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2011

Tempered Representations (Seminar)

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Jantzen, Chris, "Tempered Representations (Seminar)" (2011). *Miscellaneous (presentations, translations, interviews, etc)*. Paper 48. http://opensiuc.lib.siu.edu/math_misc/48

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TEMPERED REPRESENTATIONS

CHRIS JANTZEN (NOTES TAKEN BY JOE HUNDLEY)

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1. Tempered Representations

Tempered representations play important role in automorphic forms, harmonic analysis, and also representation theory itself. We consider tempered representations of some p-adic group G.

The category of admissible representations of G is filtered according to the growth properties of matrix coefficients. This can be described in terms of inclusions of the corresponding duals.

$$\widehat{G}_{sc} \subset \widehat{G}_{ds} \subset \widehat{G}_{temp} \subset \widehat{G}_{adm}$$

supercuspidal \Rightarrow discrete series \Rightarrow tempered \Rightarrow admissible
compact support \Leftarrow $L^2 \Leftarrow L^{2+\varepsilon} \forall \varepsilon \Leftarrow$ (no condition)

Representations in each class are described in terms of representations induced from the previous step. Supercuspidals are treated as "atomic."

The tempered to admissible step is the Langlands classification, and is general. For the others, we want to work specifically with classical groups. Moeglin-Tadic have a description of that first step.

The discrete series to tempered step has a description in terms of R-groups, based on work of Goldberg. But it's not convenient for certain applications. The goal of this work is to produce an alternate description which is more convenient for his purposes.

2. Classical groups

2.1. GL(n, F). If P = MU is a standard parabolic subgroup, then $M \cong GL(n_1) \times \cdots \times GL(n_k)$, for some partition $n = n_1 + \cdots + n_k$ of n.

Classical groups, for us, means Sp(2n, F), SO(2n, F) and O(2n, F).

We'll write everything out tonight only in the case of the symplectic group.

Date: February 17, 2011.

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A parabolic subgroup P = MU satisfies $M \cong GL(n_1) \times \cdots \times GL(n_k) \times Sp(2n_0)$, for some partition $n = n_0 + n_1 + \cdots + n_k$ of n. We define Sp relative to the form for the matrix

$$J' = \begin{pmatrix} & & & & & & 1 \\ & & & & & & 1 \\ & & & 1 & & & \\ & & -1 & & & & \\ & & \cdots & & & & \\ -1 & & & & & \end{pmatrix}$$

and take the torus to consist of all the diagonal elements in the group and the Borel to consist of all the upper triangular ones.

Then standard parabolics are block upper triangular, standard Levis are block diagonal.

3. Discrete series

Take ρ an irreducible unitary supercuspidal of GL(n,F) and $b-a\in\mathbb{N}\cup\{0\}$ and then the representation

$$\operatorname{Ind}_{P}^{G}\left(|\det|^{b}\rho\otimes|\det|^{b-1}\rho\dots|\det|^{a}\rho\right)$$

of GL(kn, F) has a unique irreducible subrepresentation denoted $\delta([\nu^a \rho, \nu^b \rho])$, which is essentially discrete series, and every discrete series is of this form.

For Sp(2n, F), Moeglin and Tadic have classified discrete series in terms of triples, (Jord, σ, ε).

Roughly, $\delta \hookrightarrow \operatorname{Ind}_P^G \delta([\nu^{-a_1}\rho_1, \nu^{b_1}\rho]) \otimes \cdots \otimes \delta([\nu^{-a_k}\rho_k, \nu b_k\rho_k]) \otimes \sigma$, where σ is the σ above, and is unique.

Jord consists of $\{(\rho_i, 2a_i + 1), (\rho_i, 2b_i + 1)\}$, plus some terms which depend on σ . Note that each can appear at most once.

Suitably interpreted, ε is a function Jord $\to \{\pm 1\}$ (in order to view it this way "there is an equivalence relation floating around in the background").

Key property: for $(\rho, a) \in \text{Jord}$, let a_- be the next smaller number such that $(\rho, a_-) \in \text{Jord}$. Then $\varepsilon(\rho, a) = \varepsilon(\rho, a_-) \iff \delta \hookrightarrow \delta([\nu^{\frac{-a_-+1}{2}}\rho, \nu^{\frac{a-1}{2}}\rho]) \rtimes \theta$ for some irreducible representation θ .

4. Tempered Representations

Suppose T irreducible tempered of Sp(2n, F). Then

$$T \hookrightarrow \delta\left(\left[\nu^{\frac{-c_1+1}{2}}\rho_1,\nu^{\frac{c_1-1}{2}}\right]\right) \times \dots \times \delta\left(\left[\nu^{\frac{-c_k+1}{2}}\rho_k,\nu^{\frac{c_k-1}{2}}\rho_k\right]\right) \rtimes \delta$$

some supercuspidals and some discrete series δ of a smaller symplectic group.

We would prefer to have a characterization more along the lines of Moeglin-Tadic.

To extend Moeglin-Tadic, we attach to tempered T a quadruple (Jord, σ , ε , m) with σ being the σ for δ , Jord(T) = Jord(δ) \cup {(ρ_1, c_1), ..., (ρ_k, c_k)}, and

$$m(\rho, d) = 2 \left(\#\{i \mid (\rho_i, c_i) \cong (\rho, d)\} \right) + \begin{cases} 1 & (\rho, d) \in \operatorname{Jord}(\delta) \\ 0 & \operatorname{not} \end{cases}$$

Extend ε_{δ} to Jord(T) so that

$$\varepsilon(\rho, a) = \varepsilon(\rho, a_{-}) \iff T \hookrightarrow \delta(\rho, a_{-}, a)^{m(\rho, a)} \rtimes \theta \text{ for some } \theta.$$