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**Abstract** – *We consider the problem of signal parameter estimation using a collection of distributed sensors. Each sensor quantizes its data to one-bit information and sends it to a fusion processor for the estimation of the parameter. Estimation of a constant signal in additive noise is considered. Since the emphasis is for the case of a moderately large number of sensors, we consider in this study two cases of estimation with 8 sensors and 20 sensors. We formulate several estimators based on one-bit sensor data and evaluate their mean squared error performances through simulation studies. Two parametric noise densities are simulated to ascertain the efficacies of various estimators. Results from this study show that robust estimation of parameter is possible by using a moderately large number of one-bit quantized sensor data.*

## 1. Introduction

Fusing data from multiple sensors has been done in radar imaging, radar target detection and tracking, process control in industries, and structural health monitoring. For the past two decades, researchers have worked on multiple sensor data fusion for target detection application (see review articles [1-2], books [3-4] ). There has been limited research in the area of parameter estimation using quantized data from multiple sensors. Distributed estimation using unquantized data from multiple sensors has been analyzed in a number of applications. Chong, Mori *et.al.* have considered target tracking with multiple sensor data [5-6]. Ming, Cheng, and Viswanathan [7] have considered the fusion of several local estimates to obtain a final estimate of a parameter. A local estimate is obtained from a small group of sensor data. In that paper, the mean squared error performances of some fusion rules were evaluated. Mahajan, *et.al.* [8] developed a generic fusion model using a Fuzzy Inference System (FIS) to fuse redundant data from three different sensors attached on a cantilever beam which was actuated by a pair of piezoceramic patches. The fusion scheme uses additional information on the sensor's performance characteristics (e.g. temperature, frequency range, life cycles, etc.) rather than simple numerical estimation and probabilistic models, though these can also be used to further enhance the model.

With recent interests in wireless sensor networks, there is a need for design of low-complexity encoding schemes for wireless sensor networks [9]. In future sensor networks, sensor dynamic range and resolution may be severely limited due to either physical limitations in sensor design, or power and bandwidth constraints in communication link back to the central site. In such cases, optimal encoding of sensor data for transmission to central site becomes important. In this paper we consider the estimation of parameter representing a physical quantity using one-bit quantized data from multiple sensors. A moderately large number of sensors measure the parameter under investigation and report to a central site whether each measured value exceeds its pre-set threshold value or not. Based on the one-bit information from these sensors, the fusion center estimates the value of the parameter. It is assumed that the fusion center has the knowledge of threshold values used by each sensor. Several nonparametric estimation schemes are proposed and their performances are evaluated using mean squared error criterion. Whereas we assume the data at sensors to be statistically independent, future work is planned to consider correlation among sensor data. In section 2 we state the estimation problem and propose several nonparametric estimators for one-bit quantized sensor data. Section 3 presents the performance evaluation of these estimators through simulation study. We conclude our paper in section 4.

## 2. Parameter Estimation Using Multiple Quantized Data

Consider a large number of sensors of similar types. For the moment, let us assume that the reliability and confidence level of each sensor is known. Assume that this collection of sensors (presumably inexpensive) monitor certain physical quantities denoted by $\theta_i$. Denote the observed quantity at sensor $i$ as $Y_i = \theta_i + X_i$, $i$=1,2,...,$L$, where $X_i$ accounts for the reliability and the noise affecting $i^{th}$ sensor measurement. A sensor observation is assumed to be statistically independent of other sensors' observations. In certain situations where sensors may be placed in close proximity, it is conceivable to approximate all $\theta_i$ to be a constant $\theta$. Based on the sensor observations the problem could be to

(i) estimate $\theta$ or

(ii) decede on a hypothesis involving $\theta$,

for example, $H_0$: $\theta = \theta_0$ Vs $H_1$: $\theta = \theta_1$. In a monitoring (health or a process) application, $\theta_0$ may correspond to normal operating conditions whereas $\theta_1$ may indicate a failure or an abnormal situation. Hypothesis testing problems involving sensor data or sensor decisions have been widely studied in the literature on decentralized detection and decision fusion [1-4].

