2010

Dynamic Models of Mobilization in Political Networks

Navid Hassanpour
Yale University, navid.hassanpour@yale.edu

Dynamic Models of Mobilization in Political Networks

Navid Hassanpour

navid.hassanpour@yale.edu

August 15, 2010

Abstract

The Eastern European color revolutions, and the recent post-election unrest in Iran pose a pressing question: how can local organization networks facilitate large-scale collective action? The final result of a collective action is contingent upon two factors, the relational structure of the network of the individuals involved, and their mutual learning, imitation, and belief-updating dictated by the network structure. I propose a formalization of the Granovetter threshold model for participation in collective action in networks, which takes both the network structure and belief updating into account. In order to make verifiable predictions, I outline a graph theoretical model for threshold updating using the DeGroot learning model. I demonstrate that full connectivity in a social network sometimes can hinder collective action. Later I will show that with some assumptions on the structure of the social network, repeated threshold updating takes the network to an equilibrium on the network graph; hence, the updating procedure acts as an equilibrium selection mechanism based on network parameters and initial participation thresholds. When these assumptions do not hold, cycles of participation and disengagement can occur. Furthermore, using this model one could find the network structure that brings about a particular asymptotic action equilibrium. Unlike the Granovetter/Kuran model, this model predicts non-monotone participation levels and heterogeneous outcomes at the final equilibrium, where some individuals act and some do not. Hence, it provides a more realistic model of mobilization dynamics, which can explain the ebb and flow in large-scale political demonstrations.

Keywords: cascade, collective action, diffusion, mobilization, network, structure, threshold

1 Introduction

Historical examples of mass political action as well as the Eastern European color revolutions, and more recently the post-election unrest in Iran, ongoing antigovernment protests in Bangkok and the takeover in Kyrgyzstan are examples of mass collective action resulting from decentralized planning and local mobilization. On the domestic level, the Tea Party mobilization techniques have attracted much attention. A common factor among all these political demonstrations is the role of a small core of radical elements. Because of the disproportionate size of the core of mobilization, structural considerations are of utmost importance. Because the relation between the radicals and the rest of the social network define the scope and ultimate result of the collective action. Instead of a structural analysis, the existing formalized scholarship of collective action has mainly emphasized either strategic thinking in the context of token games, or equilibrium studies based on population statistics. When the scope of political action is small and possible strategies are common knowledge, it is plausible to describe the outcome as a strategic equilibrium among multiple players. In large-scale political acts, these assumptions are unrealistic. The access to information is limited and the interaction between individuals is dictated by their local social network and the influence of the media and focal political players. One needs to imitate, trust, and update his/her beliefs based on the acts of his/her neighbors in the local network. Such modes of behavior based on bounded rationality result in cascade effects such as fast paced contagion of political participation.[1]

How could decentralized planning and communication through local interactions result in such large scale shows of political unrest? If radical instigators are a small part of the mass mobilization, then the structure of communication and the mass learning and imitation mechanisms are the defining factors of the collective action. Such structural considerations link political events as diverse as local Tea Party protest mobilization, ongoing Redshirt protests in Bangkok, and street protests in the post-election Tehran.

Tilly (1978) notes that mass collective actions happen when the population becomes aware of a window of opportunity. Mass political movements arise in cycles, expanding until the ancien regime collapses. This is an accurate observation rather than an explanation. The cycle is the cascade effect inherent in a mass political action. The diffusion of activism starts from a small number of radicals and spreads through the society. When inactive individuals see the growing participation of others, they themselves decide to join in; hence they contribute to the size of the action and empower the signal to be sent to the remaining population for the action in the next round. The expansion Tilly alludes to can be formalized by modeling diffusion and learning in large-scale network. It is possible to show that cascades are more likely to happen in some configurations than others (Watts (2002)). The structure of the underlying network also dictates the speed of the diffusion. Identifying cascade of events in recent successful mass political and social movements can confirm the theory using pieces of evidence such as a few recent successful political movements including Eastern Germany (1989-91) and Iran (1978-79). For example, Lohmann notes that the inability of the GDR to counter the initial gatherings signaled the vulnerability

[1]For the political examples other than mass movements including identity switch and language reversal see Laitin (1998).

of the Eastern Germany's regime and helped to augment the protests. The knowledge of an opportunity associated with the unrest flows around through social links, and depending upon the social network structure, can result in cascades. Beissinger (2002) notes the role of tidal diffusion of nationalist sentiments across the former states of the soviet union. According to Beissinger, a study of such cascades of events is necessary for the understanding of the the events leading to the fall of the soviet union.[2] One could further explain social cascades by analyzing the social structure and belief dynamics.

