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Data Rate Determination for Fixed, Matched Filter Channels

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Abstract

In today's world, with ever rising data rate requirements and ever shrinking budgets, the use of existing equipment to perform new tasks is highly desirable. Often the need to increase data transmission rates through an existing system is clear, yet financial or logistical constraints do not permit complete redesign of the system. Other situations do not allow the designer access to all of the system components, as in satellite systems. Whatever the circumstances, the desirable result is to maximize data rates through existing channels.

This paper will present a method for determination of the allowable data rates in existing channels. In addition, a measure of the expected degradation associated with the use of a pre-existing receive filter and matched transmit filter pair at increased transmit data rates is determined. Examples will be given for different filter types and data modulation formats.

1.0 Introduction

In an engineer's ideal world, every generation of a communication system would consist of the latest cutting edge technology and would be completely rebuilt each time a modification was to be made. This is clearly not the case in our world. Often, the simple replacement of filters in the channel can be difficult due to inaccessibility or budgetary and logistic constraints. This, however, does not affect the ever increasing need for higher data transmission rates through almost every communication channel.

Recently, the focus has been directed to higher order modulation formats for increase data throughput. Higher order modulation formats, such as 8PSK, 16QAM and (12,4) allow more information throughput while maintaining the spectral properties of lower order modulation schemes such as QPSK, thus accommodating the existing transponder bandwidth.

The use of such modulation schemes does not guarantee that, for the existing transmit/receive filters, the throughput has been maximized. The data transmission rate must also be maximized. However, simply increasing the transmit rate will not accomplish the goal, since inter-symbol interference will be introduced and will drastically effect performance.

The National Aeronautics and Space Administration (NASA) in particular is interested in increasing the data rate capabilities of their Tracking and Data Relay Satellite System (TDRSS), which provides a data relay for Low Earth Orbiting satellites throughout their orbit. In such a situation, the replacement or modification to filters in the transponder is prohibitively expensive.

Currently, research is being conducted at New Mexico State University into the use of the first satellite transponder filter in the receive filter role — this technique involves the linearization of the downlink channel at the ground terminal, in effect making the satellite part of the receiver. It has been shown that, for an ideal channel void of ISI and in which the transponder filter is perfectly matched to the transmitted pulse, performance very near theory can be achieved for a range of modulation schemes, for very high data rates [1].

In the actual channel, however, the transmitted pulse must be matched to the transponder filter, which is unchangeable for obvious reasons. Thus, when a matched filter channel is desired in order to maximize the SNR at the output of the transponder filter, some degree of ISI will be present. The amount of ISI will vary as the symbol rate is increased, thus there will be data rates for which the system works well and data rates for which the system will not work at all.

This paper will present one technique which can be used to determine the highest possible data rate for an arbitrary matched filter channel. The approach taken will be to evaluate the amount of ISI introduced over a broad range of transmission rates, and then choose the highest transmission rate which yields a low ISI distortion. Examples will be given for butterworth and elliptical filters. The increased data rate capabilities will be demonstrated through simulation of 8PSK and 16QAM in such a channel, over a range of data rates.

2.0 Channel Model

As mentioned in the first section, this paper will focus on the matched filter channel. It will be assumed that the channel between the transmit and receive filter is wideband and the only distortion introduced consists of additive white Gaussian noise (AWGN). A block

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diagram of this channel model is shown in Figure 2.1 below.

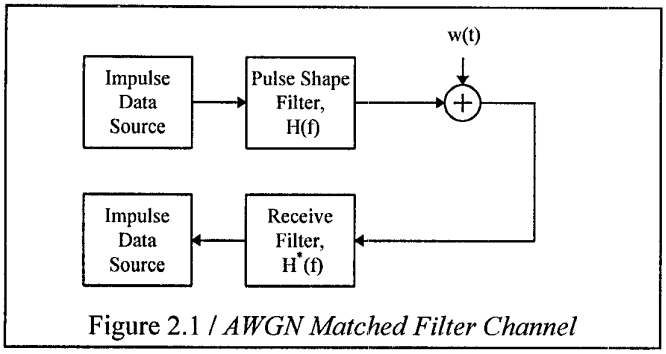


Figure 2.1 / *AWGN Matched Filter Channel*

As shown in this figure, the matched filter channel is one in which the receive filter is the complex conjugate of the transmit pulse-shaping filter. This model provides the maximum allowable signal-to-noise ratio (SNR) to the receiver at the sample points [3]. The overall channel impulse response, $x(t)$, is the combination of the two filter responses, i.e.