Our approach here explains how it may be possible to obtain robust and reliable estimate of $\theta$ based on one-bit information from each sensor. As pointed out earlier, generation of one-bit information is motivated by either physical limitations in sensor design or the need to achieve extremely simple signal processing circuitry at a sensor or the need to reduce the volume of data emanating from each sensor. For simplicity let us assume all $X_i$ to be distributed with a density $p(.)$, having zero mean and variance $\sigma_i^2$. To illustrate the reliability aspect of sensor pack networks, let us consider the following situation. All sensor reliability and measurement noise are identical, leading to all $\sigma_i^2$ being identical to a constant. Sensor $i$ makes one-bit quantization based on its own observation as follows: Set a variable $U_i = 1$ if $Y_i > t_i$, otherwise set $U_i = 0$. Since $U_i$ is a binary variable, it can be conveniently described as a decision as to whether the sensor $i$ data exceeded its threshold or not. Here $t_i$ denotes the threshold of sensor $i$. Even though the observations at all sensors have identical distribution, in order to estimate $\theta$, the sensors judge their observations against non-identical thresholds. Similar non-identical thresholds have appeared in decision fusion problems [1-4] and in an estimation problem [6] (In [6] the authors employ multi-bit quantization, but the quantization with fixed thresholds is carried out after adding a dithering noise to the sensor signal). Each sensor sends their decisions to a central processing site where these decisions are fused to produce an estimate of $\theta$.

If the density function of the sensor observation, $p(.)$ is known, except for the parameter $\theta$, it would be possible to obtain the probability distributions $p_j = P(U_j) = P(Y_j > t_j \mid \theta)$ and hence the likelihood function $L(\theta;\mathbf{U}) = \prod_{j=1}^{L} p_j^{u_j}(1-p_j)^{1-u_j}$. The likelihood function can then be maximized to obtain the maximum likelihood estimate (MLE) of $\theta$ based on the sensor decisions [10]. For example, sensor noise under most conditions may be assumed to be Gaussian distributed. In such cases, for a given set of threshold values, for each of the possible $2^L$ combinations of **U**, the likelihood function can be maximized off-line and the corresponding estimates stored in a look-up table. However, in some situations, the density of sensor observations may be unknown and therefore, a nonparametric estimation procedure would be useful. Here we consider few nonparametric procedures based on one-bit decisions. Estimator 1 is given by

$$\hat{\theta}_1 = \frac{(M_1 + M_2)}{2} \tag{1}$$

where $M_1 = \underset{all_{U_j}=0}{median}(t_j)$, $M_2 = \underset{all_{U_j}=1}{median}(t_j)$

If no sensor noise is present, the actual parameter $\theta$ would fall in the interval $\left(V_1 = \max_{all_{u_j}=1}(t_j), W_1 = \min_{all_{u_j}=0}(t_j)\right)$. Because the noises at sensors are statistically independent, it is possible that $W_1$ may exceed $V_1$ and vice-versa.

Estimator 2 based on this criterion could be

$$\hat{\theta}_2 = \frac{(W_1 + V_1)}{2} \tag{2}$$

Estimator 1 uses the average of the two medians of the thresholds in the sets of all 1 decisions and all 0 decisions, respectively. Median as an estimator of parameter is usually preferred when the additive noise happens to have a heavy tail density function. However, the choice of medians of thresholds, which are indexed by one-bit decisions, may be overly conservative in terms of noise effects and therefore, may lead to inaccurate estimates under a majority of conditions. This motivates us to examine Estimator 3 shown in Table 1, which depends on the variables $V_1$, $W_1$ defined earlier and on

$V_2$=second largest$\underset{\text{all }u_j=1}{(t_j)}$, $W_2$=second smallest$\underset{\text{all }u_j=0}{(t_j)}$

$C_1$= Cardinality(all $_{u_i=1}$), $C_2$ = Cardinality(all $_{u_i=0}$).

