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***Abstract* – A recent algorithm of Modiano selects packet sizes in a selective repeat ARQ protocol based on the acknowledgement history of the most recently transmitted packets. In this paper we modify this algorithm so that the choice of packet size is restricted to one of two pre-specified values. We provide a strategy for switching between these packet sizes and show that is optimal in the sense of maximizing the one step efficiency. The throughput efficiency of the proposed adaptive scheme is analyzed for a constant bit-error-rate channel and for two state Gilbert-Elliot channel. The results show that the throughput efficiencies of this scheme under high and moderate bit-error-rates are slightly less than that of Modiano's algorithm. However the scheme is attractive because of its simplicity.**

I. INTRODUCTION

A major concern in data communication is the control of transmission errors caused by the channel degradations. In wireless links, there is a high probability of bit errors in a packet because of multi-path signal fading and interference. In order to improve the throughput efficiency of an ARQ protocol in these channels, a number of adaptive schemes have been devised [1-5]. A scheme could adapt modulation and/or coding [1], adapt between ARQ protocols [2-4], or adapt the packet size [5]. Here we consider the adaptation of packet size as in [5]. Too large a packet size results in increased retransmissions and too small a packet would be inept due to the fixed overhead required per packet. Therefore, a packet size should be selected based on the error-rate encountered in the channel. In [5] an algorithm has been developed which chooses the packet size based on the history of transmitted packets. If the number of retransmitted packets is known, a packet size can be chosen which maximizes the one-step efficiency of the protocol. A Markov chain analysis, which evaluates the steady state performance of the algorithm for static channel conditions, was also done. The algorithm assumes that the transmitter uses a framing mechanism to accommodate variable packet sizes. Hence, no additional communication is required between the transmitter and the receiver for the purpose of coordinating the packet size.

In this paper we modify the algorithm of Modiano so that the proposed scheme adapts between one of only two pre-specified packet sizes. Our analysis follows closely the analysis of [5], but in the process we also provide a simplified solution to equation (8) in [5]. The rest of the paper is organized as follows. In section II, the algorithm of Modiano is briefly explained. In section III we discuss the proposed algorithm and derive its optimal parameters for maximizing the one-step efficiency of the protocol. In section IV, numerical and simulation results obtained in this study are discussed. We conclude this study in section V.

II. ADAPTIVE SCHEME OF MODIANO

Throughput, which is a measure of the efficiency of the protocol for delivering useful data, is dependent on the packet size, the overhead bits and the channel bit error rate. The efficiency of a selective repeat ARQ (SR-ARQ) protocol for a given channel error rate can be given by [5]

$$EFF = \left(\frac{k}{k+h}\right)\frac{1}{(1-p)^{-(k+h)}} \tag{1}$$

where k is the number of information bits, h is the number of header bits in the packet and p is the channel bit error rate. The algorithm in [5] selects the packet size based on the retransmission history of the most recently transmitted packets. When the number of packets that required retransmission is known, a packet size can be chosen to maximize the expected efficiency of the protocol. If R is the number of retransmissions out of M packet transmissions, then the average efficiency can be expressed by averaging equation (1) with respect to the conditional density of p given R, $f(p|R)$:

$$EFF_R(k) = \int_p \frac{k(1-p)^{k+h}}{(k+h)} f(p|R)\, dp \tag{2}$$

If a uniform prior distribution for p is assumed then (2) can be simplified to yield [5]

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$$EFF_R(k) = \int_p \left[\frac{k(1-p)^{k+h}}{(k+h)} \times \frac{\binom{M}{R} E^R (1-E)^{M-R}}{\int \binom{M}{R} E^R (1-E)^{M-R}} \right] dp \qquad (3)$$

where

$$E = 1-(1-p)^{\hat{k}+h} \qquad (4)$$

and $\hat{k}$ is the packet size used in the previous M transmissions. This derivation assumes that the packet errors are independent from packet to packet. This is valid in a static channel or in a fading channel where the channel conditions do not change within the duration of M packets. The optimal value of k for next transmission can be found by choosing the value of k that maximizes the efficiency (3).