Therefore, the distinctive characteristics of mass collective action including cascades and the tipping point mechanism are products of two factors, first the structure of the social network and the characteristics of the network relations underlying the political action, and second the dynamics of inter-personal learning, imitation, and influence. In the following, I examine these two factors in more detail to elucidate the insights a structural analysis of social phenomena can bring.

In the following I review the existing scholarship of the dynamics of mass collective action and identify the structural consideration inherent in these explanations. After identifying qualitative explanations based on structure, I will propose a model to capture the relational nature of a mass political act.

2 Dynamic Models of Collective Action

In making a decision to risk arrest, incarceration, and death on streets, an individual is not making a merely personal decision. He joins a larger group who have made the same decision before him. Similarly his decision sets a precedent for the others.

The number of individuals involved in the situation complicates a fully strategic model. Modeling the conditions in the form of a giant prisoners' dilemma would overlook inter-personal connections and the sequential nature of the acts resulting in a mass uprising. In other words, the action originates from a few and diffuses in the network. Strategic modeling does not fully capture this. Limited available information based on the shape of the social network results in decisions based on bounded rationality rather than full deliberation. Herd behavior and imitation is more prevalent than utility calculations. Instead it is possible to consider the percentage of participants in the collective action as a defining parameter in recruiting new members, see Schelling (1978), Granovetter (1978). Each individual has a personal level of risk taking. Some are willing to engage in risky behavior irrespective of other participants, these are the initiators. Some might never join in. Each individual is embedded in a local network. She can learn from them or influence their threshold levels.

[2] Rasler (1996) is another study of repression and mobilization in the context of the Iranian revolution of 1979. Rasler finds spatial diffusion and group dynamics to be paramount factors.

Individuals have a personal perception of the turnout level. The participation information can be of two kinds. First, the level of participation might be well known to everybody in the society. For example one could assume a comprehensive coverage of protests by the media that is accessible to the majority of the population. Nevertheless the global perceived level of participation can be an under or overestimate of the real level. An authoritarian regime would try to downplay the size of the demonstrations and their significance, while the rumors circulating among hopeful dissidents often overestimate the size and the level of participation. There are abundant examples of successful collective acts following shocking rumors.[3] Second, The amount of available information could be quite limited. Consider a case where the level of total participation is withheld from the public, or it might be simply very difficult to estimate the participation levels. Each individual should decide based on the level of participation she detects in her own personal network. As it was mentioned above, this can happen if there is no central media for channeling information or if details are deliberately withheld from the populace. In both cases individuals constantly update their belief about the level of participation; I model this process using a tractable updating mechanism. In the course of adapting their thresholds, every individual is strongly influenced by his neighbors in his social network. These relations are modeled using graph-theoretical tools.

An attempt at such an explanation was set forward by Granovetter (1978) and Schelling (1978) and later expanded by Kuran (1989), Lohmann (1994), Gould (1993), and Siegel (2009) among others. Granovetter noted that for bringing about a collective action cascade, all the group members do not have to be agitators. Radical instigators are inclined toward participation with no restraints; nevertheless, sometimes few radicals are enough to "start a prairie fire." Individuals have different reservations and risk taking habits.

One way of tying these personal reservations to a collective action is through representing the level of risk taking by an other regarding threshold. For example, some might be cautious and join a collective action only if 70 percent of their acquaintances have joined already. While some might be more daring and be willing to join in when only 30 percent of the others have joined in. What we are interested in is the rate of participation. The individuals either participate or they don't. So the easiest way to link the mass behavior to recruitment is by defining such a threshold for each individual. Note that one could use other measures as well. For example the threshold could be defined as the speed of the spread of the collective action instead of the rate of participation. The potential participants might care more about the speed of the contagion than its real percentage. They might join in if they see a large number joining in at high rates. In the following, I will only consider the first order dynamics, i.e. the rate of participation.

Consider n participants, i'th threshold is given as $(i-1)/n$. The agent $i=1$, always acts, setting off a chain of action starting from $i=2$ and encompassing all the agents. Note that if the first instigator does not act, nothing happens, but once the first agent, $i=1$, acts; all others join

[3] See a recent report on the Czech velvet revolution in 1989 here in the New York Times: `http://www.nytimes.com/2009/11/18/world/europe/18czech.html`

the sequential chain reaction one after another.

Granovetter's model can be improved upon by adding two components, first a model of social structure; note that the above example is concerned with only the distribution of thresholds and not the structure of connections. One could envision a case where the person with a $1/n$ threshold does not see the person with the 0 threshold. In other words, they might not be directly connected in the representative graph of the network. In that case, again no one other than the agitator would act. As this example and the structured sequel show, the connections between the individuals change the ultimate outcome of a collective action based on individual thresholds. Also the theory predicts how the individuals act with fixed thresholds, there is little discussion of how these thresholds might evolve through time. One could imagine that in the course of collective action the thresholds change as well. It is plausible to think that (1) while interacting with others one would be influenced by his social neighbors' beliefs (2) when participating becomes prevalent, one's threshold could decrease.

