3-2005

Distributed Detection with Channel Errors

M. Madishetty
Southern Illinois University Carbondale

V. Kanchumarthy
Southern Illinois University Carbondale

R. Viswanathan
Southern Illinois University Carbondale, viswa@engr.siu.edu

Chandrakanth H. Gowda
Tuskegee University

Published in Madishetty, M., Kanchumarthy, V., Viswanathan, R., & Gowda, C.H. (2005). Distributed detection with channel errors. Proceedings of the Thirty-Seventh Southeastern Symposium on System Theory, 2005 (SSST '05), 302-306. doi: 10.1109/SSST.2005.1460926

Distributed Detection with Channel Errors

M. Madishetty, V. Kanchumarthy, R. Viswanathan
Department of Electrical & Computer Engineering
Southern Illinois University Carbondale
Carbondale IL 62901-6603

Chandrakanth H. Gowda
Department of Electrical Engineering
Tuskegee University
Tuskegee, AL, 36088.

***Abstract*—Performances of distributed detection (DD) systems employing a set of geographically dispensed sensors have been investigated for the past two decades. In this paper we study the variations in the false alarm and detection probabilities of a DD system due to the errors caused by the links between sensors and the fusion center. Both asymptotic and finite sample performances are studied. The results bring out the exact dependence of these probabilities on the link reliability. Such a study is meaningful because of the recent research interests in wireless sensor networks.**

I. INTRODUCTION

In a wireless sensor network with distributed sensors, each sensor makes measurement with regard to a phenomenon of interest (POI) in order to make a decision on the presence or the absence of POI. The POI might be a biological spill or the sighting of a vehicle of an adversary. Each sensor processes its own information and passes the condensed information to a cluster head (or fusion center) through a wireless channel. The data arriving from various sensors are fused together appropriately in order to make a final decision on the presence or the absence of a POI.

Several papers have addressed a myriad of signal processing issues in sensor networks [1-4]. A number of problems in detection, classification, and tracking of targets are discussed in [1]. The allocation of optimum number of quantization bits at the sensors for a rate constrained communication channel and a large number of sensors was addressed in [2]. References [3-4] discuss the performances of different fusion rules that could be formulated based on varied knowledge of communication channel statistics. Another asymptotic (large number of sensors) optimization of wireless sensor networks for decentralized detection was addressed in [5]. A relatively old contribution [6-7] on optimal detection with faulty processors has relevance to distributed detection problem with channel errors. Binary symmetric channel was considered as the model for the sensor-to-fusion center link in [8], but the emphasis was on the person-by-person optimization of local sensor rule and the fusion rule.

Due to the hostile nature of a wireless channel, a sensor data might not be received reliably at the fusion center. Hence, sensor quantization rules designed for a specific false alarm probability would not produce a fixed false alarm probability at the fusion center. In this study we derive analytical expressions for the false alarm and detection performances of a distributed detection system at the fusion center and examine how much variation of false alarm probability can be anticipated. To our knowledge, a study of the variation of false alarm and detection probabilities due to changes in channel statistics has not been addressed in the literature. A study of the changes in these probabilities due to a randomized data selection strategy was addressed in [9].

II. EFFECT OF CHANNEL ERRORS ON THE RELIABILITY OF SENSOR DECISION

Consider a wireless sensor network consisting of N sensors. The network is deployed to assess the presence or the absence of a phenomenon of interest (POI) in a geographical area of interest. Sensor i gathers information pertaining to the POI and makes a decision u_i ($u_i = 1$ for deciding the presence of POI and $u_i = 0$ otherwise). Each sensor sends its decision to a fusion center through a communication link, which is not totally reliable. Let u_{i0} denote the decision of the i^{th} sensor as received at the fusion center.

