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# Performance Loss Computation in Distributed Detection

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Abstract - The loss associated with a distributed signal detection system as compared to a centralized scheme is evaluated with respect to probability of error. Such a loss is numerically computed for several members of the exponential family.

## I. INTRODUCTION

An important problem in a Distributed Signal Detection (DSD) scheme is the loss associated with the system. Hence, error analysis plays a significant role in the design of DSD processors. Here, we make an attempt to quantify the loss associated with a DSD system as compared to a centralized scheme by providing an easily computable probability of error expression.

Consider a network of  $n$  distributed sensor communicating with a fusion center. Let  $\{U_1, U_2, \dots, U_k\}$  represent the quantized data passed from the sensors numbered 1 through  $k$  to the fusion center. Let  $\{X_{k+1}, X_{k+2}, \dots, X_n\}$  represent the observations at the remaining sensors, which are passed directly on to the fusion center without any quantization. Let us assume that  $U_i$ 's,  $i = 1, 2, \dots, k$  are binary valued and that the problem is to decide between two hypotheses  $H_0$  and  $H_1$ . Denoting the density of the  $i$ th sensor as  $f(x_i | H_j)$ ,  $j = 0, 1$ , and assuming that sensor observations given the hypothesis are independent and identical, we can formulate an optimum fusion center test based on a Likelihood Ratio Test (LRT) [1]. The LRT is given by the following

$$\Lambda_k = C_k \cdot D_k \begin{matrix} H_1 \\ \geq \\ < \\ H_0 \end{matrix} t_k \quad (1)$$

where

$$C_k = \frac{f(x_{k+1}, \dots, x_n | H_1)}{f(x_{k+1}, \dots, x_n | H_0)}, \text{ and } D_k = \frac{P(U_1, \dots, U_k | H_1)}{P(U_1, \dots, U_k | H_0)} \quad (2)$$

and  $t_k$  is an appropriate threshold.

## II. AVERAGE PROBABILITY OF ERROR

The average probability of error corresponding to (1) can be written as

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$$P_e(k) = P(H_0)P\left(C_k \geq \frac{t_k}{D_k} | H_0\right) + P(H_1)P\left(C_k < \frac{t_k}{D_k} | H_1\right) \quad (3)$$

In many problems of practical interest, sufficient statistics of fixed low dimensions exist. Hence, the probability sets involving the  $C_k$  in (3) can be replaced by appropriate sets involving the sufficient statistic. Moreover, the  $D_k$  in (3) can only take discrete number of values, a maximum of  $k + 1$  different values.

These possible values are  $r^j s^{k-j}$ ,  $j = 0, 1, \dots, k$ , where

$$r = \frac{P(U_i = 1 | H_1)}{P(U_i = 1 | H_0)}, \text{ and } s = \frac{P(U_i = 0 | H_1)}{P(U_i = 0 | H_0)} \quad (4)$$

Therefore, the probabilities of the type (3) can be very easily computed as a function of  $k$ . Such computations are carried out for the case when the density of observation belongs to an exponential family.

## III. PERFORMANCE ANALYSIS

Closed form error expressions for gamma, exponential (for testing scale parameter) and normal (for testing location parameter) densities are derived. Table 1 shows the ratio of the error probabilities when  $n = 5$  and Signal power to Noise power Ratio (SNR) is 10 dB.  $\alpha$  is the shape parameter of the gamma density. As  $\alpha$  increases the ratio of the error probabilities also increases.

	Exponential	Normal	Gamma, $\alpha = 3$
$\frac{P_e(2)}{P_e(1)}$	1.2	1.6	2.0
$\frac{P_e(4)}{P_e(1)}$	1.8	4.4	8.0

Table 1

Numerical results indicate that for normal and gamma (with large  $\alpha$ ) densities the loss due to quantization is more significant than for exponential density.

## REFERENCE

- [1] J.N. Tsitsiklis, "Decentralized Detection," in *Advances in Statistical Signal Processing*, Vol. 2., Signal Detection, H.V. Poor and J.B. Thomas, Eds., Greenwich, CT: JAI press, 1990.