrangian w.r.t. the different sensors assuming that the thresholds of previously optimized sensors are set so that the sensors operate at either zero or one probability of detection. The two algorithms will be refered as SOFA 1 and SOFA 2 respectively and are presented next.

SOFA 1 ALGORITHM: Let 1, 2, ..., N be an arbitrary ordering of N sensors. Starting from the N-th sensor, the threshold of the k-th sensor as determined by SOFA 1 is given by

$$\lambda_k = \lambda_0 \frac{C_0^{N,N-1,\ldots,k}}{C_1^{N,N-1,\ldots,k}} \tag{1}$$

where

$$C_i^{N,N-1,\ldots,k} = \sum_{U_{N,N-1,\ldots,k}} [d(0,0,\ldots,0,1,U_{N,N-1,\ldots,k}) - d(0,0,\ldots,0,0,U_{N,N-1,\ldots,k})]p(U_{N,N-1,\ldots,k}|H_i) \tag{2}$$

where λ_k designates the threshold of the k-th sensor, λ_0 is the threshold at the fusion,

$$d(u_1, u_2, \ldots, u_N) = \Pr(u_0 = 1 \mid u_1, u_2, \ldots, u_N) \tag{3}$$

is the decision function at the fusion center with u_i designating the binary decision of the i-th sensor and u_0 the decision at the fusion, H_i, i = 1, 0 is the true hypothesis and the alternative, and $U_{N,N-1,\ldots,k}$ is the set of decisions of all the sensors excluding those (decisions) of the N, N-1, ..., k sensors whose thresholds have already been determined. Furthermore, for the first sensor

$$\lambda_1 = \lambda_0 \tag{4}$$

where λ_0 is the threshold at the fusion center.

SOFA 2 ALGORITHM: Let 1, 2, ..., N be an arbitrary ordering of N sensors. Starting from the N-th sensor, the threshold of the k-th sensor as determined by SOFA 2 is given by

$$\lambda_k = \lambda_0 \frac{D_0^{N,N-1,\ldots,k}}{D_1^{N,N-1,\ldots,k}} \tag{5}$$

where

$$D_i^{N,N-1,\ldots,k} = \sum_{U_{N,N-1,\ldots,k}} [d(1,1,\ldots,1,1,U_{N,N-1,\ldots,k}) - d(0,0,\ldots,0,0,U_{N,N-1,\ldots,k})]p(U_{N,N-1,\ldots,k}|H_i) \tag{6}$$

$i = 0,1$

where λ_k designates the threshold of the k-th sensor, λ_0 is the threshold at the fusion,

$$d(u_1, u_2, \ldots, u_N) = \Pr(u_0 = 1 \mid u_1, u_2, \ldots, u_N) \tag{7}$$

is the decision function at the fusion center with u_i designating the binary decision of the i-th sensor and u_0 the decision at the fusion, H_i, i = 1, 0 is the true hypothesis and the alternative, and $U_{N,N-1,\ldots,k}$ is the set of decisions of all the sensors excluding those (decisions) of the N, N-1, ..., k sensors whose thresholds have already been determined. Furthermore, for the first sensor

$$\lambda_1 = \lambda_0 \tag{8}$$

where λ_0 is the threshold at the fusion center.

The derivation of the equations that define the two algorithms can be found in [7].

The Lagrange multipliers method fails when the decision at the fusion involves logical (Boolean) products of the sensor decisions. This is due to the fact that the permissible decision functions are monotonic functions of their arguments [7]. Hence, presence of a logical product in the decision function forces the decision to lie at the boundaries of the decision space at which point the differentiability required by the Lagrange multipliers method seizes to exist. The SOFA 1 Algorithm exhibits the same singular behavior as the Lagrange multipliers method when the decision lies at the boundaries of the decision space. However, the SOFA 2 Algorithm exhibits stable behavior even when the decision rule is singular. The stable behavior of SOFA 2 is attributed to the fact that, in the determination of the sensor thresholds, sensors whose thresholds have been previously determined, are neglected in the determination of the remaining thresholds by setting their operational points at probability of false alarm P_F = probability of detection $P_D = 1$. Thus, a logical product in the decision rule does not affect the fusion rule since the effect of a product term is always eliminated by setting the operating point of the particular sensor at $P_F = P_D = 1$.

NUMERICAL RESULTS

Several numerical results from the application of the two algorithms in distributed decision fusion with various numbers of sensors are given and the performance of the algorithms is compared with the globally optimal solution obtained by direct optimization. Figures 2 through 6 summarize the performance of SOFA 1 whereas Figures 7 through 11 summarize the performance of SOFA 2. It is seen that the two algorithms yield almost identical results very close to the optimal ones. However, the thresholds obtained by SOFA 2 are asymmetric as opposed to the thresholds of SOFA 1 and that of the optimal solution. Figures 6 and 11 depict a singular case [7] where the Lagrange multipliers method and SOFA 1 fail to yield the correct answer, whereas SOFA 2 remains robust and gives a solution which is close to the optimal one. Additional numerical results can be found in [7].

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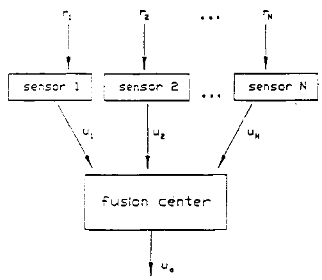

Figure 1 Distributed Sensor Fusion System

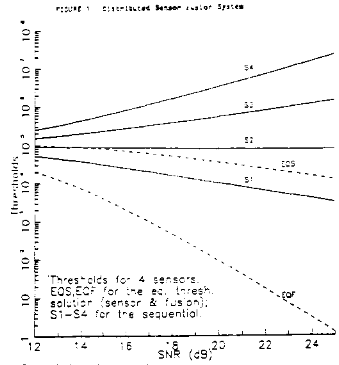

Figure 4 Optimal and SOFA 1 thresholds for four equal SNR sensor fusion. SOFA 1 yields symmetric thresholds.