A. A Simplified Expression for (3)

Let $$C = \int_p \binom{M}{R} E^R (1-E)^{M-R}\, dp \qquad (5)$$

Using (4)-(5), (3) becomes

$$EFF_R(k) = \frac{k \times \binom{M}{R}}{C \times (k+h)} \cdot \int_p (1-p)^{k+h} (1-(1-p)^{\hat{k}+h})^R ((1-p)^{\hat{k}+h})^{M-R}\, dp \qquad (6)$$

Using the beta function integral $\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ in (6) and employing routine manipulations yield the following simple expression:

$$EFF_R(k) = \frac{k}{k+h} \times \frac{\prod_{x=0}^{R} \left(\frac{1}{\hat{k}+h} + M - x \right)}{\prod_{y=0}^{R} \left(\frac{k+h+1}{\hat{k}+h} + M - y \right)} \qquad (7)$$

This expression is a simplified equivalent of (8) in [5].

III. PROPOSED ALGORITHM

The proposed adaptive algorithm uses only two packet sizes, which can be termed as the short packet size (S) and the long packet size (L). We consider two thresholds for switching between the short packet and the long packet. If the previous packet size is $\hat{k} = S$ and if $R \le t_1$, a switch from short to long is made. Otherwise, the transmission of short packets continues for the next M consecutive packets. If $\hat{k} = L$ and $R \ge t_2$, a switch from long to short is made. Otherwise, the long packet size is continued for the next M transmissions.

When $\hat{k} = S$ the efficiency of the protocol can be given as

$$EFF_R(\hat{k} = S) = \int_p \left[\frac{S(1-p)^{S+h}}{(S+h)} \cdot f(p|R > t_1) \right] dp + \int_p \left[\frac{L(1-p)^{L+h}}{(L+h)} \cdot f(p|R \le t_1) \right] dp \qquad (8)$$

Using the procedure similar to the one employed in section II to arrive at (7), we get

$$EFF_R(\hat{k} = S) = \frac{S}{S+h} \cdot \frac{\sum_{R=t_1+1}^{M} \binom{M}{R} \cdot \beta(b_s, R+1)}{\sum_{R=t_1+1}^{M} \binom{M}{R} \cdot \beta(b_{s0}, R+1)} + \frac{L}{L+h} \cdot \frac{\sum_{R=0}^{t_1} \binom{M}{R} \cdot \beta(b_{s1}, R+1)}{\sum_{R=0}^{t_1} \binom{M}{R} \cdot \beta(b_{s0}, R+1)} \qquad (9)$$

where $b_{s0} = \frac{1}{S+h} + (M-R)$,

$b_s = b_{s0} + 1$, $b_{s1} = \frac{L+h+1}{S+h} + (M-R)$.

Similarly,

$$EFF_R(\hat{k} = L) = \frac{S}{S+h} \cdot \frac{\sum_{R=t_2}^{M} \binom{M}{R} \cdot \beta(b_l, R+1)}{\sum_{R=t_2}^{M} \binom{M}{R} \cdot \beta(b_{l0}, R+1)} + \frac{L}{L+h} \cdot \frac{\sum_{R=0}^{t_2-1} \binom{M}{R} \cdot \beta(b_{l1}, R+1)}{\sum_{R=0}^{t_2-1} \binom{M}{R} \cdot \beta(b_{l0}, R+1)} \qquad (10)$$

where

$b_l = \frac{S+h+1}{L+h} + (M-R)$, $b_{l0} = \frac{1}{L+h} + (M-R)$,

$b_{l1} = b_{l0} + 1$

It can be shown theoretically that $t_1 = 0$ ($t_2 = 1$) maximizes the expression (9) ((10)). To prove this we show that each of the two factors in (9) is maximized at $t_1 = 0$ and that each of the two factors in (10) is maximized at $t_2 = 1$ (complete proof is given in [6]). Therefore, the optimum algorithm switches the packet size from short to long, if and only if, while in short packet mode, the number of re-transmissions in *M* successive transmissions is zero. Similarly, the optimum algorithm switches the packet size from long to short, if and only if, while in long packet mode, the number of re-transmissions in *M* successive transmissions is one or more. The *M* value used while in *S* mode and the value used while in *L* mode need not be identical. In fact, in our simulation, the transmission history is kept at 10, 000 bits, which translates to an *M* value of $M_S = \lceil 10{,}000/S \rceil$ in the *S* mode and an *M* value of $M_L = \lceil 10{,}000/L \rceil$ in the *L* mode.