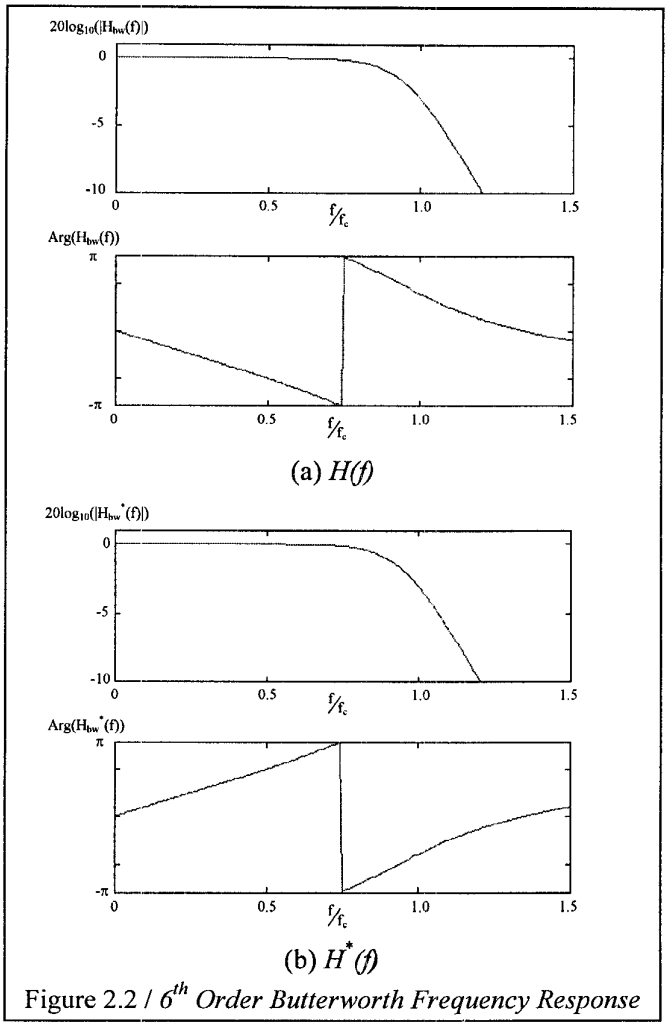
$$X(f)=H(f)H^*(f),$$

and,

$$x(t)=\mathcal{F}^{-1}\{X(f)\}.$$

Such a channel can be completely specified by either the pulse-shaping filter, $H(f)$, or the receive filter, $H^*(f)$. In the TDRSS channel study, $H^*(f)$ is specified and the transmit filter is chosen to be it's conjugate, $H(f)$. However, the results will be identical for a channel in which $H(f)$ is fixed and the receive filter is chosen.

