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Network Selection and "Path-Dependent" Coevolution

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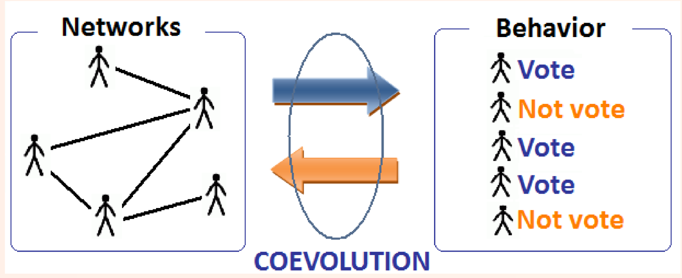
Coevolution Is Everywhere

Coevolution is:



Examples in political science:

Networks		Behavior
Peer groups	⇌	Political behavior
Cosponsorship	⇌	Legislative behavior
Party coalitions	⇌	Manifesto content
Alliances	⇌	**Conflict behavior**

Coevolution and Multiple Steady States

What about the long-run characteristics of systems with coevolution?

- Consider a longitudinal process where N **actors** choose **behavior** in each time period t.
- Define **state** (roughly) by the set of outcome-behaviors of all the actors.
- We can define the **equilibrium** of the system as the steady-state distribution of N actors' types.

▶ **Our goal (1): Establish theoretically that systems with coevolution can more easily generate multiple equilibria than systems without coevolution.**
(Due to violation of assumptions for the Ergodic Theorm.)

▶ **Our goal (1'): Establish theoretically that systems with coevolution are more likely history-dependent than systems without coevolution.**

History Dependence

Rigorous definitions of "history matters".

- **3 Types of History-Dependent processes** (Page 2006):
  1. **State dependence:** the outcome at t depends *only* on the currently observable state, implying Markov processes.
  2. **"Phat" dependence:** the outcome at t depends on the past states (history) but the order of past states doesn't matter.
  3. **Path dependence:** the outcome at t depends on the past states *and* the sequence of the past states.

▶ **Our goal (2): Assess the empirical significance of coevolutionary political dynamics.**

▶ **Our goal (3): Develop an empirical strategy to estimate and evaluate history-dependence, merging the theoretical and empirical models of coevolution.**

Selected References and Funding

(1) Hays, Jude C., Aya Kachi and Robert Franzese Jr. 2010. "A Spatial Model Incorporating Dynamic, Endogenous Network Interdependence: A Political Science Application." *Statistical Methodology* 7(3):406–428.
(2) Page, Scott E. 2006. "Path Dependence." *Quarterly Journal of Political Science* 1:87-115.
(3) Page, Scott E. 2007. "Type Interaction Models and The Rule of Six." *Economic Theory* 30(2):223-241.
(4) Walker, Robert W. 2007. "Path/Phat/State Dependence in Observation-Driven Markov Models." *Working paper*

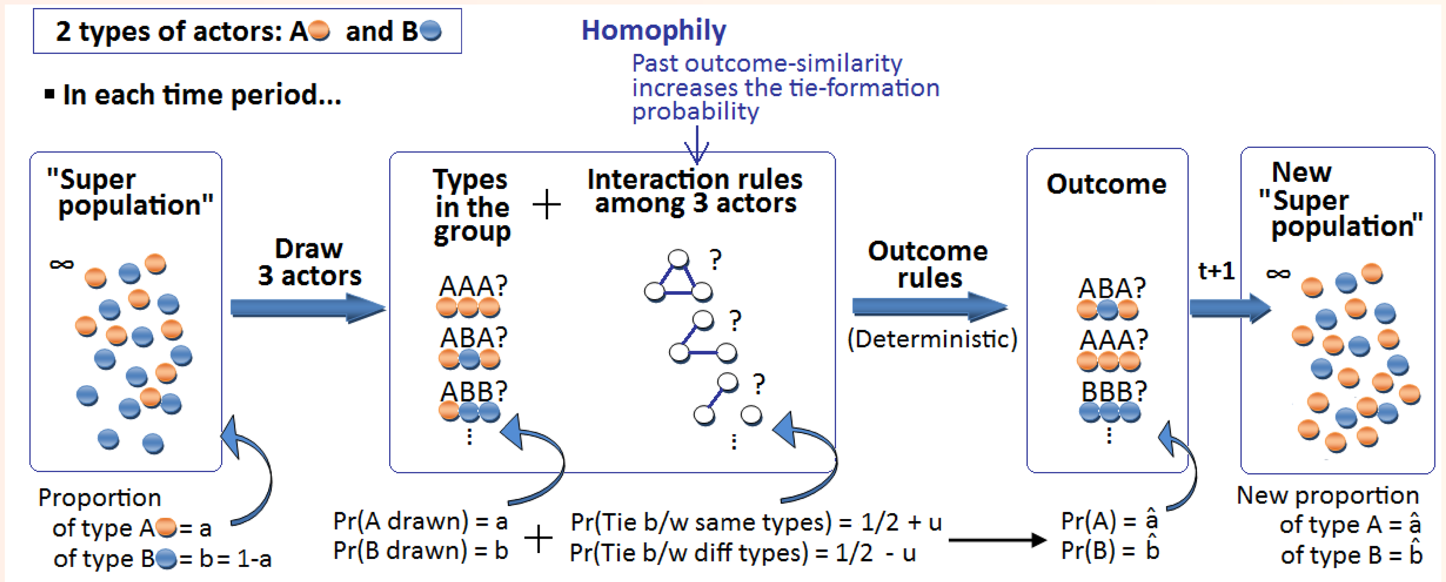
(5) Supported by NSF Grant SES-0851084, in part by NSF Grant 0318045.