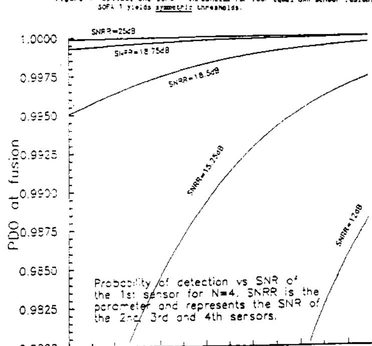

Figure 10 SOFA 2 solutions for three unequal SNR sensor fusion.

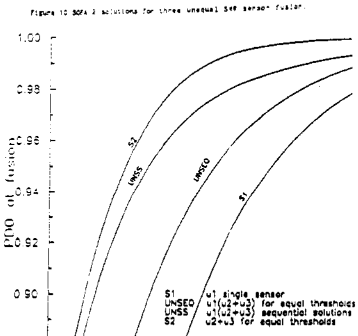

Figure 11 Singular Case for three sensors with equal SNR's: Fusion Rule is $u_1(u_2+u_3)$. The Lagrange multipliers method fails to yield the optimal solution u_2+u_3. However, SOFA 2 yields a solution close to the optimal one (robustness).

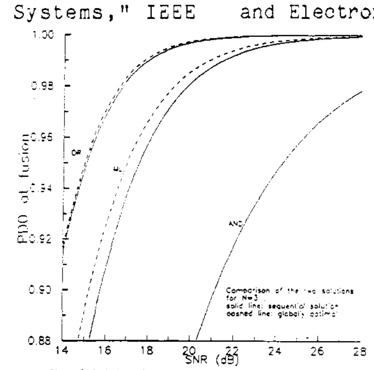

Figure 2 Optimal and SOFA 1 solutions for three equal SNR sensor fusion

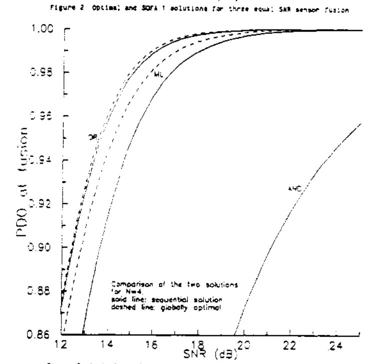

Figure 3 Optimal and SOFA 1 solutions for four equal SNR sensor fusion

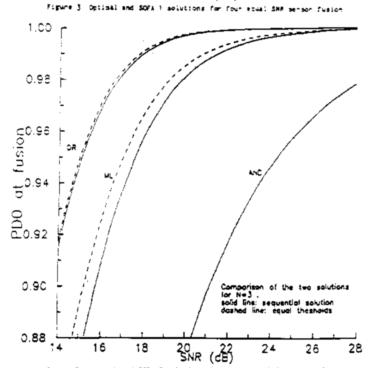

Figure 7 Optimal and SOFA 2 solutions for three equal SNR sensor fusion

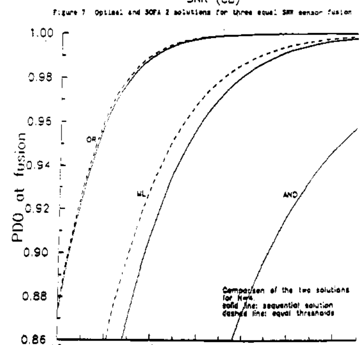

Figure 8 Optimal and SOFA 2 solutions for four equal SNR sensor fusion

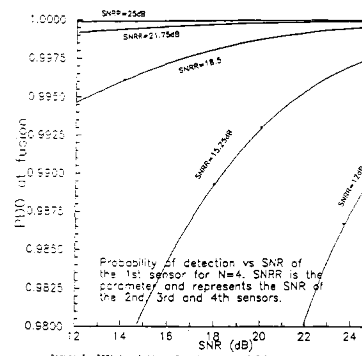

Figure 5 SOFA 1 solutions for three unequal SNR sensor fusion. The *'s represent optimal solutions.

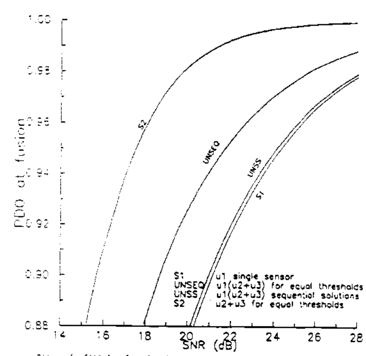

Figure 6 Singular Case for three sensors with equal SNR's: Fusion Rule is $u_1(u_2+u_3)$. The Lagrange multipliers method and SOFA 1 both fail to yield the optimal solution u_2+u_3.

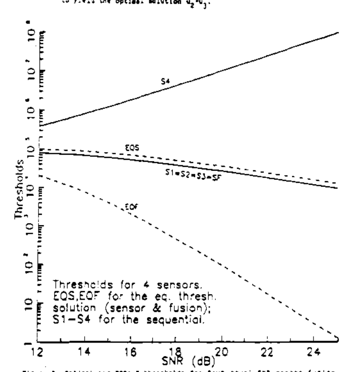

Figure 9 Optimal and SOFA 2 thresholds for four equal SNR sensor fusion. SOFA 2 yields non-symmetric thresholds.