Based on Granovetter's insights, Kuran (1989) proposes a formalized model of personal participation thresholds based on a utility function comprised of two parts: a private and a public one. Ultimately the utility function is contingent upon private interests and the perceived level of opposition, therefore gives a threshold function based on these two parameters. Each individual chooses opposition if the utility of doing so is greater than supporting the incumbent regime. Based on their expected share of the opposition, a certain number of individuals join the opposition. In the next round of choosing to join the opposition or not, the decisions are made based on the number of the people who joined in the previous round. The system is in equilibrium when the perceived share of the opposition reflects the real levels of participation.

After the first round of action based on the expected share, the second round of activity is based on the actual share and so forth. Hence Kuran–as well as Granovetter–assume that the level of participation is fully visible to all others in the society, which can be an unrealistic assumption. The individual's perceived levels of participation can differ from the real levels and be quite myopic. These local observations are strongly influenced by the network of local connections and spatial confines of the situation in which the individual is situated. Also the distribution of the thresholds can predict which equilibrium is the final result of the contagion, but does not reveal how it is reached, or how an equilibrium is dislodged. In other words, the model is static, the distribution of thresholds does not change; furthermore the initial participation of some T percent of the population is taken for granted without explaining how they could have been mobilized.

Lohmann (1994) finds the Kuran/Granovetter threshold model unrealistic, because the participation rates are decreasing or increasing monotonically until they stagnate at an equilibrium. She notes that such a model can not simply explain the occasional ebb and flow of unrest. Instead she proposes a dynamic model with individual utility functions similar to Kuran's model. At each time t, everybody observe the number of participants in the collective action and perform a Bayesian update on their beliefs regarding the level of participation. While Lohmann duly recognizes several problems with the threshold model and proposes a Bayesian dynamics, she does not study the *structure* of the contextual network.

Gould (1993) proposes a different dynamic mechanism for collective action, in which individuals' total contribution to the collective action is dictated by an average of their neighbors' contributions plus their own personal offer. He assumes at the beginning there is only one contributor and solves for the steady state of the network. He also contrasts some special network configurations including fully connected and star networks. In Gould's model contributions are still ever-increasing; hence, Lohmann's critique is still in place.

Recently Siegel (2009) has proposed a dynamic model for collective action in a social network. Similar to Kuran and Lohmann's model each individual's motivation level has two parts, one is personal and the other is directly contingent upon the level of participation among the neighbors in the network. At each time unit, each individual superimposes these two parts and acts if the the sum is above zero. The initial values of these parameters are randomly generated. Siegel studies the qualitative behavior of four types of networks under these dynamics, small world networks (networks with small diameter and large size), opinion-leader networks (star shaped), clique networks (several cliques with links among them), and hierarchical networks. For the four classes of studied networks the results are qualitatively illuminating. Nevertheless one can benefit from a study of basic network topologies. Some of Siegel's simulation assumptions, including Gaussian random thresholds for individuals, might not be plausible in the context of the intermediate and small scale networks.

In the following, I introduce a formal model to study two major factors in network collective action, i.e. structure, and updating beliefs on thresholds. I will use the formal results of the study to make predictions that are empirically verifiable. I will show that this model can explain heterogeneous participation outcomes and the ebb and flow of the thresholds, hence the fluctuating incidents of mass collective action.

3 Designing Structural Mobilization

In section (2), I outlined the building blocks of a dynamic explanation of the collective action in networks. In this section, I propose a number of hypotheses about the dynamic nature of collective action and proceed to investigate their validity. The progression of mass political acts including protests, mass shift of political allegiance, and revolutions provide a rich set of evidence against which the formal speculations can be confirmed or falsified.

3.1 Hypotheses

A realistic model of dynamics of collective action needs to address not only the questions of "what" the equilibria are, but should also answer "how" they are arrived at. Consider the following.

It is usually assumed that full connectivity among the participants in a collective action is beneficial to the act of mobilization. But consider the cases where there are a few radicals who are to recruit ordinary individuals. Further connections among the individuals can foster apathy, because an individual who considers both a radical leader and a likeminded citizen is less likely to take risks than somebody who is solely in contact with a radical opinion leader. In the following section I use a dynamic formalization to show that establishing separation among individual in a social network can effectively help the mobilization. This is inline with similar observations in Gould (1993) and Siegel (2009).

Dynamics are not always monotone. Protests and mass political acts ebb and flow through time. Kuran (1989)'s model predicts a monotonically increasing or decreasing level of participation. Instead we expect to observe non-monotone progression if the individuals are influenced by their network of acquaintances. Because the inaction of a faction of the society influences others, specially if that faction is well connected. Using the same logic, the aforementioned faction would be more prone to act, if their neighbors are active at the present time. The oscillations should be more evident, if the communication between the groups involved in a collective act is not fast paced, hence there is enough time lag for the asynchronous acts to sequentially follow each other. Most examples of popular protests, revolutions, and mobilization pass through several phases of ebbs and flows before culminating in victory or failure.

