4-2008

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April 18, 2008

Abstract

We consider a principal-agent relationship where a buyer contracts with a risk-averse supplier for the production of certain good. At the time of contracting, both parties share incomplete information on cost of production. However, after contracting and before production, the supplier privately discovers its cost of production. We study the optimal contract between the two parties in the presence of cost uncertainty when the supplier is privately informed of its risk preference at the time of contracting.

Keywords: Risk Sharing; Adverse Selection; Risk Preference

JEL classification: D81; D82; D86

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1 Introduction

It is well understood that principal-agent relationships often involve simultaneous consideration of risk sharing, effort motivation (moral hazard), and information revelation (adverse selection). For example, Zeckhauser (1970), Spence and Zeckhauser (1971), Holmstrom (1979), Shavell (1979); Grossman and Hart (1983) among others consider optimal risk sharing under moral hazard; Salanie (1990) studies optimal risk sharing under adverse selection; Laffont and Rochet (1998), Theilen (2003), and Dai (2007) study optimal risk sharing under both adverse selection and moral hazard. In all these studies, the equilibrium contracts closely depend on both the principal's and the agent's degree of risk aversion.

In reality, principals often do not have precise information on agents' risk preference. For example, the owner of a firm typically does not know either a manager's or a worker's degree of risk aversion. Similarly, a regulator seldom has perfect information on how risk-averse a firm is. In those cases, the agent conceivably can manipulate the principal's perception of his risk preference. The purpose of this study is to extend the adverse selection model to settings where the agent is privately informed about his degree of risk aversion.

We consider a principal-agent relationship where a buyer contracts with a risk-averse supplier for the production of certain good. At the time of contracting, both the buyer and the supplier share the same incomplete information about cost of production. However, after signing the contract and before the production, the supplier can privately discover the cost of production. We study the optimal contract between the two parties in the presence of cost uncertainty when the supplier is privately informed of its risk preference at the time of contracting.

When both parties share the same information on cost of production at the time of contracting, the efficient supply schedule can be achieved by a fixed-price contract which makes the supplier the residual claimant of the production, if the supplier is risk-neutral.

However, when the supplier is risk-averse, the optimal supply schedule must balance risk sharing and the incentive for the supplier to truthfully reveal its private information on cost of production. Consequently, a supplier of small degree of risk aversion supplies less than the efficient level of output except for the lowest and the highest realizations of the cost. The production distortion increases as the supplier becomes more risk-averse. When the supplier becomes sufficiently risk-averse, bunching arises in the supply schedule–the supplier is required to produce a constant level of output for high realizations of cost. When the supplier becomes infinitely risk-averse, the supply schedule converges to one where the supplier is privately informed about its cost of production at the time of contracting.

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its marginal cost of production but also by its degree of risk aversion. When the buyer is risk-neutral, the optimal contract balances risk sharing and the incentive for the supplier to truthfully reveal both the realization of cost and its degree of risk aversion. Consequently, the supply schedule for the more risk-averse supplier is further distorted towards a cost-plus contract in order to limit a less risk-averse supplier's incentive to mimick a more risk-averse one. However, the supply schedule for the less risk-averse supplier is the same as when the supplier's degree of risk aversion is common information.

When the buyer is also risk-averse, the optimal contract must simultaneously balance the buyer's profits with different types of suppliers, risk sharing between the two parties and the supplier's incentives for truthful information revelation. The downward distortion in production decreases for both types of suppliers as a risk-averse buyer allocates more risk towards the suppliers. Moreover, a risk-averse buyer also reduces the production distortion for a more risk-averse supplier to smooth its profits with different types of suppliers. When the buyer is sufficiently risk-averse, both types of suppliers produce above the efficient level of output.

de Mezza and Webb (2000) and Jullien, Salanie and Salanie (2007) study the optimal insurance contracts under moral hazard when insurance customers are privately informed of their risk preference. Landsberger and Meilijson (1994) consider a principal-agent setting with one risk neutral monopolistic insurer and one risk-averse agent who is privately informed about his degree of risk aversion. Smart (2000) studies a screening game in a competitive insurance market in which insurance customers differ with respect to both accident probability and degree of risk aversion. In contrast to the above studies, we consider a principal-agent relationship where suppliers differ with respect to both cost of production and degree of risk aversion.

The rest of the paper is organized as follows. Section 2 describes the central elements of the model. As a benchmark, Section 3 presents the optimal contract when the supplier's degree of risk aversion is common information. Section 4 examines the optimal contract when the supplier is privately informed of its degree of risk aversion. Section 5 summarizes our main findings and concludes the paper with future research directions. The proofs of all formal conclusions are in the Appendix.

2 The model

A buyer contracts with a supplier to obtain some quantity, $q \geq 0$, of a good. The buyer's valuation of q is $V(q)$, and $V(\cdot)$ is a smooth, increasing, and concave function. The buyer's net surplus is $W = V(q) - T$, where T is the buyer's payment to the supplier. The supplier's total cost of producing q is $C = cq$, where c is the supplier's marginal/average cost of production. Hence, the supplier's profit is $\pi = T - cq$.

The utility function of the supplier, $U(\cdot)$, belongs to some smooth one-dimensional family of utility functions that is ranked according to the Arrow-Prat measure of risk aversion: for any wealth level π, $-U'(\rho, \pi)/U''(\rho, \pi)$ is increasing with ρ. Thus, ρ measures the the supplier's degree of risk aversion. The supplier's degree of risk aversion is unknown to the buyer. However, it is common knowledge that the supplier's degree of risk aversion, ρ, belongs to the two point support $\{\rho_l, \rho_h\}$ with $\rho_h > \rho_l$ and $\Pr(\rho = \rho_l) = \alpha$ (therefore $\Pr(\rho = \rho_h) = 1 - \alpha$).

The supplier's marginal cost of production, c, is uncertain at the time of contracting. However, both the buyer and the supplier know that the realization of c follows a uniform distribution between $\underline{c}$ and $\overline{c}$. After contracting with the buyer and before the production takes place, the supplier privately discovers the realization of c.

The timing and contractual relation between the buyer and the supplier are as follows: (1) the supplier privately learns its degree of risk aversion ρ; (2) the buyer offers the supplier a set of contract menus $M_n = \{T_n(c), q_n(c)\}$ conditional on the supplier's degree of risk aversion n, where $n = l$, h, and its eventual marginal cost c; (3) the supplier selects its preferred menu M_n given its private information on ρ; (4) the supplier discovers c, and selects a desired option $(T_n(c), q_n(c))$ from the selected menu M_n; (5) exchange takes place according to the contract terms.

3 Common Information on Risk Preference

As a benchmark, in this section we discuss the optimal contract when the supplier's degree of risk aversion is common information. When the buyer is risk-neutral, its optimization problem is choosing $\{T(c), q(c)\}$ to maximize

$$\int_{\underline{c}}^{\overline{c}} [V(q(c)) - T(c)] f(c) dc, \tag{1}$$

where $f(c)$ is the probability density function of c. Denote $\Delta c = \overline{c} - \underline{c}$, then $f(c) = 1/\Delta c$.