Let

$P_{fi} = P(u_i = 1|\text{POI absent})$, $P_{di} = P(u_i = 1|\text{POI present})$,

$P_{fi}^c = P(u_{i0} = 1|\text{POI absent})$, $P_{di}^c = P(u_{i0} = 1|\text{POI present})$,

P_{ci} = Probability of bit error of the i^{th} link. Assuming that the link performance is statistically independent of the decision made by the sensor, the reliability parameters of the sensor decision as received by the fusion center can be describe by the following set of equations:

$$P_{fi}^c = P_{fi}(1 - P_{ci}) + (1 - P_{fi})P_{ci} \tag{1}$$

$$P_{di}^c = P_{di}(1 - P_{ci}) + (1 - P_{di})P_{ci} \tag{2}$$

Rewriting the above equations yields the following results:

$$P_{fi}^c = P_{fi}(1 - 2P_{ci}) + P_{ci} \tag{3}$$

$$P_{di}^c = P_{di}(1 - 2P_{ci}) + P_{ci} \tag{4}$$

Therefore, the reliability of the decision received u_{i0} could be different from that of the decision u_i made at the sensor.

0-7803-8808-9/05/$20.00 ©2005 IEEE

Assume that the link bit error, $P_{c\,i} < 1/2$ (if it is greater than ½, then the decision rule of receiver for the i^{th} link at the fusion center could be complimented to achieve it to be less than ½). If $P_{f\,i} < 1/2$ then $P^c_{f\,i} > P_{f\,i}$. That is, the false alarm probability of the decision received at the fusion center is higher than the false alarm probability of the decision made by the sensor. As the link becomes very unreliable, both the link bit error probability and the probability, $p^c_{f\,i}$, approach ½. Similarly, when the probability of detection at the sensor is greater than ½, the detection probability of the decision received at the fusion center is less than the detection probability at the sensor. Only when P_{di} <1/2, the link error "increases" the probability of detection, $P^c_{d\,i}$, to be above that of $P_{d\,i}$ (of course this is achieved with a concomitant increase in the false alarm probability). Given the unreliable nature of the communication link between a sensor and the fusion center, we examine in the next section its impact on the reliability of the decision made by the fusion center.

III. PERFORMANCE OF THE FUSION CENTER

Let us assume that each sensor in the network makes a decision independent of others such that each exhibits an identical performance. That is, each sensor decision is independent and identically distributed given the true state of nature with regard to the presence or the absence of POI. Also, assume that each link between a sensor and the fusion center exhibits, on an average, an identical link error performance. Given a large number of sensors in the network, it is well known that, under very general conditions, an optimum fusion rule for combining the decisions from the sensors takes the form of a counting rule. That is, the fusion center declares that POI is present when the number of sensors declaring that the POI is present exceeds a certain pre-determined threshold *t*. We first examine what choice of *t* would be reasonable for the asymptotic condition of large ($N \to \infty$) number of sensors. Because of the assumptions mentioned above, hereafter we can drop the subscript *i* that identifies a particular sensor.

A. Asymptotic Condition:

When the number of sensors is large, we can apply the Gaussian approximation to the sum of binomial probabilities. Let P_{F0} ,P_{D0} denote the false alarm probability and the detection probability, respectively, of the final decision made by the fusion center and let the count arrived at the fusion center be denoted by $Z = \sum_{i=1}^{N} u_{i0}$. Since the false alarm probability of the decision received from a sensor at the fusion center depends on the link error probability, it is reasonable to assume that its value can be bounded below an upper bound corresponding to a minimum reliability of the communication link. If we do not impose such a minimum reliability measure, then the false alarm probability could approach ½, in the worst scenario, as indicated in the previous section. Denoting $P^c_f = \alpha < \alpha_u$, we can write

$$P_{F0} = P\left(Z \ge t \,\middle|\, \text{POI absent}\right) = Q\left(\frac{t - N\alpha}{\sqrt{N\alpha(1-\alpha)}}\right) \quad (5)$$

If we let

$$t = N\alpha_u, \quad (6)$$

$$P_{F0} = Q\left(\sqrt{N}\,\frac{\alpha_u - \alpha}{\sqrt{\alpha(1-\alpha)}}\right) \quad (7)$$

tends to zero as $N \to \infty$, as long as $\alpha_u > \alpha > 0$. Moreover,

$$P_{D0} = Q\left(-\sqrt{N}\,\frac{\beta - \alpha_u}{\sqrt{\beta(1-\beta)}}\right) \quad (8)$$

tends to 1 as $N \to \infty$, provided $\beta = P^c_d > \alpha_u$. If $\beta < \alpha_u$, then P_{D0} tends towards zero, and hence it is important that $\beta > \alpha_u$ be satisfied. This can be guaranteed as long as the signal-to-noise ratio (SNR) at the sensor is above a certain minimum value and the link bit error rate is below a certain value. For example, when detecting a constant signal in AWGN, the detection probability and the false alarm probability at a sensor are related by

$$P_d = Q\left(Q^{-1}(P_f) - \sqrt{SNR}\right) \quad (9)$$

Using (3), (4) and (9) the required values of SNR and P_c to guarantee $\beta > \alpha_u$ can be arrived at.