III.A Steady-state efficiency in fixed error rate channel

To calculate the steady-state performance of the proposed algorithm, we consider a two-state Markov chain model where the state of the system is described by the packet size being used. Transitions between the two packet sizes take place according to the transition probabilities, p_{SL} for transition from short to long and p_{LS} for transition from long to short. These transition probabilities can be given as follows:

$$p_{SL} = \sum_{R=0}^{t_1} \binom{M}{R} E_S^R (1-E_S)^{M-R} \tag{11}$$

where $E_S = 1-(1-p)^{S+h} = 1-q^{S+h}$

$$p_{LS} = \sum_{R=t_2}^{M} \binom{M}{R} E_L^R (1-E_L)^{M-R} \tag{12}$$

where $E_L = 1-(1-p)^{L+h} = 1-q^{L+h}$

By denoting the steady-state probability of short (long) packet state as $P_S\,(P_L)$, we have $P_L = 1 - P_S$ and

$$P_S = \frac{p_{LS}}{p_{SL} + p_{LS}} \tag{13}$$

Therefore, the efficiency of the proposed protocol is

$$EFF(p) = \frac{S}{S+h} \cdot P_S \cdot q^{S+h} + \frac{L}{L+h} \cdot P_L \cdot q^{L+h} \tag{14}$$

Using (11) through (15) along with $t_1 = 0\,(M = M_s)$, $t_2 = 1\,(M = M_L)$ yield

$$EFF(p) = \frac{1-q^{(L+h)M_L}}{1-q^{(L+h)M_L} + q^{(S+h)M_S}} \cdot \frac{S}{S+h} \cdot q^{(S+h)} + \frac{L}{L+h} \cdot q^{(L+h)} \cdot \frac{q^{(S+h)M_S}}{1-q^{(L+h)M_L} + q^{(S+h)M_S}} \tag{15}$$

III.B PERFORMANCE OF THE PROPOSED ALGORITHM UNDER CHANGING BIT-ERROR-RATE CONDITIONS

We assume a Gilbert-Elliot model for the changing bit-error-rate (BER) conditions of a time varying channel. The channel may be in good state (low BER) or the bad state (high BER) and the optimal packet size may be either short or long. In this model we assume that the transitions between the two states occur according to the exponential random process of rate μ_G for transitions from the good state to the bad state and μ_B for transitions from the bad state to the good state. As in [5], we assume the rate of transition between the two states to be same in both directions $(\mu = \mu_G = \mu_B)$. Therefore, the amount of time the channel spends in the good state (and the bad state) is exponentially distributed with an average value of $\rho = 1/\mu$. For a channel rate R_C, the average number of bits between channel transitions is simply $b = \rho R_C$. We resort to simulation in order to study the throughput performance under the above channel model. It is to be noted that the switch between short and the long packets could happen only after *M* successive transmissions whereas the channel state could switch within an *M* transmission cycle. The throughput efficiency is calculated as

$$EFF = P_{gs}\frac{S}{S+h}\left(1-p_g\right)^{S+h} + P_{gl}\frac{L}{L+h}\left(1-p_g\right)^{L+h} + P_{bs}\frac{S}{S+h}\left(1-p_g\right)^{S+h} + P_{bl}\frac{L}{L+h}\left(1-p_b\right)^{L+h} \tag{16}$$

where P_{xy} is the steady-state probability with first subscript indicating good (g) or bad (b) channel state and the second subscript indicating short (s) or long (l) packet size and p_g (p_b) is the good (bad) channel BER.

IV. NUMERICAL AND SIMULATION RESULTS

For constant BER channel, the steady-state throughput efficiency results for the proposed algorithm are computed using (15). For studying the adaptive algorithm of Modiano, simulation with 200 packet sizes, identical to the one in [5] is employed. A simulation study was employed to compute the efficiency for the two-state channel. Figures 1-5 show the efficiency performances vs. the channel BER for a transmission history of 10,000 bits and h = 40 bits. The efficiencies obtained with the optimal algorithm (with optimal packet size of a perfect retransmission algorithm [5]) and fixed packet size schemes with sizes 200 and 2000, respectively, are also shown in these figures. In Fig. 1-3 where L =2000 and S is varied, it can be observed that the efficiency at high and medium BERs are affected. As the short packet size increases, the efficiency of the proposed algorithm gradually decreases at high BERs, but at medium BERs the efficiency increases. At low BERs, the efficiency is quite close to that of the optimum algorithm. In Fig. 4-5 we show results for L=1500, S=150 and S=200, respectively. Though not shown here, we examined values of L down to 750 and values of S up to 250. The steady-state efficiency of the proposed algorithm is somewhat close to that of the adaptive algorithm of Modiano when S=150 and L=1500 or 2000. The efficiency is lower than that of Modiano's for BERs in the vicinity of 10^{-4} and in the vicinity of 10^{-2}. The reduction at high BER is due to the fact that Mdiano's scheme considers short packet sizes starting from 10 bits whereas the proposed procedure has only a fixed short packet size. The performance at very low BERs is closer to that of the optimal and is slightly better than that of Modiano's scheme.