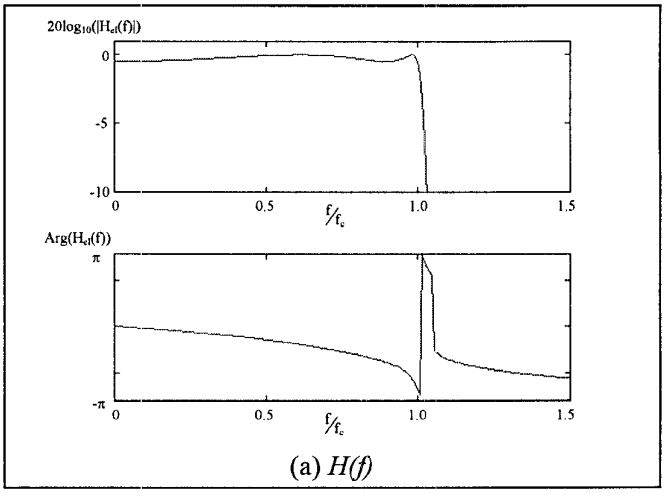
To demonstrate the procedure presented here for selecting the proper data rate, $H^*(f)$ will be modeled as a sixth order butterworth filter and a fourth order elliptical filter — these filters have been found to exhibit performance similar to that of the TDRSS channel [2]. However, the procedure can be performed on any filter pair. The impulse responses of a sixth order butterworth filter and a fourth order elliptical filter are shown in Figures 2.2(a) and 2.3(a) respectively. Their complex conjugates, and therefore their respective matched pair, are shown in Figures 2.2(b) and 2.3(b). Each of these plots have been normalized by the 3 dB cutoff frequency f_c, for generality. The elliptical filter was chosen to have .5 dB of ripple and 10 dB of attenuation in the passband.



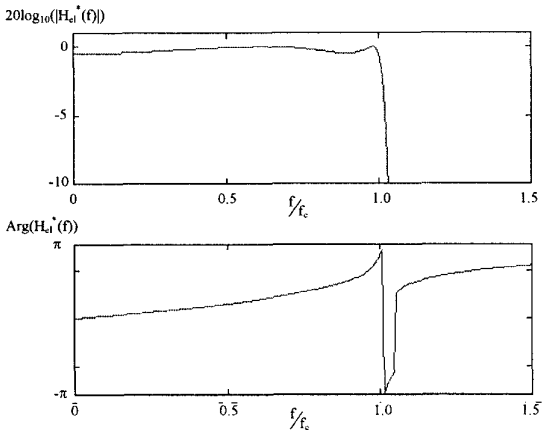
(a) $H(f)$

(b) $H^*(f)$

Figure 2.2 / *6th Order Butterworth Frequency Response*



(a) $H(f)$



(b) $H^*(f)$

Figure 2.3 / *4th Order Elliptical Frequency Response*

The channel impulse response, *x(t)*, will determine the ISI seen by the receiver and therefore will be the focus here. The impulse response of the matched sixth order butterworth filter channel, $x_{bw}(f)$, and the matched fourth order elliptical filter channel, $x_{el}(f)$, are shown below in Figures 2.3 and 2.4. Due to the nature of the matched filter channel, the phase response of the combined response is zero across the filter bandwidth. Therefore, the impulse response is all real for a real impulse — hence only the real component of the impulse response is plotted.

These impulse responses will be used in the next sections to determine the best data rates for operation through these channels. Again, these plots have been normalized by the filter's 3 dB cutoff frequency, f_c. Similar plots can be generated for any matched filter pair, *H(f)* and $H^*(f)$.

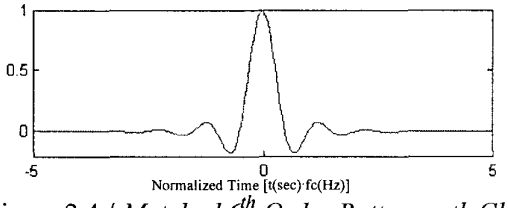


Figure 2.4 / *Matched 6th Order Butterworth Channel Impulse Response*

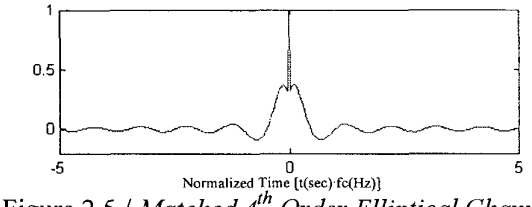


Figure 2.5 / *Matched 4th Order Elliptical Channel Impulse Response*

3.0 Choosing a Data Rate Visually

One method of choosing a data rate for the system is to read it directly from a plot similar to Figures 2.3 and 2.4. Although this method may not guarantee minimum distortion, it may be close enough for a particular application.

