# Performance of a Rank Sum Combiner for FFH-MFSK Signaling in Partial Band Interference

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**PERFORMANCE OF A RANK SUM COMBINER FOR FFH-MFSK SIGNALING IN PARTIAL BAND INTERFERENCE**

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**Abstract** - We consider the performance of a fast frequency hopping $M$-ary frequency shift keying spread spectrum rank sum diversity combiner. The spread signals are received in partial band interference and the parameters of this intentional interference are unknown. For the BFSK ($M = 2$) case and a Rayleigh fading channel, the analytical performance of the rank sum receiver is compared to that of the linear receiver. Simulations are carried out for the rank sum receiver in a non-fading channel and compared to simulated performances of the clipper receiver and product combiner receiver (PCR). The performance of the rank sum combiner, in the non-fading channel, is comparable to the product combiner receiver and almost always is worse than the clipper receiver. In the Rayleigh fading channel, the rank sum receiver performs considerably better than the linear receiver when the jamming fraction is relatively small.

## I. Introduction

In this paper, we consider the performance of a rank sum diversity combiner for detecting a fast frequency hopping $M$-ary frequency shift keying (FFH-MFSK) signal received in partial band interference. Parallel fast frequency hopping with the number of hops per bit exceeding one is assumed. Parallel refers to the fact that the data modulation tones are placed contiguously within a hop band. The receiver structure is well known and can be found in [1-3]. We assume ideal acquisition and synchronization of the PRBS at the receiver. The number of hops per bit or symbol is referred to as the diversity order, denoted by $L$, and relates the symbol duration to the hop duration by $T_S = L\,T_h$. The maximum likelihood receiver, which is optimal in the sense of minimum probability of error, is unrealizable since it requires the knowledge of the jammer state and jammer parameters [1]. Therefore, several sub-optimal receivers have been discussed in literature [1-5]. Rank sum test has been used in other hypothesis testing applications[7].

Some relevant system parameters are: (1) spread spectrum bandwidth, $W_{SS}$ Hz., (2) hopping rate, $B$ Hz., (3) data rate, $R_b = 1/T_b$ bits/sec. or a symbol rate $R_s = \dfrac{R_b}{\log_2 M}$ symbols/sec., (4) thermal noise is AWG with two sided spectral height, $N_0/2$ and (5) partial band Gaussian jammer of average power $J$ watts, jamming fraction, $\gamma$, and two-sided power spectral density, $\dfrac{J}{2\gamma W_{ss}} = \dfrac{N_J}{2\gamma}$.

The block diagram of a non-coherent FFH-MFSK receiver with a rank sum diversity combiner is shown in Fig. 1. The sum of the squared in phase and quadrature phase envelopes, corresponding to the $M$ modulation bins, are sampled every $T_h$ seconds to form the observations, $r_{jk}$, $j = 1, 2, ..., M$, $k = 1, 2, ..., L$. The combined ordering of these observations are replaced with their ranks and then summed in each of the $M$ frequency bins to form the rank sums statistics, $S_j$, $j = 1, 2, ..., M$. $\arg\max_j \{S_j\}$ is taken as the bit/symbol decision.

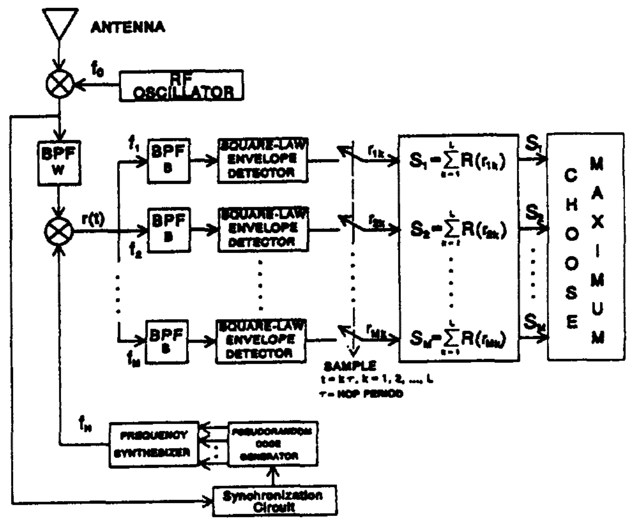


Figure 1 Rank Sum Receiver Structure for FFH-MFSK Signaling

## II. Analysis

The error rates of the rank sum combiner for the non-fading channel have been simulated using the Monte Carlo method since it is not possible to obtain the error rate analytically(for details see [6]).