A contract is feasible (or implementable) provided if it is *incentive compatible* and *individually rational.* Incentive compatibility requires that the contract induces the supplier to truthfully report its realization of marginal cost, i.e.,

$$\pi(c_i \mid c_i) \geqslant \pi(c_i \mid c_j) \text{ for } c_i \neq c_j, \tag{2}$$

where $\pi(c_i \mid c_i)$ and $\pi(c_i \mid c_j)$ denote the supplier's respective profits from choosing options $(T(c_i), q(c_i))$ and $(T(c_j), q(c_j))$ when the realization of its marginal cost in fact is c_i. Individual rationality requires that the supplier's expected utility from entering the contract must be nonnegative, i.e.,

$$E[U] = \int_{\underline{c}}^{\overline{c}} U(T(c) - cq(c))f(c)dc \geqslant 0. \tag{3}$$

When the supplier is risk-neutral, it is well known that the optimal contract $\{T(c), q(c)\}$ takes the following form:

$$T(c) = V(q(c)) - \overline{T}, \text{ where} \tag{4}$$

$$\overline{T} = \arg\max_{q(c)} \int_{\underline{c}}^{\overline{c}} [V(q(c)) - cq(c)]f(c)dc. \tag{5}$$

The optimal contract in essence makes the supplier the residual claimant of its production. Under the contract, the supplier produces the efficient amount of goods (i.e., $V'(q(c)) = c$,) based on the realization of its marginal cost. Moreover, the supplier receives zero rent in expectation as $E[U] = \underset{q(c)}{Max} \int_{\underline{c}}^{\overline{c}}[(V(q(c)) - \overline{T}) - cq(c)]f(c)dc = 0$ under the contract.

Under the above optimal contract, the supplier bears the entire risk of cost uncertainty as the residual claimant of the production. When the supplier is risk-averse, however, the optimal contract must balance risk sharing and the incentive for the supplier to truthfully reveal its marginal cost of production.

Lemma 1 describes the general properties of the optimal contract when the supplier's degree of risk aversion is common information.

Lemma 1 *The optimal contract must be of the following form, for some c^* in $[\underline{c}, \overline{c}]$ and $q^* > 0$:*

(a) $E[U] = \frac{1}{\Delta c}\int_{\underline{c}}^{\overline{c}} U(\pi(c))dc = 0$;

(b) $q(c)$ is given by

$$\frac{1}{\Delta c}[V'(q(c)) - c] = \frac{c - \underline{c}}{\Delta c} - \frac{\int_{\underline{c}}^{c} U'(\pi(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)} \tag{6}$$

on $[\underline{c}, c^*)$ and $q(c) = q^*$ on $[c^*, \overline{c}]$.

Proof. See appendix. ■

At the time of contracting both the supplier and the buyer face the same uncertainty regarding the cost of production. Consequently, although the supplier can capture information rent from its private information on the realization of c after signing the contract, the buyer can fully extract the expected information rent at the time of contracting by reducing the level of transfer payments $T(c)$ for all realization of c. (Note that it is the difference in $T(c)$ that provides the incentive for the supplier to truthfully reveal its marginal cost.) Consequently, the supplier receives zero expected utility under the optimal contract.

Given that the buyer can fully extract the supplier's ex post information rent at the time of contracting, the buyer does not face the traditional trade-off between rent extraction and production efficiency as in Baron and Myerson (1982). As we have shown earlier, the supplier's ex post information rent would be costless to the buyer and the efficient outcome would be achieved if the supplier were risk-neutral. However, when the supplier is risk-averse, the optimal supply schedule must balance risk sharing and the incentive for truthful information revelation. Equation (6) demonstrates the intuition.

When the supplier's realization of marginal cost is $\widehat{c}$, raising $q(\widehat{c})$ by δq will in expectation increase the supplier's production efficiency by $[V'(q(\widehat{c})) - \widehat{c}]\delta q/\Delta c$ where $1/\Delta c$ is the probability that $c = \widehat{c}$. However, the increase in $q(\widehat{c})$ will also raise the supplier's ex post information rent by δq when $c < \widehat{c}$. Consequently, in expectation the increase in $q(\widehat{c})$ raises the supplier's ex post information rent by $(c - \underline{c})\delta q/\Delta c$, where $(c - \underline{c})/\Delta c$ is the probability that $c < \widehat{c}$. When the supplier is risk-averse, the buyer can only reduce $T(c)$ for all realization of c by $\delta q \int_{\underline{c}}^{c} U'(\pi(x))dx / \int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx$ in order to keep the supplier's expected utility unchanged. Notice that $\delta q \int_{\underline{c}}^{c} U'(\pi(x))dx$ is the increase in the supplier's expected utility as a result of the increased ex post information rent, and $\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx$ is the supplier's additional expected utility as a result of one unit of increase in $T(c)$ for all realization of c. Therefore, $\delta q \int_{\underline{c}}^{c} U'(\pi(x))dx / \int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx$ is the certainty equivalent of the supplier's increased additional information rent. At the optimum, the supplier's marginal benefit of raising $q(\widehat{c})$ must equal its marginal cost of doing so, which yields equation (6).

When the supplier is risk-neutral, i.e., $u'' = 0$, $\int_{\underline{c}}^{c} U'(\pi(x))dx / \int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx = (c - \underline{c})/\Delta c$, which means the the certainty equivalent of the supplier's increased additional information rent is the same for both the buyer and the seller. Consequently, the buyer can fully extract the supplier's expected ex post information rent by reducing the transfer payments under all realization of c by exactly $(c - \underline{c})/\Delta c$. In that case, the right-hand side of equation (6) becomes zero, and $V'(q(c)) = c$. The optimal contract would be a fixed price contract, and the supplier would always supply the efficient level of goods.

Denote the term on the left-hand side of equation (6) as $D(c)$. When the optimal supply schedule is strictly decreasing in c in $[\underline{c}, \overline{c}]$, i.e., $c^* = \overline{c}$ (no bunching), equation (6) suggests that $D(c) = 0$ and $V'(q(c)) = c$ at $\underline{c}$ and $\overline{c}$. Therefore, the supplier delivers the efficient amount of goods at $\underline{c}$ and $\overline{c}$. Furthermore, $D''(c) = U''(\pi(c))q(c) / \int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx < 0$, which implies that $D(c)$ is concave on $(\underline{c}, \overline{c})$. Since $D(\overline{c}) = D(\underline{c}) = 0$, the concavity suggests that $D(c) > 0$ on $(\underline{c}, \overline{c})$. Consequently, the supplier delivers less than the efficient amount of goods on $(\underline{c}, \overline{c})$.

To fully demonstrate the effect of risk aversion on the optimal contract, we assume that the supplier has a constant absolute risk aversion (CARA) utility function, $u(x) = 1 - e^{-\rho x}$ with $\rho > 0$ and the buyer have a quadratic value function, $V(q) = aq - bq^2$, with $a > \overline{c} > 0$ and $b > 0$.[1]

Lemma 2 demonstrates the effect of risk aversion on the optimal supply schedule in the no-bunching region.

