A method for evaluating MPSK performance using a (M/2)PSK signal set

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Abstract
As data rate demands on existing satellite systems increase, the need to evaluate more bandwidth efficient modulation schemes such as M-ary Phase Shift Keying (MPSK) becomes important. However, the unavailability of higher order modems operational at high data rates and the construction of such a complex device for testing purposes can make evaluation difficult. In support of NASA grant NAG 5-1491, New Mexico State University has developed a method of evaluating the performance of 8PSK in a satellite channel using a QPSK signal set. This paper will discuss this technique and will generalize the results to the use of any (M/2)PSK signal set to evaluate MPSK modulation. Verification by both computer simulation and hardware implementation will also be discussed.

Background

The rapidly increasing demands on NASA's Tracking and Data Relay Satellite System (TDRSS) has presented the need for more bandwidth efficient modulation scheme to increase data rate capabilities in the existing TDRSS channel. One such scheme is Octal-Phase Shift Keying (8PSK) with Trellis Coded Modulation (TCM). However, before 8PSK TCM is considered, the performance of uncoded high rate 8PSK must be measured. Due to time constraints and the unavailability of a 8PSK modem capable of operating at rates high enough to stress the TDRSS channel, the research team at New Mexico State University's Manuel Lujan Jr. Center for Space Telemetering and Telecommunications Systems was forced to develop a method of testing 8PSK with the existing TDRSS QPSK equipment.

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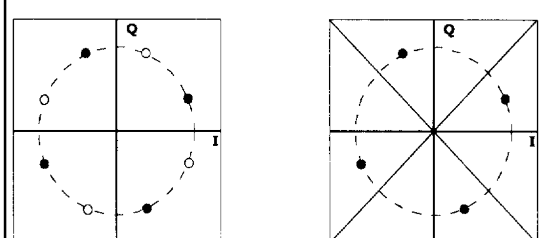
Approach

The method developed uses the QPSK signal set viewed by the receiver as a limited 8PSK signal set. The 8PSK constellation is divided into four vacant and four occupied decision regions (see Figure 1). Received signal vectors are declared to be correct or in error based on their location in these decision regions. A symbol error rate is measured by counting those vectors found to be in vacant region as symbol errors. Likewise, any received vector located in any of the four occupied regions are said to be correct. Symbol error rate calculations were performed with custom software on a personal computer which processed the in-phase and quadrature vector components, supplied by a commercial vector analyzer.



Simplified Constellation **Decision Regions**

Figure 1 / Constellation and Decision Regions

In 1968, Gooding presented a method for monitoring the error rate for digital receivers which is similar to this

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technique [1]. His extrapolated error rate technique employs modified decision regions, similar to those shown in Figure 1, to generate a pseudo-error rate, and based on this rate, the actual error rate is extrapolated. Similar approaches were presented by Scholz et. al. [2] and Kostic [3]; however, none of these techniques apply to performance evaluation of M-ary modulation without an M-ary modem.

The hardware required to implement the reduced symbol set technique is relatively simple — consisting of the PC, vector analyzer with filters and MPSK demodulator — compared to the construction of a high rate 8PSK modem. The signal is first demodulated, then matched filtered at the input to the vector analyzer. The analyzer samples the input and provides the personal computer quantized in-phase and quadrature information for processing. The vector analyzer must be triggered with a clock synchronous with the data. Such a clock is available in most communication systems at the output of a bit-synchronizer. Due to the capabilities of a personal computer, only a very small fraction of the received data can be processed as data rates approach and exceed hundreds of millions of symbols per second. This increases the time required for evaluation, especially as the error rates decline.

In addition to the theoretical verification of this approach, simulation and hardware modeling results will be presented. The method will also be generalized for performance evaluation of any MPSK system using a (M/2)PSK signal set, for M>2.

Theory

It can be shown by applying the union bound, that the symbol error rate of 8PSK in additive white Gaussian noise can be bounded by [4]

$$Q\left(\sqrt{\frac{2Es}{N_0}}\sin\frac{\pi}{8}\right) \le P_E \le 2Q\left(\sqrt{\frac{2Es}{N_0}}\sin\frac{\pi}{8}\right),$$

where Es is symbol energy, N_0 is the single-sided noise power spectral density in Watts/Hertz, and

$$Q(x) \equiv \int_x^{\infty} \frac{1}{2\pi\sigma^2} e^{-\frac{u^2}{2}} du.$$

The above equation for probability of error assumes that the full 8PSK signal set was transmitted and that errors are declared any time received symbols fall outside the corresponding decision region. In using the limited symbol set of QPSK, the 8PSK constellation is divided into four occupied and four vacant regions. (Figure 1). The occupied regions are those areas of the constellation that the QPSK signal set would normally occupy. A symbol error is then declared if a symbol falls in the empty regions of the constellation. Notice that this excludes errors that occur when a signal vector falls in an incorrect occupied region.