The error rates for the Rayleigh fading channel are analytically obtained. Before we discuss the analytical method, let us introduce some relevant notations. We write the probability density functions of all $r_{jk}$, $j = 1, 2, ..., M$ and $k = 1, 2, ..., L$, as, for $\lambda > 0$ and $r_{jk} \geq 0$,

$$f_{R_{jk}}(r_{jk}) = \lambda e^{-\lambda r_{jk}} \quad (1)$$

where the parameter, $\lambda$, depends on the noise and (possible) signal, $s_f(t)$, components. Let $E\{s_f^2(t)\}=1$, for a normalized signal power of one watt. Therefore, $\frac{1}{\lambda_{sj}}=E\{s_f^2(t)\}+E\{n^2(t)\}+E\{J^2(t)\}$, where $n(t)$ is thermal noise and $J(t)$ is the jamming noise. The first term on the right side of (9) is absent when the signal is not present and the third term is absent when the particular frequency bin is not jammed. We define the notation, $\lambda_{SJ}, \lambda_{SU}, \lambda_J$ and $\lambda_U$, as the $\lambda' s$ corresponding to, the bin with a signal component and jamming noise, the bin with a signal component and no jamming noise, the bin with jamming noise and no signal component and the bin with no jamming noise and no signal component, respectively.

$\frac{E\{n^2(t)\}}{E\{s_f^2(t)\}}$ can be defined as $\frac{1}{SNR}$, where $SNR$ is the signal to thermal noise power ratio and

$\frac{E\{j^2(t)\}}{E\{s_f^2(t)\}}$ can be defined as $\frac{1}{SJR}$, where $SJR$ is the signal to jammer noise power ratio .

Therefore, all the $\lambda$'s can be written in terms of $SNR$ and $SJR$. In the analytical derivation, we consider BFSK signaling with three and five hops. We illustrate the procedure for $L$=3. Let us start with six samples, $z_1, z_2, \ldots, z_6$, obtained from three hops of the mark and space frequency bins. We calculate the probability of the event $P\{Z_1<Z_2<Z_3<Z_4<Z_5<Z_6\}$. All of the random samples, $Z_1, Z_2, \ldots, Z_6$, are distributed exponentially with appropriate scale parameters. Let us write the density of any $Z_i$ as,

$$f_{Z_i}(z_i)=\lambda_i e^{-\lambda_i z_i}, \tag{2}$$

where $z_i>0$, $\lambda_i>0$ and $i\in\{1,2,\ldots,6\}$. Since $Z_i$'s are independent,

$$\begin{aligned}P\{Z_1<Z_2<Z_3<Z_4<Z_5<Z_6\}\\ &=\int_0^\infty\int_{z_1}^\infty\int_{z_2}^\infty\int_{z_3}^\infty\int_{z_4}^\infty\int_{z_5}^\infty f_{Z_6}(z_6)dz_6 f_{Z_5}(z_5)dz_5 f_{Z_4}(z_4)dz_4\\ &\qquad f_{Z_3}(z_3)dz_3 f_{Z_2}(z_2)dz_2 f_{Z_1}(z_1)dz_1\\ &=\int_0^\infty\int_{z_1}^\infty\int_{z_2}^\infty\int_{z_3}^\infty\int_{z_4}^\infty\int_{z_5}^\infty \lambda_6 e^{-\lambda_6 z_6}dz_6\lambda_5 e^{-\lambda_5 z_5}dz_5\\ &\qquad\cdots\lambda_1 e^{-\lambda_1 z_1}dz_1\end{aligned} \tag{3}$$

