3-1995

Quantization of Signals and Metrics in the Decoding of 8-PSK TCM

Michael D. Ross
New Mexico State University - Main Campus

William P. Osborne
osborne@engr.siu.edu

Quantization of Signals and Metrics in the Decoding of 8-PSK TCM

Michael D. Ross William P. Osborne

Manuel Lujan Jr. Center for Space Telemetering
and Telecommunications Systems,
New Mexico State University,
Las Cruces, New Mexico 88003

ABSTRACT The performance of rate-2/3 8-PSK Trellis Coded Modulation was investigated as a function of the numerical resolution of the metrics and signal vectors used in decoding by the Viterbi Algorithm. It was found that a 16-state Ungerboeck code using 5-bit I and Q with 7-bit metrics achieved roughly the same performance as pragmatic TCM. The findings were applied to the digital logic design of a TCM decoder.

1. Introduction

Trellis Coded Modulation (TCM), pioneered by Ungerboeck [1,2] has received widespread interest as a bandwidth efficient forward error correction scheme. Phase shift keying (PSK) offers the advantage of constant amplitude modulation, required on satellite channels, and other channels consisting of non-linearities. TCM is optimally decoded using the Viterbi Algorithm [3] with floating point signal vectors and metrics. Because of the fact that in any technology it is impossible for the speed of floating point calculations to match the speed of integer arithmetic, some form of quantization will be used, representing the required quantities as finite-bit numbers. Furthermore, in the construction of application specific hardware to implement the algorithm, the numerical precision of the involved quantities should be carefully considered, as this will significantly affect the size of the hardware. Quantization will always result in some degradation of error correcting performance, but given an appropriate quantization scheme, performance can be made arbitrarily close to unquantized performance by making quantization sufficiently fine.

Quantization of the received signal vector may take a number of forms, the most prevalent being phase only quantization, phase radius quantization, and rectangular coordinate (I and Q) quantization, as reasonable implementations of these forms can be built in hardware. Phase only quantization for PSK has been discussed in other works [4-9]. This paper addresses the effects of quantization of the signal vectors and metrics on the performance as well as the architecture of the Viterbi decoder. The findings were applied to the logic-level design of a complete TCM decoder at NMSU.

2. The Structure of the Viterbi Algorithm

A top level diagram for a Viterbi decoder is shown in figure 1. The branch metric calculator calculates a metric for each symbol in the symbol set, relative to the received symbol. The received signal could be hard or soft-decision binary, or a signal vector from a QAM signal set. The Add-Compare-Select unit maintains a state metric to each path, and revises the state metrics by adding in the branch metrics, and selecting the minimum metric at each state. The path memory unit maintains a running history of the most likely path to each state, accepting and rejecting paths according to the decisions made by the add-compare-select-unit. The issues of signal set quantization and branch metric quantization, discussed in this paper do not affect the architecture of the path memory unit, but profoundly affect the architecture of the branch metric calculator and the add-compare-select unit.

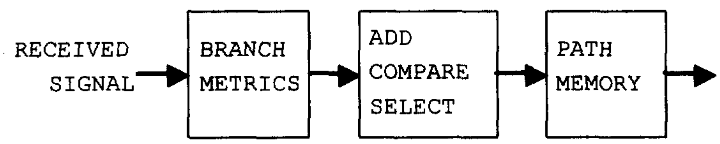

Figure 1 *Top level diagram of a Viterbi Decoder.*

3. Signal Vector Quantization.

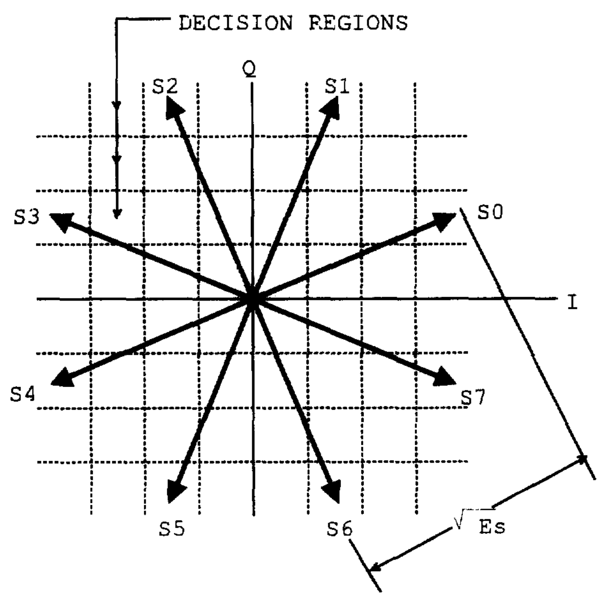

Figure 2 *8-PSK signal constellation with I and Q quantization*

The 8-PSK signal constellation is shown in figure 2. The analog to digital conversion of the I and Q components imposes quantization to rectangular grid points, also shown in the figure.