The belief updating process is another parameter of the model. If an individual's impression of her neighbors' thresholds is accurate, then the dynamics will be different from the case where the only information available is a distorted version of neighbors' activities. An exact or approximate knowledge of thresholds is usually unavailable. Instead, the actions of one's neighbors can give clues on the their proclivity for participating in the mobilization. One expects that inference based on only previous actions (not the thresholds themselves) slow down the convergence toward a final steady state. In other words, oscillations are more likely when the updating is based on speculations instead of accurate knowledge. Ermakoff (2008)-Ch.9- observes series of vacillations in the process of voting in the Vichy parliament in July 1940. He takes this oscillations to be a product of insufficient communication between the members of the parliament. Later in the next section I will show that when only actions are the basis of inference, even in the context of regular network structures, one can observe severe oscillations.

Another important question to be answered is the size of the critical mass of radicals needed to incite a resurrection in a specific network configuration. The importance of a body of radical actors who unconditionally engage in mobilization is well known (see Marwell and Oliver (1993)). A network model can predict the smallest size of such a group needed for engaging the whole population.

It is plausible to assume that there are multiple equilibria for the network game defined based on the threshold mechanism. In the next section, I show that the proposed dynamics acts as an equilibrium selection mechanism, singling out a specific outcome out of many potential ones. Also the same formalization can be used to find the type of structure that is needed to bring about a particular outcome. Hence we can perform a *structural* mechanism design. Knowing the final outcome, we can find a network construct that induces the desired equilibrium.

4 The Model

In order to address the issues raised in section (2), I present a dynamic model of network threshold updating that merges Granovetter's model with the DeGroot updating mechanism (see Jackson (2008)).

The network is represented by a graph $\mathcal{G}(\mathcal{I}, \mathcal{E})$, where $\mathcal{I}$ is the set of all nodes in the network $i = 1, \ldots, n$, and $\mathcal{E}$ is the set of all edges connecting these nodes. Each node represents an agent, and each link is a social connection. Edges can be directed, i.e. some can not see others acting, while they can be seen by others.[4]

Each of the agents is deciding between taking (A) or not taking (N) action–this is a binary choice between N and A. The decision is made based on the proportion of the network neighbors who are acting, according to the following rules. Take $p_i(t)$ to be the proportion of i's neighbors acting at time $t = 1, \ldots, T$ (self included), and $\Gamma_i(t)$ to be the i's threshold at time t. At each time t, i acts if $p_i(t) \geq \Gamma_i(t)$, and does not act otherwise. This defines a game in which each agent, based on her threshold, has to choose between A and N.

4.1 Equilibria in Network Threshold Games

An equilibrium on this network game is defined similarly to conventional games. Each network agent should not have an incentive to deviate. There can be more than one network equilibrium. Consider the following example. Note that I have not included self-loops in these figures, but it is implicit in the model; i.e. each agent includes her own action in equilibrium calculations.

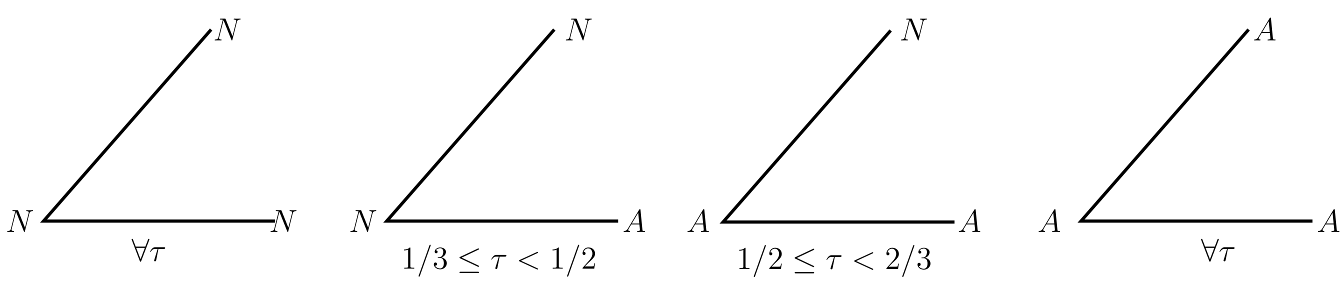

Figure 1: Equilibria for a network game, all agents have a common threshold τ

All players share a common threshold τ. Contingent upon τ, the game in figure (1) can have multiple equilibria. Note that for any value of $0 < \tau < 1$, all nodes acting (A, A, A), and none

[4]There is always a self-loop, because everybody is aware of what he himself does, or what threshold one has.

of them acting (N, N, N) are two equilibria of the game. There exist two other asymmetrical ones as well. For example when $1/2 \leq \tau < 2/3$, there is another equilibrium (N,A,A), the third configuration from left in figure (1). The central player is in equilibrium because $\tau < 2/3$ (A), the peripheral ones are as well, because for the first player $\tau \geq 1/2$ (N), and for the third $\tau < 1$ (A).