Since the symbols in the set are assumed to be equally probable, without loss of generality, the two dimensional probability density function can be described for a received vector assuming signal S_1 (θ=0) was transmitted. The pdf for the received signal vector can be expressed as [4, pg. 261]

$$p(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-2\alpha Es)^2+y^2}{2\sigma^2}},$$

where α is the channel attenuation, and

$$X = 2\alpha E_s + \mathrm{Re}(N)$$

$$Y = \mathrm{Im}(N).$$

The joint probability density function can be expressed in terms of phase and magnitude by a change of variables:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Integration of the resulting pdf $p(r,\theta)$, over the range of r leads us to an expression for $p(\theta)$ [5],

$$p(\theta) = \frac{1}{2\pi} e^{-\gamma}(1 + \sqrt{4\pi\gamma}\cos\theta e^{\gamma\cos^2\theta}(1 - Q(\sqrt{2\gamma}\cos\theta))$$

where

$$\gamma \equiv \alpha^2 Es/No$$

is the signal-to-noise ratio per symbol.

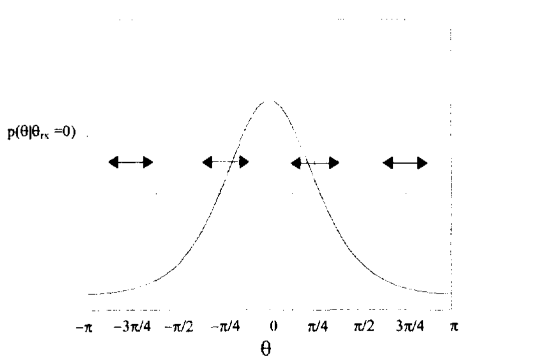

Figure 2 / P(θ) w/ vacant constellation regions

The probability of error for the reduced signal set can be calculated as the integral of $p(\theta)$ over the ranges of θ that correspond to vacant regions (Figure 3) in the 8PSK constellation:

$$P_{Er} = 2\int_{\pi/8}^{3\pi/8} p(\theta)d\theta + 2\int_{5\pi/8}^{7\pi/8} p(\theta)d\theta$$

If the signal-to-noise ratio is varied, the probability of symbol error, P_{er}, is subject to the same bounds as pure 8PSK signaling. That is,

$$Q\left(\sqrt{\frac{2Es}{N_0}}\sin\frac{\pi}{8}\right) \le P_{Er} \le 2Q\left(\sqrt{\frac{2Es}{N_0}}\sin\frac{\pi}{8}\right)$$

Figure 3 shows the performance of the reduced symbol set versus the theoretical bounds of 8PSK. At relatively high SER (.1 Err/Sym), it can be seen that the SNR for the reduced symbol set is within half a dB of the performance of true 8PSK. Therefore, the exclusion of errors that occur when a signal vector falls in an incorrect occupied region has a negligible effect on performance at signal-to-noise ratios of interest. It should also be noted that as SNR decreases ($SNR \longrightarrow -\infty$), the received symbol set becomes uniformly distributed on the constellation and the performance approaches the lower bound of 8PSK signaling which is $P_{Er} \rightarrow 1/2$.

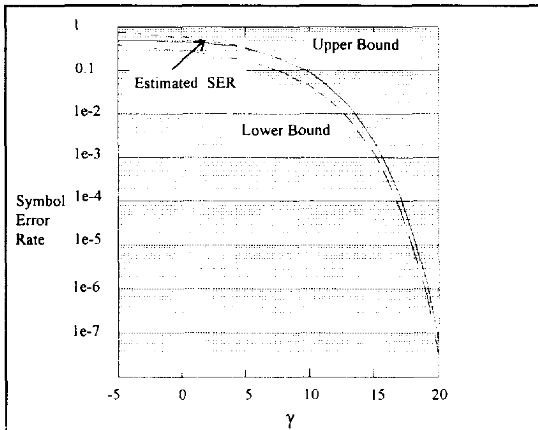

Figure 3 / Symbol Error Rate

Generalization to MPSK

These results can easily be generalized to the use of (M/2)PSK symbol sets to estimate the performance of any MPSK system. The probability of error for such a system (P_{EMr}) can be calculated as,

$$P_{EMr} = 2\sum_{i=1}^{M/4}\left[\int_{(4i-3)\pi/M}^{(4i-1)\pi/M} p(\theta)d\theta\right].$$

The resulting symbol error rate curve is bounded by the lower and upper error bounds of the corresponding MPSK signal set:

$$Q(\sqrt{\frac{2Es}{N_0}}sin\frac{\pi}{M}) \le P_{EMr} \le 2Q(\sqrt{\frac{2Es}{N_0}}sin\frac{\pi}{M}).$$

It can be seen that for symbol error rates below .1 errors/symbol, the approximated performance is within one half dB of the theoretical performance of the MPSK signal set. This is demonstrated in the following curve, which shows the theoretical upper bound and the approximated (M/2) performance for M=4,8, and 16 PSK symbol sets.