Lemma 2 *When there is no bunching, the supply schedule, $q(c)$, decreases with the supplier's degree of risk aversion, ρ, on $(\underline{c}, \overline{c})$.*

Proof. See Appendix. ■

Lemma 2 suggests that, when the supplier becomes more risk-averse, the buyer optimally reduces the supplier's exposure to cost uncertainty by distorting the supply schedule on $(\underline{c}, \overline{c})$ downwards. As ρ becomes increasingly large, the monotonicity condition that requires $q'(c) \leqslant 0$ — a necessary condition for the supplier to truthfully reveal its realization of marginal cost — may become constraining. Consequently, the optimal supply schedule may involve bunching as the buyer becomes sufficiently risk-averse.

Lemma 3 fully characterizes the effect of the supplier's degree of risk aversion on the optimal contract.

Lemma 3 *There exists ρ^* with $\rho^* > 0$, such that*

(a) For $\rho < \rho^*$, there is no bunching and $q(c)$ is given by

$$\frac{1}{\Delta c}[V'(q(c)) - c] = \frac{c - \underline{c}}{\Delta c} - \frac{\int_{\underline{c}}^{c} e^{-\rho\pi} dx}{\int_{\underline{c}}^{\overline{c}} e^{-\rho\pi} dx} \tag{7}$$

[1] Technically, our analysis in this case is similar to Salanie (1990) where a risk-neutral producer contracts with a risk-averse retailer.

for all c on $[\underline{c}, \overline{c}]$;

(b) For $\rho > \rho^*$, $q(c)$ is given by equation (7) on some interval $[\underline{c}, c^*)$ and is constant on $[c^*, \overline{c}]$.

Proof. See Appendix. ■

Lemma 3 suggests that bunching arises in the optimal contract when the supplier becomes sufficiently risk-averse. In the optimal contract, the supply schedule is strictly decreasing in the realization of marginal cost for small value of marginal cost but is constant for all realizations of marginal cost above c^* whose value depends on ρ.

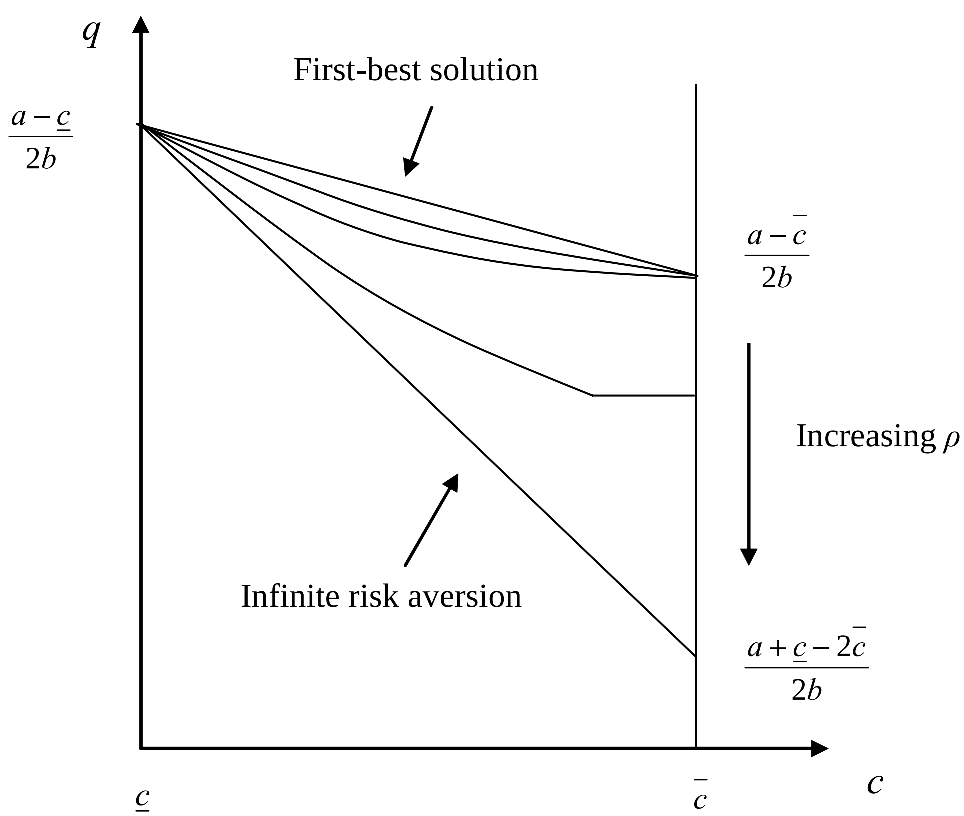

Figure 1. The comparative statics of the second-best supply schedule as ρ increases.

Notice that when the supplier is infinitely risk-averse, i.e., ρ converges to infinity, equation (7) becomes

$$\frac{1}{\Delta c}[V'(q(c)) - c] = \frac{c - \underline{c}}{\Delta c}. \tag{8}$$

It is the well known solution for a standard adverse selection problem where the supplier is privately informed about its marginal cost of production at the time of contracting. This is because the supplier will participate in the contract only if he is guaranteed nonnegative profit for all realization of c when he is infinitely risk-averse. Consequently our model becomes equivalent to one that the supplier is perfectly informed about its marginal cost at the time of contracting.

For later use, we call the optimal supply schedule when the supplier's degree of risk aversion is common information the second-best supply schedule.

4 Asymmetric Information on Risk Preference

4.1 A Risk-Neutral Buyer

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its marginal cost of production but also by its degree of risk aversion.

When the buyer is risk-neutral, the buyer's optimization problem is choosing a set of contract menus $M_n = \{T_n(c), q_n(c)\}$ for $n = l$, h to maximize

$$\int_{\underline{c}}^{\overline{c}} \{\alpha[V(q_l(c)) - T_l(c)] + (1-\alpha)[V(q_h(c)) - T_h(c)]\} f(c)dc, \tag{9}$$

subject to

$$E[U(\rho_n, M_n)] = \int_{\underline{c}}^{\overline{c}} U(T_n(c) - cq_n(c))f(c)dc \geqslant 0; \tag{10}$$

$$\pi_n(c_i \mid c_i) \geqslant \pi_n(c_i \mid c_j) \text{ for } c_i \neq c_j; \text{ and} \tag{11}$$

$$E[U(\rho_n, M_n)] \geqslant E[U(\rho_s, M_s)], \tag{12}$$

where $n = l, h$, $s = l, h$, and $n \neq s$.

While conditions (10) and (11) ensure the supplier's participation and truthful report of its marginal cost regardless of its degree of risk aversion, condition (12) guarantees that the supplier truthfully reveals its degree of risk aversion.

Proposition 1 describes the general properties of the optimal contract when the supplier is privately informed of its degree of risk aversion.

