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S. Gollakota
Southern Illinois University Carbondale

R. Viswanathan
viswa@engr.siu.edu

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Order Statistics Based Diversity Combining for Fading Channels*

S. Gollakota
Dept. of Electrical Engineering
Southern Illinois University at Carbondale
Carbondale, IL 62901-6603

R. Viswanathan
Division of Engineering
University of Texas at San Antonio
San Antonio, TX 78249-0665

Abstract

In this paper we present a new order statistics based diversity combining scheme (OSDC) for combining a set of independently fading signal amplitudes. The OSDC orders all the received signal amplitudes and uses only the two strongest signals in the combining process. The decision as to whether to use only the strongest or both the strongest and the next strongest is made depending on the relative strengths of these two highest order statistics. Signal-to-noise ratio performance of the new scheme is compared with that of the traditional schemes such as, selection combining, maximal ratio combining, equal gain combining, and a second order selection combining (SC2), for three channels, namely Rayleigh, Nakagami and exponential. The results show that OSDC performs as well as SC2.

1. Introduction

Diversity has long been recognized as an effective technique for combating the detrimental effects of channel fading [1-3]. The traditional diversity combining techniques include selection, equal gain, and maximal ratio combining. Another recently proposed scheme is called the second order selection combining (SC2)[4]. In selection combining (SC), the branch signal with the largest signal to noise ratio (SNR) is selected for demodulation. For coherent modulation with independent branch fading, the optimal linear combining technique to use is the maximal ratio combining (MRC), in which the SNR at the output is given by the sum of each individual branch SNR's [2]. Though MRC is the optimal diversity combiner, it is difficult to implement in a multipath fading channel, as the receiver complexity for MRC is directly proportional to the number of resolvable paths, M [4].

Another conventional technique is the equal gain combining (EGC), where all the available branches are equally weighted and added [1-3]. The receiver complexity is dependent on the number of branches, M. In the recently proposed scheme called SC2, the two branch signals, which have two largest SNR's, are combined coherently [4]. The receiver complexity is much more simplified as we need only two signals to combine irrespective of the number of branches. In [4], it is shown that SC2 performs comparably to EGC, and in some cases performs better than EGC, when the bit error rate is 10^{-3} or greater. This method may not be easily realizable under fast fading conditions, as extracting phase is difficult under these conditions.

In this paper, we present a new diversity combining method based on order statistics, in which, all the received signal amplitudes are ordered and only the two strongest signals are used in the combining process. The decision as to whether to use only the strongest or both the strongest and the next strongest is made depending on the relative strengths of these two highest order statistics. This particular idea of combining is motivated by the statistical literature on integrated selection and ranking procedures [7].

In section II we introduce the new order statistics based diversity combining scheme (OSDC). Section III discusses the SNR performance of OSDC under various fading conditions. Discussion and conclusions of the results from this study are presented in section IV.

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2. Order statistics based diversity combining scheme

Let $x_{(1)}, x_{(2)}, \ldots, x_{(M)}$ be the ranked signal amplitudes (r.m.s.) of the received signals in the M available branches where $x_{(1)}$ is the minimum order statistic and $x_{(M)}$ is the maximum order statistic of the signal amplitudes $x_1, x_2, \ldots, x_M$ [6]. $x_{(M)}$ and $x_{(M-1)}$ are used to provide the combined signal amplitude s:

$$s = \begin{cases} x_{(M)} & r \geq t \\ x_{(M)} + x_{(M-1)} & r < t \end{cases} \tag{1}$$

where

$$r = \frac{x_{(M)}}{x_{(M-1)}} \tag{2}$$

Here the value of t (a real number, $t \geq 1$) is to be chosen suitably.

The output SNR, q, of OSDC is given by

$$q = \begin{cases} \dfrac{x_{(M-1)}^2 r^2}{\eta} & r \geq t \\ \dfrac{x_{(M-1)}^2 (r+1)^2}{2\eta} & r < t \end{cases} \tag{3}$$

where $\eta = E(n^2(t))$ is the noise power and is assumed to be the same in all the M branches.