For the Gilbert-Elliott channel, Fig. 6 shows the average efficiency vs. the average amount of time between channel state transitions. To obtain this graph we used a fixed transmission history of M = 50 (for both short and long packets modes), a channel transmission rate (R_C) of 100, 000 bps, the good channel BER of 10^{-5}, the bad channel BER of 10^{-3}, and ρ values from 0 to 5. The average efficiency of the proposed algorithm in good state is 0.94 and in the bad state is 0.6 (giving an average efficiency of 0.77). Corresponding values for Modiano's algorithm are 0.96 and 0.65, respectively. The performance of the proposed algorithm is quite close to the performance of the Modiano algorithm in the Gilbert-Elliott channel.

V. CONCLUSIONS

In this paper we proposed a modification to the adaptive algorithm of Modiano, for adapting packet sizes in an ARQ protocol depending on the channel error rate. The proposed adaptive algorithm adapts one of two packet sizes, achieving nearly the efficiency of the Modiano algorithm. A slight reduction in efficiency, as compared to the Modiano algorithm, does occur at high and moderate channel bit error rates, but the simplicity counter balances the deterioration in performance.

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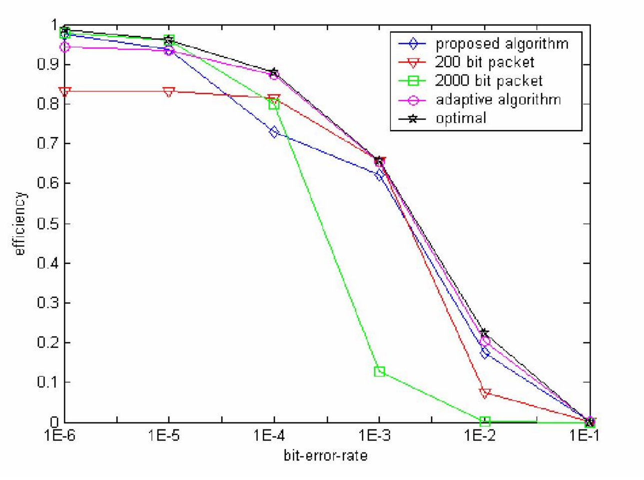

Fig. 1. Steady State performance comparison of optimal, fixed, adaptive (Modiano), and proposed algorithms with M=10,000 bits, S = 100, L = 2000

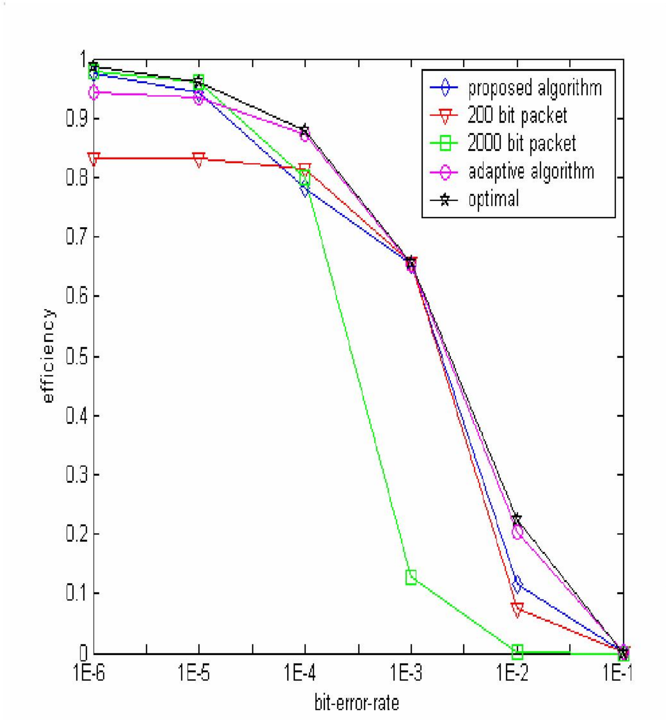

Fig..2. Steady State performance comparison of optimal, fixed, adaptive (Modiano), and proposed algorithms with M=10,000 bits, S = 150, L =2000