Proposition 1 *The optimal contract has the following properties :*

(a) $E[U(\rho_l, M_l)] > E[U(\rho_h, M_h)] = 0$;

(b) In no bunching region, the optimal supply schedule for the less risk-averse supplier is characterized by

$$\frac{1}{\Delta c}[V'(q_l(c)) - c] = \int_{\underline{c}}^{c} \left\{ \frac{1}{\Delta c} - \frac{e^{-\rho_l \pi_l}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx} \right\} dz; \text{ and} \tag{13}$$

the optimal supply schedule for the more risk-averse supplier is characterized by

$$\frac{1-\alpha}{\Delta c}[V'(q_h(c)) - c] = \int_{\underline{c}}^{c} \{ \frac{1-\alpha}{\Delta c} - \frac{e^{-\rho_h \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx} + \frac{\alpha e^{-\rho_l \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx} \} dz. \tag{14}$$

Proof. See Appendix. ■

Under the optimal contract, the buyer can fully extract the more risk-averse supplier's ex post information rent by adjusting the level of payments for all realizations of marginal cost as in the case of common information on risk aversion. However, the utility function of a less risk-averse supplier is an increasing and convex transformation of that of a more risk-averse supplier, and in equilibrium the less risk-averse supplier can enjoy positive expected utility by mimicking a more risk-averse one. Consequently, the optimal contract provides a less risk-averse supplier positive expected utility to induce its truthful revelation of its degree of risk aversion.

Under the optimal contract, the supply schedule for the less risk-averse supplier optimally balances risk sharing and the incentive for the supplier to truthfully reveal its realization of marginal cost, as in the case of common information on risk aversion. Consequently, the less risk-averse supplier produces according to the second-best supply schedule.

However, the supply schedule for the more risk-averse supplier now must simultaneously trade-off risk sharing, the supplier's incentives to truthfully reveal its marginal cost of production, and the less risk-averse supplier's incentive to truthfully reveal its degree of risk aversion. To demonstrate the trade-off, we rewrite equation (14) as

$$\frac{1-\alpha}{\Delta c}[V'(q_h(c)) - c] = (1-\alpha)[\frac{(c-\underline{c})}{\Delta c} - \frac{\int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx}] + \alpha G(c), \tag{15}$$

where $G(c) \equiv \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx - \int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx$.

When the more risk-averse supplier's realization of marginal cost is $\widehat{c}$, raising $q_h(\widehat{c})$ by δq will in expectation increase the production efficiency by $(1-\alpha)[V'(q(\widehat{c})) - \widehat{c}]\delta q/\Delta c$ where $1-\alpha$ is probability that the supplier is more risk-averse. However, the increase in $q_h(\widehat{c})$ will also raise the more risk-averse supplier's ex post information rent by δq when $c < \widehat{c}$. Consequently, in expectation it increases the more risk-averse supplier's ex post information rent by $\delta q \int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz$. In addition, the increase in $q_h(\widehat{c})$ will also increase the less risk-averse supplier's rent from mimicking the more risk-averse one by $\delta q \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz$

in expectation.

The certainty equivalents of the above ex post information rents for both types of suppliers are $\delta q \int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx$ and $\delta q \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$, respectively. Notice that $\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx$ and $\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$ are the marginal utilities of one unit of increase in certainty equivalent for both types of suppliers, respectively.

In anticipation of the supplier's information rent, at the time of contracting the buyer can reduce both types of suppliers' payments by $\delta q \int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx$ for all realizations of marginal cost. Doing so fully extracts the more risk-averse supplier's ex post information rent and provides the less risk-averse supplier just enough incentive to truthfully reveal its degree of risk aversion. At the optimum, the supplier's marginal benefit of raising $q_h(\widehat{c})$ must equal its marginal cost of doing so, which yields equation (15).

Notice that $\alpha G(c)$ (which is positive on $(\underline{c}, \overline{c})$ as shown in the proof of Proposition 2) is the effect of asymmetric information on the supplier's risk aversion. In order to limit a less risk-averse supplier's rent of exaggerating its degree of risk aversion, the buyer further distorts the more risk-averse supplier's contract towards a cost-plus contract. Consequently, as we show in Proposition 2, the more risk-averse supplier produces below the second-best supply schedule.

Proposition 2 *Under the optimal contract, the more risk-averse supplier's supply schedule is below the second-best level.*

Proof. See Appendix. ■

Similar to the case of common information on risk aversion, bunching arises in the optimal contract as either type of supplier becomes increasingly risk-averse. In the optimal contract with bunching, the supply schedule is strictly decreasing in the realization of marginal cost for small value of marginal cost but is constant for all realizations of marginal cost above certain critical value of marginal cost. The critical value of marginal cost depends on the degree of risk aversion of both types of suppliers.

Proposition 3 describes the properties of the optimal contract in that case.

Proposition 3 *When both types of suppliers become sufficiently risk-averse, there exist some c_h^* and c_l^* in $[\underline{c}, \overline{c}]$ that the optimal supply schedule is constant over $[c_h^*, \overline{c}]$ for the more risk-averse supplier and $[c_l^*, \overline{c}]$ for the less risk-averse supplier; The supply schedules for the non-bunching regions are determined by equation (14) for the more risk-averse supplier and by equation (13) for the less risk-averse supplier.*

Proof. The proof is similar to that of Lemma 3 and is therefore omitted. ■

Suppose that one type of supplier is risk-neutral and the other type of supplier is infinitely risk-averse. Then equation (15) becomes

$$\frac{1-\alpha}{\Delta c}[V'(q_h(c)) - c] = \frac{c - \underline{c}}{\Delta c}. \tag{16}$$

A direct comparison between equations (8) and (16) demonstrates the effect on the optimal contract of asymmetric information on the supplier's risk aversion. An increase in $q_h(\widehat{c})$ by δq increases the more risk-averse suppliers' production efficiency by $[V'(q(\widehat{c})) - \widehat{c}]\delta q/\Delta c$ regardless whether he is privately informed about its risk aversion. However, with asymmetric information on risk aversion, an increase in $q_h(\widehat{c})$ by δq increases the ex post information rent for not only the more risk-averse supplier but also the less risk-averse supplier by $(c - \underline{c})/\Delta c$. The certainty equivalent of the ex post information rent is zero for the more risk-averse supplier, which means that the buyer cannot extract any of the ex post rent at the time of contracting. Consequently, with asymmetric information on risk aversion, the more risk-averse supplier's supply schedule is further distorted towards a cost plus contract.

4.2 A Risk-Averse Buyer

When the buyer is also risk-averse, the optimal contract must balance the buyer's profits with different types of suppliers, in addition to the tradeoff among risk sharing and the incentives for the supplier to truthfully reveal both its marginal cost of production and its degree of risk aversion. Suppose that the buyer has a constant absolute risk aversion (CARA) utility function, $u(x) = 1 - e^{-\rho_b x}$ with $\rho_b > 0$. The buyer's optimization problem is choosing a set of contract menus $M_n = \{T_n(c), q_n(c)\}$ for $n = l,\ h$ to maximize

$$E[U_b] = \int_{\underline{c}}^{\overline{c}} \left\{\alpha[1 - e^{-\rho_b W_l}] + (1-\alpha)[1 - e^{-\rho_b W_h}]\right\} f(c)dc \tag{17}$$

subject to conditions (10), (11), and (12), where $W_l = V(q_l(c)) - T_l(c)$ and $W_h = V(q_h(c)) - T_h(c)$.