B. Finite N:

If *N* is only finite, then the above asymptotic results are not valid. Moreover, the false alarm probability at the fusion, for a designed value of *t*, could increase to a large unacceptable value as the sensor-to-fusion link becomes unreliable. We next examine the variations in the fusion rule performance as a function of the reliability of the sensor-to-fusion link. Towards this effort we characterize the link to be the result of transmitting orthogonal binary FSK signals in a Rayleigh fading channel. For a wireless sensor network, the Rayleigh fading channel is an appropriate model to assume. For simplicity, we do not assume any error control coding for this link. Certainly, an error control code would make the link more reliable. But, to remind the reader, our aim here is to observe the impact of a less reliable link on the performance of the fusion rule. Using standard results for noncoherent detection of binary FSK in slow Rayleigh fading channels [10], we can write the following relations:

$$P_c = \frac{1}{2+\gamma_0} \quad (10)$$

$$\alpha = P_f + \frac{1-2P_f}{2+\gamma_0} \quad (11)$$

$$\beta = P_d + \frac{1-2P_d}{2+\gamma_0} \tag{12}$$

where γ_0 is the average SNR of the Rayleigh fading channel.

Above, we used an average value of P_e, averaged with respect to the fading distribution. We show in III. C that, for independent trials and a counting rule at the fusion center, this is a correct procedure for calculating the overall fusion probability of False Alarm (Detection).

In Figures 1-4 we show the variations of P_{F0} and P_{D0} as a function of γ_0 for some values of P_f, P_d, t and N. The γ_0 axis (average channel SNR) shows values from a minimum value, which guarantees that $\alpha \le \alpha_u$, for a specific α_u. As γ_0 approaches infinity, α approaches P_f and P_{F0} approaches the value that would be obtained had the links been error free. Depending on the values of P_f, t, and N, the fusion false alarm probability could be two or three decades higher than the desired value. This is in contrast to the asymptotic case where perfect detection ($P_{F0} \to 0$, $P_{D0} \to 1$) is possible. For finite N, it is essential that the link reliability is greater than a certain minimum value in order that an acceptable P_{F0} is achieved. In general, except for weak sensor signal conditions, the effect of link errors on the detection probability is less severe, because the sensor detection probability will be larger than 0.1. Notice that $P(Z \ge t|\text{POI present})$ is a monotonic increasing function of β. Hence, interestingly, when P_d<0.5, better detection probability, P_{D0}, is achieved when the link is less reliable! (see (4)).

C. False Alarm Probability of Fused Decision With Independent but Identical Fading Links

In this section we provide a proof to show that the average link error probability can be used for each link while computing the overall false alarm (and detection) error probability. Let $\gamma_1, \gamma_2, .., \gamma_N$ be the instantaneous SNR of the received signal corresponding to the individual links between a sensor and the fusion center and let P_f be the false alarm probability of the decision made by a sensor. Then, for a specified counting rule at the fusion center, the false alarm probability of the fused decision can be written as

$$P_{F0} = E\left(f\left(P_f, \gamma_1, \gamma_2, .., \gamma_N, t\right) \middle| \gamma_1, \gamma_2, .., \gamma_N\right) \tag{13}$$

where the expectation operation with respect to the distribution of the instantaneous SNRs and $f(.)$ describes the function that determines the conditional false alarm probability for a given fusion rule, conditioned on the instantaneous SNRs. For a counting rule with threshold t,

$$P_{F0} = E\left(\sum_{s=t}^{N} P\begin{pmatrix} s \text{ successes out of } N \text{ trials with } p_j \\ \text{as the success probability for the } j^{\text{th}} \text{ trial} \end{pmatrix}\right) \tag{14}$$