## 3. Performance of Estimators

In order to estimate the performances of different estimators, we wrote a MATLAB simulation program. Two representative noise distributions, namely, Gaussian and double exponential (Laplace) are considered in the simulation study. Laplace noise was included to represent the situations where sensors may encounter heavy tail noise disturbances. $L$ sensors ($L = 8$ or $20$)

**Table 1 Algorithm for Esimator 3**

| Condition | | | Estimator value |
|---|---|---|---|
| $C_1 = 0$ | | | $(W_1 + W_2)/2$ |
| $C_1 = 1$ | $V_1 > W_2$ | | $W_2$ |
| | $V_1 < W_2$ | | $(W_1 + W_2)/2$ |
| $C_2 = 0$ | | | $(V_1 + V_2)/2$ |
| $C_2 = 1$ | $W_1 < V_2$ | | $V_2$ |
| | $W_1 > V_2$ | | $(V_1 + V_2)/2$ |
| $C_1 \geq 2$ $C_2 \geq 2$ | $V_1 < W_1$ | $C_1 > C_2$ | $V_1$ |
| | | $C_1 < C_2$ | $W_1$ |
| | $V_2 < W_1 < V_1 < W_2$ | | $(V_1 + W_1)/2$ |
| | $W_1 < V_2 < W_2 < V_1$ | | $(V_2 + W_2)/2$ |
| | $W_1 < V_2 < V_1 < W_2$ | | $(V_1 + V_2)/2$ |
| | $V_2 < W_1 < W_2 < V_1$ | | $(W_1 + W_2)/2$ |
| | $W_2 < V_2$ | $C_1 > C_2$ | $V_2$ |
| | | $C_1 < C_2$ | $W_2$ |

generate *iid* samples from a noise distribution with mean $\theta$ and variance $\sigma^2$. Two choices for sensor thresholds were considered, including a uniform spacing and a non-uniform spacing, where the spacing between thresholds is increased as the threshold values increase in magnitude. Without any loss of generality, it is assumed that the sensor thresholds are all different (the case of two sensor thresholds being same can be handled by assuming one sensor threshold to be extremely close to the other ). Mean squared errors (MSE) of different estimators were obtained by simulating sensor samples over 1000 iterations. This large sample size of 1000 ensures a good accuracy in the estimation of MSE values.

Figures 1-8 show the MSE performances of different estimators for the Gaussian noise case and Figures 9-12