Hence,

$$P\{Z_1<Z_2<Z_3<Z_4<Z_5<Z_6\}=\prod_{i=1}^{5}\frac{\lambda_i}{\sum_{k=i}^{6}\lambda_k} \tag{4}$$

where each $\lambda_i$ can take one of four values, $\lambda_{SJ}, \lambda_{SU}, \lambda_J$ and $\lambda_U$. The above probability of error expression applies for one out of 6! or 720 events, since 6! is the number of ways 6 rank integers can permute. The expression is very dependent on the actual values of the $\lambda' s$, therefore, all permutations and combinations must be considered. Recall that the frequency bins are placed contiguously and when one modulation bin is jammed for a particular hop, the other modulation bin is jammed as well. Therefore, when calculating the probability of all possible events including jamming, we must consider $2^3$ or 8 possible ways of being jammed, since there are three hops and each hop can be jammed or unjammed independently of the others.

If $S_1\le 10$, an error occurs, assuming a tone $f_1$ corresponding to a space or binary "0" was transmitted. We now must determine all the vectors, which are the actual rank integers in the space bin, that yield a sum less than or equal to 10. For example, [1 2 3] is an error vector since $S_1=1+2+3<10$. In the error probability calculation, all permutations of [1 2 3] are identified with the error vector designation, [1 2 3]. The corresponding vectors in the mark frequency bin must also be identified. Since there are six possible rank integers to choose from and three of these form the samples in the space frequency bin, we have a total of $\binom{6}{3}=20$ combinations in which these samples can appear. A computer program was written to identify all combinations whose sum was less than or equal to ten. By finding the space rank sum corresponding to 20 vector combinations, we observe that there are 10 combinations out of 20 total combinations whose sum is less than or equal to ten. Each of these ten error vectors has its own distinct probability of error and therefore, each one must be calculated individually. We must also consider all permutations of the elements of each of the ten error vectors and all eight possible jamming events for all six of the samples in the space and mark frequency bins. If we calculate each probability of error in a brute force manner, we will have (6!)/2 or 360 error expressions to evaluate for each possible jamming event. To reduce the number of these calculations, we consider permutations of these combinations that result in equivalent errors. Further reduction is achieved by recognizing equivalent errors for different jamming events. These reductions are explained in detail in [6].

Then the probability of error is,

$$\begin{aligned}P\{e\}&=(1-\gamma)^3\cdot P\{e|\text{no hops jammed}\}+\\ &\gamma(1-\gamma)^2\cdot P\{e|\text{1 hop jammed}\}\\ &+\gamma^2(1-\gamma)\cdot P\{e|\text{2 hops jammed}\}+\\ &\gamma^3\cdot P\{e|\text{all hops jammed}\}.\end{aligned} \tag{5}$$

The logic behind the derivation of the probability of error for $L=5$ is the same as in the case when $L=3$[6]. We observe that the number of distinct probability of error expressions increases combinatorially with respect to the number of hops, which is why the analysis is performed for only three and five hops.

### 2.1 *Clipper, PCR and Linear Receivers*

We also evaluated the performances of the product combiner receiver (PCR), the clipper receiver (CLP) and the linear receiver that have been discussed in the literature [1-4]. The error rates of the PCR and CLP are simulated for BFSK and 4-ary FSK modulations for three and four hops in a non-fading channel. The input to each of these combiners are the samples, $r_{jk}$, obtained from the square-law envelope detector.

Assuming a normalized noise power of 1 watt, the clipping level for the clipper is set at signal power. The error rate of the linear receiver is obtained analytically for BFSK signaling and a Rayleigh fading channel. Details can be found in [6]. We can write the conditional probability of error, conditioned on $\ell$ hops jammed, when $\ell = 1, 2, \ldots, L-1$, as,