For the case where all agents have an identical threshold τ, Morris (2000) provides an extensive equilibrium analysis. Here I do not limit the thresholds to be all equal. Consider the following case where the central actor has a negligible but larger than zero threshold ($\epsilon \rightarrow 0$), in other words she is a radical, while the two other players have identical thresholds τ, hence the threshold triplet is (τ, ϵ, τ). In this case the equilibria of the game are different from the previous case, see below.

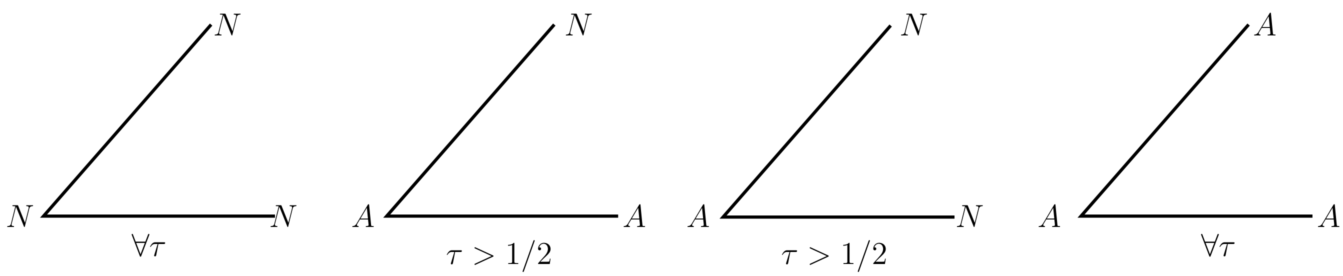

Figure 2: Equilibria for a network game, agents have thresholds (τ, ϵ, τ)

Here when $\tau < 1/2$, there are only two equilibria (N, N, N) and (A, A, A). When $\tau > 1/2$, there are four, (N, N, N), (A, A, A), (N, A, A), and (N, A, N) (see figure (2)).

It is plausible to take the final result of a collective political act to be an equilibrium in the underlying network. In the contention process, each of these equilibria can be a realization of mass political action based on the threshold mechanism. In particular Lohmann (1994) emphasizes the changes in the participation levels in the course of the Leipzig protests. One could take each participation scheme to represent an equilibrium in the corresponding network game. In the following subsection I outline a dynamic model to explain transitions between two distinct equilibria. For example, how does a network from the type depicted in figures (1), and (2) transition from a (N, N, N) equilibrium to (A, A, A). Or why is a certain equilibrium the result of the dynamics? These question motivate the following dynamic model.

4.2 A Dynamic Model

The previous section was an examination of equilibria in network games based on the threshold model. One expects that at each action period, agents revise their action thresholds based on the history of previous actions. If the majority of their neighbors in the network are eagerly active in the collective action and have low participation thresholds, the agents update their thresholds accordingly and will be more prone to participation in the next round. In the case of inaction, the same mechanism is at work. The threshold update mechanism is contingent upon the model and is chosen based on the information available to the actors and their rationalization levels. For example Lohmann (1994) takes the updates to be Bayesian. I instead adopt an averaging model based on the DeGroot model. The reason for such a choice is the that during large scale political acts, usually full information and optimal inference apparatus are unavailable. Political actors need to approximate the situation and coarsely extrapolate about the future. It is plausible to assume that they could simply adopt a weighted average of their own thresholds with their neighbors'. These weights are proportional to one's influence on the neighbors. This updating relationship does not need to be symmetric. i can take j's recommendation very seriously, while the opposite might not be true. An individual may closely watch the acts of an *opinion leader*, while the leader does not care as much about a single follower's beliefs. The weights individual i assigns to person j are taken to be α_{ij}s. We normalize the αs so that $\sum_j \alpha_{ij} = 1$, again note that α_{ij} is not necessarily equal to α_{ji}. Also note that one's neighbors' thresholds are not always fully known, and are hard to exchange in the course of a fast paced mobilization act. Hence, the actors have to infer the real value of thresholds from the acts of each of their neighbors. For example they can take i s threshold to be the the proportion of the times she has failed to act. Or if keeping a detailed history is implausible, one can make a coarse approximation and take i's threshold to be 1 if i does not act, and 0 if she does.

In the following, I examine two updating mechanism. One is based on averaging the neighbors' thresholds, and the other infers a neighbor's threshold from his act in the previous round of collective action. I will show that these two dynamics result in quite different asymptotic outcomes.