where p_j depends on γ_j and P_f. Rewriting (14) yields

$$\begin{aligned} P_{F0} &= E\left(\sum_{s=t}^{N} \sum_{\substack{i_1+i_2+..+i_N=s \\ i_k\in\{0,1\}}} \prod_{j=1}^{N} p_j^{i_j}\left(1-p_j\right)^{1-i_j}\right) \\ &= \sum_{s=t}^{N} \sum_{\substack{i_1+i_2+..+i_N=s \\ i_k\in\{0,1\}}} \prod_{j=1}^{N} E(p_j^{i_j}\left(1-p_j\right)^{1-i_j}) . \end{aligned} \tag{15}$$

since $\left(\gamma_i, i = 1,2,..,N\right)$ are independent. Using the following relation,

$$E\left(p_j^{i_j}\left(1-p_j\right)^{1-i_j}\right) = \begin{cases} E\left(p_j\right) \text{ if } i_j = 1 \\ E\left(1-p_j\right) \text{if } i_j = 0 \end{cases} \tag{16}$$

(15) can be simplified to yield the following

$$P_{F0} = \sum_{s=t}^{N} \sum_{\substack{i_1+i_2+..+i_N=s \\ i_k\in\{0,1\}}} \left(E\left(p_j\right)\right)^{i_j}\left(1-E\left(p_j\right)\right)^{1-i_j} \tag{17}$$

$E\left(p_j\right)$ for a slow Rayleigh fading channel and noncoherent FSK detection can be written as

$$E\left(p_j\right) = P_f + \left(1-2P_f\right)P_e \tag{18}$$

where P_e is given by (10). (17) shows that the average probability $E\left(p_j\right)$ can be used for the j^{th} link in order to arrive at P_{F0}. If the links are identical, as we have assumed here, then $E\left(p_j\right)$ is independent of j, as shown by (18). It can be seen that a similar result for the detection probability, P_{D0}, is valid.

IV. CONCLUSION

In this paper we considered a wireless sensor network, which gathers information in order to make inference on a binary hypothesis. Assuming identical sensors and a counting rule at a fusion center, the exact dependence of the fusion false alarm and detection probabilities on the reliability of sensor-to-fusion center link was examined. Whereas perfect decision is possible in the asymptotic case of an infinite set of sensors, for the case of finite number of sensors, depending on the noisyness of the link, the fusion false alarm probability could increase by several-fold.

ACKNOWLEDGMENT

This work was supported in part by a grant from Material Technology Center, SIUC, Carbondale, IL.

V. REFERENCES

[1] D.Li,.D.Wong, H. Hu, and A.M. Sayeed, "Detection, classification, and tracking of targets," *IEEE Signal Process. Magazine*, March 2002, pp. 17-30.

[2] J.F. Chamberland and V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Trans. Signal Process.*, Feb. 2003, pp. 407-416.

[3] R. Niu, B. Chen, and P.K. Varshney, "Decision fusion rules in wireless sensor networks using fading channel statistics," in *2003 Conference on Information Sciences and Systems*, The Johns Hopkins University, March 2003.

[4] B. Chen, R. Jiang, T. Kasetkasem, and P.K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, Dec. 2004, pp. 3454-3458.

[5] J.F. Chamberland and V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE Joural Select. Areas Commun.*, Aug. 2004, pp. 1007-1015.

[6] A.R. Reibman and L.W. Nolte, "Optimal fault-tolerant signal detection," *IEEE Trans. ASSP*, Jan. 1990, pp. 179-180.

[7] A.R. Reibman and L.W. Nolte, "Optimal design and performance of distributed signal detection systems with faults," *IEEE Trans. ASSP*, Oct. 1990, pp. 1771-1782.

[8] S.C.A. Thomopoulos and L. Zhang, "Distributed decision fusion with networking delays and channel errors," *Inform. Sci.*, Dec. 1992, pp. 91-118.

[9] C.K. Sestok, M.A. Said, and A.V. Oppenheim, "Randomized data selection in detection with applications to distributed signal processing," *Procee. IEEE*, Aug. 2003, pp. 1184-1198.

[10] J. Proakis, *Digital Communications*, McGraw Hill, NY, 2002.