Proposition 4 describes the properties of the optimal contract when both the buyer and the supplier are risk-averse.

Proposition 4 *When both the buyer and the supplier are risk-averse, the optimal contract has the following properties:*

(a) $E[U(\rho_l, M_l)] > E[U(\rho_h, M_h)] = 0$;

(b) In no bunching region, the optimal supply schedule for the less risk-averse supplier is characterized by

$$\frac{e^{-\rho_b W_l}}{CE}[V'(q_l(c)) - c] = \int_{\underline{c}}^{c} \left\{ \frac{e^{-\rho_b W_l}}{CE} - \frac{e^{-\rho_l \pi_l} \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx}{CE \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx} \right\} dz; \tag{18}$$

and the optimal optimal supply schedule for the more risk-averse supplier is characterized *by*

$$\frac{(1-\alpha)e^{-\rho_b W_h}}{CE}[V'(q_h(c))-c] = \int_{\underline{c}}^{c}\{\frac{(1-\alpha)e^{-\rho_b W_h}}{CE} + \frac{\alpha e^{-\rho_l \pi_h}\int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l}dx}{CE\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l}dx} - \frac{e^{-\rho_h \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h}dx}\}dz, \tag{19}$$

where $CE \equiv \alpha\int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l}dx + (1-\alpha)\int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_h}dx.$

Proof. See Appendix. ■

Under the optimal contract, the more risk-averse supplier still receives zero expected utility, and the less risk-averse supplier still receives positive expected utility due to its private information on its degree of risk aversion. However, the optimal supply schedule is profoundly different compared with the case when the buyer is risk neutral.

For the less risk-averse supplier, when the realization of marginal cost is $\widehat{c}$, raising $q_l(\widehat{c})$ by δq will in expectation increase $W(\widehat{c})$ by $[V'(q_l(\widehat{c}))-\widehat{c}]\delta q$ which increases the buyer's certainty equivalent by $\delta q\alpha[V'(q_l(\widehat{c}))-\widehat{c}]e^{-\rho_b W}/CE$. Note that CE is the increase in the buyer's expected surplus as a result of one unit increase in its profits for all possible events. On the other hand, the increase in $q_l(\widehat{c})$ will also raise the less risk-averse supplier's ex post information rent by δq when $c < \widehat{c}$. The certainty equivalent of the additional ex post information rent is $\delta q\int_{\underline{c}}^{c} e^{-\rho_h \pi_h}dz/\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h}dx$ for the supplier. Therefore, therefore at the time of contracting the buyer can optimally reduce the supplier's payments under all realization of c by $\delta q\int_{\underline{c}}^{c} e^{-\rho_h \pi_h}dz/\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h}dx$.

However, the buyer's certainty equivalent of the supplier's additional ex post information rent is $\delta q\alpha\int_{\underline{c}}^{c} e^{-\rho_b W_l}dz/CE$ and its certainty equivalent of the amount that can be extracted from the supplier at the time of contracting is $\delta q\alpha\int_{\underline{c}}^{c} e^{-\rho_l \pi_l}dz\int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l}dx/CE\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l}dx$. Hence depending on the relative sizes of these two certainty equivalents for the buyer, which in turn depends on the relative degree of risk aversion between the two parties, the optimal supply schedule can be either above or below the efficient level.

For example, when ρ_l converges to zero, i.e., the supplier converges to risk-neutral, $\int_{\underline{c}}^{c} e^{-\rho_l \pi_l} dx / \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$ converges to $(c - \underline{c})/\Delta c$ and the latter certainty equivalent converges to $\delta q \alpha (c - \underline{c}) \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx / (CE) \Delta c$. In the optimal contract, W_l must be non-increasing on $[\underline{c}, \overline{c}]$. Then, we have $(c - \underline{c}) \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx / \Delta c \geqslant \int_{\underline{c}}^{c} e^{-\rho_b W_l} dz$ and the right-hand side of equation (18) is non-positive. Consequently, the optimal supply schedule is above or at the efficient level on $[\underline{c}, \overline{c}]$.

On the other hand, based on our analysis of a risk-neutral buyer in the previous section, by continuity the optimal supply schedule must be below the efficient level when the buyer converges to risk-neutral.

The buyer's risk aversion has a different impact on the more risk-averse supplier's supply schedule. Equation (19) demonstrates how the optimal supply schedule for the more risk-averse supplier balances risk sharing, incentives for truthful revelation, and the buyer's profits with different types of suppliers.

The certainty equivalent of the additional profits of increasing $q_h(c)$ by δq is $(1 - \alpha)[V'(q_h(c)) - c] e^{-\rho_b W_h} / CE$. However, the increase in $q_h(c)$ also increases the ex post information rent for both types of suppliers. The certainty equivalent for the buyer of the more risk-averse supplier's additional ex post information rent is $(1 - \alpha) \delta q \int_{\underline{c}}^{c} e^{-\rho_b W_h} dz / CE$. We have shown earlier that the less risk-averse supplier's certainty equivalent of the additional ex post information rent is $\delta q \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$, the certainty equivalent of which for the buyer is $\alpha \delta q \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx / CE \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$. Note that $\alpha \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx / CE$ is the certainty equivalent for the buyer of one unit increase in surplus under a less risk-averse supplier for all realizations of marginal costs. Therefore, it measures how the risk-averse buyer values additional surplus under the less-risk averse supplier.

Recall that, in anticipation of the additional ex post information rents for both types of suppliers, the buyer can reduce both types of suppliers' payments for all realizations of marginal cost by the amount equal to the more risk-averse supplier's certainty equivalent of the additional rent. Therefore, as indicated by the right-hand of Equation (19), the marginal cost of increasing $q_h(c)$ is the sum of the certainty equivalents for the buyer of both types of suppliers' additional rents (which are $(1-\alpha)\delta q \int_{\underline{c}}^{c} e^{-\rho_b W_h} dz/CE$ and $\alpha \int_{\underline{c}}^{c} e^{-\rho_l \pi_h} dz \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dx/CE \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx$, respectively) minus the more risk-averse supplier's certainty equivalent of the additional rent $\int_{\underline{c}}^{c} e^{-\rho_h \pi_h} dz / \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dx$. Nonetheless, the less risk-averse supplier receives positive information rent from its private information on its degree of risk aversion. Similar to the case of risk-neutral buyer, in order to restrict a less risk-averse supplier's rent of exaggerating its degree of risk aversion, the buyer distorts the more risk-averse supplier's contract towards a cost plus contract compared with the contract for the less risk-averse supplier. However, the distortion is smaller compared with the case of a risk-neutral buyer as the distortion for a more risk-averse supplier becomes more costly and the information rent for a less risk-averse supplier becomes less important to a risk-averse buyer.

It can be readily shown that the optimal supply schedule for a more risk-averse supplier can also be either below or above the efficient level depending on the relative degree of risk aversion between the buyer and the supplier.

We summarize this property in Proposition 5.