4.2.1 First Model, Full Threshold Knowledge

First type of dynamics is to replace one's threshold with a weighted average of one's own and the neighbors' thresholds; and act at each time t according to the threshold and the level of activism in one's neighborhood. Because of the continuous nature of political action, I take this process to be repeated multiple times. Ideally the objective is to use the model to arrive at the asymptotic acts and thresholds of each individual in the network.

Dynamics of Thresholds: At time t, i updates his threshold to be a linear combination of his neighbors' thresholds and his own. Define Matrix $A = [\alpha_{ij}]$ equal to the weight that i gives to neighbor j's threshold. Take $\Gamma_{n\times 1}(t)$ to be the vector of thresholds for individuals 1 to n at time t.

$$\begin{aligned}\Gamma(t) &= \alpha.\Gamma(t-1)\\ \Gamma_i(t) &= \sum_{j:ij\in\mathcal{G}} \alpha_{ij}\Gamma_j(t-1)\end{aligned}$$

Dynamics of Action: take $A_{n\times 1}(t)$ as the vector of individuals' action. 1 implies action, and 0 non-action. At each time t, individuals either join in the collective action or refrain. The perceived level of participation for individual i, can be a product of her personal network, $p_i(t)$, or a number known to everybody and the same for all, $p_i(t) = p(t), \forall i$. The decision to *act* or *not act* is made based on the comparison of $p_i(t)$ and $\Gamma_i(t)$. Person i acts if $p_i(t-1) \geq \Gamma_i(t)$ and would not act if $p_i(t-1) < \Gamma_i(t)$. For the purpose of analysis in this paper I assume that $p_i(t)$ is the percentage of i's neighbors acting at time t. Note that there could be various ways of modeling $p_i(t)$. For example we could assume a universally accepted $p(t)$ or perceived participation levels, $\tilde{p}_i(t)$, that are different from reality $p_i(t)$.

We are interested in the dynamics of both Γ and A, specifically their asymptotic behavior when $T \to \infty$

Note that in this model, the action vector A is a derivative of the threshold vector Γ. Define

$$D \sim D_{ij} \doteq \frac{1}{\deg(i \in \mathcal{I})} \quad \text{if} \quad ij \in \mathcal{E},$$

D is fixed for all t,

$$\begin{aligned} A(t) &= \operatorname{sgn}(p(t-1) - \Gamma(t)) \\ &= \operatorname{sgn}(D.A(t-1) - \Gamma(t)) \end{aligned}$$

sgn(t) is the sign function, sgn(t)=0 if $t < 0$, sgn(t)=1 if $t > 0$. Therefore these two equations together characterize the Markov dynamics of this system,

$$\Gamma(t) = \alpha\Gamma(t-1) \tag{1}$$

$$A(t) = \operatorname{sgn}(D.A(t-1) - \Gamma(t)). \tag{2}$$

In the following examples we take $\alpha = D$, hence

$$\Gamma(t) = D.\Gamma(t-1) \tag{3}$$

$$A(t) = \operatorname{sgn}(D.A(t-1) - \Gamma(t)) \tag{4}$$

Initial conditions: $\Gamma_i(0)$ is given. $A_i(0) = 1$ if $\Gamma_i(0) = 0$, otherwise $A_i(0) = 0$.

Example: Consider the following two networks. In this case, there is a radical with initial threshold 0 and two normal individuals with identical thresholds $0 < \tau < 1$. At each time unit, players update their thresholds and decide to act or not. The relations are symmetric. Which means $\alpha_{ij} = \frac{1}{\text{degree of } i}$. The initial thresholds and the configurations are as follows,

Figure 3: Initial Thresholds

Note that for the fully connected graph, threshold updating gives $(2/3\tau, 2/3\tau, 2/3\tau)$ for time $t = 1$ onwards. For action set, if $\tau < 1/2$ the final action profile will be (A, A, A) for $t = 1$ onwards; otherwise it is (N, N, N) for $t = 1$ and afterwards.

For the other network, the dynamics are not trivial. Applying the dynamic model in equation (1) gives the following progression in table (1),

Thresholds	Actions
$\tau(1) = (2\tau/3, \tau/2, \tau/2)$	$A(1) = (A, N, N)$
$\tau(2) = (5\tau/9, 7\tau/12, 7\tau/12)$	$A(2) = (A, A, A)$ if $\tau < 1/2$; o.w. $= (N, A, A)$
$\tau(3) = (31\tau/54, 41\tau/72, 41\tau/72)$	$A(3) = (A, A, A)$ if $\tau < 6/7$; o.w. $= (A, N, N)$
$\vdots$	$\vdots$
$\tau(\infty) = (0.57\tau, 0.57\tau, 0.57\tau)$	$A(\infty) = (A, A, A)$ if $\tau < 0.875$; o.w. $= (N, N, N)$

Table 1: Dynamics of the star network in figure (3)

The asymptotic thresholds for both networks are reflected in figure (4).