The data rate must be chosen to minimize the ISI introduced by the pulse-shaping and receive filters. A well chosen symbol rate then is one which positions the sample point over nulls in the preceding and following symbol's impulse response. A poorly chosen symbol rate, conversely, is one which aligns the sample point with relative high points in the neighboring impulse responses. Since most of the ISI will be due to the nearest neighbors in the time domain, and due to the symmetry of the response, the choice can be made based only on two neighboring impulse responses. Figure 3.1 demonstrates a well chosen symbol rate for the Butterworth matched channel while a poorly chosen symbol rate for the Elliptical matched channel is shown in Figure 3.2.

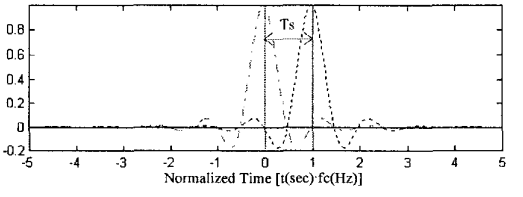


Figure 3.1 / *Butterworth Channel Impulses, Sample Rate=Cutoff Frequency*

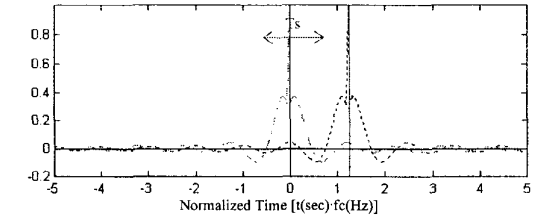


Figure 3.2 / *Elliptical Channel Impulses, Sample Rate=80%(Cutoff Frequency)*

It can be seen from these figures that symbol rates, R_s, of the form

$$R_s = 2f_c/k,$$

where k a positive integer, will yield relatively low amounts of ISI at the receiver input. Thus, the highest allowable symbol rate for both of these channels is $2f_c$. The results may not be this clear for all channels.

4.0 Calculation of the Mean Squared Distortion

Choosing the data rate through visual inspection of the channel impulse response may provide a good estimate of the ideal symbol rate. However, a situation may arise

in which a more rigorous derivation is required — the channel impulse response may be more complicated or the desire for a more precise answer may exist. In such a case, the *mean-square distortion* will provide a tractable measure of the degree of ISI present.

Referring to Figure 2.1, the signal provided to the receiver, denoted *q(t)*, is sampled every *Ts* seconds, where

$$T_s = 1/R_s,$$

and R_s is the symbol rate. Thus, the receiver output sample at time t_0 is

$$q_0 = \sum_n d_n x_n + n(t_0),$$

where d_n is the n^{th} data symbol, $x_n = x(t_0 + nT_s)$, and $n(t)$ is the filtered noise process ($\mathcal{F}\{n(t)\} = \mathcal{F}\{w(t)\}H^*(f)$).

The ISI is defined to be the portion of the above summation independent of x_0. Thus,

$$ISI = \sum_{n \neq 0} d_n x_n .$$

When this term is zero, the channel is deemed ISI free. However, when this term is non-zero it will degrade the performance of the communication system. The mean-square distortion (MSD) is a measure of the distortion the ISI term will contribute. The MSD is defined to be the ratio of the energy in the ISI term to that of the desired symbol in a binary system ($d_n \in \{-1,1\}$) [3, p. 292]:

$$MSD = \frac{1}{x_0^2} \sum_{n \neq 0} x_n^2 .$$

This equation can be written as a function of the symbol rate, R_s, by substituting

$$x_n = x(t_0 + nT_s) = x(t_0 + n/R_s)$$

into the above equation. Without loss of generality, we can let $t_0 = 0$. Then,

$$MSD(R_s) = \frac{1}{(x(0))^2} \sum_{n \neq 0} \left(x\left(n/R_s \right) \right)^2 .$$

This function can now be plotted for any channel impulse response, *x(t)*. Figures 4.1 and 4.2 demonstrate the MSD for both of the matched filter channels described in this chapter.