Figs. 1-3 show the plots of $\dfrac{q}{x_{(M-1)}^2/\eta}$ Vs r, for three different values of t, namely $t < 1+\sqrt{2}$, $t > 1+\sqrt{2}$, and $t = 1+\sqrt{2}$. From these figures we observe that q is maximized when $t = 1+\sqrt{2}$, for every value of $x_{(M-1)}$. Therefore, the SNR performance of OSDC is maximized with $t = 1+\sqrt{2}$.

The cumulative distribution function (CDF) of the output SNR is given by

$$\begin{aligned} F_Q(q) &= P\left(Q \leq q \,\middle|\, \frac{x_{(M)}}{x_{(M-1)}} > t\right) P\left(\frac{x_{(M)}}{x_{(M-1)}} > t\right) + \\ &\quad P\left(Q \leq q \,\middle|\, \frac{x_{(M)}}{x_{(M-1)}} \leq t\right) P\left(\frac{x_{(M)}}{x_{(M-1)}} \leq t\right) \\ &= P\left(x_{(M)} \leq \sqrt{\eta q}, \frac{x_{(M)}}{x_{(M-1)}} > t\right) + \\ &\quad P\left(x_{(M)} + x_{(M-1)} \leq \sqrt{2\eta q}, \frac{x_{(M)}}{x_{(M-1)}} \leq t\right) \end{aligned} \tag{4}$$

Using appropriate density functions, (4) can be simplified as

$$\begin{aligned} F_Q(q) = &\int_0^{\frac{1}{t}\sqrt{\eta q}} \int_{tx}^{\sqrt{\eta q}} f_{(X_{(M-1)}, X_{(M)})}(x, y)\,dy\,dx + \\ &\int_0^{\sqrt{\frac{\eta q}{2}}} \int_{\frac{y}{t}}^{y} f_{(X_{(M-1)}, X_{(M)})}(x, y)\,dx\,dy + \\ &\int_{\sqrt{\frac{\eta q}{2}}}^{\frac{\sqrt{2\eta q}}{(1+1/t)}} \int_{\frac{y}{t}}^{\sqrt{2\eta q}-y} f_{(X_{(M-1)}, X_{(M)})}(x, y)\,dx\,dy \end{aligned} \tag{5}$$

where the joint probability density is given by [6]

$$f_{(X_{(M-1)}, X_{(M)})}(x, y) = 4M(M-1)\left[F_{X_i}(x)\right]^{M-2} \cdot f_{X_i}(x) f_{X_i}(y) \tag{6}$$

and $F_{X_i}(.)$ denotes the CDF of X_i

The normalized SNR γ is defined as the ratio of the combined SNR and the single branch average SNR:

$$\gamma = \frac{q}{E(X_i^2)/\eta} \tag{7}$$

3. SNR performance of OSDC under different types of fading

In this section we explain how to compute CDF of Q for various fading signal statistics such as Rayleigh, Nakagami and exponential.

3.1 Rayleigh fading

Under Rayleigh fading conditions, the probability density function of X_i is given by

$$f_{X_i}(x_i) = \frac{2x_i}{\beta} \exp\left(\frac{-x_i^2}{\beta}\right) \qquad x_i \geq 0 \tag{8}$$

where $\beta = E(X_i^2)$ and is the same for all the branches.

Using (6-8) in (5), we can numerically obtain $F_Q(q)$ as a function of γ. Standard integral routines, such as IMSL, can be used to carry out the numerical integration.

3.2 Nakagami fading

Nakagami m-distribution is a versatile statistical model, which can model a variety of fading environments, including those modeled by the Rayleigh and the one-sided Gaussian distributions [8,9].

Under Nakagami fading conditions, the probability density function of X_i is given by [3]

$$f_{X_i}(x_i) = 2\left(\frac{m}{\Omega}\right)^{m_i} \frac{1}{\Gamma(m)}(x_i)^{2m-1} \exp\left(\frac{-m}{\Omega}x_i^2\right) \quad x_i \geq 0 \quad (9)$$

where m is called the fading parameter and $E\left(X_i^2\right) = \Omega$ is the average signal power of the i^{th} branch. Rayleigh density (m =1) and one-sided Gaussian (m =0.5) are two special cases of Nakagami. $m = \infty$ corresponds to a non-fading channel.

