9-1995

Performance Loss Computation in Distributed Detection

Hamid Amirmehrabi
Southern Illinois University Carbondale

R. Viswanathan
Southern Illinois University Carbondale, viswa@engr.siu.edu

Performance Loss Computation in Distributed Detection

Hamid Amirmehrabi and R. Viswanathan*
Electrical Engineering Dept., Southern Illinois Univ.
Carbondale, IL 62901-6603

Abstract - The loss associated with a distributed signal detection system as compared to a centralized scheme is evaluated with respect to probability of error. Such a loss is numerically computed for several members of the exponential family.

I. INTRODUCTION

An important problem in a Distributed Signal Detection (DSD) scheme is the loss associated with the system. Hence, error analysis plays a significant role in the design of DSD processors. Here, we make an attempt to quantify the loss associated with a DSD system as compared to a centralized scheme by providing an easily computable probability of error expression.

Consider a network of n distributed sensor communicating with a fusion center. Let $\{U_1, U_2, \ldots, U_k\}$ represent the quantized data passed from the sensors numbered 1 through k to the fusion center. Let $\{X_{k+1}, X_{k+2}, \ldots, X_n\}$ represent the observations at the remaining sensors, which are passed directly on to the fusion center without any quantization. Let us assume that U_i's, $i = 1,2,\ldots,k$ are binary valued and that the problem is to decide between two hypotheses H_0 and H_1. Denoting the density of the ith sensor as $f\left(x_i \middle| H_j\right)$, $j = 0,1$, and assuming that sensor observations given the hypothesis are independent and identical, we can formulate an optimum fusion center test based on a Likelihood Ratio Test (LRT) [1]. The LRT is given by the following

$$\Lambda_k = C_k . D_k \underset{H_0}{\overset{H_1}{\gtrless}} t_k \qquad (1)$$

where

$$C_k = \frac{f(x_{k+1}, \ldots, x_n | H_1)}{f(x_{k+1}, \ldots, x_n | H_0)}, \text{ and } D_k = \frac{P(U_1, \ldots, U_k | H_1)}{P(U_1, \ldots, U_k | H_0)} \qquad (2)$$

and t_k is an appropriate threshold.

II. AVERAGE PROBABILITY OF ERROR

The average probability of error corresponding to (1) can be written as

$$P_e(k) = P(H_0) P\left(C_k \geq \frac{t_k}{D_k} \middle| H_0 \right) + P(H_1) P\left(C_k < \frac{t_k}{D_k} \middle| H_1 \right) \qquad (3)$$

In many problems of practical interest, sufficient statistics of fixed low dimensions exist. Hence, the probability sets involving the C_k in (3) can be replaced by appropriate sets involving the sufficient statistic. Moreover, the D_k in (3) can only take discrete number of values, a maximum of $k+1$ different values. These possible values are $r^j S^{k-j}$, $j = 0,1,\ldots,k$, where

$$r = \frac{P(U_i = 1 | H_1)}{P(U_i = 1 | H_0)}, \text{ and } S = \frac{P(U_i = 0 | H_1)}{P(U_i = 0 | H_0)} \qquad (4)$$

Therefore, the probabilities of the type (3) can be very easily computed as a function of k. Such computations are carried out for the case when the density of observation belongs to an exponential family.

III. PERFORMANCE ANALYSIS

Closed form error expressions for gamma, exponential (for testing scale parameter) and normal (for testing location parameter) densities are derived. Table 1 shows the ratio of the error probabilities when $n = 5$ and Signal power to Noise power Ratio (SNR) is 10 dB. α is the shape parameter of the gamma density. As α increases the ratio of the error probabilities also increases.

	Exponential	Normal	Gamma , $\alpha = 3$
$\frac{P_e(2)}{P_e(1)}$	1.2	1.6	2.0
$\frac{P_e(4)}{P_e(1)}$	1.8	4.4	8.0

Table 1

Numerical results indicate that for normal and gamma (with large α) densities the loss due to quantization is more significant than for exponential density.

REFERENCE

[1] J.N. Tsitsiklis, "Decentralized Detection," in *Advances in Statistical Signal Processing* , Vol. 2., *Signal Detection*, H.V. Poor and J.B. Thomas, Eds., Greenwich, CT: JAI press, 1990.

*This work was supported by BMDIO/IST and managed by the office of naval research under contract N00014-94-1-0736.