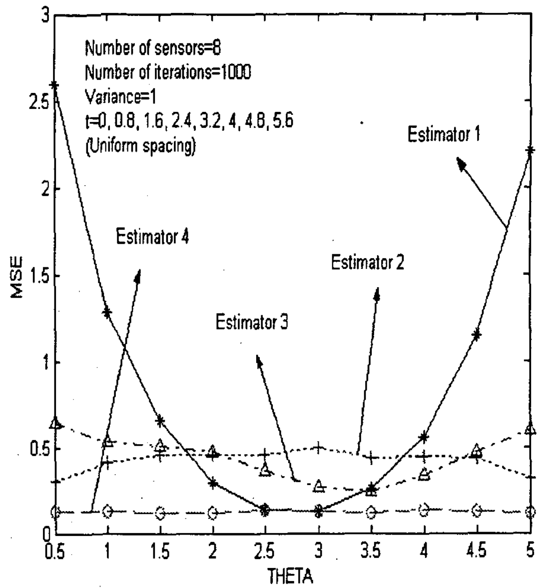


Figure 1. MSE Performance of Estimators for Gaussian Noise

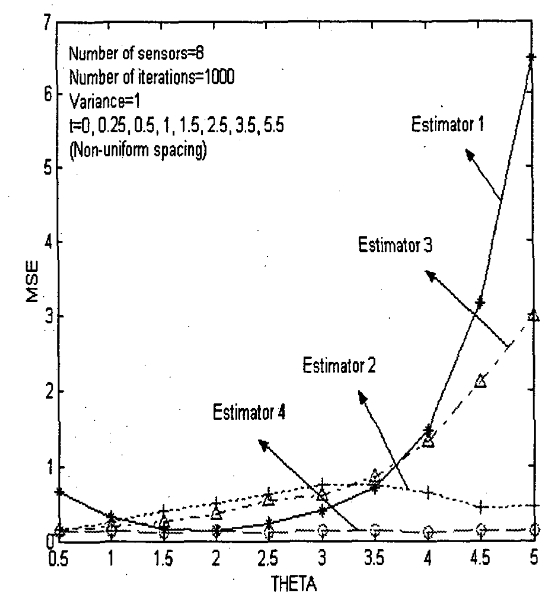


Figure 2. MSE Performance of Estimators for Gaussian Noise

show corresponding graphs for the double exponential case. Gaussian case has results for $L = 8$, $20$, two noise variances of $1$ and $8$, and for two threshold sets, namely, uniform spacing and a non-uniform assignment. For