Proposition 5 *Depending on the relative degree of risk aversion between the buyer and the supplier, the optimal supply schedule for both types of suppliers can be either below or above the efficient level.*

5 Conclusion

We extend the standard adverse selection model to settings where the supplier is privately informed of its degree of risk aversion. The optimal contract simultaneously balances risk sharing, incentives for information revelation, and the buyer's expected profits with different types of suppliers. A supplier with small degree of risk aversion produces below the efficient levle of output le except for the lowest and the highest realizations of cost. The production distortion increases as the supplier becomes more risk-averse. The asymmetric information on risk preference further distorts the supply schedule of a more risk-averse supplier towards a cost-plus contract. However, when the buyer is also risk-averse, both types of supplier may produce above the efficient supply schedule.

Our research could be extended in several directions. For example, although the supplier's information on cost of production is imperfect at the time of contracting, the supplier could be better informed of its potential costs than the buyer. Then the optimal contract must screen the firm not only by its degree of risk aversion but also by its information regarding cost of production at the time of contracting. The optimal contract in this situation merits further investigation.

6 Appendix

6.1 Proof of Lemma 1

A well known characterization of feasible contracts is the following: *(a)* $T'(c) = cq'(c)$; *(b)* $q(c)$ *is non increasing;* (c) $EU \geq 0$.

Therefore, we can rewrite the buyer's optimization problem as an optimal control problem with state variables $T(c)$ and $q(c)$ and control variable $q'(c) = z$:

$$Max \frac{1}{\Delta c} \int_{\underline{c}}^{\overline{c}} [V(q(c)) - T(c)] dc, \tag{A1}$$

subject to

$$q'(c) = z \tag{A2}$$

$$T'(c) = cz \tag{A3}$$

$$q'(c) \leq 0; \text{ and} \tag{A4}$$

$$\frac{1}{\Delta c}\int_{\underline{c}}^{\overline{c}} U(T(c) - cq(c))dc \geqslant 0. \tag{A5}$$

The Hamiltonian is

$$H = [V(q) - T] + \mu cz + \lambda z + \theta U(\pi). \tag{A6}$$

The necessary conditions are given by

$$\frac{\partial H}{\partial z} = \mu c + \lambda \geqslant 0,\ z \leqslant 0,\ \text{and } (\mu c + \lambda)\ z = 0; \tag{A7}$$

$$\lambda' = -\frac{\partial H}{\partial q} = -[V'(q) - \theta U'(\pi)c]; \tag{A8}$$

$$\mu' = -\frac{\partial H}{\partial T} = -[-1 + \theta U'(\pi)];\ \text{and} \tag{A9}$$

$$\lambda(\underline{c}) = \lambda(\overline{c}) = \mu(\underline{c}) = \mu(\overline{c}) = 0. \tag{A10}$$

From the transversality condition (A8) and equation (A9),

$$\mu(\overline{c}) - \mu(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [1 - \theta U'(\pi)]dc = 0. \tag{A11}$$

Therefore,

$$\theta = \frac{\Delta c}{\int_{\underline{c}}^{\overline{c}} U'(\pi(c))dc}. \tag{A12}$$

Define $h(c) = \mu c + \lambda$. From condition (A7), on any interval where q is strictly decreasing, $h(c)$ must be zero. So $h'(c) = \mu + \mu' c + \lambda' = 0$, which leads to

$$\mu = -\mu' c - \lambda'. \tag{A13}$$

Substituting equations (A8) and (A9) into the above equation for μ' and λ', we have

$$\mu = \int_{\underline{c}}^{c} [1 - \theta U'(\pi)]dx = V'(q) - c. \tag{A14}$$

Substituting equation (A12) into the above equation for θ, we have

$$\frac{1}{\Delta c}[V'(q) - c] = \frac{c - \underline{c}}{\Delta c} - \frac{\int_{\underline{c}}^{c} U'(\pi(x))dx}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dx}. \tag{A15}$$

Next we show that $q'(c) = 0$ can only occur on some interval $[c^*, \overline{c}]$, and the solution is strictly decreasing on $[\underline{c}, c^*)$. Suppose that there exist c_1, c_2, and c_3 such that q is constant on (c_1, c_2) and strictly decreasing on (c_2, c_3).

Since q is constant on (c_1, c_2) on (c_2, c_3), $h(c_2^+) = h'(c_2^+) = h''(c_2^+) = 0$. Furthermore,

$$\begin{aligned} h'(c) &= \mu + \mu' c + \lambda' \\ &= \int_{\underline{c}}^{c} [1 - \theta U'(\pi)]dx - [V'(q) - c]. \end{aligned} \tag{A16}$$

Hence,

$$\begin{aligned} 0 &= h''(c_2^+) = [1 - \theta U'(\pi(c_2^+))] - [V''(q(c_2^+))q'(c_2^+) - 1] \\ &< 2 - \theta U'(\pi(c_2^-)) = h''(c_2^-), \end{aligned} \tag{A17}$$

as $V''(q(c_2^+)) < 0$, $q'(c_2^+) < 0$, and $q'(c_2^-) = 0$.

Moreover,

$$h'''(c) = \theta U''(\pi(c))q(c) < 0 \tag{A18}$$

on (c_1, c_2), which together with (A17) implies that $h''(c)$ is positive (i.e., $h(c)$ is convex,) on (c_1, c_2). Since $h(c_2^+) = 0$, $h(c)$ is convex on (c_1, c_2) means $h(c_1) > 0$. As $h(c)$ is continuous, it $h(c) > 0$ must be true for some $c < c_1$. Since $\mu(\underline{c}) = 0$ by the transversality condition, there is a contradiction.

6.2 Proof of Lemma 2

With CARA utility function, when there is no bunching, $q(c)$ is given by equation (7) for all c on $[\underline{c}, \overline{c}]$. Differentiating both sides of equation (7) with respect to c, we have $1 - e^{-\rho\pi(c)} = -2bq'(c) - 1$ and $q'(c) = (e^{-\rho\pi(c)} - 2)/2b$. Since $\pi'(c) = -q(c)$,

$$\pi''(c) = -q'(c) = (2 - e^{-\rho\pi(c)})/2b. \tag{A19}$$

Assume for some (ρ_0, c_0), $A \equiv \partial q/\partial\rho = -\partial^2\pi(c)/\partial c\partial\rho > 0$. Since $A(\rho_0, \underline{c}) = A(\rho_0, \overline{c}) = 0$ and $A(\rho_0, \cdot)$ is smooth, A must admit an interior positive maximum on $(\underline{c}, \overline{c})$, i.e.,

$$A(\rho_0, c_m) > 0; \tag{A20}$$

$$\frac{\partial A(\rho_0, c_m)}{\partial c} = 0; \text{ and} \tag{A21}$$

$$\frac{\partial^2 A(\rho_0, c_m)}{\partial c^2} \leqslant 0. \tag{A22}$$

From equation (A17),

$$\frac{\partial A(\rho_0, c_m)}{\partial c} = -\frac{\partial^3 \pi(c_m)}{\partial c^2 \partial \rho} = -\frac{e^{-\rho\pi}}{2b}\frac{\partial \rho\pi}{\partial \rho}; \text{ and} \tag{A23}$$

$$\frac{\partial^2 A(\rho_0, c_m)}{\partial c^2} = -\frac{\partial^4 \pi(c_m)}{\partial c^3 \partial \rho} = -\frac{1}{2b}[e^{-\rho\pi}\rho q\frac{\partial \rho\pi}{\partial \rho} + e^{-\rho\pi}\frac{\partial^2(\rho\pi)}{\partial \rho \partial c}]. \tag{A24}$$