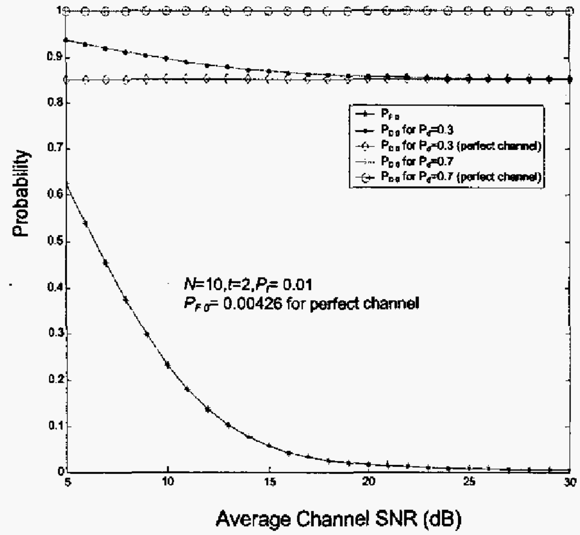

Fig 1. Probability of False Alarm/Detection Vs Average Channel SNR

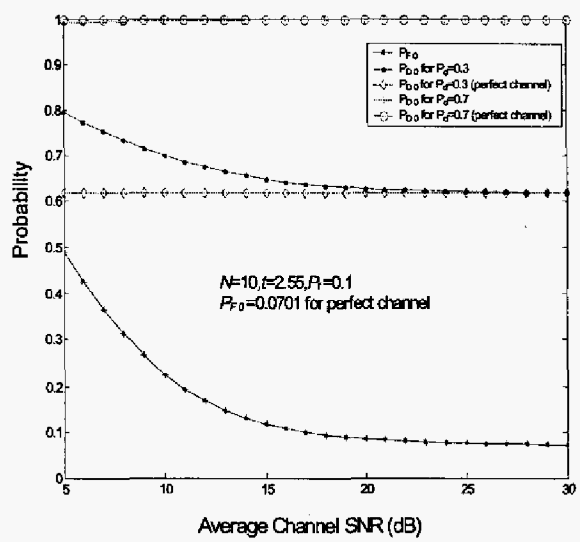

Fig 2. Probability of False Alarm/Detection Vs Average Channel SNR

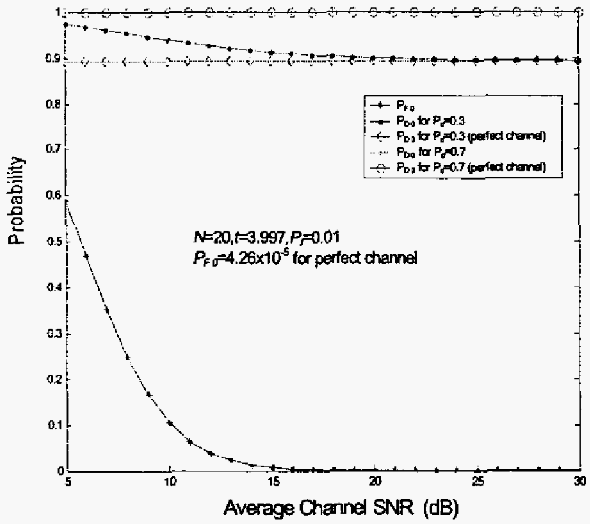

Fig 3. Probability of False Alarm/Detection Vs Average Channel SNR

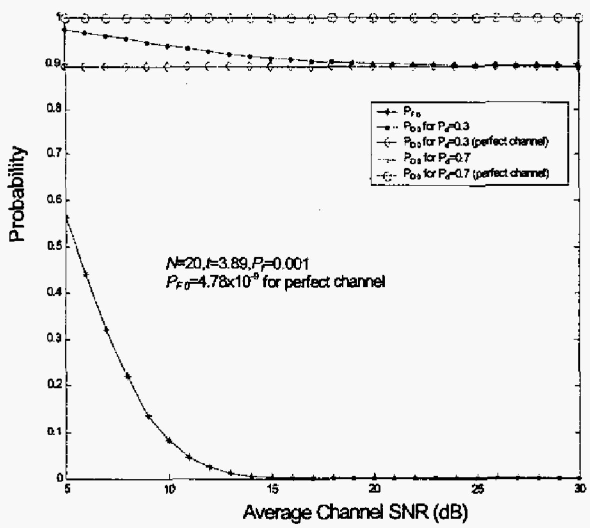

Fig 4. Probability of False Alarm/Detection Vs Average Channel SNR