$$P_{e|\ell} = \sum_{j=0}^{\ell-1}\sum_{k=0}^{j}\sum_{p=0}^{\ell-1}\frac{\binom{j}{k}\binom{\ell-1}{p}(-1)^{k+p}\lambda_U^{L-\ell}\lambda_{SJ}^{\ell}\lambda_S^{L-\ell}\lambda_J^{j}}{j!\left[\Gamma(L-\ell)\right]^2\Gamma(\ell)(\lambda_U-\lambda_J)^{k+L-\ell}(\lambda_S-\lambda_{SJ})^{p+L-\ell}}\cdot\{I_1+I_2+I_3+I_4\}$$
$$+\sum_{m=0}^{L-\ell-1}\sum_{p=0}^{\ell-1}\frac{\lambda_{SJ}^{\ell}\lambda_S^{L-\ell}\lambda_U^{m}\binom{\ell-1}{p}(-1)^p}{m!\,\Gamma(\ell)\Gamma(L-\ell)(\lambda_S-\lambda_{SJ})^{p+L-\ell}}\cdot K \quad (6)$$

where,

$$I_1 = \frac{(\ell-1-p+j-k)!(k+L-\ell-1)!(p+L-\ell-1)!}{(\lambda_J+\lambda_{SJ})^{\ell-p+j-k}}$$

$$I_2 = -(k+L-\ell-1)!(p+L-\ell-1)!\sum_{r=0}^{p+L-\ell-1}\frac{(\lambda_{SU}-\lambda_{SJ})^r(\ell-1-p+j-k+r)!}{r!(\lambda_J+\lambda_{SU})^{\ell-p+j-k+r}}$$

$$I_3 = -(k+L-\ell-1)!(p+L-\ell-1)!\sum_{q=0}^{k+L-\ell-1}\frac{(\lambda_U-\lambda_J)^q(\ell-1-p+j-k+q)!}{q!(\lambda_{SJ}+\lambda_U)^{\ell-p+j-k+q}}$$

$$I_4 = (k+L-\ell-1)!(p+L-\ell-1)!\sum_{r=0}^{p+L-\ell-1}\sum_{q=0}^{k+L-\ell-1}\frac{(\lambda_{SU}-\lambda_{SJ})^r(\lambda_U-\lambda_J)^q(\ell-1-p+j-k+q+r)!}{r!q!(\lambda_{SU}+\lambda_U)^{\ell-p+j-k+q+r}}$$

$$K = (p+L-\ell-1)!\cdot\left[\frac{(\ell-1-p+m)!}{(\lambda_U+\lambda_{SJ})^{\ell-p+m}}-\sum_{t=0}^{p+L-\ell-1}\frac{(\lambda_{SU}-\lambda_{SJ})^t}{t!}\frac{(\ell-1-p+m+t)!}{(\lambda_{SU}+\lambda_U)^{\ell-p+m+t}}\right].$$

For $\ell = L$ or $\ell = 0$, we can write,

$$P_{e|L} = \sum_{j=0}^{L-1}\binom{L+j-1}{j}\frac{\lambda_J^j\lambda_{SJ}^L}{(\lambda_J+\lambda_{SJ})^{L+j}} \quad (7)$$

$$P_{e|0} = \sum_{j=0}^{L-1}\binom{L+j-1}{j}\frac{\lambda_U^j\lambda_{SU}^L}{(\lambda_U+\lambda_{SU})^{L+j}}. \quad (8)$$

In the literature, the error rate for the linear receiver has been obtained for (i) non-fading case [2] and (ii) for Rayleigh fading, $M$-ary case [5]. The error expression obtained here, for the Rayleigh fading and $M = 2$ case, is much simpler than the one given for arbitrary $M$ in [5].