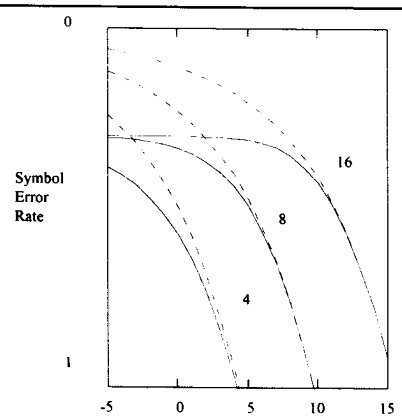

Figure 4 / Approximate SER and Upper Bounds for M=4, 8, 16PSK

Technique Verification

Two methods for verification of this technique, hardware modeling and computer simulation, were applied. This section will present these results.

Simulations were performed using the Signal Processing WorkSystem® (SPW™) software package on a Sun Sparc 10 workstation. Performance of 8PSK modulation was measured over an AWGN channel, using a QPSK signal set, as described above. The system model is shown in Figure 5, in which n(t) represents AWGN.

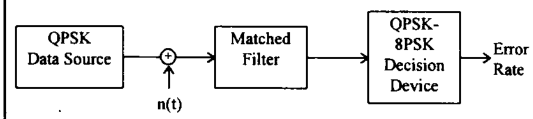

Figure 5 / System Model

The resulting simulated symbol error rate curve is presented in Figure 6 below. Note that, in accordance with the theory developed previously, system performance quickly approaches that of true 8PSK.

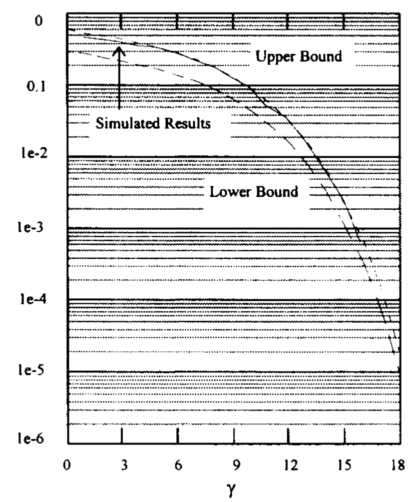

Figure 6 / Simulated System

The system represented in Figure 5 was also implemented in hardware. Data was provided by a commercial link analyzer. The white noise was created by a variable wideband noise source. Matched filtering was performed by a programmable transversal filter (PTF), and the symbol error decisions were performed through the combination of a vector analyzer and personal computer with custom software. Figure 7 shows the hardware configuration, and the measured performance curve is compared to theory in Figure 8. Note that the measured performance is slightly worse than the theoretical upper bound due to implementation losses in the test hardware.

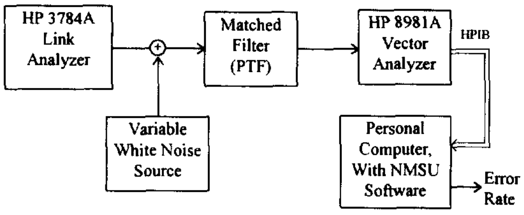

Figure 7 / Hardware Configuration

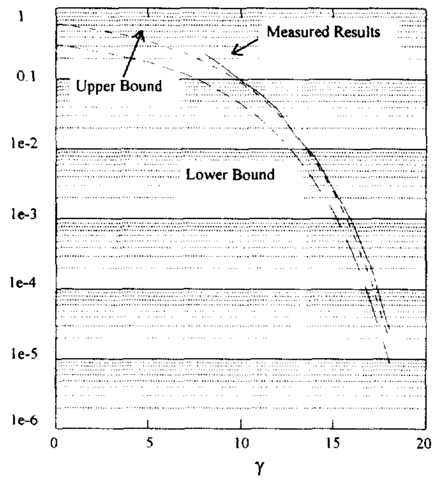

Figure 8 / Hardware Performance

Conclusion

A technique has been described allowing the performance verification of 8PSK modulation using a QPSK signal set. This method allows high data rate investigation of the more bandwidth efficient modulation scheme without the need for a high rate 8PSK modem. The technique was also extended for application to MPSK, using an (M/2)PSK signal set for M>2.

Such an approach only allows approximation to the performance of a true MPSK system. However, due to the dominance of nearest-neighbor errors at high signal-to-noise ratios, the approximation is excellent at error rates of interest.

Special consideration must be made when applying this technique to nonlinear channels on account of the decreased number of possible data transitions. Due to the nature of the method, however, the worst case transitions are included; hence, very little degradation in the approximation occurs.

It must be noted that the full generality of this approach has not been explored. With very little modification to the theory, higher order modulation schemes could be evaluated with even smaller signal sets — 16PSK with a QPSK signal set for example. However, the loss of accuracy is unknown, especially when applied to nonlinear channels.

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