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Honors Thesis

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NUMERICAL SIMULATION OF SUBCRITICAL AND SUPERCRITICAL FLOW IN A CONVERGING CHANNEL

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Abstract

Subcritical and supercritical flow in a contraction is simulated by numerically solving the 2D depth-averaged equations using a finite difference model DASH (Molls 1992; Molls and Chaudhry 1995) and a finite element model HIVEL2D (Stockstill and Berger 1994). In the finite difference model, time differencing is accomplished using a second-order accurate Beam and Warming approximation and spatial derivatives are approximated by second-order accurate central differencing. The equations are solved using an alternating-direction-implicit (ADI) scheme. HIVEL2D uses linear basis functions for depth and unit discharge and incorporates a SUPG type test function weighted along characteristics.

The numerical models are compared with experimental data reported by Ippen and Dawson (1951) for flow in a straight-walled contraction. The models demonstrate the effectiveness of the numerical schemes in simulating the 2D depth-averaged equations, but also reveal the weakness of these equations under severe conditions.

Introduction

Hydraulic engineers are commonly confronted with problems involving channel transitions. For example, the design of supercritical channel contractions is an important and complex problem. Due to the contracting sidewalls, standing waves appear in (and downstream of) the transition. Thus, the velocity and water depth vary considerably across the channel. On the other hand, a subcritical channel contraction

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does not exhibit standing waves. Consequently, the velocity and water depth are relatively constant across the channel and only vary longitudinally.

When designing a channel transition, it is important to estimate the velocity and water depth throughout the transition. A computer model can quickly assess the effect of varying design parameters (i.e. geometry, flow, channel material, etc.) on the velocity and water depth in the contraction. However, to apply the computer model with confidence, it must be demonstrated that the numerical results are sufficiently accurate.

Experimental Verification Data

The data used to verify the numerical models was obtained by Coles and Shintaku (1943) and reported by Ippen and Dawson (1951) for flow in a rectangular straight-walled contraction. The upstream and downstream channel widths were 0.610 m and 0.305 m, respectively. The transition section was 1.45 m long with walls angled in at 6°. Neither the channel slope or material were reported. The water depth, in and downstream of the contraction, was recorded for various flow conditions. To verify the computer models, a subcritical and supercritical data set were chosen. In both cases, the flow rate was 0.0411 m^3/s. For the subcritical case, the Froude number and water depth 0.305 m upstream of the contraction were 0.315 and 0.168 m, respectively. For the supercritical case, the Froude number and water depth 0.305 m upstream of the contraction were 4.0 and 0.0305 m, respectively.

Model Parameters

The numerical models were run using no-slip sidewall boundary conditions. Since the channel bottom slope and channel roughness (i.e. friction) information was not reported, the models assumed a horizontal channel and frictionless flow. The subcritical computations were performed on a 30x21 grid with the inflow boundary 0.305 m upstream of the contraction entrance and the outflow boundary 0.305 m downstream from the end of the contraction. The 72x21 grid used for the supercritical case was similar to the subcritical grid except the outflow boundary was extended to 3.66 m downstream from the end of the contraction. In both cases, Δx was 0.0726 m and Δy was variable.

Results

The computed water depths are compared with the experimental data in Figures 1-3. It should be noted that the experimental data was obtained from very small contour plots, which inevitably resulted in some error.

For the subcritical case, the depth was relatively constant across the channel and decreased in the longitudinal direction. From Fig. 1, it is evident that the computed solutions closely resemble the experimental data, with only a slight discrepancy at the downstream boundary. Interestingly, the flow became supercritical ($F_r \approx 1.1$) a few nodes upstream of the outlet boundary. The DASH model was more accurate than HIVEL2D near the outlet boundary. The computed solutions were not particularly sensitive to variations in the grid spacing (Δx), channel bottom slope, or channel roughness.

Figures 2 and 3 present results for the supercritical case. This flow is very complex, due to the formation of cross waves in the contraction, and discrepancies exist between the computed results and experimental data. From the upstream boundary to the middle of the contraction agreement between the computed results and experimental data is favorable. However, near the end of the contraction, the experimental and computed results differ significantly. HIVEL2D computed the water depth magnitude more accurately than DASH; but, neither model accurately predicted the location of the cross waves. Both models were sensitive to changes in channel bottom slope and channel roughness. In addition, reducing Δx caused both models to predict increased water depths.