Theoretical Model (*Type-Interaction*)

Purpose of the Theoretical Model

- A simple model to derive conditions under which systems have multiple equilibria.
- As stated in Page (2007), the potential for multiple equilibria is important:
  1. The set of conditions for multiple equilibria is a reasonable measure of systemic complexity.
  2. Allows us to analyze how initial/past states affect the attainment of one of the equilibria. = A simple model of phat/path-dependent processes.

Type-Interaction Model



Difference Equations and Equilibria

$$\begin{pmatrix} a_{t+1} & b_{t+1} \end{pmatrix} = \begin{pmatrix} a_t & b_t \end{pmatrix} \begin{pmatrix} z_1 & 1-z_1 \\ z_2 & 1-z_2 \end{pmatrix},$$

where z's are polynomials of a and b.
⇒ Multiple steady-states (where $a_{t+1} = a_t$ and $b_{t+1} = b_t$) for a and b.

Take-Away Points (Theory)

Key Finding & Claim

- With coevolution, a very simple type-interaction model (with deterministic outcome rules) can generate multiple equilibria.
- If the coevolutionary dynamic exist in given data (= an empirical question), then the system is most likely phat/path-dependent, b/c the type-switching probabilities are changing over time.

Comparison with Page (2007)'s "Rule of Six"

- Page derives, in his non-coevolutionary interaction model, that at least 3 types are necessary for a system of 3 actors to have multiple equilibria, if the outcome-rules are deterministic. Hence the "rule of six" (3+3).
- We show that, with coevolution, multiple equilibria arise even with 2 types, 3 actors, and deterministic outcome rules. Hence a "rule of less-than-six"–showing the level of complexity.

Where to Go with This?

- An important and difficult empirical question arises;
  To what extent does history matter?–How sensitive are equilibrium distributions to the past states?)

Our Statistical Model (*Spatial-Logit + P-Star*)

[Discrete-Time Markov Model]

- **To explain the behavioral-type switching,** *a simple spatial-logit model*

$$Pr(\text{type}_{i,t} = 1) = logit(\beta_0 + \beta_1 \text{type}_{i,t-1} + \beta_2 \mathbf{W} \cdot \textbf{type}_{t-1}).$$

- **To explain the tie-formation,** *a simple p-star model (independent dyads)*

$$Pr(\text{tie}_{dyad-i,t} = 1) = logit(\gamma_0 + \gamma_1 \text{tie}_{dyad-i,t-1} + \gamma_2 I(\text{type match}_t)),$$

where $I(\text{type match}_t)$ indicates whether the types in the given dyad were the same in the previous period. The term captures "homophily".

⇒ **Estimated $\hat{\beta}_2$ and $\hat{\gamma}_2$ indicate the existence of coevolutionary dynamics.**

NB: SIENA (Snijders et al.) = Continuous-time Markov model.

Application

Alliances and the Conflict Behavior of Major Powers (1900-1950)

[Markov Model]	Discrete-Time (MATLAB) Ours		Cont.-Time (SIENA) Snijders et al.	
	Alliance Networks	MIDs Behavior	Alliance Networks	MIDs Behavior
Temp lag	4.93*** (0.27)	1.45*** (0.27)		
Previous MIDs similarity	**-0.51*** (0.27)		**-3.52***** (0.60)	
Previous Alliance tie		**0.85***** (0.31)		**1.06**** (0.53)
Loglikelihood	-237.4	-179.5		

- (1) Evidence of heterophily–pacific powers are more likely to ally with aggressive powers, and (2) conflict behavior diffuses through alliances.

Take-Away Points (Empirical Strategy)

- The combined spatial-logit and p-star (ERGM) model provides a relatively simple way to assess the empirical significance of coevolutionary dynamics.
- The combined spatial-logit and p-star (ERGM) model is directly related to the theoretical Markov interaction model.
  ⇒ This connection is crucial to analyze empirically history dependence.
- Connecting theory and empirics would be much more difficult in a continuous-time framework.

Empirical Analyses of History Dependence

Next Steps

- **Short Run:** To add stochastic outcome rules to the theoretical model in form conducive to generating the statistical model.
- **Medium Run:** To theorize more fully the relationship between multiple equilibria and history dependence.
- **Long Run:** To develop statistical tests for various types for history dependence, including **path dependence**.