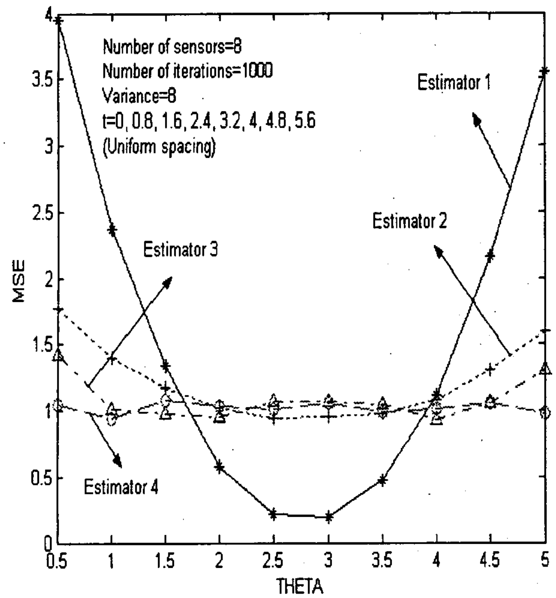


Figure 3. MSE Performance of Estimators for Gaussian Noise

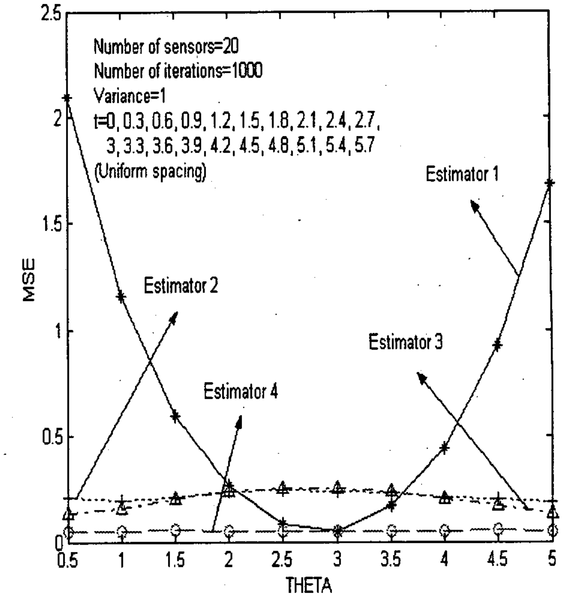


Figure 5. MSE Performance of Estimators for Gaussian Noise

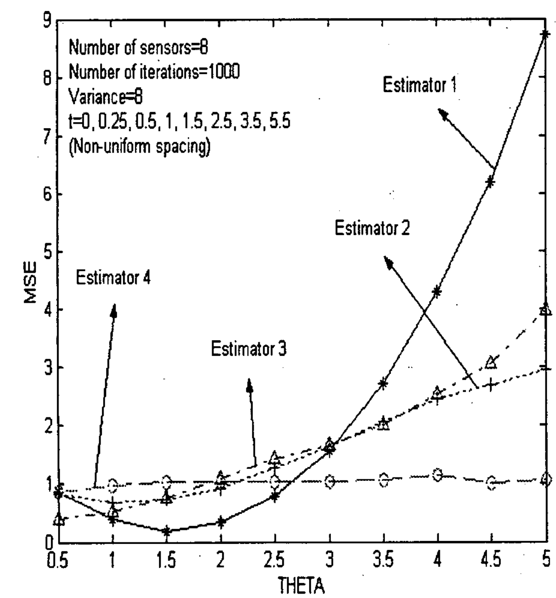


Figure 4. MSE Performance of Estimators for Gaussian Noise

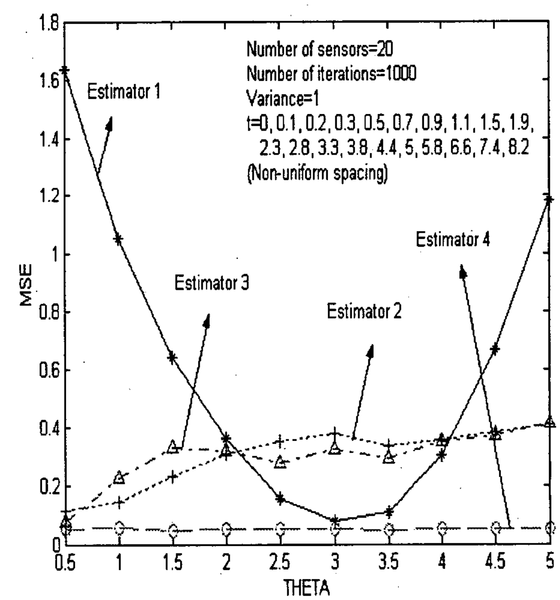


Figure 6. MSE Performance of Estimators for Gaussian Noise

Laplace, we show results for $L = 20$ only. The unknown parameter value is assumed to be in the range $(0.5,\ 5)$. In Gaussian case, by looking at Figures 1-8, we can make the following observations. For uniform threshold spacing and low noise variance, among the three estimators based on one-bit data, Estimator 3 performs the best over most of the range of $\theta$. For large noise variance, Estimator 2 performs better than Estimator 3 over medium values of $\theta$. Certainly, the sample mean (denoted Estimator 4 in

