5-2010

A Network Model of Violent Elections and Clientilism

Christoher J. Haid
University of Chicago, haid@uchicago.edu

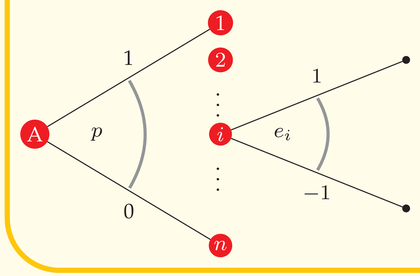
A Network Model of Violent Elections and Clientilism

Christopher J. Haid haid@uchicago.edu Yale University & University of Chicago

Contribution

1. Analyze a model where voters observe and can obtain indirect benefits from their neighbors efforts on behalf of one candidate or another.
2. Characterize equilibrium as a Horizontal Linear Complementarity Problem in a subgame where voters select into one of two patronage networks.
3. Provide an initial analysis of how the substitute-complement relationship between violence and patronage varies with a campaign's knowledge of the voters' social networks and preferences.

Stylized Sequence



Voters' Problem

$$\max_{(x_i,y_i)} u_i = a_i x_i + b_i y_i - \frac{1}{2}c_i(x_i^2 + y_i)^2 + \sum_{j\in N\setminus\{i\}} A_{ij}x_j x_i + \sum_{j\in N\setminus\{i\}} B_{ij}y_j y_i \quad \text{(LP)}$$

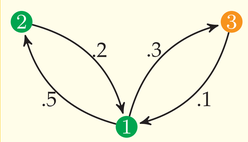
$$\text{s.t.} \quad x_i, y_i \geq 0 \text{ and } \underbrace{x_i y_i = 0}_{\text{effort for A } \textit{or} \text{ B}}$$

Compact matrix algebra recasts (LP) as a Horizontal Linear Complementarity Problem (HLCP), which is to find the vector pair $(\boldsymbol{x}, \boldsymbol{y})$ satisfying:

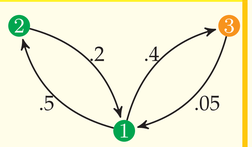
$$\begin{aligned} \boldsymbol{e_0} &= \boldsymbol{B}\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x} \\ \boldsymbol{x} \geq, \boldsymbol{y} &\geq 0 \\ \boldsymbol{x}^\mathsf{T}\boldsymbol{y} &= 0, \end{aligned} \quad \text{(HLCP)}$$

where $\boldsymbol{e_0}$ is the vector with entries that are difference in optimal efforts when each $i \in N$ ignores her neighbors $x_i^0 - y_i^0$; $\boldsymbol{e}_0$ measures i's *intrinsic* intensity of preference for Candidate A over B. The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ contain the (cost-adjusted) individual cross-effects for Candidates A and B that accrue when any pair of voters i and j exert effort on behalf of the same candidate, e.g. individual entries in $\boldsymbol{A}$ equal A_{ij}/c_i.

3 Voters, 2 Patronage Networks



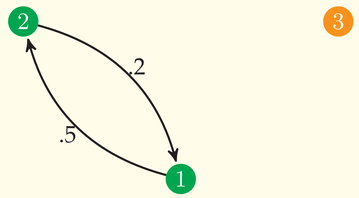
$$\boldsymbol{A} = \begin{pmatrix} 0 & .2 & .1 \\ .5 & 0 & 0 \\ .3 & 0 & 0 \end{pmatrix}$$



$$\boldsymbol{B} = \begin{pmatrix} 0 & .2 & .05 \\ .5 & 0 & 0 \\ .4 & 0 & 0 \end{pmatrix}$$

Outcomes

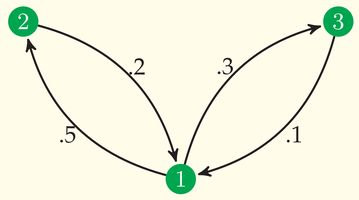
Voters 1 and 2 each exert effort for Candidate A in equilibrium, forming a 2 person patronage network and reaping indirect benefits form each other. Voter 3 forms a singleton network by herself on behalf of Candidate B. NB that voters are supporting the candidate they intrinsically prefer (i.e., without considering their neighbors' choices).



$$(x_1^0, y_1^0) = (2, 0) \implies (x_1^*, y_1^*) = (2.44, 0)$$
$$(x_2^0, y_2^0) = (1, 0) \implies (x_2^*, y_2^*) = (2.22, 0)$$

$$(x_3^0, y_3^0) = (1, 2) \implies (x_3^*, y_3^*) = (0, 2)$$

Slighting increasing voter 3's preference for Candidate A ($x_3^0 = 1 \to 1.3$) results in a new equilibrium where voter 3 joins A's patronage network and provides A with positive support, contrary to 3's intrinsic preference for Candidate B.