Here, 16-level I and 16-level Q are illustrated. Because the I and Q components will be represented as binary numbers in the decoding hardware, it is necessary that the number of quantization values (for I or Q) be a power of two. From previous studies [10], it is known that 8-level (3-bit) quantization seriously degrades the performance of pragmatic TCM, whereas the performance of a system using 8-bit I and Q is close to that of an unquantized system. Therefore, for 8-PSK TCM decoders, we expect that the I and Q components should be quantized to a resolution of no more than 4 bits, but no more than eight.

4. Simulations

The block-oriented systems simulator (BOSS), a commercially available software package was used to test the complete logic design of a Viterbi decoder for the 16-state rate 2/3 8-PSK Ungerboeck code. Of interest were the effect of signal vector and metric quantization, as well as the verification of the decoder logic, and the overall bit-error-rate performance of the decoder. Table I shows the bit error rates of various the resolutions of signal vectors and metrics, obtained at by simulating at Es/N0 = 10dB. As can be seen, the performance of any particular combination cannot be easily predicted by studying the effect of I and Q quantization and metric quantization independently. This decoder was designed to have a bit error rate comparable to the emerging "pragmatic" standard of Viterbi [11]. To do this, a bit error rate of less than 3×10^{-6} at $E_s/N_0 = 10$dB was necessary. As can be seen from the table, the 16-state Ungerboeck code accomplishes this with 5-bit I and Q and 7-bit metrics. Unfortunately, it was difficult to obtain statistically tight results, since a trial of 5 million symbols is barely sufficient to measure a bit error rate of 10^{-6}. For 8-bit I and Q, the simulation detected no errors in a trial of one million symbols, showing that it is not unreasonable to expect performance which is slightly better than that of pragmatic TCM.

Signal vector quantization and metric quantization are not interchangeable. Usually the requirement for metric quantization is driven by the degree of signal set quantization. For example, the use of N-bit I and Q components results in 2N-bit square difference terms, two of which are added to produce a 2N+1 bit metric. Therefore, if 4-bit I and Q quantization were decided on, a 9-bit metric represents no further compromise of performance, that is nine bits is the maximum useful metric resolution for 4-bit I and Q, whereas 11-bit I and Q is the maximum useful metric resolution for 5-bit I and Q.

Table I shows the bit error rate as a function of metric resolution and I and Q resolution, at E_s/N_0=10dB. The results of Table I show that if the metrics are quantized to a low level of resolution, an increase in I and Q resolution will not necessarily result in an improvement in performance unless also accompanied by an increase in metric resolution. As can be seen, with 5-bit metrics the performance of 5-bit I and Q is worse than the performance of 4-bit I and Q. Also, we can see from the chart that with 4-bit I and Q, the performance with maximum metrics is 3×10^{-6}, which is comparable to the performance of the multimode codec [10], which used the pragmatic standard with 4-bit I and Q and 4-bit metrics. By using 5-bit I & Q, the 16-state Ungerboeck code improves its performance by approximately a factor of two, achieving performance comparable to unquantized pragmatic TCM. These results were based on trials of 5 million symbols, except for the three results presented for 4- and 5-bit metrics, which were based on 1 million symbols. One of the problems encountered is that 5 million symbols may not have been a sufficient simulation length to obtain statistically tight results. In running the final performance tests for the decoder, a different random sequence was used and a bit error rate of 3.8×10^{-6} was obtained. The variance for the final performance trial, which also used 5 million symbols was calculated at 1.2×10^{-6}. In light of this, a decision to use 5-bit I and Q and 7-bit metrics probably represents over-design. However, since the logic has been worked out for these parameters, designing a simplified version of the circuit, if desired, should not be a problem.

METRIC RESOLUTION	I AND Q RESOLUTION 4-BIT	5-BIT
4-BIT	4.15E-5	
5-BIT	6.5E-6	1.25E-5
6-BIT	3.7E-6	4.8E-6
7-BIT	3.4E-6	2.1E-6
8-BIT	2.8E-6	1.8E-6
9-BIT	3.1E-6	1.2E-6
10-BIT		
11-BIT		1.2E-6

Table I *Decoder bit error rate at Es/N0=10dB.*

The complete logic for a Viterbi decoder to decode the rate 2/3 8-PSK 16-state Ungerboeck TCM code, using 5-bit I and Q signal vectors, and 7-bit branch metrics was designed and simulated. Combinational logic was designed to calculate the branch metrics. The performance of the decoder is shown in Figure 3. The variance of the result was calculated by dividing the simulation time into ten equal intervals, and calculating the sample variance as

$$\sigma_s = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(x_i-\bar{x})^2}.$$

The variance of the mean was calculated as $\sigma=\frac{\sigma_s}{\sqrt{10}}$.

5. Conclusion

From table 1, we see that the performance of 4-bit I and Q with 4-bit metrics is not even within the order of magnitude of the performance of the pragmatic code. The use of 4-bit I and Q with 6-bit metrics could be an acceptable option, although the performance falls slightly short of pragmatic TCM. The use of 5-bit I and Q with 7-bit metrics achieves performance comparable to pragmatic TCM. At E_s/N_0 = 10dB, it can be seen from figure 3 that the decoder has nearly approached the asymptotic error rate for pragmatic TCM, and achieves performance equivalent to quantized pragmatic TCM.