3.3 Exponential fading

The density of X_i under exponential fading conditions is given by

$$f_{X_i}(x_i) = \lambda \exp(-\lambda x_i) \qquad x_i \geq 0 \qquad (10)$$

More disperse fadings, such as exponential fading, are often encountered only at the frequencies below ultra high frequencies (UHF), which are not so common in practice. However, it is known that EGC may not be a better method than SC for more disperse distributions [2]. Therefore, it is of interest to compare the performances of these and OSDC under this fading.

4. Discussion and Conclusion

The SNR performance of the OSDC (with $t = 1+\sqrt{2}$, which is optimal as shown earlier), is compared with SC, SC2, and MRC under Rayleigh and Nakagami fading conditions and with SC, SC2, and EGC under exponential fading conditions. For $M = 2$ and $t = \infty$, OSDC becomes EGC. This allows us to verify the results for $M = 2$ and $t = \infty$ with the results of EGC under $M = 2$, for all the three fading conditions. $t = 1$ corresponds to SC.

A diversity scheme performs better if its CDF at any given γ assumes small values. An equivalent statement is that a better combining scheme assumes large values with high probabilities. Fig. 4 shows the performances of OSDC, SC, SC2 and MRC, for $M = 2$ and $M = 4$, under Rayleigh fading conditions. We observe that the new method performs as well as SC2. Fig. 5 shows the performance comparison of various methods, for $m = 2$, $M = 2$ and 4, under Nakagami fading conditions. Figs. 6 (M =2) and 7 (M =4) show the performances of various methods under Nakagami fading conditions, for m equal to 4. We again observe that the two curves marked SC2 and OSDC are very close to each other. Under both Rayleigh and Nakagami fading, the OSDC performs better than SC for any M.

Fig. 8 shows the performances of various schemes under exponential fading for $M = 2$ and $M = 4$. For all M, OSDC performs better than SC and EGC and its performance is equivalent to that of SC2. Another observation is that SC performs slightly better than EGC for $M = 2$, for all values of γ, and that for other values of M, it performs slightly better only when γ is large. In conclusion, the OSDC performs as well as SC2.

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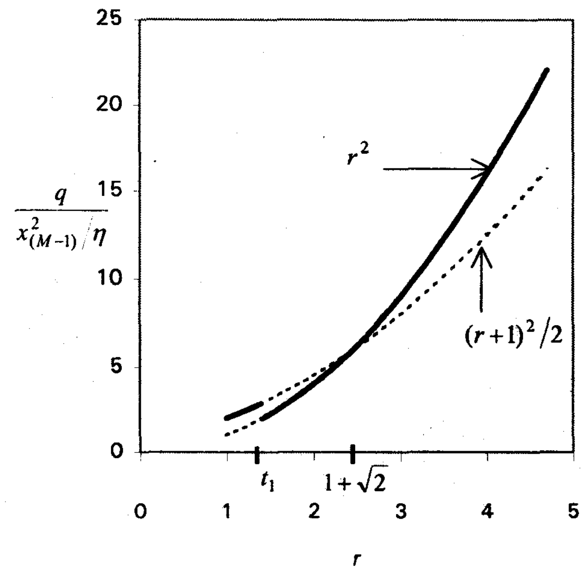