## III. Discussion and Conclusions

We consider the performances of the different receivers for different values of $E_b/N_0$, $E_b/N_J$ and jamming fraction, $\gamma$. Figures 2-5 show the performance of the rank sum, PCR and clipper receivers in a non-fading channel. In Fig. 2 for a jamming fraction of 0.1, we see that the performance of the rank sum is competitive with the PCR and the clipper receiver performs better than both the rank sum and the PCR for the majority of the range considered. Also the performances of all three receivers for three hops are very close to their respective performances for four hops. Figure 3 illustrates that the performances of all three receivers are relatively close for a jamming fraction of 1.0. In Fig. 4, we see that the rank sum performs better than the PCR for three hops, but performs worse for four hops. This may be a result of the possible randomization occurring because of ties among rank sums for four hops. Figure 5 shows the performances of each receiver relative to the jamming fraction. Again we see that the performances of the rank sum and the PCR are competitive over the range of $E_b/N_J$. Figures 6 through 10 illustrate the performance of the rank sum and linear receivers in a Rayleigh fading channel. Figure 6 shows that the rank sum performs better than the linear receiver for a wide range of $E_b/N_J$ and that the performance improvement of the rank sum is much better for five hops. Figure 7 illustrates a jamming fraction of 1.0, and show that the linear receiver performs better than the rank sum receiver for the entire range. In Fig. 8, we see that the error rate of the rank sum receiver is about two decades below that of the linear for three hops and is just short of four decades for five hops. Figure 9 depicts the performance of the rank sum receiver for different jamming fractions ranging from 0.001 to 1.0. We have found, from Fig. 9 and other performance curves, that the optimal jamming fraction, for the rank sum receiver in the Rayleigh fading channel, is 1.0. In contrast to this, we see that for the linear receiver, the optimal jamming fraction changes over the range of $E_b/N_j$ and forms an envelope as shown in Fig. 10.

In non-fading channel, we find that the rank sum receiver performs slightly better in certain situations as compared to the PCR and a little worse in others. Since the clipper requires the knowledge of *SNR*, its performance is almost always better than the rank sum and the PCR. The performance of the rank sum receiver for BFSK signaling and a Rayleigh fading channel was compared to that of the linear receiver. In fading channel, the results exhibit that the rank sum receiver performs better than the linear receiver when the jamming fraction is small. However, when we consider their performances under optimal (worst case) jamming fraction conditions, we see that the performance of the linear receiver is somewhat better than the rank sum receiver. Therefore, the rank sum combiner is preferable over the linear combiner, when partial band jamming with a relatively low jamming fraction is anticipated.

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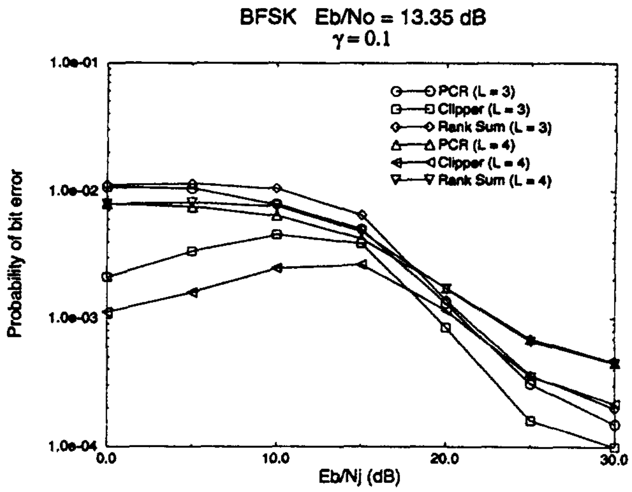


Fig. 2 Performance of the PCR, Clipper and Rank Sum Receivers in a Non-Fading Channel

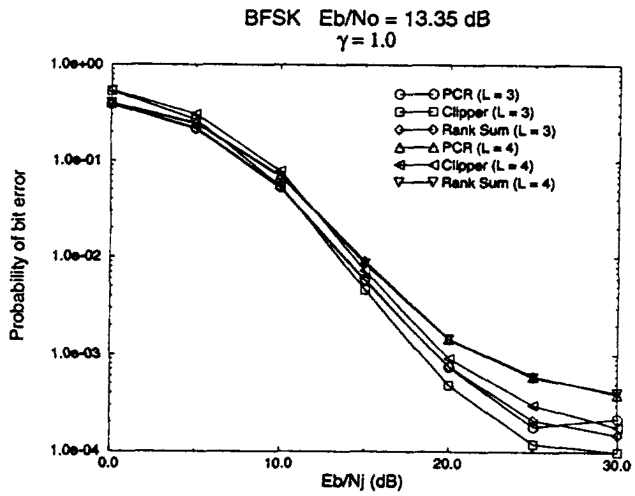


Fig. 3 Performance of the PCR, Clipper and Rank Sum Receivers in a Non-Fading Channel