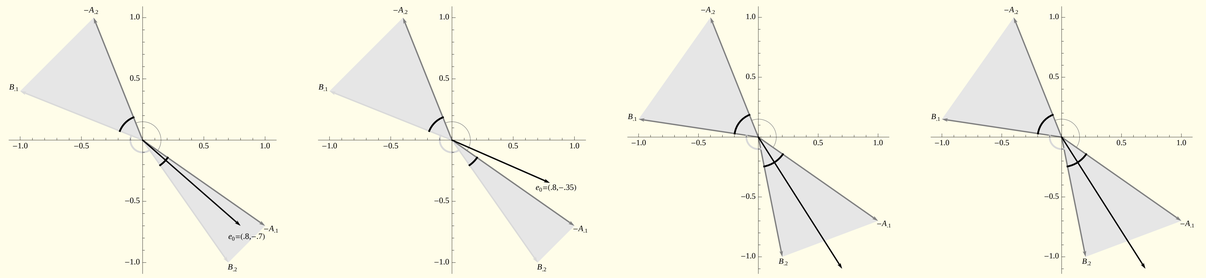


$$(x_1^0, y_1^0) = (2, 0) \implies (x_1^*, y_1^*) = (2.68, 0)$$
$$(x_2^0, y_2^0) = (1, 0) \implies (x_2^*, y_2^*) = (2.34, 0)$$

$$(x_3^0, y_3^0) = (1.3, 2) \implies (x_1^*, y_1^*) = (2.10, 0)$$

HLCP Solution & Effects of Violence with 2 Voters

A solution to HLCP exists and is unique so long as the matrix pair $(\boldsymbol{A}, \boldsymbol{B})$ are $\mathcal{W}_0$-matrices, a class of matrices which generalizes the well known class of $\mathcal{P}_0$-matrices and ensure that positive feedback loops cannot expand indefinitely. With two voters in a complete network we can examine the effects of different types of violence on levels of campaign effort by voters. In this example we start with a configuration where the matrices of cross-effects for Candidates A and B are $\boldsymbol{A} = \left(\begin{smallmatrix} -1 & .4 \\ .7 & -1 \end{smallmatrix}\right)$ and $\boldsymbol{B} = \left(\begin{smallmatrix} -1 & .7 \\ .4 & -1 \end{smallmatrix}\right)$, respectively.



(i) $(x_1^*{=}.608, y_1^*{=}0)$ $(x_2^*{=}0, y_2^*{=}.275)$ (ii) $(x_1^*{=}.917, y_1^*{=}0)$ $(x_2^*{=}.291, y_2^*{=}0)$ (iii) (iv)

Low local information $\implies$ **Violence-as-substitute**: Panels (i) and (ii) illustrate a situation where Candidate A is *not* socially embedded in a community. Campaign violence is conditioned on observable efforts by each $i \in N$. Here violence serves to increase the individual costs of effort on behalf of candidate B (e.g., by increasing the probability of targeted violence). At a constant level of electoral support violence (implying $b_i' < b_i$ in (LP)) and clientilism (a_i in (LP)) are substitutes. An increase in the cost to the campaign of one activity results in an increase by the campaign of the other activity. In Panel (i) each party receives positive support since $\boldsymbol{e}_0$ can be given as a positive linear combination of $-A_{\cdot 1}$ and $B_{\cdot 2}$ (given below figure). Increasing 2's costs of supporting B by 0.35 moves the own-efforts vector $\boldsymbol{e}_0$ into the cone defined by $-A_{\cdot 1}$ and $-A_{\cdot 2}$; increasing 2's benefit of supporting A by 0.35 results in an equiivilant shift of $\boldsymbol{e}_0$. Although voter 2 intrinsically supports Candidate B, the disutility of violence or gain from patronage combined with indirect benefits from voter 1's support of A result in both voters providing positive efforts for Candidate A. Also note 1's higher efforts for A in Panel (ii).

High local information $\implies$ **Violence-as-complement**: Panels (iii) and (iv) demonstrate this case. Here A's campaign can use local knowledge of relationships to decrease the amount of benefits derived from B that flow from one voter to another. A lower cost of violence leads to direct targeting of voter 2's ability to share proceeds from supporting B with voter 1. Consequently, $\boldsymbol{B} = \left(\begin{smallmatrix} -1 & .2 \\ .4 & -1 \end{smallmatrix}\right) \to \left(\begin{smallmatrix} -1 & .2 \\ .15 & -1 \end{smallmatrix}\right)$, decreasing the region were both voters support B to a region of split support. In Panel (iv) the marginal cost of patronage is less than panel Panel (iii), resulting in an increase in the level patronage as a result of a decrease in the cost of violence.

Next Steps

Relax assumptions in model:

- Decouple political and violent organizations
- Change information conditions (who knows more about social network)
- Add electoral borders
- Add patronage and violence decisions by a second candidate/compaign.

References

[1] C. Ballester and Y. Zenou. Who's who in Networks. Wanted: The Key Player. *Econometrica*, 74(5):1403–1417, 2006.

[2] C. Ballester and A. Calvó-Armengol. Moderate Interactions in Games with Induced Complementarities. Unpublished manuscript, Universitat Autonoma de Barcelona, 2007.

[3] R. Cottle, J. Pang, and R. Stone. *The Linear Complementarity Problem*. Society for Industrial Mathematics, 2009.

[4] M. O. Jackson. *Social and Economic Networks*. Princeton University Press, Princeton, NJ, 2008.

[5] R. Sznajder and M. Gowda. Generalizations of P_0- and P-Properties; Extended Vertical and Horizontal Linear Complementarity Problems. *Linear Algebra and its Applications*, 223:695–715, 1995.