Fig. 1. $\frac{q}{x^2_{(M-1)}/\eta}$ Vs r, $t_1 < 1+\sqrt{2}$

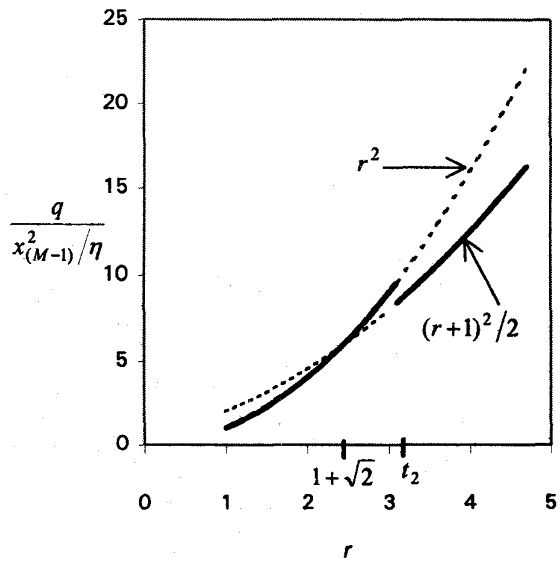

Fig. 2. $\frac{q}{x^2_{(M-1)}/\eta}$ Vs r, $t_2 > 1+\sqrt{2}$

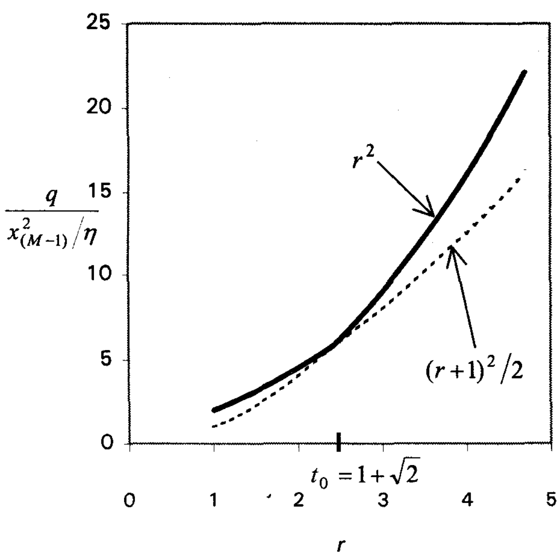

Fig. 3. $\frac{q}{x^2_{(M-1)}/\eta}$ Vs r, $t_0 = 1+\sqrt{2}$

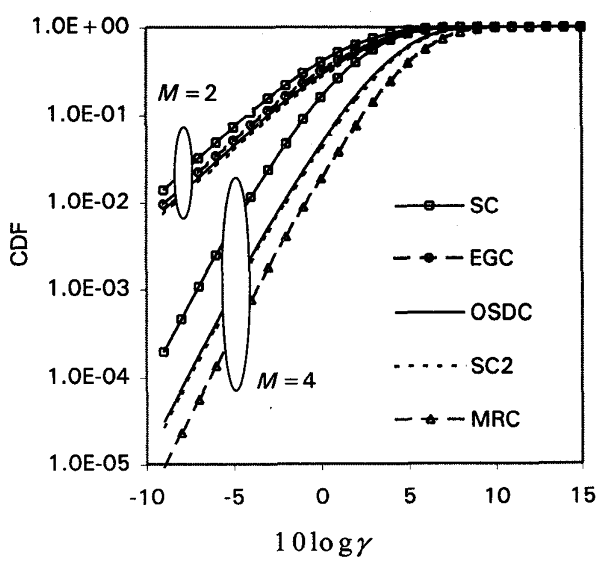

Fig. 4. SNR performance, Rayleigh fading

