Optimal Serial Distributed Decision Fusion

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Abstract

The problem of distributed detection involving N sensors is considered. The configuration of sensors is serial in the sense that the $(j-1)^{th}$ sensor passes its decision to the j^{th} sensor and that the j^{th} sensor decides using the decision it receives and its own observation. When each sensor employs the Neyman-Pearson test, the probability of detection is maximized for a given probability of false alarm, at the N^{th} stage. With two sensors the serial scheme is better than the parallel fusion scheme analyzed in the literature. For certain distributions of observations, the serial scheme performs better for all N. Numerical examples illustrate the global optimization by the selection of operating thresholds at the sensors.

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Introduction

The theory of distributed detection is receiving a lot of attention in the literature [1-6]. Typically, a number of sensors process the data they receive and decide in favor of one of the hypotheses about the origin of the data. In a two class decision problem, the hypotheses would be signal present (H_1) or the signal absent (H_0). These decisions are then sent to a fusion center where a final decision regarding the presence of the signal is made. This scheme can be termed parallel decision making. In this paper, we consider a serial distributed decision scheme (Fig. 1). Though the performance of this configuration is susceptible to link failures, the performance of the serial scheme can exceed that of the parallel scheme. Also, the geographical closeness of some of the sensors might make a serial or serial-parallel configuration desirable.

Development of Key Equations

Consider the serial configuration of distributed sensors shown in Fig. 1. Denote the sensor decisions as $u_1, u_2, \ldots, u_N$. The j^{th} sensor receives the decision u_{j-1} and its own observation Z_j to make its decision u_j. The decision u_N at the N^{th} sensor is the fused decision about the hypotheses. We assume that the data at the sensors, conditioned on each hypothesis, are statistically independent. This implies that Z_j and u_{j-1} are also conditionally independent. The j^{th} sensor employs an N-P test using the data (Z_j, u_{j-1}). The optimality of this assumption is shown by Theorem 1, discussed later.

Denoting the distribution of Z_j as $p(Z_j|H_1)$ and $p(Z_j|H_0)$, the likelihood ratio test becomes

$$\frac{p(Z_j|H_1)}{p(Z_j|H_0)}\,\frac{P_{D,j-1}}{P_{F,j-1}} \mathop{\gtrless}_{H_0}^{H_1} t \quad \text{if } u_{j-1} = 1$$
$$\frac{p(Z_j|H_1)}{p(Z_i|H_0)}\,\frac{1-P_{D,j-1}}{1-P_{F,j-1}} \mathop{\gtrless}_{H_0}^{H_1} t \quad \text{if } u_{j-1} = 0 \tag{1}$$

Many times it is convenient to use the log likelihood ratio, $\ell n\, \Lambda(Z_j) = \Lambda^*(Z_j)$. Hence,

$$\Lambda^*(Z_j) \mathop{\gtrless}_{H_0}^{H_1} \begin{cases} t^*_{j,1} & \text{if } u_{j-1} = 1 \\ t^*_{j,0} & \text{if } u_{j-1} = 0 \end{cases} \tag{2}$$

and

$$t^*_{j-1} = t^*_{j,0} + \ell n\left(\frac{P_{F,j-1}}{1-P_{F,j-1}}\,\frac{1-P_{D,j-1}}{P_{D,j-1}}\right) j = 1, 2, \ldots, N.$$

For the first stage, $t^*_{1,1} = t^*_{1,0}$.

At the j^{th} stage, the false alarm probability is given by

$$P_{F,j} = Pr(\Lambda^*(Z_j) > t^*_{j,0}|H_0, u_{j-1} = 0)\, Pr(u_{j-1} = 0|H_0)$$
$$+ Pr(\Lambda^*(Z_j) > t^*_{j,1}|H_0, u_{j-1} = 1)\, Pr(u_{j-1} = 1|H_0) \tag{3}$$

Let

$$a_j = Pr(\Lambda^*(Z_j) > t^*_{j,0}|H_0)$$
$$b_j = Pr(\Lambda^*(Z_j) > t^*_{j,1}|H_0)$$
$$c_j = Pr(\Lambda^*(Z_j) > t^*_{j,0}|H_1) \tag{4}$$
$$d_j = Pr(\Lambda^*(Z_j) > t^*_{j,1}|H_1)$$

Using (3), (4) and the conditional independence assumption, we have

$$P_{F,j} = a_j(1 - P_{F,j-1}) + b_j\, P_{F,j-1} \tag{5}$$

Similarly,

$$P_{D,j} = c_j(1 - P_{D,j-1}) + d_j\, P_{D,j-1} \tag{6}$$

Knowing the distribution of the observations Z_j and using (2) and (4 through 6), it is possible to compute the $P_{D,j}$'s recursively provided the $P_{F,j}$'s are specified. If the $P_{F,j}$'s are kept the same, the serial configuration exhibits some nice properties [7]. However, for a given $P_{F,N}$ at the N^{th} stage, this procedure does not guarantee a maximum $P_{D,N}$. In order to globally optimize the performance, that is to maximize $P_{D,N}$ for a given $P_{F,N}$, we need a multidimensional search with respect to the variables $P_{F,j}$'s, j = 1,2,...,(N-1). The results obtained using the numerical search procedure will be presented in the next Section. The Theorem 1 stated below shows that the N-P tests at the sensors is optimum for the serial distributed decision problem. The proof can be found in [8].