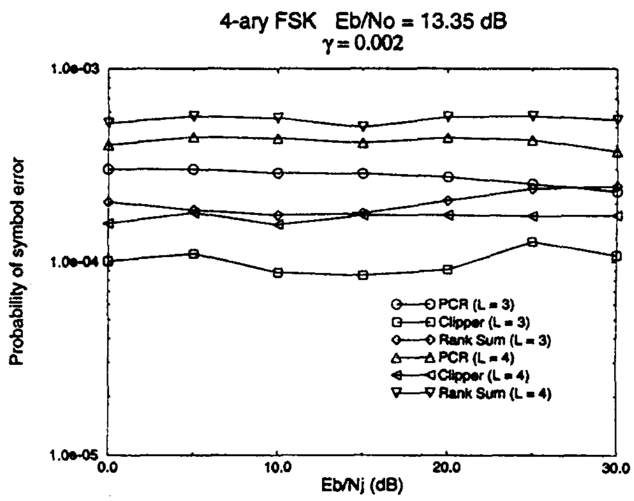


Fig. 4 Performance of the PCR, Clipper and Rank Sum Receivers in a Non-Fading Channel

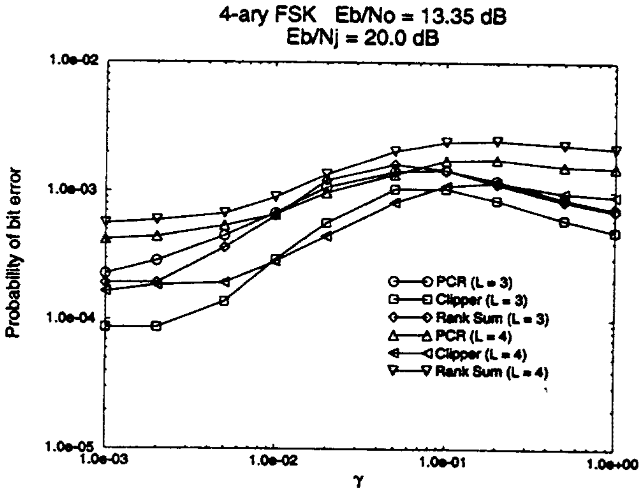


Fig. 5 Performance of the PCR, Clipper and Rank Sum Receivers in a Non-Fading Channel