Summary and Conclusions

It is obvious that both models performed better for the subcritical case. In this case, the models accurately predicted the water depth throughout the contraction. This suggests that the depth-averaged equations can be used with confidence to design subcritical channel contractions.

On the other hand, the supercritical computed results and experimental data differed significantly. In general, the models more closely predicted the water depth magnitude, but the predicted cross wave location did not coincide with the experimental data. This was probably due to the presence of substantial vertical accelerations resulting in a non-hydrostatic pressure distribution in the experimental flume. The depth-averaged equations become suspect under such conditions, since they assume negligible vertical accelerations and hydrostatic pressure. Although the models did not precisely match the supercritical experimental data, the authors believe they can still be useful design tools. For example, it is usually desirable to minimize the size and occurrence of cross waves in a channel contraction. The models can be used to identify a poor design (i.e. one with many cross waves), even though the model results will not be extremely accurate under these conditions. As the design is refined, the vertical accelerations and cross waves will diminish, and the model results will become more accurate.

Finally, the channel bottom slope and channel roughness were not reported. These are important parameters and the authors found that varying the bottom slope and roughness significantly affected the computed results for the supercritical flow. However, to avoid accusations of model "tuning", both models were run assuming frictionless flow and a horizontal channel. This inevitably contributed to the discrepancy between the computed and experimental results. In addition, no attempt was made to quantify errors in the experimental data. Even though the flow is assumed steady, the cross waves are not stationary in a supercritical contraction and a statistical estimate of the uncertainty in the experimental data would be very useful. In view of these consideration, a more complete experimental analysis of supercritical flow in a channel transition would be valuable.

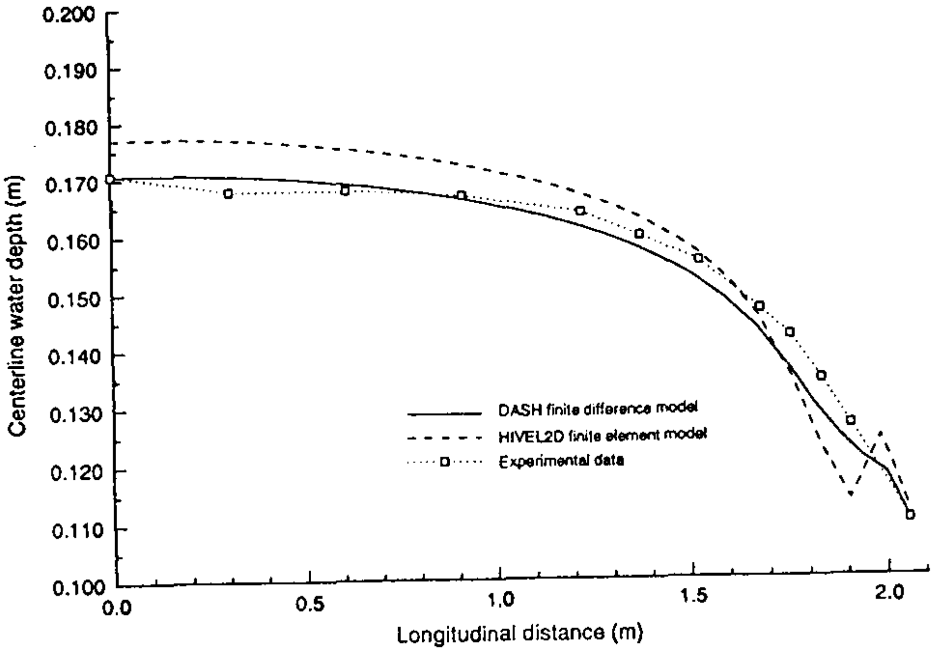

Figure 1 - Centerline flow depth in a contraction (subcritical flow).