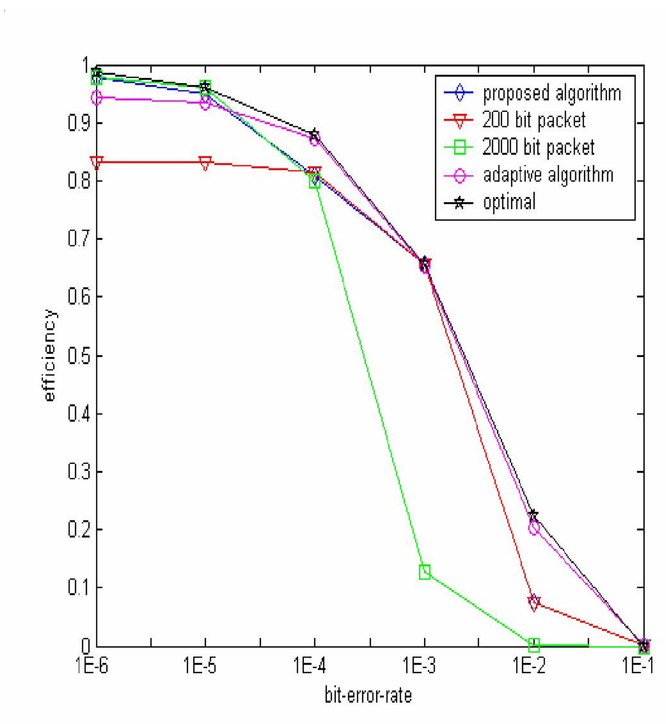

Fig.3. Steady State performance comparison of optimal, fixed, adaptive (Modiano), and proposed algorithms with M=10,000 bits, S = 200, L =2000

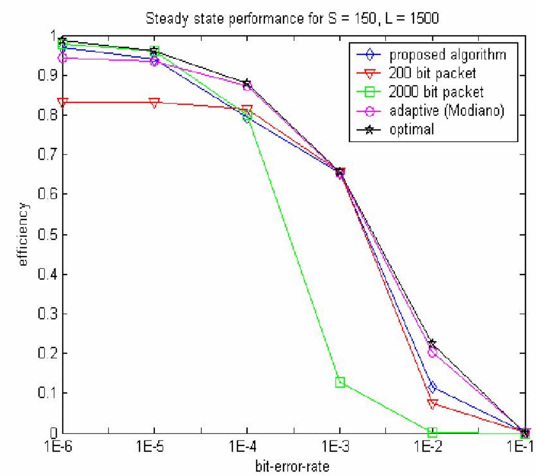

Fig. 4. Steady State performance comparison of optimal, fixed, adaptive (Modiano), and proposed algorithms with M=10,000 bits, S = 150, L =1500

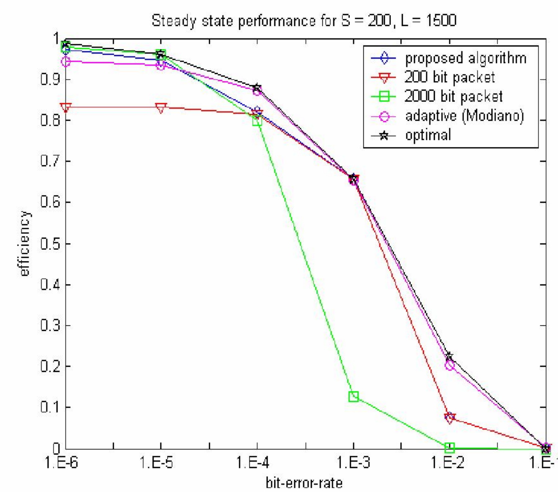

Fig. 5. Steady State performance comparison of optimal, fixed, adaptive (Modiano), and proposed algorithms with M=10,000 bits, S = 200, L =1500

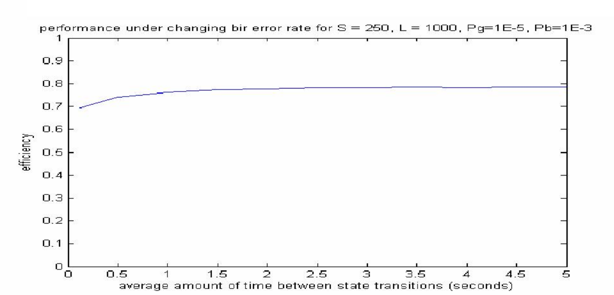

Fig. 6. The efficiency of the proposed protocol vs. the average amount of time between channel state transitions for S = 250, L =1000, P_g= 10^{-5}, P_b = 10^{-3}.