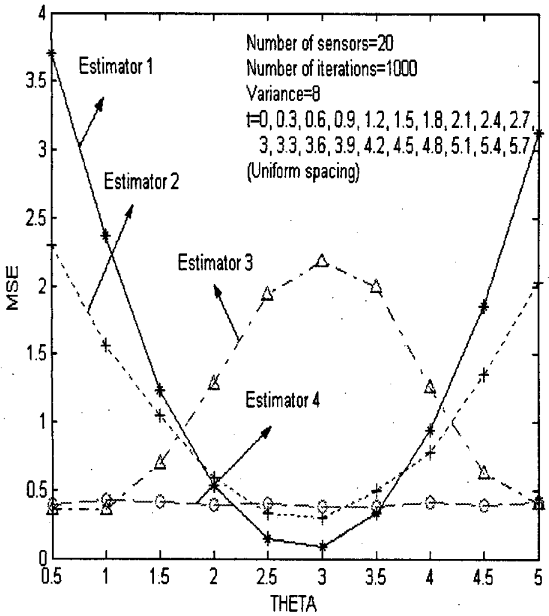


Figure 7. MSE Performance of Estimators for Gaussian Noise

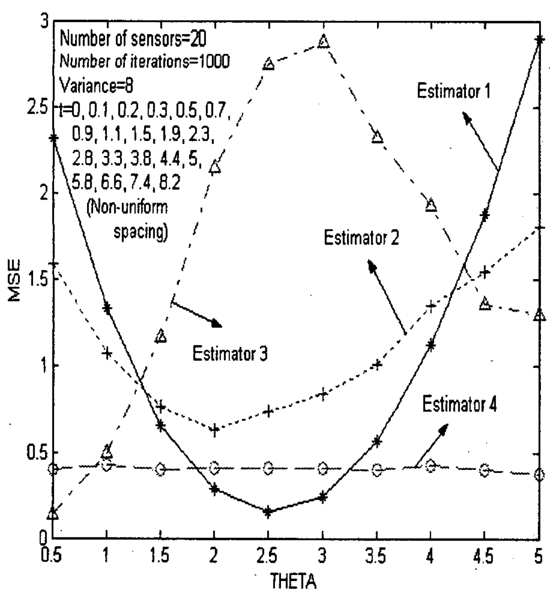


Figure 8. MSE Performance of Estimators for Gaussian Noise

these figures) based on all the samples performs the best for the Gaussian case. Recall that the sample mean is a uniformly minimum variance unbiased estimator for estimating the mean of Gaussian. Of course, this does not preclude some estimators to have lower MSE than the

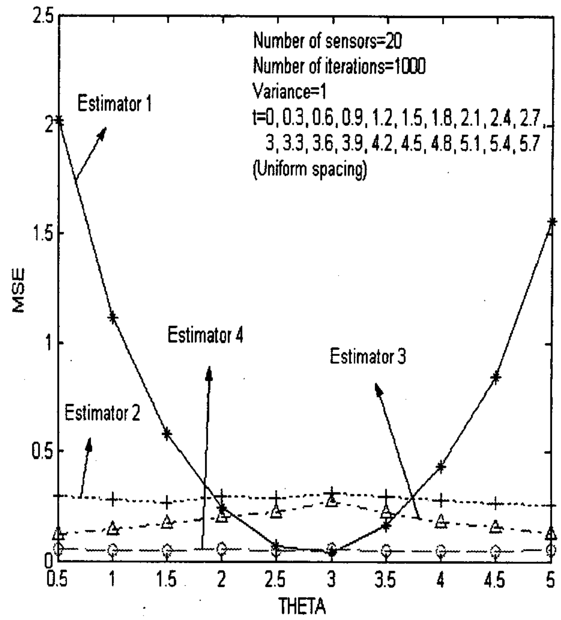


Figure 9. MSE Performance of Estimators for Laplace Noise

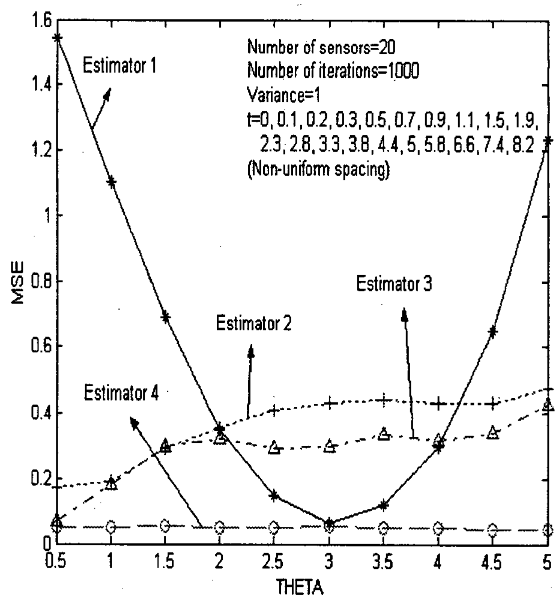


Figure 10. MSE Performance of Estimators for Laplace Noise

sample mean at some values of $\theta$ (see Estimator 1 in Figures 3-8 and Estimator 3 in Figures 7-8). The MSE of sample mean is the ratio variance to sample size, which, for unity variance, is $0.5$ for a size of $2$, $0.25$ for a