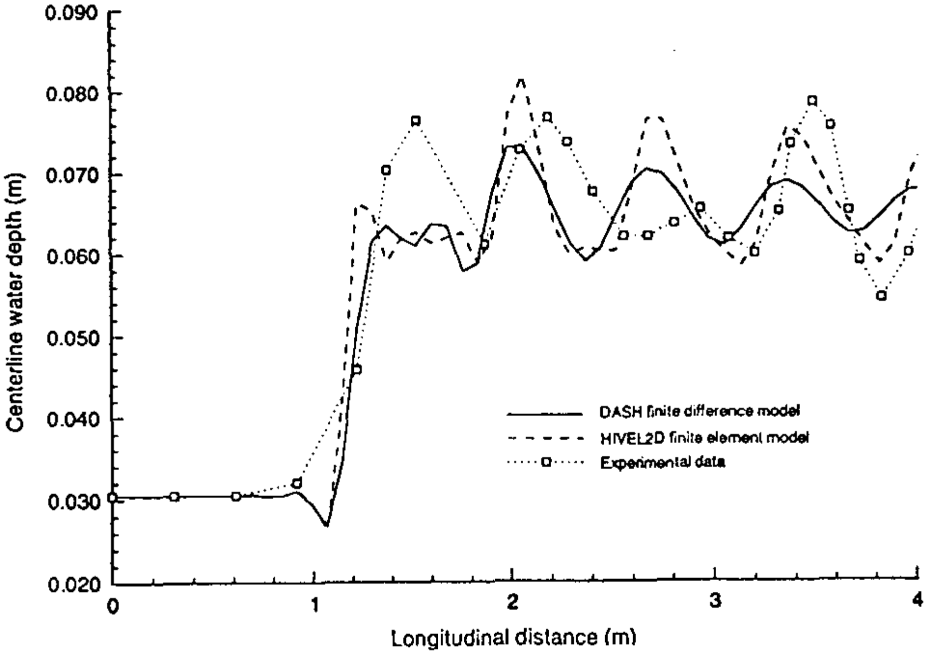

Figure 2 - Centerline flow depth in a contraction (supercritical flow).

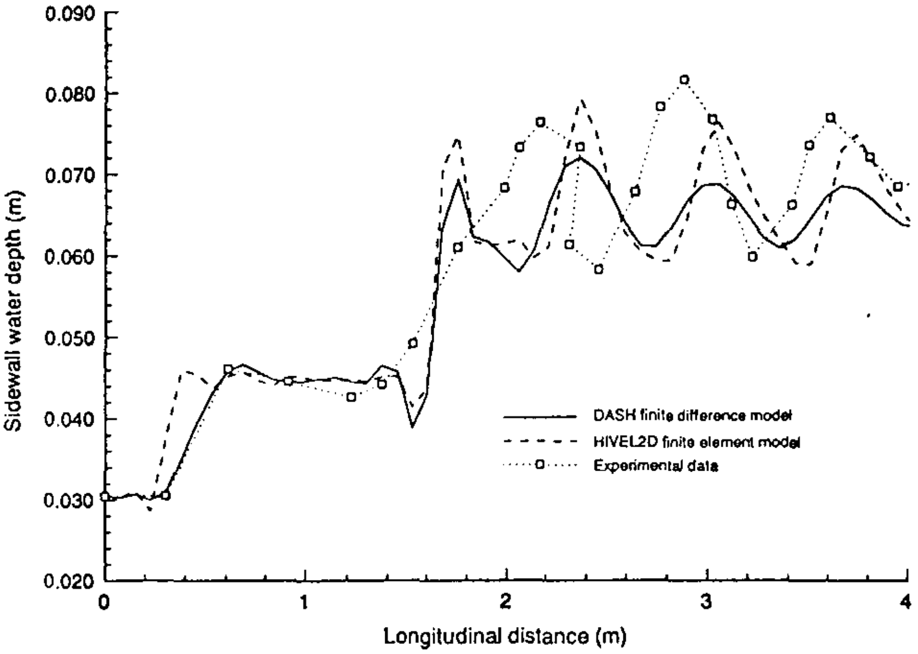

Figure 3 - Sidewall flow depth in a contraction (supercritical flow).

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