Theorem 1

Given that the observations at each stage in a serial distributed detection environment with N sensors are i.i.d., the probability of detection is maximized for a given probability of false alarm, at the N^{th} stage, when each stage employs the Neyman-Pearson test.

Performance Evaluation

Using standard numerical procedure, we evaluated the performance of a serial scheme for the case of the detection of a constant signal in additive white Gaussian noise and compared it with the parallel scheme. The result for two sensors is shown in Fig. 2. In general, for 2 sensors, the serial scheme is not inferior to the parallel scheme. The proof of this follows from Theorem 2 [8].

Theorem 2

If the switching function corresponding to the optimal parallel fusion can be realized in terms of a sequence of two variable functions with single output, then the optimal serial scheme is better than the optimal parallel scheme.

Conclusion

A serial distributed network of N sensors detecting the presence or absence of a signal is analyzed in this paper. When the sensor observations conditioned on the hypothesis, are statically independent, the sensors employ Neyman-Pearson test for maximizing the detection probability for a given false alarm probability at the N^{th} stage (Theorem 1). For certain noise distributions, the parallel structure requiring its fusion scheme to belong to certain class of switching functions, is inferior to the serial scheme (Theorem 2). As a drawback, any serial network is vulnerable to link failures. Some numerical examples illustrate the performance of the optimal serial decision scheme.

References

1] Tenney, R. R. and Sandell, N. R., Jr., "Detection with Distributed Sensors," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-17, July 1981, pp. 501-510.

2] Chair, Z. and Varshney, P. K., "Optimal Data Fusion in Multiple Sensor Detection Systems," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-22, No. 1, January 1986, pp. 98-101.

[3] Srinivasan, R., "Distributed Radar Detection Theory," IEE Proceedings, Vol. 133, PtF, No. 1, February 1986, pp. 55-60.

[4] Thomopoulos, S. C. A., Viswanathan, R. and Bougoulias, D. P., "Computable Optimal Distributed Decision Fusion," under preparation.

[5] Thomopoulos, S. C. A., Viswanathan, R. and Bougoulias, D. P., "Optimal Decision Fusion in Multiple Sensor Systems," to appear in IEEE Transactions on Aerospace and Electronic Systems.

[6] Sadjadi, F. A., "Hypotheses Testing in a Distributed Environment," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-22, No. 2, March 1986, pp. 134-137.

[7] Viswanathan, R., Thomopoulos, S. C. A. and Tumuluri, R., "Serial Decision in Multiple Sensor Fusion," to appear in the Proceedings of the 1987 CISS Conference, John Hopkins University.

[8] Viswanathan, R., Thomopoulos, S. C. A. and Tumuluri, R., "Optimal Serial Distributed Decision Fusion," submitted to IEEE Transactions on Aerospace and Electronic Systems.

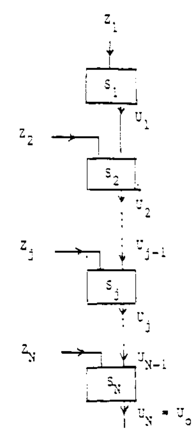

Figure 1 Serial Decision Fusion

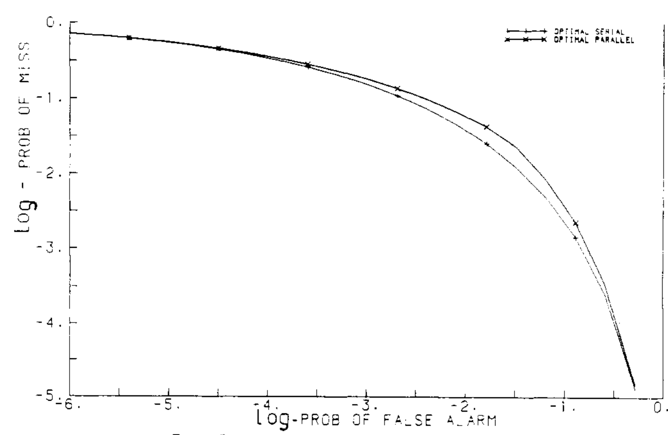

FIG 2

ADDITIVE GAUSSIAN NOISE.2 SENSORS.10-DB SNR