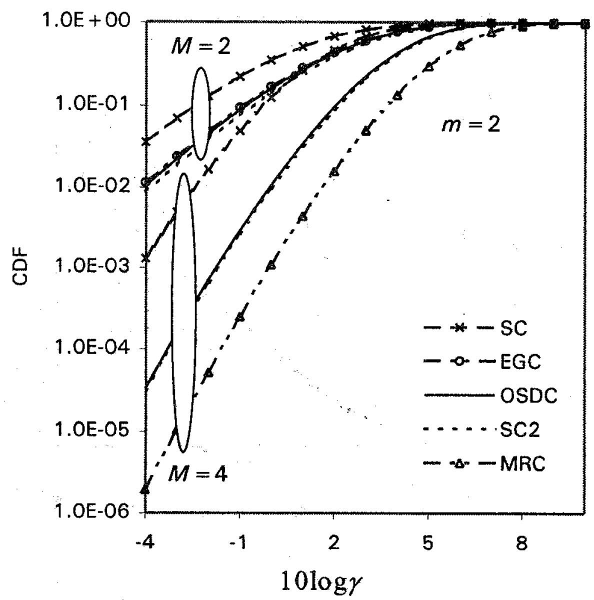

Fig. 5. SNR performance, Nakagami fading

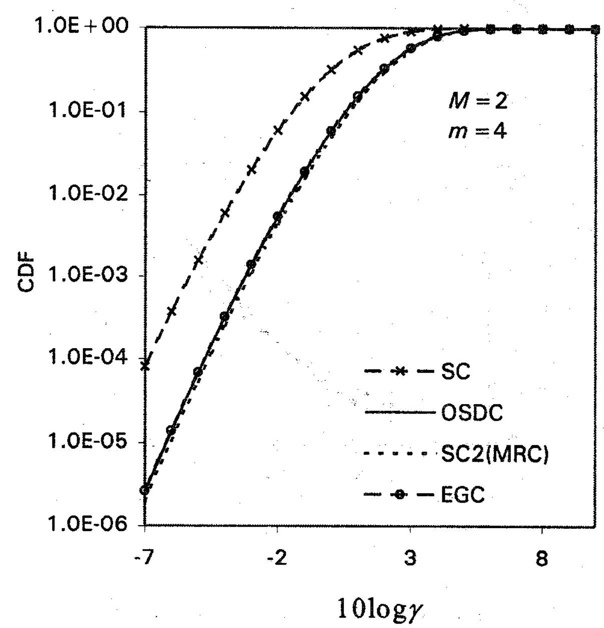

Fig. 6. SNR performance, Nakagami fading

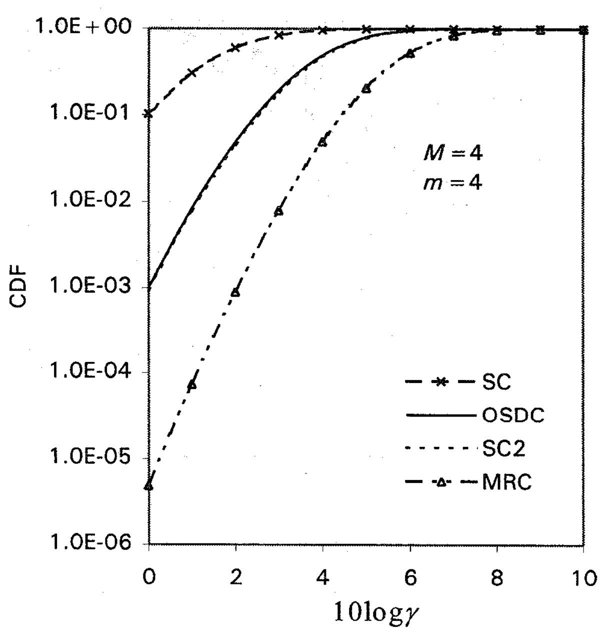

Fig. 7. SNR performance, Nakagami fading

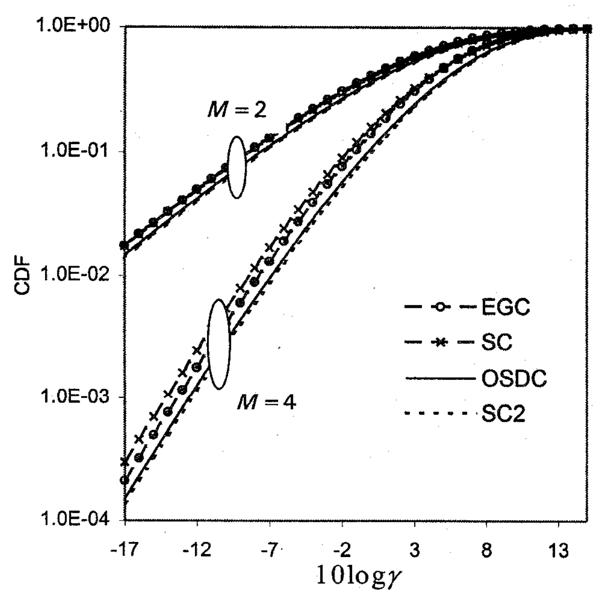

Fig. 8. SNR performance, Exponential fading