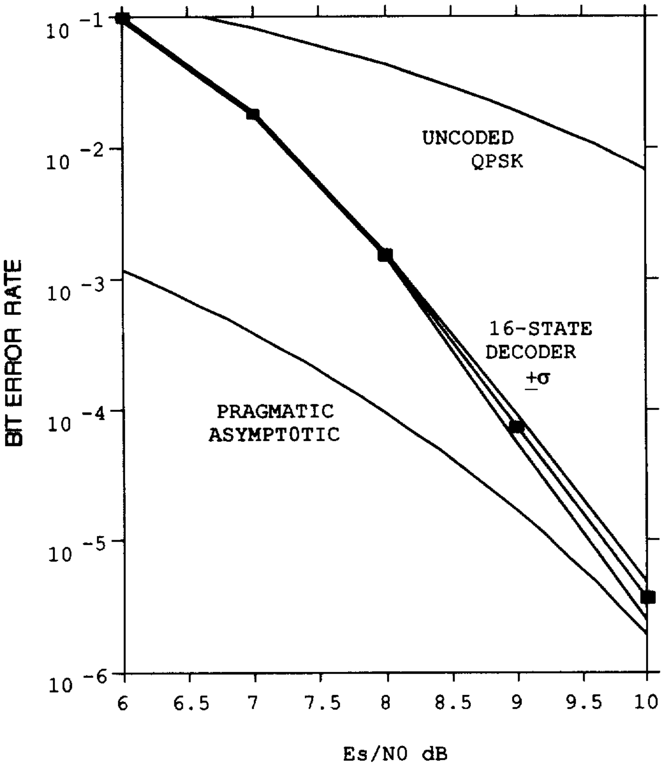

Figure 3. *Performance of 16-state 8-PSK TCM using 5-bit I and Q and 7-bit branch metrics.*

5. Conclusion

From table 1, we see that the performance of 4-bit I and Q with 4-bit metrics is not even within the order of magnitude of the performance of the pragmatic code. The use of 4-bit I and Q with 6-bit metrics could be an acceptable option, although the performance falls slightly short of pragmatic TCM. The use of 5-bit I and Q with 7-bit metrics achieves performance comparable to pragmatic TCM. At E_s/N_0 = 10dB, it can be seen from figure 3 that the decoder has nearly approached the asymptotic error rate for pragmatic TCM, and achieves performance equivalent to quantized pragmatic TCM.

REFERENCES

[1] Ungerboeck, Gottfried, "Trellis Coded Modulation with Redundant Signal Sets, Part I: Introduction," *IEEE Communications Magazine*, Vol. 25, No. 2, pp. 5-11, February 1987.

[2] Ungerboeck, Gottfried, "Trellis Coded Modulation with Redundant Signal Sets, Part II: State of the Art," *IEEE Communications Magazine*, Vol. 25, No. 2, pp. 12-21, February 1987.

[3] Viterbi, A.J., "Convolutional Codes and their performance in Communication Systems," *IEEE Transactions on Communication Technology*, vol. CT-19, pp. 751-771, October 1971.

[4] Parsons, R.D., and S. G. Wilson, "Polar Quantizing for Coded PSK Transmission," *IEEE Transactions on Communications*, vol. 38, no. 9, pp. 1511-1519, September 1990.

[5] Frank Carden, Michael Ross, Brian Kopp and William Osborne, Fast TCM Decoding: Phase Quantization and Integer Weighting, *IEEE Transactions on Communications*, vol. 42,no. 3 pp. 808-812, Mar 1994.

[6] Michael Ross and William P. Osborne, "On the Calculation of the Exponential Bound Parameter for Phase Quantized 8-PSK," *IEEE Transactions on Communications, to be published.*

[7] Frank Carden, "A Quantized Euclidean Soft-Decision Maximum Likelihood Sequence Decoder: A Concept for Spectrally Efficient TM Systems," *Proceedings of the International Telemetering Conference*, Vol. XXIV, pp. 375-384, October 1988.

[8] Frank Carden and Brian Kopp, "A Quantized Euclidean Soft Decision Maximum Likelihood Sequence Decoder for TCM," *IEEE Military Communications Conference*, Vol. 2, pp. 679-682, October 1988.

[9] Frank Carden and Michael Ross, "A Spectrally Efficient Communication System Utilizing a Quantized Euclidean Decoder," *Proceedings of the International Telemetering Conference*, Vol. XXV, pp. 575-582, October 1989.

[10] William Osborne, Frank Carden, Brian Kopp, and Michael Ross, "Multi-Mode Modem/Codec Designs", *AIAA Communication Satellite System Conference*, 1991.

[11] Viterbi, Andrew J., Jack K. Wolf, Ephraim Zehavi, Roberto Padovani, "A Pragmatic Approach to Trellis-Coded Modulation," *IEEE Communications Magazine*, Vol. 27, No. 7, pp. 11-19, July 1989.