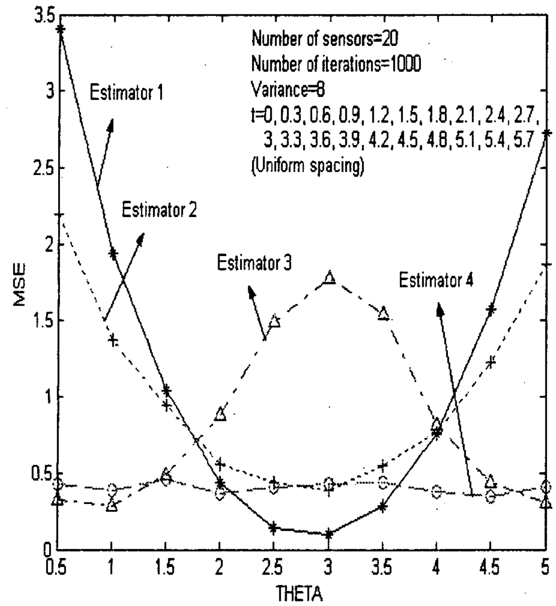


Figure 11. MSE Performance of Estimators for Laplace Noise

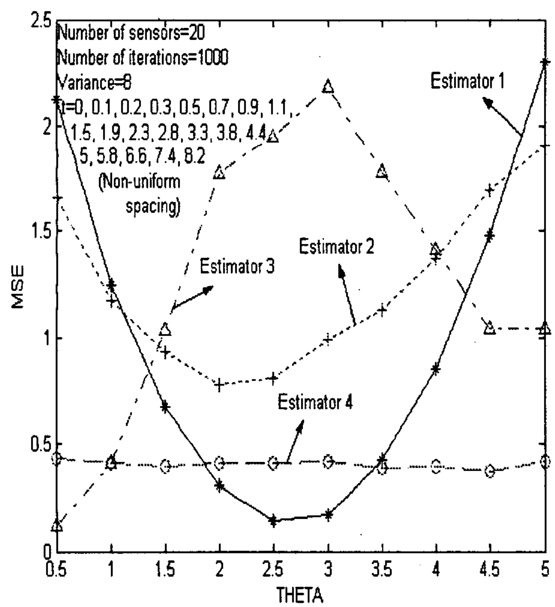


Figure 12. MSE Performance of Estimators for Laplace Noise

sample size of 4, and 0.05 for a sample size of 20. For uniform sampling, when we compare the MSE of Estimator 3 with a sample mean estimator based on 4 samples, we observe that the Estimator 3 performs uniformly better (see Figures 1-3, 5-7). If one assumes that a 12 bit quantization produces a good representation of a sample, then 4 samples require 48 bits for representation of all. Comparing this with $L = 20$, we observe that we are able to obtain better accuracy estimation with 20 bits of information through Estimator 3 and an appropriate choice of threshold. With uniform threshold spacing, all the observations we have made for the Gaussian case also hold good for the Laplace case (Figures 9,11).

When we make comparison between the two threshold choices, namely, uniform spacing and a non-uniform assignment as shown, we observe that, in general, the uniform spacing produces lower MSE for Estimators 2 and 3. If we consider very large values of $L$, say in hundreds, then the average MSE for all random choices of thresholds could be of interest. Certainly, such an asymptotic analysis for MSE would be interesting and useful for wireless sensor system with a very large number of sensors.

Extensions of the above-simplified situation are many, such as:

(i) Reliability of each sensor could be different. This is captured by differences in the variances of sensor observations. In this case, a sensor observation needs to be normalized with the standard deviation of reliability of sensor before the threshold comparison is carried out.

(ii) Sensors may be measuring slightly different physical quantities. Differences in $\theta_i$ s account for this. Knowledge of the model of the quantities observed may provide additional information about the relations among $\theta_i$.

(iii) Optimization of sensor thresholds. This is in general a difficult problem because of the dependence of performance on several parameters. Expected range of $\theta$ as well as the knowledge of the variance of sensor noise can be used to determine the range over which the thresholds are to be assigned.

(iv) Large sample (very large $L$) performance, as well as the effect of sensor data correlation on the performances of different estimators.

## 4. Conclusions

In this paper we considered a signal parameter estimation based on one-bit quantized data from $L$ multiple sensors. Attention was restricted to constant parameter estimation in additive sensor noise. Data at various sensors are assumed to be statistically independent. The mean squared error performances of various estimators reveal that Estimators 2 and 3 give robust performances over a wide range of parameter values. Future work may involve asymptotic analysis, sensor noise correlation, non-constant signal estimation, and estimation with sensors of different reliability.

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