These figures demonstrate that visual inspection of the impulse responses provided a fair estimate of the operational data rates for both of the channels presented here. However, the calculation of the MSD for each channel provides additional information, not directly apparent when inspecting the impulse response. At a symbol rate of $2f_c$, the MSD for the butterworth and elliptical channel is -36 dB and -28 dB, respectively. Thus, neither channel will be distortion free at this symbol rate, and it can be expected that the elliptical channel will not perform as well as the butterworth channel when operated with this symbol rate.

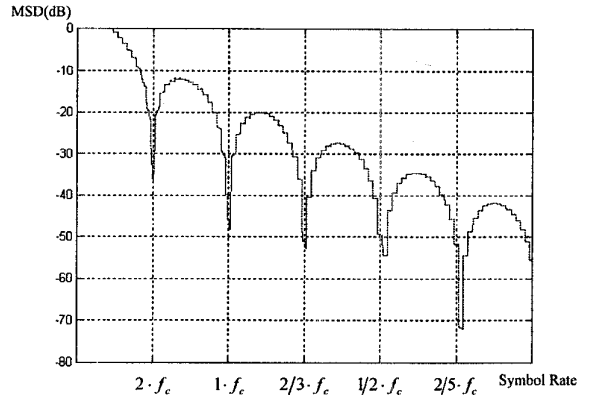


Figure 4.1 / *Mean-Square Distortion vs. Symbol Rate for the Matched Butterworth Channel*

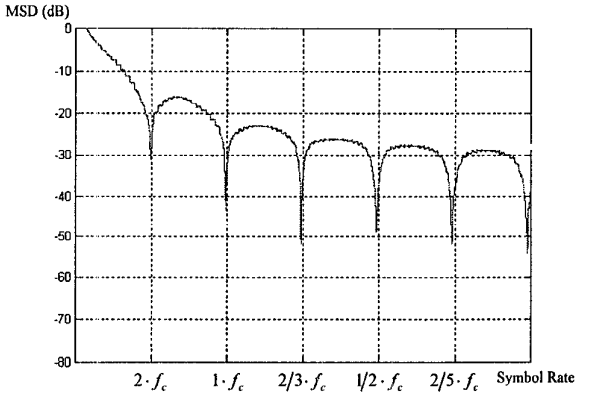


Figure 4.2 / *Mean-Square Distortion vs. Symbol Rate for the Matched Elliptical Channel*

5.0 Simulated 8PSK and 16QAM Performance

To demonstrate the performance of such a system operated at data rates selected using this procedure, the performance of both channels operated with 8PSK and 16QAM were approximated through Monte Carlo simulation. The data rates chosen for both the matched Elliptical channel and the matched Butterworth channel are *Rs/fc=1.90, 2.0* and *2.10*. These data rates were chosen to demonstrate the effects of overshooting and undershooting the optimum symbol rate of *Rs/fc=2.0* and to emphasize the importance of choosing the symbol rate wisely. The performance curves are presented in Figures 5.1 through 5.4.

As expected the performance of the butterworth channel, at a symbol rate of $2f_c$ was better than that of the elliptical channel. In fact, the butterworth channel performed very near theory while the elliptical channel exhibits significant degradation. If, such degradation is unacceptable in a particular application, a different data rate with a lower MSD must be chosen. This choice is

greatly simplified by Figure 4.2, which indicates those rates whose MSD is above and below -28 dB.

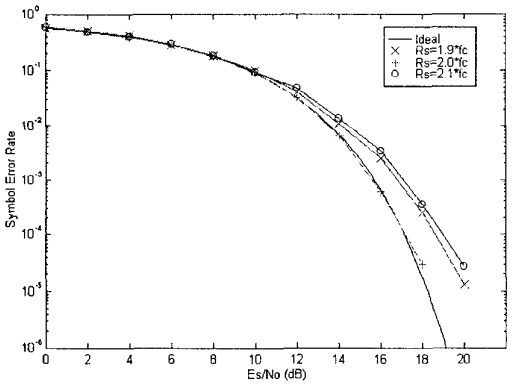


Figure 5.1 / *Monte Carlo Simulation Results for 8PSK in a Butterworth Matched Filter Channel*