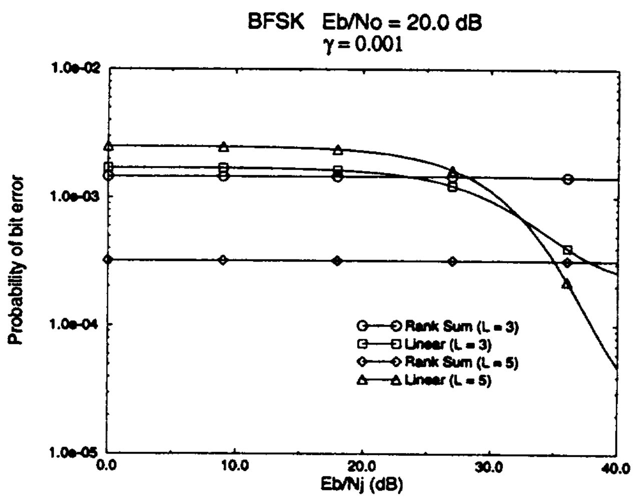


Fig. 6 Performance of the Rank Sum and Linear Receivers in a Fading Channel

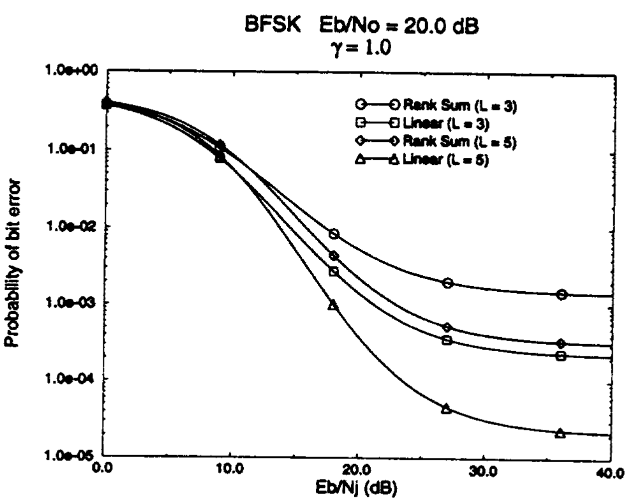


Fig. 7 Performance of the Rank Sum and Linear Receivers in a Fading Channel

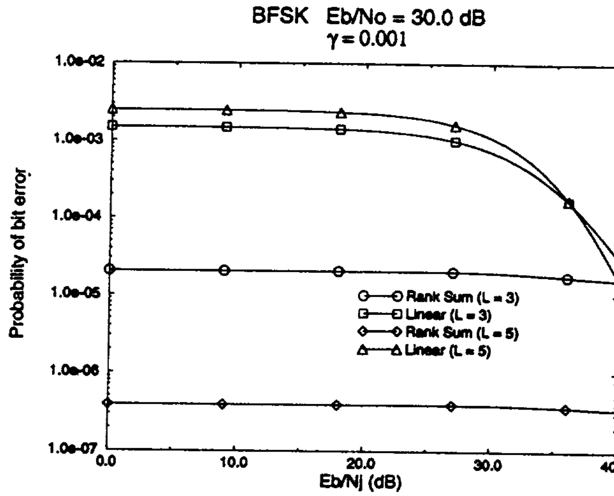


Fig. 8 Performance of the Rank Sum and Linear Receivers in a Fading Channel

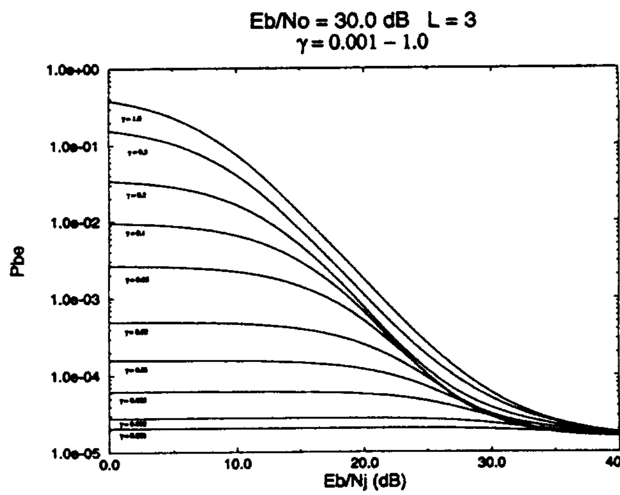


Fig. 9 Performance of the Rank Sum Receiver in a Fading Channel

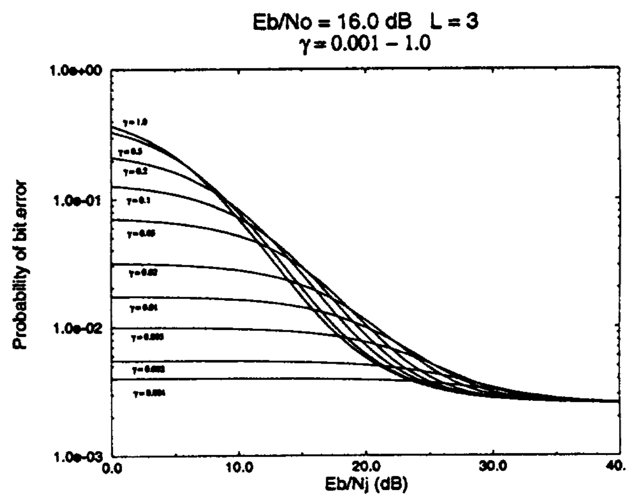


Fig. 10 Performance of the Linear Receiver in a Fading Channel