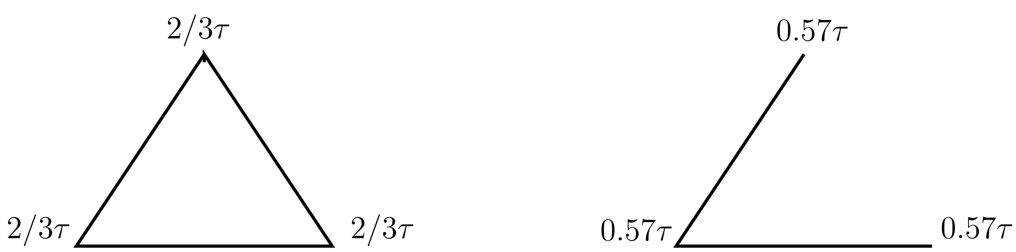

Figure 4: Final thresholds, full connectivity is not always helpful

Several observations are due consideration here. First, the final value of the protest threshold is the same for everybody, this is an analytical fact to be demonstrated shortly. There is also a possibility of cycles in general. Second, the completely connected graph has a larger final threshold compared to the other one, i.e. full connectedness is not always helpful. Third, in the progression of dynamics, an individual can switch from action to non-action and vice versa. Hence, Lohmann's main objection to Kuran's model is not applicable here. Fourth, the final action vector is either all zeros (no action in the network), or all ones (all individuals active) depending on the value of the parameter τ; therefore, contingent upon the risk-taking level of the population, both outcomes are possible. There also exist cases where assigning different values of initial thresholds to each actor makes the asymptotic actions heterogeneous.

Consider the thresholds at time t, $\tau(t) = \alpha\tau(t-1) = \alpha^t\tau(0)$, with the condition that $\exists i, \tau_i(0) = 0$.If the network's graph $\mathcal{G}$ is aperiodic[5] and irreducible,[6] the steady state thresholds will be the same for all the individuals in the network and is equal to v where v is the normalized[7] unit left eigenvector of the matrix α (the left eigenvector with eigenvalue equal to 1) and the final threshold for each individual is

$$\Gamma_i(\infty) = v^T.\tau(0).$$

For more information on $\Gamma_i(\infty)$ for specific network configurations such as star networks see the Appendix section (6).

Note that while the threshold perceptions are changing indefinitely (converging though), the actions might reach the steady state and remain the same. Here we have the private preferences changing while the actions remain the same.

4.2.2 Dynamics, Second Model, Only Action Knowledge

Dynamics: Consider a case where there is not enough information about the personal thresholds of one's neighbors. It is usually the case that the agent has to infer the neighbors' thresholds based on their actions. Take the coarse estimation of a neighbor j's threshold at time t to be 1 if j does not act at time $t-1$, and 0 if he does act.

To update the threshold one takes an average of his own threshold and an estimation of his neighbors' thresholds based on their prior acts. The dynamics of this new updating mechanism is

$$\Gamma_i(t) = D_{ii}.\Gamma_i(t-1) + \sum_{j\neq i} D_{ij}.(1 - A_j(t-1)) \tag{5}$$

$$A(t) = \text{sgn}(D.A(t-1) - \Gamma(t)) \tag{6}$$

The initial conditions are the same as the previous case in section (4.2.1). There are a number of noteworthy points. First, the dynamics based on inference from the actions results in oscillations, even in conventional topologies such as star networks. The periphery and the central agent switch between action and inaction. In the case of threshold based updating, such oscillations do not happen. Remember Ermakoff (2008) 's insight on the possibility of oscillation when the inference

[5] A graph is aperiodic if the greatest common divisor of all of its cycles is 1. This is true of all graphs discussed in this paper, because of the self-cycle (each individual counts her own threshold in her averaging).

[6] A graph is irreducible if there is a path from each node to any other node. Again it is true of all of the graphs in this study unless it is stated otherwise.

[7] Such that the final vector is stochastic i.e. its elements add to one.

is loosely based on actions, not more thorough deliberation. These examples confirm Ermakoff's speculations. Second, in the fully connected network, the size of the critical mass is half of the actors. In any network it is possible to find the minimum size of the critical mass needed for inciting global action.

4.3 Finding Steady State Thresholds and Actions

In the above equations (1), (2), (5), and (6) analytical expressions for the steady state vectors α and Γ sometimes can be found by putting $A(t) = A(t-1)$ and $\Gamma(t) = \Gamma(t-1)$. For example one can find the steady state Γ from equation (1) and substitute it in the (2). Then one could simply try all of the 2^n possibilities for A and find the ones that satisfy the identity $A(t) = A(t-1)$. Because the size of the possible As is well bounded, finding the equilibria is feasible. Same techniques can be applied to the equations (5), and (6).