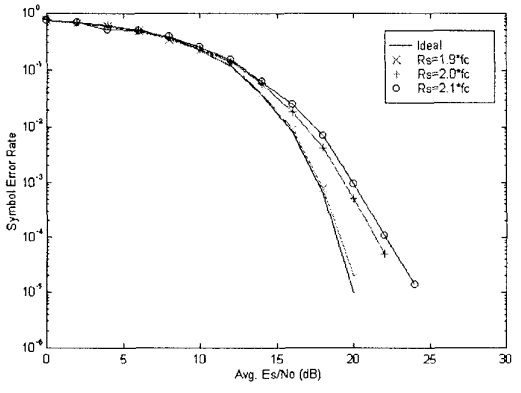


Figure 5.2 / *Monte Carlo Simulations of 16QAM in a Butterworth Matched Filter Channel*

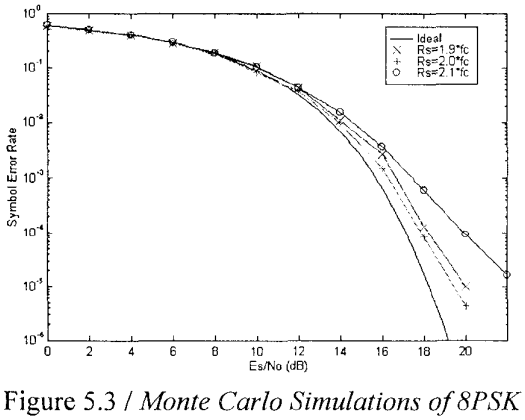


Figure 5.3 / *Monte Carlo Simulations of 8PSK in an Elliptical Matched Filter Channel*

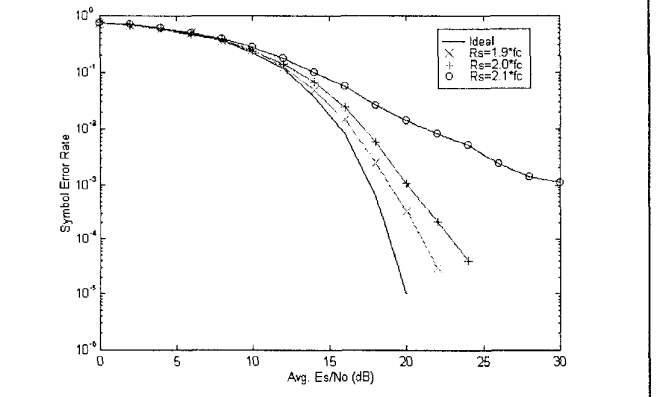


Figure 5.4 / *Monte Carlo Simulations of 8PSK in an Elliptical Matched Filter Channel*

6.0 Conclusions

Often, when the operational data rate of a communication system employing matched filtering must be increased without modification to the system filters, the inclination is to simply increase the rate until degradation results in unacceptable performance. As shown here, such an ad hoc approach in data rate selection is not optimum. Instead, a method has been proposed here which uses overall channel impulse response to better choose the rate of operation.

The rate can either be chosen by inspection of the impulse response, or through calculation of the mean square distortion (MSD). Visual inspection of the response will not provide as much information as the MSD approach, but will result in a fair estimate of the proper data rate. Although the MSD cannot be directly related to error rate performance, it provides a direct measure of the distortion to be expected from ISI, which significantly effects such performance.

This technique was demonstrated through simulation of butterworth and elliptical filters for 8PSK and 16QAM modulation. These simulation demonstrated both the ability to perform near theory with data rates significantly higher than would have been chosen ad hoc and the importance of choosing the proper rate precisely.

References

[1] Wolcott, Ted J. & W. P. Osborne "Uplink-Noise Limited Satellite Channel", *Proceedings of the IEEE Military Communications Conference*, November, 1995.

[2] Wolcott, Ted J., J. M. Gardner & W. P. Osborne, "Performance of High Rate 8PSK through NASA's TDRSS Network", NASA Report (submitted to ICC '96).

[3] Proakis, John G., *Digital Communications — 3rd Edition*, McGraw-Hill, Inc., New York, 1995.