Equations (7) and (A21) together imply that $\partial \rho\pi/\partial \rho = 0$ and

$$\frac{\partial^2 A(\rho_0, c_m)}{\partial c^2} = -\frac{1}{2b}e^{-\rho\pi}\frac{\partial^2(\rho\pi)}{\partial \rho \partial c}. \tag{A25}$$

Then equations (A22) and (A25) together require that

$$\frac{\partial^2(\rho\pi)}{\partial \rho \partial c} = -\frac{\partial \rho q}{\partial \rho} = -(q + \rho\frac{\partial q}{\partial \rho}) \geqslant 0. \tag{A26}$$

Equation (A26) implies $\partial q/\partial \rho < 0$, which contradicts with condition (A20).

6.3 Proof of Lemma 3

With CARA utility function, when there is no bunching, from equation (6) we have

$$\frac{1}{\Delta c}[V'(q(c)) - c] = \frac{c - \underline{c}}{\Delta c} - \frac{\int_{\underline{c}}^{c} e^{-\rho\pi} dx}{\int_{\underline{c}}^{\overline{c}} e^{-\rho\pi} dx} \tag{A27}$$

on $[\underline{c}, \overline{c}]$. When $\rho = 0$, $q(c)$ is strictly decreasing on $[\underline{c}, \overline{c}]$. By continuity, the optimal supply schedule is strictly decreasing on $[\underline{c}, \overline{c}]$ for small ρ.

With the quadratic value function, equation (A27) provides

$$q'(c) = (e^{-\rho\pi(c)} - 2)/2b > -1/b; \text{ and} \tag{A28}$$

$$q''(c) = e^{-\rho\pi(c)}\rho q/2b = (1 + bq'(c))\rho q/b. \tag{A29}$$

Therefore, $q''(c) > 0$, i.e., the supply schedule is strictly convex on $[\underline{c}, \overline{c}]$ when there is no bunching. Hence, the supply schedule will be non-increasing everywhere if and only if $\partial q(\overline{c})/\partial c \leqslant 0$. Suppose that $\partial q(\overline{c})/\partial c \leqslant 0$ for any ρ in the optimal contract.

Since $q'(c) > -1/b$, the graph of q must stay inside the triangle pictured in Figure 1. Therefore, we have

$$\int_{\underline{c}}^{\overline{c}} q(c)dc > \frac{a - \overline{c}}{2b}\Delta c + \frac{b}{2}[\frac{(a - \underline{c}) - (a - \overline{c})}{2b}]^2 = \frac{\Delta c}{2b}[a - \overline{c} - \frac{\Delta c}{4}]. \tag{A30}$$

Integrating both sides of equation (A29) with respect to c provides

$$\begin{aligned} q'(\overline{c}) - q'(\underline{c}) &= \frac{\rho}{b}\int_{\underline{c}}^{\overline{c}} (1 + bq'(c))q(c)dc \\ &> \frac{\rho}{b}\left\{\frac{\Delta c}{2b}(a - \overline{c} - \frac{\Delta c}{4}) + \frac{q^2(\overline{c}) - q^2(\underline{c})}{2}\right\} \\ &> \frac{\rho\Delta c}{2b^2}\left\{(a - \overline{c} - \frac{\Delta c}{4}) + \frac{2a - \overline{c} - \underline{c}}{4}\right\}. \end{aligned} \tag{A31}$$

The term on the left-hand side of inequality converges to infinity as ρ goes to infinity. However, since $0 > q'(c) > -1/b$, $q'(\overline{c}) - q'(\underline{c}) < 1/b$, which is contradict with condition (A31).

Proposition 2 implies that $\partial^2 q(\overline{c}, \rho)/\partial c\partial\rho > 0$. Therefore, there exists ρ^* with $\rho^* > 0$, such that bunching occurs when $\rho > \rho^*$.

6.4 Proof of Proposition 1

The Hamiltonian is

$$H = \alpha[V(q_l(c)) - T_l(c)] + (1-\alpha)[V(q_h(c)) - T_h(c)] \quad \text{(A32)}$$

$$+\mu_l c z_l + \mu_h c z_h + \lambda_l z_l + \lambda_h z_h - \theta e^{-\rho_h \pi_h} + \beta[e^{-\rho_l \pi_h} - e^{-\rho_l \pi_l}], \quad \text{(A33)}$$

where μ_l, μ_h, λ_l, λ_h, and β are the Lagrange multipliers.

The necessary conditions are given by

$$\frac{\partial H}{\partial z} = \mu_l c + \lambda_l \geqslant 0,\ z_l \leqslant 0,\ \text{and } (\mu_l c + \lambda_l)\ z_l = 0; \quad \text{(A34)}$$

$$\frac{\partial H}{\partial z} = \mu_h c + \lambda_h \geqslant 0,\ z \leqslant 0,\ \text{and } (\mu_h c + \lambda_h)\ z_l = 0; \quad \text{(A35)}$$

$$\lambda_l' = -\frac{\partial H}{\partial q} = -[\alpha V'(q_l) - \beta e^{-\rho_l \pi_l} \rho_l c]; \quad \text{(A36)}$$

$$\lambda_h' = -\frac{\partial H}{\partial q} = -[(1-\alpha)V'(q_h) - \theta e^{-\rho_h \pi_h} \rho_h c + \beta e^{-\rho_l \pi_h} \rho_l c]; \quad \text{(A37)}$$

$$\mu_l' = -\frac{\partial H}{\partial T} = -[-\alpha + \beta e^{-\rho_l \pi_l} \rho_l]; \quad \text{(A38)}$$

$$\mu_h' = -\frac{\partial H}{\partial T} = -[-(1-\alpha) + \theta e^{-\rho_h \pi_h} \rho_h - \beta e^{-\rho_l \pi_h} \rho_l];\ \text{and} \quad \text{(A39)}$$

$$\lambda_n(\underline{c}) = \lambda_n(\overline{c}) = \mu_n(\underline{c}) = \mu_n(\overline{c}) = 0,\ \text{where } n = l, h. \quad \text{(A40)}$$

From the transversality condition (A40) and equation (A38),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha - \beta e^{-\rho_l \pi_l} \rho_l] dc = 0, \quad \text{(A41)}$$

which provides

$$\beta = \frac{\alpha \Delta c}{\rho_l \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc}. \tag{A42}$$

From the transversality condition (A40) and equation (A39),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [(1-\alpha) - \theta e^{-\rho_h \pi_h} \rho_h + \beta e^{-\rho_l \pi_h} \rho_l] dc = 0, \tag{A43}$$

which provides

$$\theta = \frac{1}{\rho_l}[(1-\alpha) + \alpha \frac{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc}] = \frac{1}{\rho_l} \tag{A44}$$

as $\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc = \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc$, i.e., constraint (15) is binding at equilibrium.