Also note that any steady state is an *equilibrium* of the network game (excluding the cases leading to oscillation). If that is not the case, in the next period, all players choose actions that are in equilibrium, hence it is impossible to have a non-equilibrium outcome as the steady state. As it was mentioned before, the dynamics give us an equilibrium selection mechanism. At the same time, one can assume a particular action equilibrium A, and find αs that result in that equilibrium. Hence one could design a network to achieve a desired action equilibrium.

5 Conclusion

In this paper I reviewed the competing theories of network collective action and proposed a formalization of the dynamic threshold model. I demonstrated the steady state of the dynamics of the model and showed that it can describe the ebb and flow of the participation levels in a mass collective action. Then I linked the steady state of the system to the equilibria of the threshold-based participation mechanism on the network graph. There are several outstanding issues relegated to the next phase of this project, including collecting historical evidence for the theory and comparing the historical information with the theoretical predictions, in addition to the extension of the analysis to more sophisticated updating rules and more general classes of network graphs.

An empirical study can corroborate the model. Consider a panel data set containing binary data for the participation in a collective action. The unit of participation can be an individual or a group. In this empirical representation, the links between individuals and the relations (e.g. alliances, rifts) between political parties constitute the network links. The evolution of the network structure associated with the collective action during the studied time frame and the choices individuals make are both measurable and could be used for testing the theory.

Another venue for testing the model is to perform experiments. For example one could reveal the actions of some of the participants to others, while withholding some information on the actions of other individuals, hence inducing an asymmetrical network structure. The results of a repeated game of collective action vary among such configurations. A comparison can either confirm the theory or point toward more realistic revisions.

6 Appendix

We would like to find the final thresholds $\Gamma_i(\infty)$ for the first dynamic model. In a network with no loops, e.g. a star network, the elements of the stationary transition matrix are

$$\alpha^\infty = \frac{d_i}{\sum_j d_j}.$$

In the case of the example in section (4.2.1) for the star network, $v = [3/(3+2+2) \;\; 2/(3+2+2) \;\; 2/(3+2+2)]^T = [3/7 \;\; 2/7 \;\; 2/7]^T$ and the final thresholds will be $[3/7 \;\; 2/7 \;\; 2/7].[0 \;\; T \;\; T]^T = 4T/7 \approx 0.57$.

Also as the number of elements in the star increases, the threshold $\rightarrow 2/3$. Using the same technique as above, the final threshold for a $k+1$-star (the above example was a 2-star.) is $(2k)/(3k+1)$ which is always smaller than $2/3$, but approaches $2/3$ when k becomes large. Interestingly this is the final threshold for the fully connected graph with the number of nodes $n = 3$.

References

Beissinger, M. R. (2002), *National Mobilization and the Collapse of the Soviet State*, Cambridge University Press, New York, NY.

Ermakoff, I. (2008), *Ruling Oneself Out, A Theory of Collective Abdications*, Duke University Press, Durham, NC.

Gould, R. V. (1993), 'Collective action and network structure', *American Sociological Review* **58**(2), 182196.

Granovetter, M. S. (1978), 'Threshold models of collective behavior', *The American Journal of Sociology* **83**(6), 1420–43.

Jackson, M. O. (2008), *Social and Economic Networks*, Princeton University Press, Princeton, NJ.

Kuran, T. (1989), 'Sparks and prairie fires: A theory of unanticipated political revolution', *Public Choice* **61**(00), 41–74.

Laitin, D. D. (1998), *Identity in Formation, The Russian-Speaking Population in the Near Abroad*, Cornell University Press, Ithaca, NY.

Lohmann, S. (1994), 'The dynamics of informational cascades: The monday demonstrations in leipzig, east germany, 1989-91', *World Politics* **47**(1), 42–101.

Marwell, G. and Oliver, P. (1993), *The Critical Mass in Collective Action: A Micro-social Theory*, Cambridge University Press, New York, NY.

Morris, S. (2000), 'Contagion', *Review of Economic Studies* **67**, 5778.

Rasler, K. (1996), 'Concessions, repression, and political protest in the iranian revolution', *American Sociological Review* **61**(1), 132–52.

Schelling, T. C. (1978), *Micromotives and Macrobehavior*, W. W. Norton, New York, NY.

Siegel, D. A. (2009), 'Social networks and collective action', *American Journal of Political Science* **53**(1), 122–138.

Tilly, E. (1978), *From Mobilization to Revolution*, Addison-Wesley, Reading, MA.

Watts, D. J. (2002), 'A simple model of global cascades on random networks', *Proceedings of the National Academy of Sciences* **99**(9), 5766–71.