Since $h_n'(c) = \mu_n + \mu_n' c + \lambda_n' = 0$ or $\mu_l = -\mu_l' c - \lambda_l'$ when q_n is strictly decreasing in c, we have

$$\mu_l = \int_{\underline{c}}^{c} [\alpha - \frac{\alpha \Delta c e^{-\rho_l \pi_l}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dx}] dz = \alpha [V'(q_l(c)) - c], \tag{A45}$$

and

$$\begin{aligned} \mu_h &= \int_{\underline{c}}^{c} [(1-\alpha) - \theta e^{-\rho_h \pi_h} \rho_h + \beta e^{-\rho_l \pi_h} \rho_l] dx & \text{(A46)} \\ &= \int_{\underline{c}}^{c} [(1-\alpha) - e^{-\rho_h \pi_h} + \frac{\alpha \Delta c e^{-\rho_l \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dz}] dx & \text{(A47)} \\ &= (1-\alpha)[V'(q_h(c)) - c]. & \text{(A48)} \end{aligned}$$

6.5 Proof of Proposition 2

Since

$$G'(c) = \frac{e^{-\rho_l \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc} - \frac{e^{-\rho_h \pi_h}}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dc}, \tag{A49}$$

$G'(c) > 0$ if

$$\pi_h > \frac{\ln \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc - \ln \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dc}{\rho_h - \rho_l}. \tag{A50}$$

Notice that $\ln \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc - \ln \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dc < 0$ as $\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_h} dc < \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dc$.

Since π_h is strictly decreasing in c and $\int_{\underline{c}}^{\overline{c}} [1 - e^{-\rho_h \pi_h}] dc = 0$, $\pi_h(\underline{c})$ must be positive. Consequently, $G'(\underline{c}) > 0$. Since π_h is monotone in c, the sign of $G'(c)$ can change at most once. Moreover, since $G(\underline{c}) = G(\overline{c}) = 0$, the sign of $G'(c)$ must change at least once. Consequently, $G(c)$ must be increasing for some region starting from $\underline{c}$ and become decreasing for its complimentary region. As a result, $G(c) > 0$ on $(\underline{c}, \overline{c})$.

6.6 Proof of Proposition 4

The Hamiltonian is

$$H = \alpha[1 - e^{-\rho_b W_l}] + (1-\alpha)[1 - e^{-\rho_b W_h}] + \mu_l c z_l + \mu_h c z_h + \lambda_l z_l + \lambda_h z_h - \theta e^{-\rho_h \pi_h} + \beta[e^{-\rho_l \pi_h} - e^{-\rho_l \pi_l}], \tag{A51}$$

where μ_l, μ_h, λ_l, λ_h, and β are the Lagrange multipliers.

The necessary conditions are given by

$$\frac{\partial H}{\partial z} = \mu_l c + \lambda_l \geqslant 0, \; z_l \leqslant 0, \text{ and } (\mu_l c + \lambda_l) \; z_l = 0; \tag{A52}$$

$$\frac{\partial H}{\partial z} = \mu_h c + \lambda_h \geqslant 0, \; z \leqslant 0, \text{ and } (\mu_h c + \lambda_h) \; z_l = 0; \tag{A53}$$

$$\lambda_l' = -\frac{\partial H}{\partial q_l} = -[\alpha \rho_b e^{-\rho_b W_l} V'(q_l) - \beta e^{-\rho_l \pi_l} \rho_l c]; \tag{A54}$$

$$\lambda_h' = -\frac{\partial H}{\partial q_h} = -[(1-\alpha)\rho_b e^{-\rho_b W_h} V'(q_h) - \theta e^{-\rho_h \pi_h} \rho_h c + \beta e^{-\rho_l \pi_h} \rho_l c]; \tag{A55}$$

$$\mu_l' = -\frac{\partial H}{\partial T_l} = -[-\alpha\rho_b e^{-\rho_b W_l} + \beta e^{-\rho_l \pi_l}\rho_l]; \quad \text{(A56)}$$

$$\mu_h' = -\frac{\partial H}{\partial T_h} = -[-(1+\alpha)\rho_b e^{-\rho_b W_h} + \theta e^{-\rho_h \pi_h}\rho_h - \beta e^{-\rho_l \pi_h}\rho_l]; \text{ and} \quad \text{(A57)}$$

$$\lambda_n(\underline{c}) = \lambda_n(\overline{c}) = \mu_n(\underline{c}) = \mu_n(\overline{c}) = 0, \text{ where } n = l, h. \quad \text{(A58)}$$

From the transversality condition (A58) and equation (A56),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha\rho_b e^{-\rho_b W_l} - \beta\rho_l e^{-\rho_l \pi_l}]dc = 0, \quad \text{(A59)}$$

which provides

$$\beta = \frac{\alpha\rho_b \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dc}{\rho_l \int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc}. \quad \text{(A60)}$$

From the transversality condition (A58) and equation (A57),

$$\mu_h(\overline{c}) - \mu_h(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [(1-\alpha)\rho_b e^{-\rho_b W_h} - \theta e^{-\rho_h \pi_h}\rho_h + \beta e^{-\rho_l \pi_h}\rho_l]dc = 0, \quad \text{(A61)}$$

which provides

$$\theta = \frac{\alpha\rho_b \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dc + (1-\alpha)\rho_b \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_h} dc}{\rho_h \int_{\underline{c}}^{\overline{c}} e^{-\rho_h \pi_h} dc}. \quad \text{(A62)}$$

Since $\mu_l = -\mu_l' c - \lambda_l'$ when q_n is strictly decreasing in c, we have

$$\begin{aligned} \mu_l &= \int_{\underline{c}}^{c} [\alpha\rho_b e^{-\rho_b W_l} - \beta e^{-\rho_l \pi_l}\rho_l]dz && \text{(A63)} \\ &= \int_{\underline{c}}^{c} [\alpha\rho_b e^{-\rho_b W_l} - \frac{\alpha\rho_b \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dc}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc} e^{-\rho_l \pi_l}]dz && \text{(A64)} \\ &= \alpha\rho_b e^{-\rho_b W_l}[V'(q_l(c)) - c], && \text{(A65)} \end{aligned}$$

and

$$\mu_h = \int_{\underline{c}}^{c} [(1-\alpha)\rho_b e^{-\rho_b W_h} - \theta e^{-\rho_h \pi_h} \rho_h + \beta e^{-\rho_l \pi_h} \rho_l] dx \tag{A66}$$

$$= \int_{\underline{c}}^{c} [(1-\alpha)\rho_b e^{-\rho_b W_h} - \frac{CE}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dc} e^{-\rho_h \pi_h} + \frac{\alpha \rho_b \int_{\underline{c}}^{\overline{c}} e^{-\rho_b W_l} dc}{\int_{\underline{c}}^{\overline{c}} e^{-\rho_l \pi_l} dz} e^{-\rho_l \pi_h}] dx \tag{A67}$$

$$= (1-\alpha)\rho_b e^{-\rho_b W_h} [V'(q_h(c)) - c]. \tag{A68}$$

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